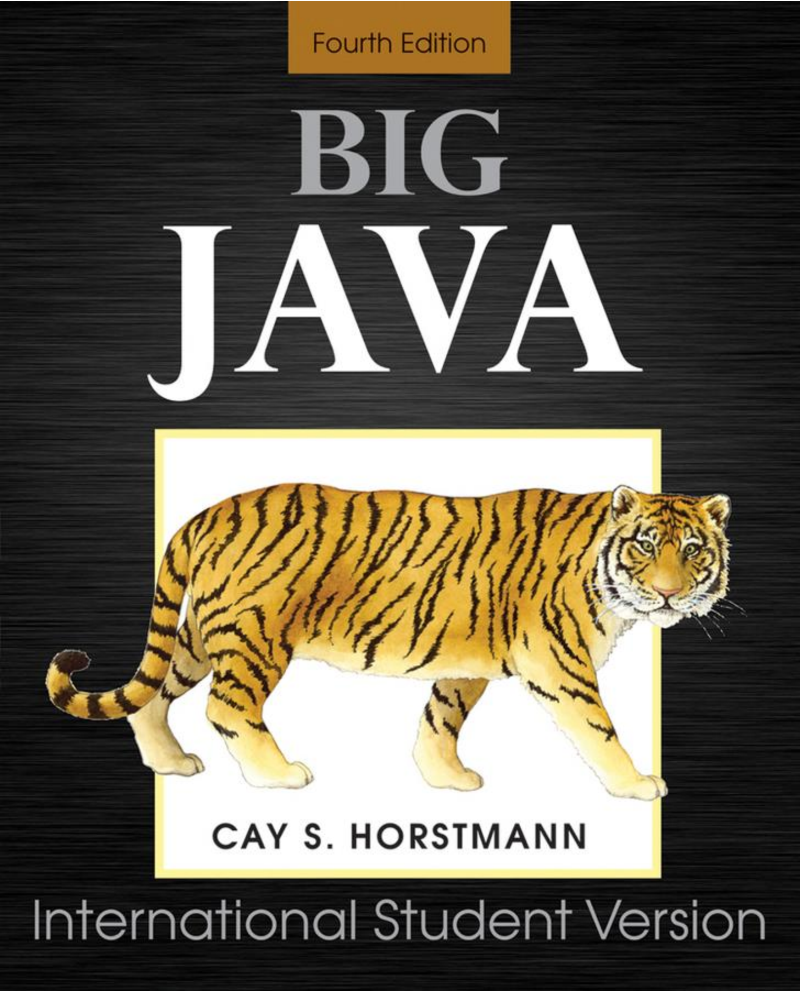

Chapter 12 – Recursion

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Chapter Goals

- To learn about the method of recursion
- To understand the relationship between recursion and iteration
- To analyze problems that are much easier to solve by recursion than by iteration
- To learn to "think recursively"
- To be able to use recursive helper methods
- To understand when the use of recursion affects the efficiency of an algorithm

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Triangle Numbers

- Compute the area of a triangle of width *n*
- Assume each `[]` square has an area of 1
- Also called the *n^{th} triangle number*
- The third triangle number is 6

```
[]
[][]
[][][]
```

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Outline of `Triangle` Class

```
public class Triangle
{
   private int width;
   public Triangle(int aWidth)
   {
      width = aWidth;
   }
   public int getArea()
   {
      ...
   }
}
```


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Handling Triangle of Width 1

- The triangle consists of a single square
- Its area is 1
- Add the code to `getArea` method for width 1

```
public int getArea()
{
   if (width == 1) { return 1; }
   ...
}
```

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Handling the General Case

- Assume we know the area of the smaller, colored triangle:

```
[]
[][]
[][][]
[][][][]
```

- Area of larger triangle can be calculated as:

```
smallerArea + width
```

- To get the area of the smaller triangle
    - *Make a smaller triangle and ask it for its area:*

```
Triangle smallerTriangle = new Triangle(width - 1);
int smallerArea = smallerTriangle.getArea();
```

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Completed `getArea` Method

```
public int getArea()
{
   if (width == 1) { return 1; }
   Triangle smallerTriangle = new Triangle(width - 1);
   int smallerArea = smallerTriangle.getArea();
   return smallerArea + width;
}
```

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Computing the area of a triangle with width 4

- `getArea` method makes a smaller triangle of width 3
- It calls `getArea` on that triangle
    - That method makes a smaller triangle of width 2
    - It calls `getArea` on that triangle
        - That method makes a smaller triangle of width 1
        - It calls `getArea` on that triangle
            - That method returns `1`
        - The method returns `smallerArea + width = 1 + 2 = 3`
    - The method returns `smallerArea + width = 3 + 3 = 6`
- The method returns `smallerArea + width = 6 + 4 = 10`

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Recursion

- A recursive computation solves a problem by using the solution of the same problem with simpler values
- For recursion to terminate, there must be special cases for the simplest inputs
- To complete our `Triangle` example, we must handle width <= 0:

```
if (width <= 0)  return 0;
```

- Two key requirements for recursion success:
  - *Every recursive call must simplify the computation in some way*
  - *There must be special cases to handle the simplest computations directly*

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Other Ways to Compute Triangle Numbers

- The area of a triangle equals the sum:

```
1 + 2 + 3 + ... + width
```

- Using a simple loop:

```
double area = 0;
for (int i = 1; i <= width; i++)
   area = area + i;
```

- Using math:

```
1 + 2 + ... + n = n × (n + 1)/2
   => width * (width + 1) / 2
```

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Animation 12.1

```
public static void main(String[] args)
{
   Triangle t = new Triangle(3);
   int area = t.getArea();
   System.out.println("Area: " + area);
}
. . .
public int getArea()
{
   if (width == 1) return 1;
   Triangle smallerTriangle = new Triangle(width - 1);
   int smallerArea = smallerTriangle.getArea();
   return smallerArea + width;
}
```

This animation demonstrates the recursive computation of the area of a `Triangle` object.

