MapReduce Algorithm for Matrix Multiplication

- **Matrix Multiplication**
  - From **high school calculus**:

    $$A \times B = C$$

    $$c_{ij} = \sum_{k=1,2,...,n} a_{ik} \times c_{kj}$$

  - **Example:**

    $$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1*6+2*5+3*4 & 1*3+2*2+3*1 \\ 4*6+5*5+6*4 & 4*3+5*2+6*1 \end{pmatrix}$$

    A B A * B

- **The reduce( ) step in the MapReduce Algorithm for matrix multiplication**
  - **Facts:**

    1. The ***final* step** in the **MapReduce** algorithm is to produce the **matrix A × B**

    2. The **unit** of **computation** of of **matrix A × B** is ***one* element** in the **matrix**:

    $$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1*6+2*5+3*4 & 1*3+2*2+3*1 \\ 4*6+5*5+6*4 & 4*3+5*2+6*1 \end{pmatrix}$$

    A B A * B

    Unit of computation

- **Conclusion:**

  - The ***input* information** of the **reduce( )** step (function) of the **MapReduce algorithm** are:

    $$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1*6+2*5+3*4 & 1*3+2*2+3*1 \\ 4*6+5*5+6*4 & 4*3+5*2+6*1 \end{pmatrix}$$

    A B A * B

    input output

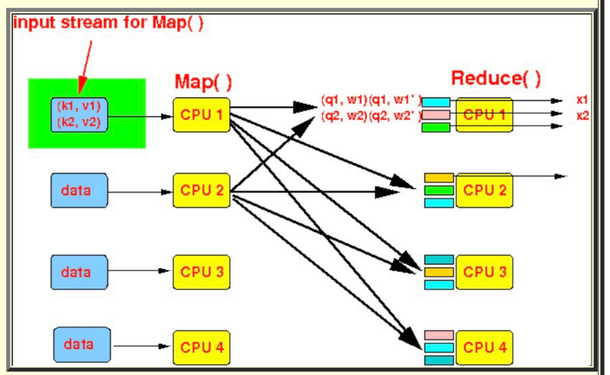
    reduce( )

    1. **One row vector** from **matrix A**
    2. **One column vector** from **matrix B**

  - The `reduce( )` function will **compute**:
    - The ***inner* product** of the
      - **One row vector** from **matrix A**
      - **One column vector** from **matrix B**

- **Preprocessing for the `map( )` function**
  - **Fact:**

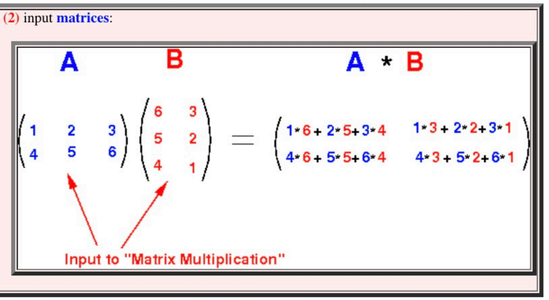
    - The `map( )` function (really) ***only*** has ***one*** **input stream**:

      of the **format** `( key_i , value_i )`

  - The **inputs** of the **matrix multiplication** are:

    - ***Tow*** **(2)** input **matrices**:

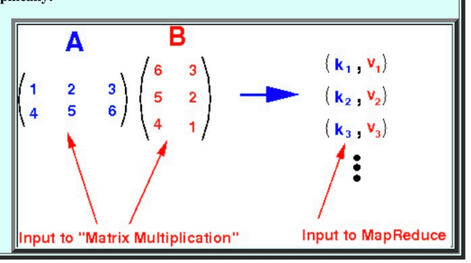
  - **Therefore:**

    - We must **insert** a ***pre-processing*** **step** to:

