No.

Date.

Ex. 1.9 | Ch. 1 | P16 | Chloe | 2025.7.14

解: (a) 归一化波函数 $\int_{-\infty}^{+\infty}|\psi(x,t)|^2dx = A^2\int_{-\infty}^{+\infty}e^{-2amx^2/\hbar}dx$ (被积函数偶)

$=2A^2\int_0^{+\infty}e^{-2amx^2/\hbar}dx$ (套用高斯积分公式) $=A^2\sqrt{\frac{\pi\hbar}{2am}}=1$

$A^2=\sqrt{2am}/\sqrt{\pi\hbar}$ $\quad A=(2am/\pi\hbar)^{\frac14}$

(b) $\psi(x,t)=Ae^{-amx^2/\hbar}e^{-iat}$

$\partial\psi/\partial t = Ae^{-amx^2/\hbar}\cdot(-ia)e^{-iat} = -ia\psi(x,t)$

$\partial\psi/\partial x = Ae^{-iat}e^{-amx^2/\hbar}\cdot(-am/\hbar)\cdot 2x = -\frac{2amx}{\hbar}\psi(x,t)$

$\partial^2\psi/\partial x^2 = \frac{\partial}{\partial x}(\partial\psi/\partial x) = (-2amA/\hbar)e^{-iat}\cdot e^{-amx^2/\hbar}[1-2amx^2/\hbar]$

$=(4a^2m^2x^2/\hbar^2 - 2am/\hbar)\psi(x,t)$

代入薛定谔方程 $i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2\partial^2\psi}{\partial x^2}+V\psi$

$i\hbar\cdot(-ia)\psi = -\hbar^2\cdot(4a^2m^2x^2/\hbar^2 - 2am/\hbar^2)\psi + V\psi$

解得 $V=2a^2mx^2$

(c) $\langle x\rangle = \int_{-\infty}^{+\infty}\psi^* x\psi dx = A^2\int_{-\infty}^{+\infty}xe^{-2amx^2/\hbar}dx$ (奇) $=0$

$\langle x^2\rangle = \int_{-\infty}^{+\infty}\psi^* x^2\psi dx = A^2\int_{-\infty}^{+\infty}x^2e^{-2amx^2/\hbar}dx$ (偶)

$=2A^2\int_0^{+\infty}x^2e^{-2amx^2/\hbar}dx$ (高斯积分) $=A^2\hbar\sqrt{\pi\hbar}/4am\sqrt{2am} = \hbar/4am$

$\langle p\rangle = \int_{-\infty}^{+\infty}\psi^*(-i\hbar\frac{\partial}{\partial x})\psi dx = \int_{-\infty}^{+\infty}\psi^*(-i\hbar)(-\frac{2amx}{\hbar})\psi dx$

$=\int_{-\infty}^{+\infty}-\frac{2amA^2}{\hbar}xe^{-2amx^2/\hbar}dx = 0$ (奇)

$\langle p^2\rangle = \int_{-\infty}^{+\infty}\psi^*(-\hbar)\partial^2\psi/\partial x^2dx = \int_{-\infty}^{+\infty}(2am - 4a^2m^2x^2/\hbar)\psi^*\psi dx$

$=\int_{-\infty}^{+\infty}(2am - 4a^2m^2x^2/\hbar)A^2e^{-2amx^2/\hbar}dx$ (偶, 高斯积分)

$=\sqrt{am\pi\hbar}\,\hbar A^2/\sqrt{2} = a\hbar m$

(d) $\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\hbar}/\sqrt{4am}$

$\sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \sqrt{\hbar am}$

$\sigma_x\cdot\sigma_p = \sqrt{\hbar}/\sqrt{4am}\cdot\sqrt{\hbar am} = \hbar/2$

满足海森伯不确定原理 $\sigma_x\cdot\sigma_p \geq \hbar/2$