13-01 Recursion

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ch12/triangle/Triangle.java

```
/**
   A triangular shape composed of stacked unit squares like this:
   []
   [][]
   [][][]
   . . .
*/
public class Triangle
{
   private int width;

   /**
      Constructs a triangular shape.
      @param aWidth the width (and height) of the triangle
   */
   public Triangle(int aWidth)
   {
      width = aWidth;
   }

```

Continued

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ch12/triangle/Triangle.java (cont.)

```
    /**
       Computes the area of the triangle.
       @return the area
    */
    public int getArea()
    {
       if (width <= 0) { return 0; }
       if (width == 1) { return 1; }
       Triangle smallerTriangle = new Triangle(width - 1);
       int smallerArea = smallerTriangle.getArea();
       return smallerArea + width;
    }
}
```

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ch12/triangle/TriangleTester.java

```java
public class TriangleTester
{
   public static void main(String[] args)
   {
      Triangle t = new Triangle(10);
      int area = t.getArea();
      System.out.println("Area: " + area);
      System.out.println("Expected: 55");
   }
}
```

Program Run:

```
Enter width: 10
Area: 55
Expected: 55
```

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Self Check 12.1

Why is the statement

```
if (width == 1) { return 1; }
```

in the `getArea` method unnecessary?

Answer: Suppose we omit the statement. When computing the area of a triangle with width 1, we compute the area of the triangle with width 0 as 0, and then add 1, to arrive at the correct area.

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Self Check 12.2

How would you modify the program to recursively compute the area of a square?

Answer: You would compute the smaller area recursively, then return

```
smallerArea + width + width - 1.
```

```
[] [] [] []
[] [] [] []
[] [] [] []
[] [] [] []
```

Of course, it would be simpler to compute

$$1 + 0 + 2 + 1 + 3 + 2 + \cdots + n + n - 1 = \frac{n(n+1)}{2} + \frac{(n-1)n}{2} = n^2.$$

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Tracing Through Recursive Methods

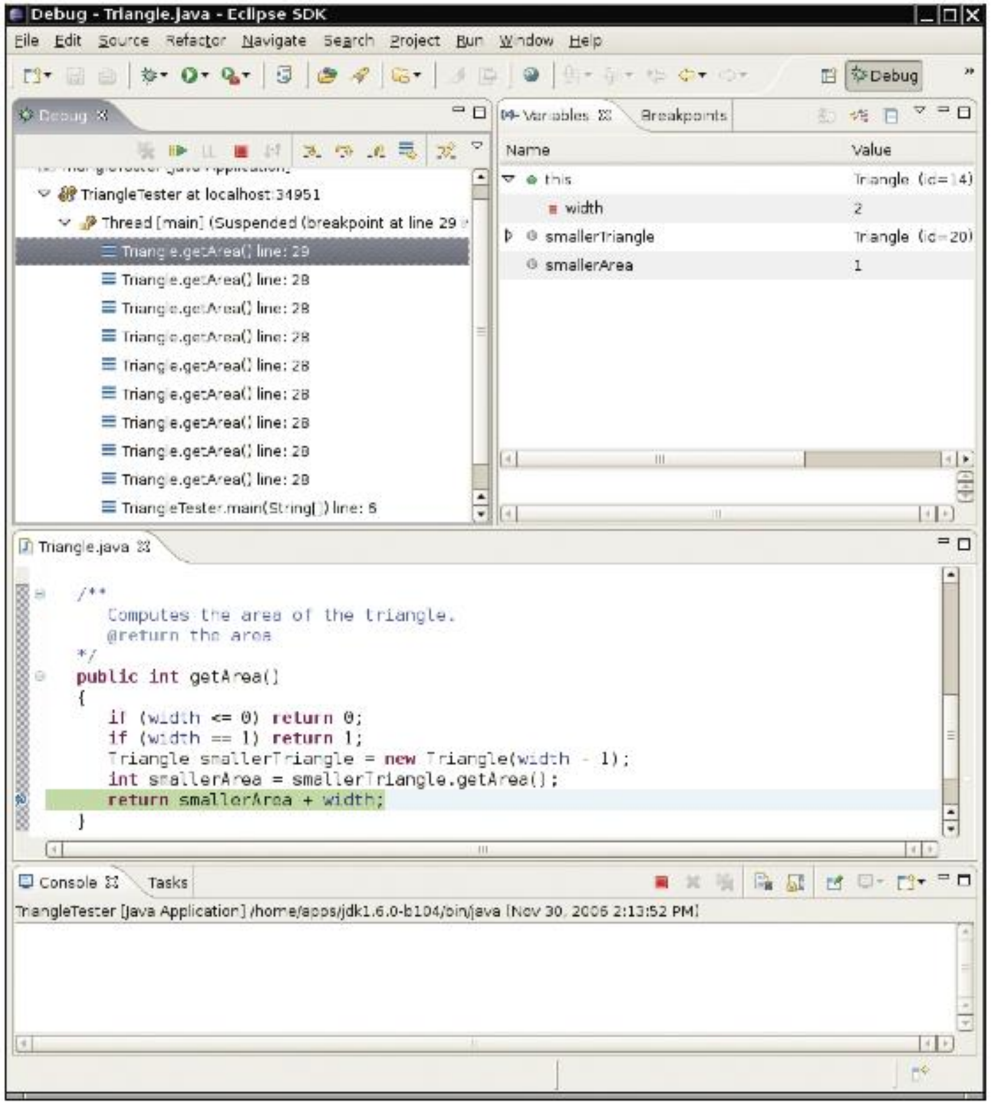

Figure 1 Debugging a Recursive Method

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Thinking Recursively

- Problem: Test whether a sentence is a palindrome
- **Palindrome:** A string that is equal to itself when you reverse all characters
  - *A man, a plan, a canal – Panama!*
  - *Go hang a salami, I'm a lasagna hog*
  - *Madam, I'm Adam*

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Implement `isPalindrome` Method

```
public class Sentence
{
   private String text;
   /**
      Constructs a sentence.
      @param aText a string containing all characters of
            the sentence
   */
   public Sentence(String aText)
   {
      text = aText;
   }

   /**
      Tests whether this sentence is a palindrome.
      @return true if this sentence is a palindrome, false
         otherwise
   */
```