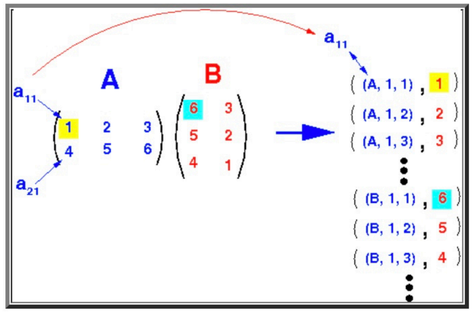
      - **Convert** the **input matrices** to the **form**:

        ```
        ( key1 , value1 )
        ( key2 , value2 )
        ( key3 , value3 )
        ...
        ```

      Graphically:

- **Pre-processing used** for **matrix multiplication**:

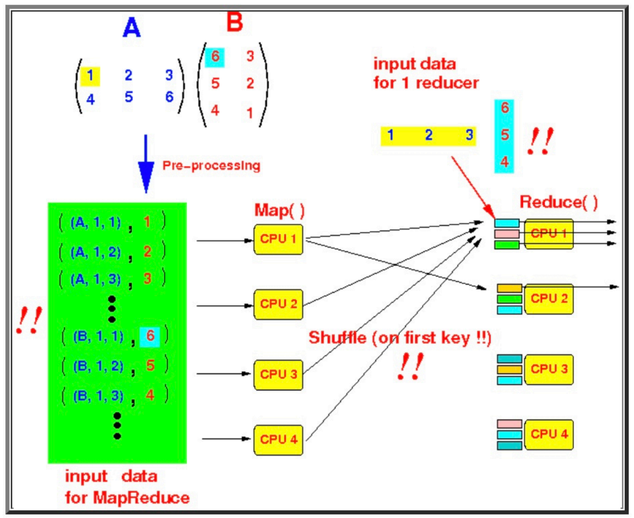


- **Overview of the MapReduce Algorithm for Matrix Multiplication**
  - **So far**, we have **discovered**:
    - The **input** to the **Map( )** is as **follows**:

```
( (A, 1, 1) , a11 )
( (A, 1, 2) , a12 )
( (A, 1, 3) , a13 )
...
( (B, 1, 1) , b11 )
( (B, 1, 2) , b12 )
( (B, 1, 3) , b13 )
...
```

    - The **input** to ***one*** **reduce( )** function is as **follows**:
      - A **row vector** from **matrix A**
      - A **column vector** from **matrix B**

  - **Graphical summary:**

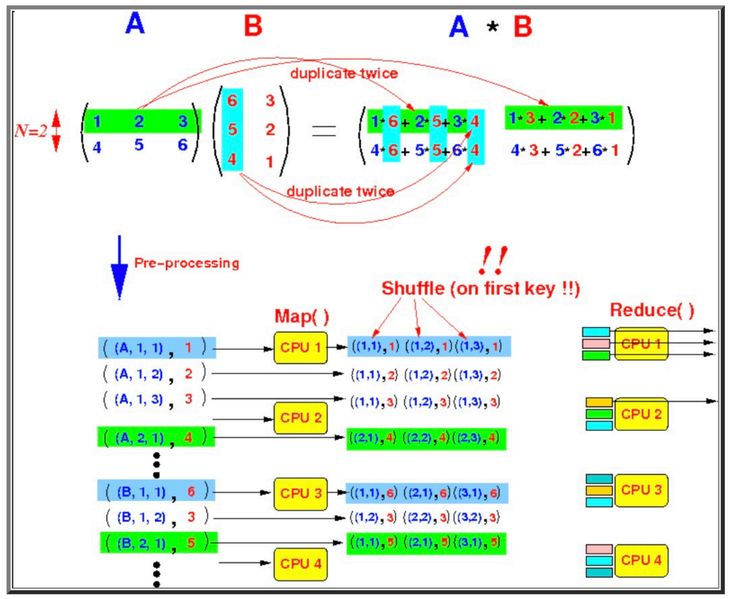


- **The MapReduce Algorithm for Matrix Multiplication**
  - The `map( )` function:
    - The `map( )` will **duplicate** ***N* times** as **follows**:

