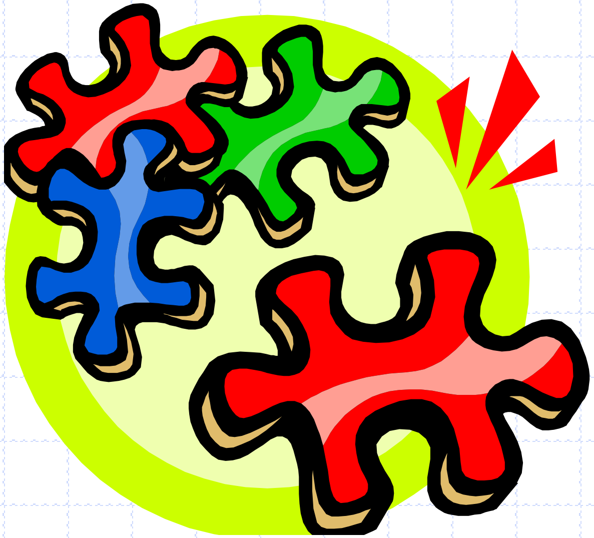
Sets



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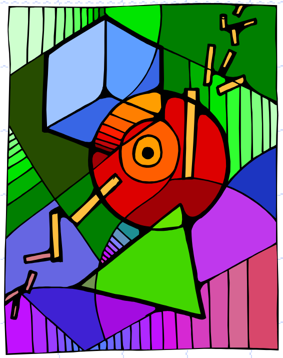
Set Operations



- We represent a set by the sorted sequence of its elements
- By specializing the auxliliary methods he generic merge algorithm can be used to perform basic set operations:
  - union
  - intersection
  - subtraction
- The running time of an operation on sets A and B should be at most $O(n_A + n_B)$

- Set union:
  - *aIsLess*(*a*, *S*)

    S.insertFirst(*a*)
  - *bIsLess*(*b*, *S*)

    S.insertLast(*b*)
  - *bothAreEqual*(*a*, *b*, *S*)

    S. insertLast(*a*)
- Set intersection:
  - *aIsLess*(*a*, *S*)

    { *do nothing* }
  - *bIsLess*(*b*, *S*)

    { *do nothing* }
  - *bothAreEqual*(*a*, *b*, *S*)

    S. insertLast(*a*)

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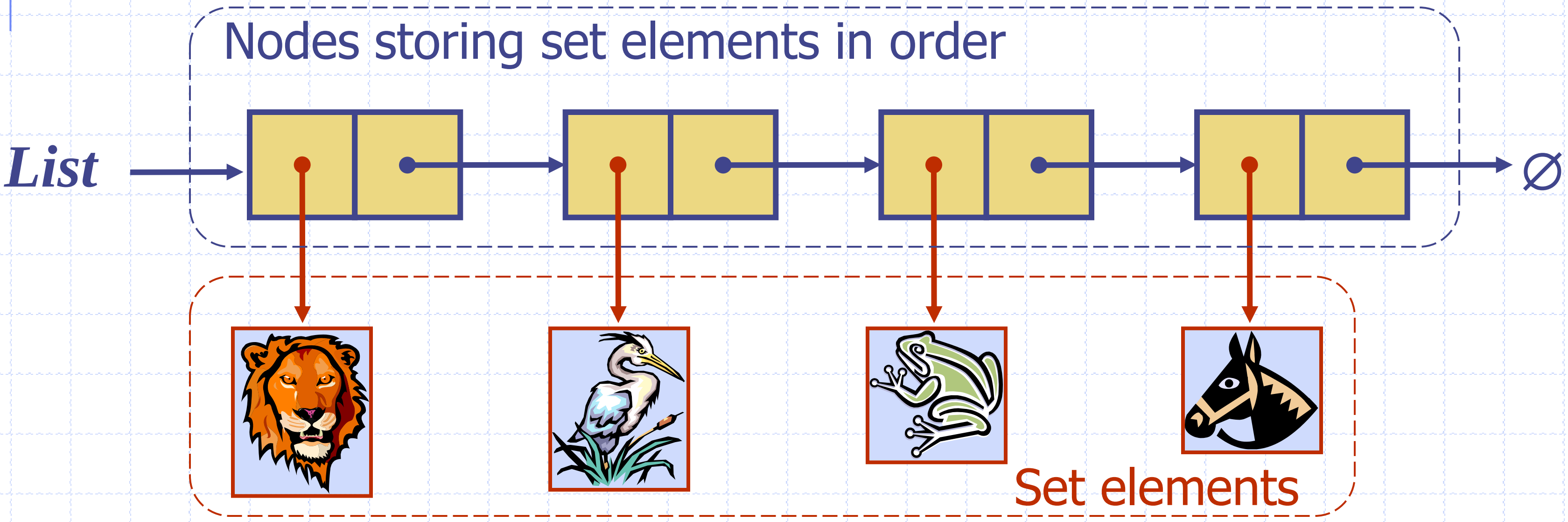
Storing a Set in a List

- We can implement a set with a list
- Elements are stored sorted according to some canonical ordering
- The space used is $O(n)$



Generic Merging

- Generalized merge of two sorted lists A and B
- Template method genericMerge
- Auxiliary methods
    - aIsLess
    - bIsLess
    - bothAreEqual
- Runs in $O(n_A + n_B)$ time provided the auxiliary methods run in $O(1)$ time

```
Algorithm genericMerge(A, B)
    S ← empty sequence
    while ¬A.isEmpty() ∧ ¬B.isEmpty()
        a ← A.first().element();  b ← B.first().element()
        if a < b
            aIsLess(a, S);  A.remove(A.first())
        else if b < a
            bIsLess(b, S);  B.remove(B.first())
        else { b = a }
            bothAreEqual(a, b, S)
            A.remove(A.first());  B.remove(B.first())
    while ¬A.isEmpty()
        aIsLess(a, S);  A.remove(A.first())
    while ¬B.isEmpty()
        bIsLess(b, S);  B.remove(B.first())
    return S
```

Using Generic Merge for Set Operations

- Any of the set operations can be implemented using a generic merge
- For example:
  - For intersection: only copy elements that are duplicated in both list
  - For union: copy every element from both lists except for the duplicates
- All methods run in linear time