Continued

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Implement `isPalindrome` Method (cont.)

```
public boolean isPalindrome()
    {
        . . .
    }
}
```

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Thinking Recursively: Step-by-Step

1. Consider various ways to simplify inputs

   Here are several possibilities:

   - *Remove the first character*
   - *Remove the last character*
   - *Remove both the first and last characters*
   - *Remove a character from the middle*
   - *Cut the string into two halves*

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Thinking Recursively: Step-by-Step

2. Combine solutions with simpler inputs into a solution of the original problem
   - *Most promising simplification: Remove first and last characters*

     "adam, I'm Ada" is a palindrome too!
   - *Thus, a word is a palindrome if*
     - *The first and last letters match, and*
     - *Word obtained by removing the first and last letters is a palindrome*
   - *What if first or last character is not a letter? Ignore it*
     - *If the first and last characters are letters, check whether they match; if so, remove both and test shorter string*
     - *If last character isn't a letter, remove it and test shorter string*
     - *If first character isn't a letter, remove it and test shorter string*

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Thinking Recursively: Step-by-Step

3. Find solutions to the simplest inputs
   - *Strings with two characters*
     - *No special case required; step two still applies*
   - *Strings with a single character*
     - *They are palindromes*
   - *The empty string*
     - *It is a palindrome*

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Thinking Recursively: Step-by-Step

4. Implement the solution by combining the simple cases and the reduction step

```
public boolean isPalindrome()
{
   int length = text.length();
   // Separate case for shortest strings.
   if (length <= 1) { return true; }
   // Get first and last characters, converted to
   // lowercase.
   char first = Character.toLowerCase(text.charAt(0));
   char last = Character.toLowerCase(text.charAt(
      length - 1));
   if (Character.isLetter(first) &&
      Character.isLetter(last))
   {
      // Both are letters.
      if (first == last)
      {
```

Continued

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Thinking Recursively: Step-by-Step (cont.)

```
         // Remove both first and last character.
         Sentence shorter = new
            Sentence(text.substring(1, length - 1));
         return shorter.isPalindrome();
      }
      else
         return false;
   }
   else if (!Character.isLetter(last))
   {
      // Remove last character.
      Sentence shorter = new Sentence(text.substring(0,
         length - 1));
      return shorter.isPalindrome();
   }
   else
   {
```

Continued

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Thinking Recursively: Step-by-Step (cont.)

```
            // Remove first character.
            Sentence shorter = new
               Sentence(text.substring(1));
            return shorter.isPalindrome();
        }
    }
```

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Recursive Helper Methods

- Sometimes it is easier to find a recursive solution if you make a slight change to the original problem
- Consider the palindrome test of previous slide

  It is a bit inefficient to construct new `Sentence` objects in every step

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Recursive Helper Methods

* Rather than testing whether the sentence is a palindrome, check whether a substring is a palindrome:

```
/**
   Tests whether a substring of the sentence is a
      palindrome.
   @param start the index of the first character of the
      substring
   @param end the index of the last character of the
      substring
   @return true if the substring is a palindrome
*/

public boolean isPalindrome(int start, int end)
```

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Recursive Helper Methods

- Then, simply call the helper method with positions that test the entire string:

```
public boolean isPalindrome()
{
   return isPalindrome(0, text.length() - 1);
}
```

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Recursive Helper Methods: `isPalindrome`

```
public boolean isPalindrome(int start, int end)
{
   // Separate case for substrings of length 0 and 1.
   if (start >= end) return true;
   // Get first and last characters, converted to
   // lowercase.
   char first = Character.toLowerCase(text.charAt(start));
   char last = Character.toLowerCase(text.charAt(end));
   if (Character.isLetter(first) &&
      Character.isLetter(last))
   {
      if (first == last)
      {
         // Test substring that doesn't contain the
         // matching letters.
         return isPalindrome(start + 1, end - 1);
      }
      else return false;
```

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Recursive Helper Methods: `isPalindrome` (cont.)

```
   }
   else if (!Character.isLetter(last))
   {
      // Test substring that doesn't contain the last
      // character.
      return isPalindrome(start, end - 1);
   }
   else
   {
      // Test substring that doesn't contain the first
      // character.
      return isPalindrome(start + 1, end);
   }
}
```

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Self Check 12.3

Do we have to give the same name to both `isPalindrome` methods?

> **Answer:** No — the first one could be given a different name such as `substringIsPalindrome`.

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Self Check 12.4

When does the recursive `isPalindrome` method stop calling itself?

Answer: When `start >= end`, that is, when the investigated string is either empty or has length 1.

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Fibonacci Sequence

- Fibonacci sequence is a sequence of numbers defined by

  $f_1 = 1$
  $f_2 = 1$
  $f_n = f_{n-1} + f_{n-2}$

- First ten terms:

  1, 1, 2, 3, 5, 8, 13, 21, 34, 55

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ch12/fib/RecursiveFib.java

```
import java.util.Scanner;

/**
    This program computes Fibonacci numbers using a recursive method.
*/
public class RecursiveFib
{
   public static void main(String[] args)
   {
      Scanner in = new Scanner(System.in);
      System.out.print("Enter n: ");
      int n = in.nextInt();

      for (int i = 1; i <= n; i++)
      {
         long f = fib(i);
         System.out.println("fib(" + i + ") = " + f);
      }
   }

```

Continued

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ch12/fib/RecursiveFib.java (cont.)

```java
   /**
      Computes a Fibonacci number.
      @param n an integer
      @return the nth Fibonacci number
   */
   public static long fib(int n)
   {
      if (n <= 2) { return 1; }
      else return fib(n - 1) + fib(n - 2);
   }
}
```

Program Run:

```
Enter n: 50
fib(1) = 1
fib(2) = 1
fib(3) = 2
fib(4) = 3
fib(5) = 5
fib(6) = 8
fib(7) = 13
...
fib(50) = 12586269025
```