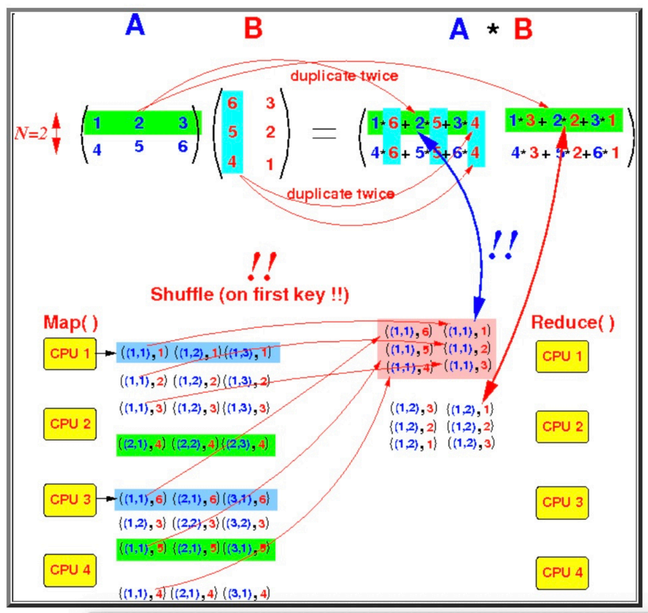
```
(A, i, j), x) --->  ( (i,1), x )  ( (i,2), x ) ....  ( (i,N), x )
(B, i, j), x) --->  ( (1,j), x )  ( (2,j), x ) ....  ( (N,j), x )
```

      wher ***N*** = **# rows** in **matrix A** (= **# columnss** in **matrix B**)

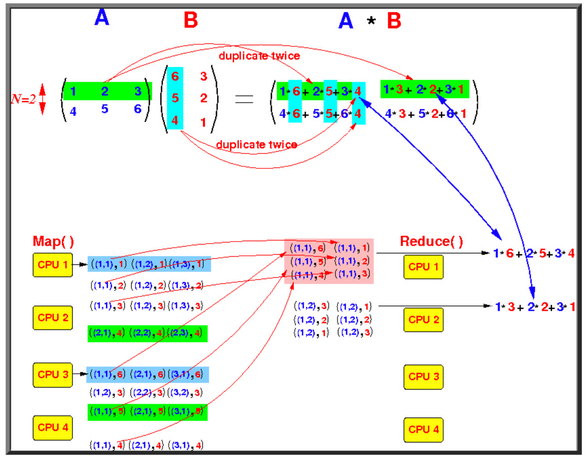
Example:



- The ***shuffle*** **mechanism** of **MapReduce** will ***re-organize*** **(group)** the `map( )` **output** as **follows**:



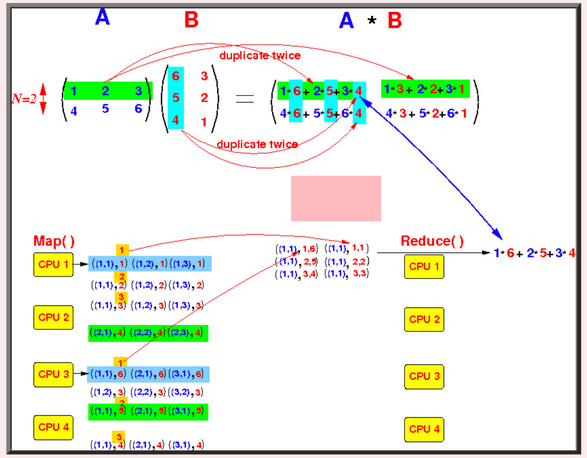
- The `reduce( )` function will compute the *inner product* of the input vectors



- **Postscript:**

  - We need to tag the map( ) function output with the position so the reduce( ) function can identify the *components* in the *different* vectors

    Example:



    (This detail was omitted for *brevity* --- **figure** is kinda *full*)

  - The `reduce( )` function is as follows:

```
sum = 0;

for ( pos = 1 to N ) do
{
   x = first  value at position pos
   y = second value at position pos

   sum = sum + x*y;
}
```