

No.

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Ex. 2.12 | Ch. 2 | P40 | chlæ | 2025.7.17

$$\text{ng. } \hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a}_+ + \hat{a}_-), \quad \hat{p} = i\sqrt{\frac{\hbar m\omega}{2}} (\hat{a}_+ - \hat{a}_-)$$

$$\begin{aligned} \langle x \rangle_n &= \int_{-\infty}^{+\infty} \psi_n^*(x) \hat{x} \psi_n(x) dx = \sqrt{\frac{\hbar}{2m\omega}} \int_{-\infty}^{+\infty} \psi_n^*(n) (\hat{a}_+ + \hat{a}_-) \psi_n(x) dx \\ &= \sqrt{\frac{\hbar}{2m\omega}} \int_{-\infty}^{+\infty} \psi_n^*(n) [\sqrt{n+1} \psi_{n+1}(x) + \sqrt{n} \psi_{n-1}] dx = 0 \text{ (正交性)} \end{aligned}$$

$$\langle p \rangle_n = i \sqrt{\frac{\hbar m\omega}{2}} \int_{-\infty}^{+\infty} \psi_n^*(n) (\hat{a}_+ - \hat{a}_-) \psi_n(x) dx = 0 \text{ (同理)}$$

$$\begin{aligned} \langle x^2 \rangle_n &= \frac{\hbar}{2m\omega} \int_{-\infty}^{+\infty} \psi_n^*(x) (\hat{a}_+^2 + \hat{a}_+ \hat{a}_- + \hat{a}_- \hat{a}_+ + \hat{a}_-^2) \psi_n(x) dx \\ &= \frac{\hbar}{2m\omega} [(n+1) + n] = \frac{\hbar}{2m\omega} (2n+1) \end{aligned}$$

$$\begin{aligned} \langle p^2 \rangle_n &= -\frac{\hbar m\omega}{2} \int_{-\infty}^{+\infty} \psi_n^*(x) (\hat{a}_+^2 - \hat{a}_+ \hat{a}_- - \hat{a}_- \hat{a}_+ + \hat{a}_-^2) \psi_n(x) dx \\ &= -\frac{\hbar m\omega}{2} (D - (M) - N + O) = \frac{\hbar m\omega}{2} (2n+1) \end{aligned}$$

$$\sigma_{x_n} = \sqrt{\langle x^2 \rangle_n - \langle x \rangle_n^2} = \sqrt{\frac{\hbar}{2m\omega} (2n+1)}$$

$$\sigma_{p_n} = \sqrt{\langle p^2 \rangle_n - \langle p \rangle_n^2} = \sqrt{\frac{\hbar m\omega}{2} (2n+1)}$$

$$\sigma_{x_n} \cdot \sigma_{p_n} = \frac{\hbar}{2} (2n+1) > \frac{\hbar}{2} \text{ (满足不确定原理)}$$