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The Efficiency of Recursion

- Recursive implementation of `fib` is straightforward
- Watch the output closely as you run the test program
- First few calls to `fib` are quite fast
- For larger values, the program pauses an amazingly long time between outputs
- To find out the problem, let's insert **trace messages**

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ch12/fib/RecursiveFibTracer.java

```java
import java.util.Scanner;

/**
   This program prints trace messages that show how often the
   recursive method for computing Fibonacci numbers calls itself.
*/
public class RecursiveFibTracer
{
   public static void main(String[] args)
   {
      Scanner in = new Scanner(System.in);
      System.out.print("Enter n: ");
      int n = in.nextInt();

      long f = fib(n);

      System.out.println("fib(" + n + ") = " + f);
   }

```

Continued

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ch12/fib/RecursiveFibTracer.java (cont.)

```
   /**
      Computes a Fibonacci number.
      @param n an integer
      @return the nth Fibonacci number
   */
   public static long fib(int n)
   {
      System.out.println("Entering fib: n = " + n);
      long f;
      if (n <= 2) { f = 1; }
      else { f = fib(n - 1) + fib(n - 2); }
      System.out.println("Exiting fib: n = " + n
            + " return value = " + f);
      return f;
   }
}
```

Continued

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Program Run:

```
Enter n: 6
Entering fib: n = 6
Entering fib: n = 5
Entering fib: n = 4
Entering fib: n = 3
Entering fib: n = 2
Exiting fib: n = 2 return value = 1
Entering fib: n = 1
Exiting fib: n = 1 return value = 1
Exiting fib: n = 3 return value = 2
Entering fib: n = 2
Exiting fib: n = 2 return value = 1
Exiting fib: n = 4 return value = 3
Entering fib: n = 3
Entering fib: n = 2
Exiting fib: n = 2 return value = 1
Entering fib: n = 1
Exiting fib: n = 1 return value = 1
```

Continued

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ch12/fib/RecursiveFibTracer.java (cont)

```
Exiting fib: n = 1 return value = 1
Exiting fib: n = 3 return value = 2
Exiting fib: n = 5 return value = 5
Entering fib: n = 4
Entering fib: n = 3
Entering fib: n = 2
Exiting fib: n = 2 return value = 1
Entering fib: n = 1
Exiting fib: n = 1 return value = 1
Exiting fib: n = 3 return value = 2
Entering fib: n = 2
Exiting fib: n = 2 return value = 1
Exiting fib: n = 4 return value = 3
Exiting fib: n = 6 return value = 8
fib(6) = 8
```

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Call Tree for Computing `fib(6)`

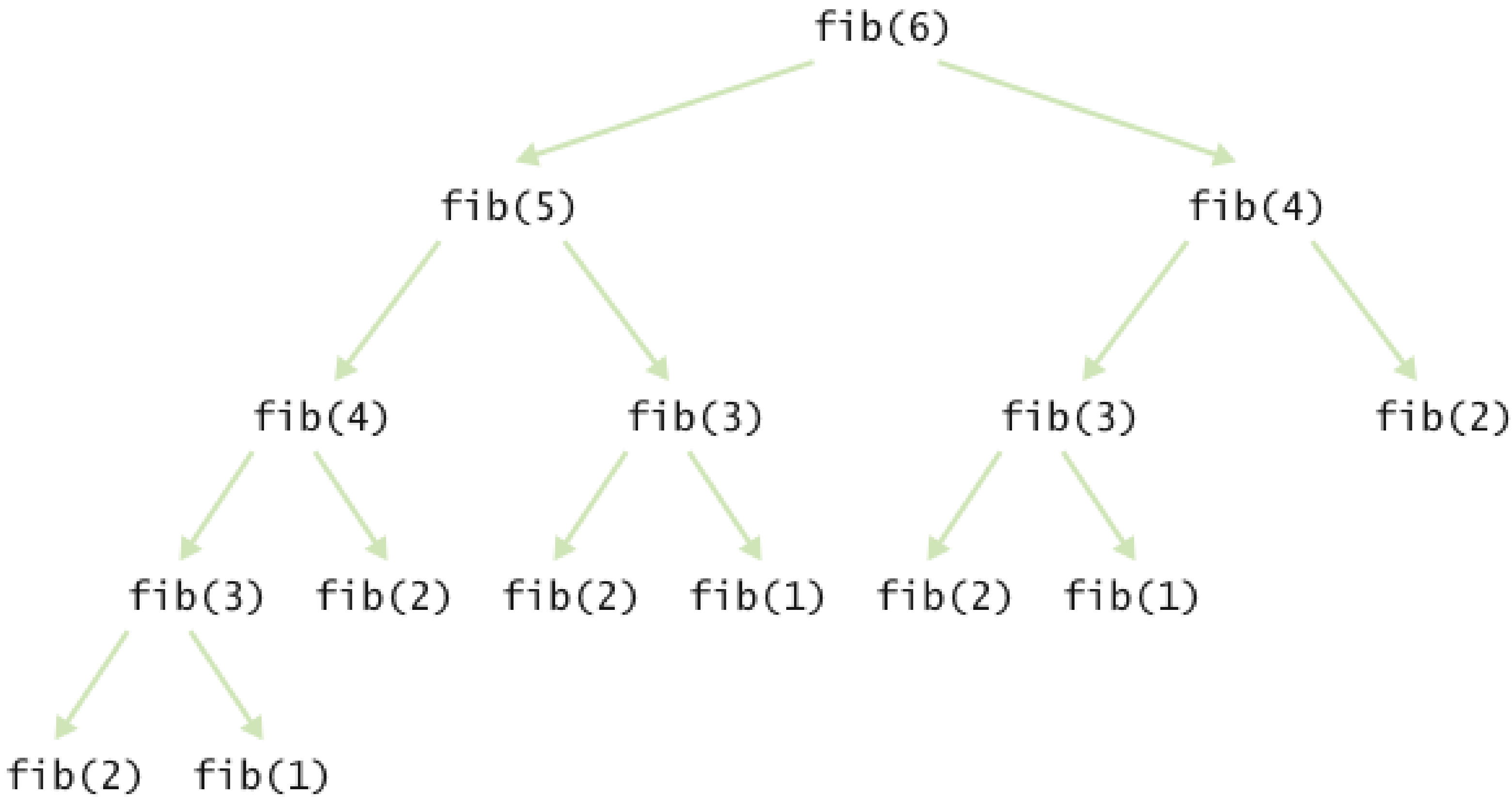

Figure 2 Call Pattern of the Recursive fib Method

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The Efficiency of Recursion

- Method takes so long because it computes the same values over and over
- The computation of `fib(6)` calls `fib(3)` three times
- Imitate the pencil-and-paper process to avoid computing the values more than once

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ch12/fib/LoopFib.java

```java
import java.util.Scanner;

/**
    This program computes Fibonacci numbers using an iterative method.
*/
public class LoopFib
{
   public static void main(String[] args)
   {
      Scanner in = new Scanner(System.in);
      System.out.print("Enter n: ");
      int n = in.nextInt();

      for (int i = 1; i <= n; i++)
      {
         long f = fib(i);
         System.out.println("fib(" + i + ") = " + f);
      }
   }

```

Continued

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ch12/fib/LoopFib.java (cont.)

```
    /**
       Computes a Fibonacci number.
       @param n an integer
       @return the nth Fibonacci number
    */
    public static long fib(int n)
    {
       if (n <= 2) { return 1; }
       long olderValue = 1;
       long oldValue = 1;
       long newValue = 1;
       for (int i = 3; i <= n; i++)
       {
          newValue = oldValue + olderValue;
          olderValue = oldValue;
          oldValue = newValue;
       }
       return newValue;
    }
}
```

Continued

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ch12/fib/LoopFib.java (cont.)

Program Run:

```
Enter n: 50
fib(1) = 1
fib(2) = 1
fib(3) = 2
fib(4) = 3
fib(5) = 5
fib(6) = 8
fib(7) = 13
...
fib(50) = 12586269025
```

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The Efficiency of Recursion

- Occasionally, a recursive solution runs much slower than its iterative counterpart
- In most cases, the recursive solution is only slightly slower
- The iterative `isPalindrome` performs only slightly better than recursive solution
  - *Each recursive method call takes a certain amount of processor time*
- Smart compilers can avoid recursive method calls if they follow simple patterns
- Most compilers don't do that
- In many cases, a recursive solution is easier to understand and implement correctly than an iterative solution
- "To iterate is human, to recurse divine." L. Peter Deutsch

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Iterative `isPalindrome` Method

```
public boolean isPalindrome()
{
   int start = 0;
   int end = text.length() - 1;
   while (start < end)
{
      char first =
         Character.toLowerCase(text.charAt(start));
      char last = Character.toLowerCase(text.charAt(end);
      if (Character.isLetter(first) &&
         Character.isLetter(last))
      {
         // Both are letters.
         if (first == last)
         {
            start++;
            end--;
         }
```

Continued

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Iterative `isPalindrome` Method (cont.)

```
        else
           return false;
     }
     if (!Character.isLetter(last))
        end--;
     if (!Character.isLetter(first))
        start++;
  }
  return true;
}
```

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Self Check 12.5

Is it faster to compute the triangle numbers recursively, as shown in Section 12.1, or is it faster to use a loop that computes 1 + 2 + 3 + . . . + width?

> **Answer:** The loop is slightly faster. Of course, it is even faster to simply compute width * (width + 1) / 2.

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Self Check 12.6

You can compute the factorial function either with a loop, using the definition that $n! = 1 \times 2 \times \ldots \times n$, or recursively, using the definition that $0! = 1$ and $n! = (n - 1)! \times n$. Is the recursive approach inefficient in this case?

> **Answer:** No, the recursive solution is about as efficient as the iterative approach. Both require $n - 1$ multiplications to compute $n!$.

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Permutations

- Design a class that will list all permutations of a string
- A permutation is a rearrangement of the letters
- The string `"eat"` has six permutations:

```
"eat"
"eta"
"aet"
"tea"
"tae"
```

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Public Interface of `PermutationGenerator`

```
public class PermutationGenerator
{
   public PermutationGenerator(String aWord) { ... }
   ArrayList<String> getPermutations() { ... }
}
```

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ch12/permute/PermutationGeneratorDemo.java

```java
import java.util.ArrayList;

/**
   This program demonstrates the permutation generator.
*/
public class PermutationGeneratorDemo
{
   public static void main(String[] args)
   {
      PermutationGenerator generator = new PermutationGenerator("eat");
      ArrayList<String> permutations = generator.getPermutations();
      for (String s : permutations)
      {
         System.out.println(s);
      }
   }
}

```

Continued

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ch12/permute/PermutationGeneratorDemo.java (cont.)

Program Run:

```
eat
eta
aet
ate
tea
tae
```


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To Generate All Permutations

- Generate all permutations that start with `'e'`, then `'a'`, then `'t'`
- To generate permutations starting with `'e'`, we need to find all permutations of `"at"`
- This is the same problem with simpler inputs
- Use recursion

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To Generate All Permutations

- `getPermutations`: Loop through all positions in the word to be permuted
- For each position, compute the shorter word obtained by removing *i*th letter:

```
String shorterWord = word.substring(0, i) +
   word.substring(i + 1);
```

- Construct a permutation generator to get permutations of the shorter word:

```
PermutationGenerator shorterPermutationGenerator
   = new PermutationGenerator(shorterWord);
ArrayList<String> shorterWordPermutations
   = shorterPermutationGenerator.getPermutations();
```

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To Generate All Permutations

- Finally, add the removed letter to front of all permutations of the shorter word:

```
for (String s : shorterWordPermutations)
{
   result.add(word.charAt(i) + s);
}
```

- Special case: Simplest possible string is the empty string; single permutation, itself

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ch12/permute/PermutationGenerator.java

```
import java.util.ArrayList;

/**
   This class generates permutations of a word.
*/
public class PermutationGenerator
{
   private String word;

   /**
      Constructs a permutation generator.
      @param aWord the word to permute
   */
   public PermutationGenerator(String aWord)
   {
      word = aWord;
   }

```

Continued

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```java
    /**
       Gets all permutations of a given word.
    */
    public ArrayList<String> getPermutations()
    {
       ArrayList<String> permutations = new ArrayList<String>();

       // The empty string has a single permutation: itself
       if (word.length() == 0)
       {
          permutations.add(word);
          return permutations;
       }

```

Continued

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ch12/permute/PermutationGenerator.java (cont.)

```
      // Loop through all character positions
      for (int i = 0; i < word.length(); i++)
      {
         // Form a simpler word by removing the ith character
         String shorterWord = word.substring(0, i)
               + word.substring(i + 1);

         // Generate all permutations of the simpler word
         PermutationGenerator shorterPermutationGenerator
               = new PermutationGenerator(shorterWord);
         ArrayList<String> shorterWordPermutations
               = shorterPermutationGenerator.getPermutations();

         // Add the removed character to the front of
         // each permutation of the simpler word,
         for (String s : shorterWordPermutations)
         {
            permutations.add(word.charAt(i) + s);
         }
      }
      // Return all permutations
      return permutations;
   }
}
```

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Self Check 12.7

What are all permutations of the four-letter word `beat`?

> **Answer:** They are `b` followed by the six permutations of `eat`, `e` followed by the six permutations of `bat`, `a` followed by the six permutations of `bet`, and `t` followed by the six permutations of `bea`.

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Self Check 12.8

Our recursion for the permutation generator stops at the empty string. What simple modification would make the recursion stop at strings of length 0 or 1?

Answer: Simply change `if (word.length() == 0)` to `if (word.length() <= 1)`, because a word with a single letter is also its sole permutation.

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Self Check 12.9

Why isn't it easy to develop an iterative solution for the permutation generator?

Answer: An iterative solution would have a loop whose body computes the next permutation from the previous ones. But there is no obvious mechanism for getting the next permutation. For example, if you already found permutations `eat`, `eta`, and `aet`, it is not clear how you use that information to get the next permutation. Actually, there is an ingenious mechanism for doing just that, but it is far from obvious — see Exercise P12.12.

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The Limits of Computation

Alan Turing

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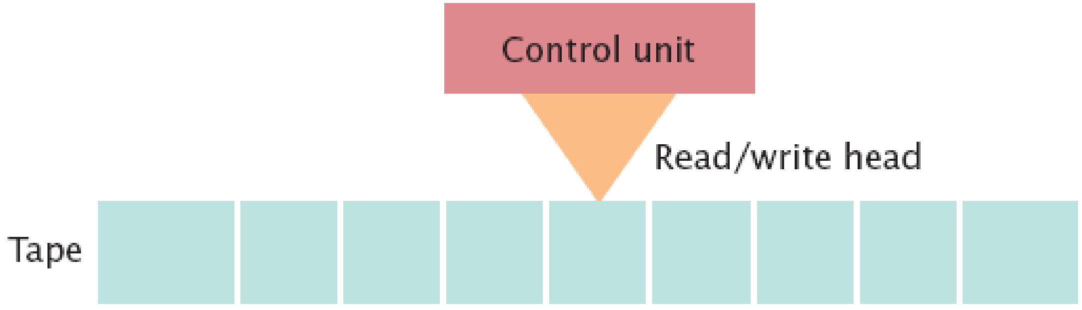
The Limits of Computation

Program

Instruction number	If tape symbol is	Replace with	Then move head	Then go to instruction
1	0	2	right	2
1	1	1	left	4
2	0	0	right	2
2	1	1	right	2
2	2	0	left	3
3	0	0	left	3
3	1	1	left	3
3	2	2	right	1
4	1	1	right	5
4	2	0	left	4



A Turing Machine

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Using Mutual Recursions

- **Problem:** To compute the value of arithmetic expressions such as

```
3 + 4 * 5
(3 + 4) * 5
1 - (2 - (3 - (4 - 5)))
```

- Computing expression is complicated
    - `*` *and* `/` *bind more strongly than* `+` *and* `-`
    - *Parentheses can be used to group subexpressions*

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Syntax Diagrams for Evaluating an Expression

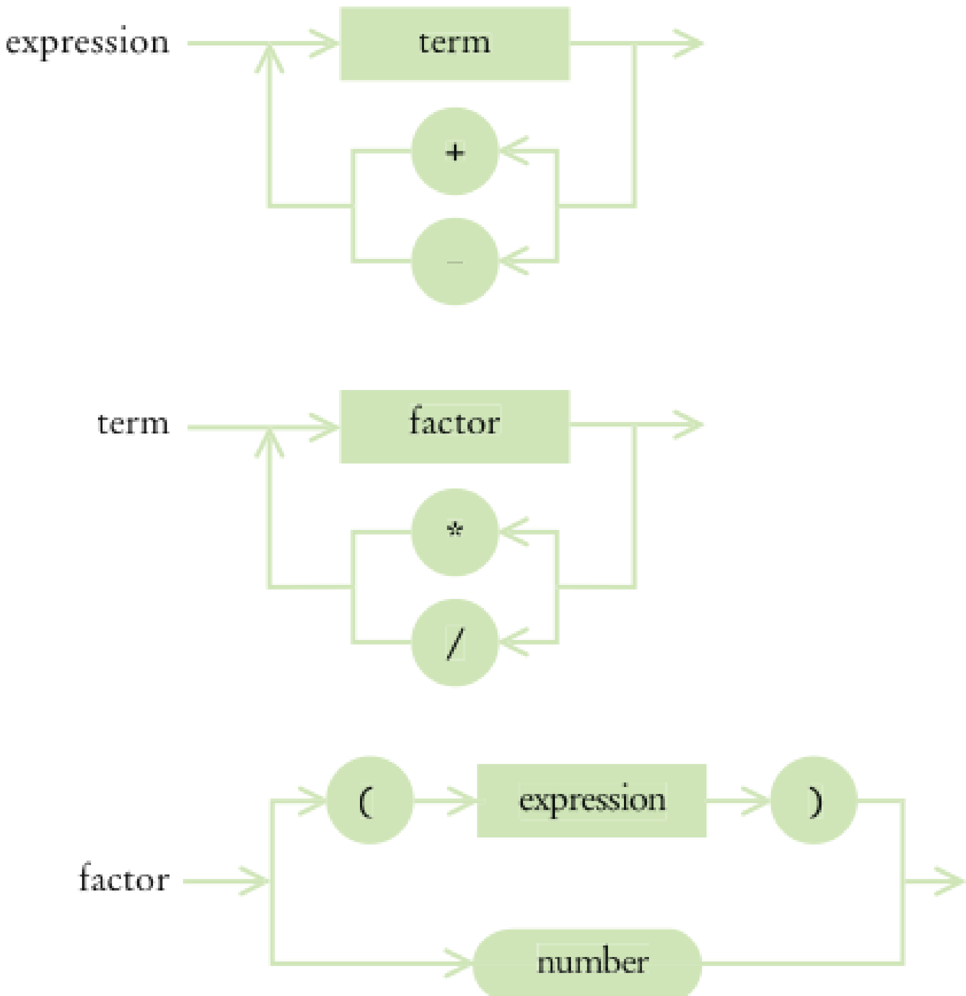

Figure 3 Syntax Diagrams for Evaluating an Expression

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Using Mutual Recursions

- An expression can broken down into a sequence of terms, separated by `+` or `-`
- Each term is broken down into a sequence of factors, separated by `*` or `/`
- Each factor is either a parenthesized expression or a number
- The syntax trees represent which operations should be carried out first

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Syntax Tree for Two Expressions

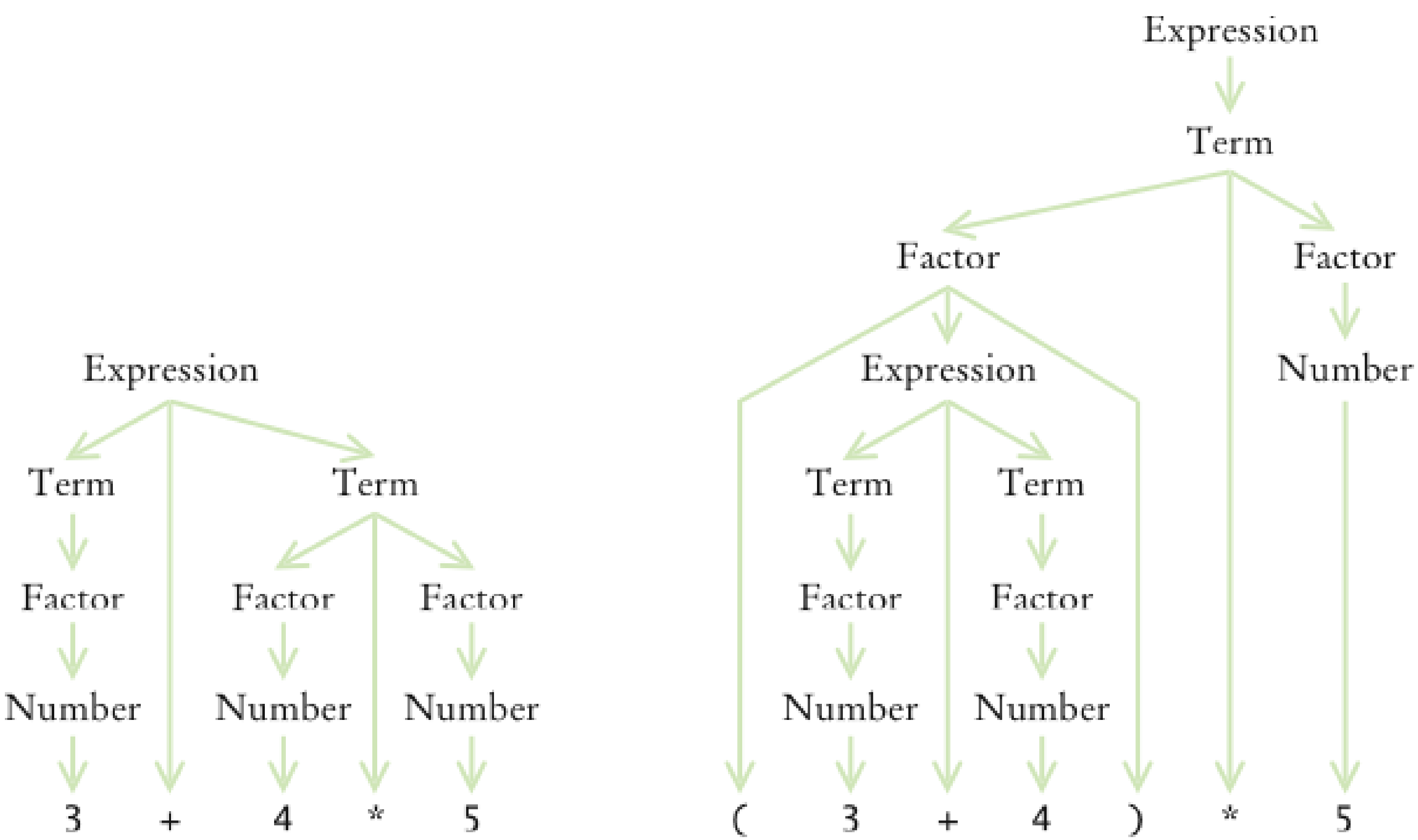

Figure 4 Syntax Trees for Two Expressions

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Mutually Recursive Methods

- In a mutual recursion, a set of cooperating methods calls each other repeatedly
- To compute the value of an expression, implement 3 methods that call each other recursively:
  - `getExpressionValue`
  - `getTermValue`
  - `getFactorValue`

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The `getExpressionValue` Method

```
public int getExpressionValue()
{
   int value = getTermValue();
   boolean done = false;
   while (!done)
   {
      String next = tokenizer.peekToken();
      if ("+".equals(next) || "-".equals(next))
      {
         tokenizer.nextToken(); // Discard "+" or "-"
         int value2 = getTermValue();
         if ("+".equals(next)) value = value + value2;
         else value = value - value2;
      }
      else done = true;
   }
   return value;
}
```

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The `getTermValue` Method

- The `getTermValue` method calls `getFactorValue` in the same way, multiplying or dividing the factor values

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The `getFactorValue` Method

```
public int getFactorValue()
{
   int value;
   String next =
   tokenpublic int getFactorValue()
{
   int value;
   String next = tokenizer.peekToken();
   if ("(".equals(next))
   {
      tokenizer.nextToken(); // Discard "("
      value = getExpressionValue();
      tokenizer.nextToken(); // Discard ")"
   }
   else
      value = Integer.parseInt(tokenizer.nextToken());
   return value;
}
```

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Using Mutual Recursions

To see the mutual recursion clearly, trace through the expression `(3+4)*5`:

- `getExpressionValue` calls `getTermValue`
  - `getTermValue` calls `getFactorValue`
    - `getFactorValue` consumes the `(` input
    - `getFactorValue` calls `getExpressionValue`
      - `getExpressionValue` returns eventually with the value of `7`, having consumed `3 + 4`. This is the recursive call.
    - `getFactorValue` consumes the `)` input
    - `getFactorValue` returns `7`
  - `getTermValue` consumes the inputs `*` and `5` and returns `35`
- `getExpressionValue` returns `35`

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ch12/expr/Evaluator.java

```
/**
   A class that can compute the value of an arithmetic expression.
*/
public class Evaluator
{
   private ExpressionTokenizer tokenizer;

   /**
      Constructs an evaluator.
      @param anExpression a string containing the expression
      to be evaluated
   */
   public Evaluator(String anExpression)
   {
      tokenizer = new ExpressionTokenizer(anExpression);
   }

```

Continued

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```
   /**
      Evaluates the expression.
      @return the value of the expression.
   */
   public int getExpressionValue()
   {
      int value = getTermValue();
      boolean done = false;
      while (!done)
      {
         String next = tokenizer.peekToken();
         if ("+".equals(next) || "-".equals(next))
         {
            tokenizer.nextToken(); // Discard "+" or "-"
            int value2 = getTermValue();
            if ("+".equals(next)) { value = value + value2; }
            else { value = value - value2; }
         }
         else
         {
            done = true;
         }
      }
      return value;
   }

```

Continued

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```
   /**
      Evaluates the next term found in the expression.
      @return the value of the term
   */
   public int getTermValue()
   {
      int value = getFactorValue();
      boolean done = false;
      while (!done)
      {
         String next = tokenizer.peekToken();
         if ("*".equals(next) || "/".equals(next))
         {
            tokenizer.nextToken();
            int value2 = getFactorValue();
            if ("*".equals(next)) { value = value * value2; }
            else { value = value / value2; }
         }
         else
         {
            done = true;
         }
      }
      return value;
   }

```

Continued

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ch12/expr/Evaluator.java (cont.)

```
    /**
       Evaluates the next factor found in the expression.
       @return the value of the factor
    */
    public int getFactorValue()
    {
       int value;
       String next = tokenizer.peekToken();
       if ("(".equals(next))
       {
          tokenizer.nextToken(); // Discard "("
          value = getExpressionValue();
          tokenizer.nextToken(); // Discard ")"
       }
       else
       {
          value = Integer.parseInt(tokenizer.nextToken());
       }
       return value;
    }
}
```

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ch12/expr/ExpressionTokenizer.java

```
/**
   This class breaks up a string describing an expression
   into tokens: numbers, parentheses, and operators.
*/
public class ExpressionTokenizer
{
   private String input;
   private int start; // The start of the current token
   private int end; // The position after the end of the current token

   /**
      Constructs a tokenizer.
      @param anInput the string to tokenize
   */
   public ExpressionTokenizer(String anInput)
   {
      input = anInput;
      start = 0;
      end = 0;
      nextToken(); // Find the first token
   }

```

Continued

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```java
    /**
       Peeks at the next token without consuming it.
       @return the next token or null if there are no more tokens
    */
    public String peekToken()
    {
       if (start >= input.length()) { return null; }
       else { return input.substring(start, end); }
    }

```

Continued

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ch12/expr/ExpressionTokenizer.java (cont.)

```
   /**
      Gets the next token and moves the tokenizer to the following token.
      @return the next token or null if there are no more tokens
   */
   public String nextToken()
   {
      String r = peekToken();
      start = end;
      if (start >= input.length()) { return r; }
      if (Character.isDigit(input.charAt(start)))
      {
         end = start + 1;
         while (end < input.length()
               && Character.isDigit(input.charAt(end)))
         {
            end++;
         }
      }
      else
      {
         end = start + 1;
      }
      return r;
   }
}
```

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ch12/expr/ExpressionCalculator.java

```java
import java.util.Scanner;

/**
    This program calculates the value of an expression
    consisting of numbers, arithmetic operators, and parentheses.
*/
public class ExpressionCalculator
{
   public static void main(String[] args)
   {
      Scanner in = new Scanner(System.in);
      System.out.print("Enter an expression: ");
      String input = in.nextLine();
      Evaluator e = new Evaluator(input);
      int value = e.getExpressionValue();
      System.out.println(input + "=" + value);
   }
}
```

Program Run:

```
Enter an expression: 3+4*5
3+4*5=23
```

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Self Check 12.10

What is the difference between a term and a factor? Why do we need both concepts?

> **Answer:** Factors are combined by multiplicative operators (`*` and `/`), terms are combined by additive operators (`+`, `-`). We need both so that multiplication can bind more strongly than addition.

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Self Check 12.12

Why does the expression parser use mutual recursion?

Answer: To handle parenthesized expressions, such as `2 + 3 * (4 + 5)`. The subexpression `4 + 5` is handled by a recursive call to `getExpressionValue`.

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Self Check 12.11

What happens if you try to parse the illegal expression `3 + 4 * ) 5`? Specifically, which method throws an exception?

Answer: The `Integer.parseInt` call in `getFactorValue` throws an exception when it is given the string `")"`.

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