No.

Date.

Ex.2.10 | Ch.2 | P40 | Chloe | 2025.7.17

解：(a) $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega}{2\hbar}x^2}$

$\psi_1(x) = \left(\frac{4m\omega}{\pi\hbar}\right)^{1/4} \cdot \frac{x}{\sqrt{\hbar/m\omega}} \cdot e^{-\frac{m\omega}{2\hbar}x^2}$

$\psi_2(x) = \frac{1}{\sqrt{2}}\hat{a}_+\psi_1(x) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2\hbar m\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)\psi_1(x)$

$= \frac{1}{\sqrt{2}} \cdot \left(\frac{4m\omega}{\pi\hbar}\right)^{1/4} \cdot \frac{1}{\sqrt{2\hbar m\omega}\sqrt{\hbar/m\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)\left(x \cdot e^{-\frac{m\omega}{2\hbar}x^2}\right)$

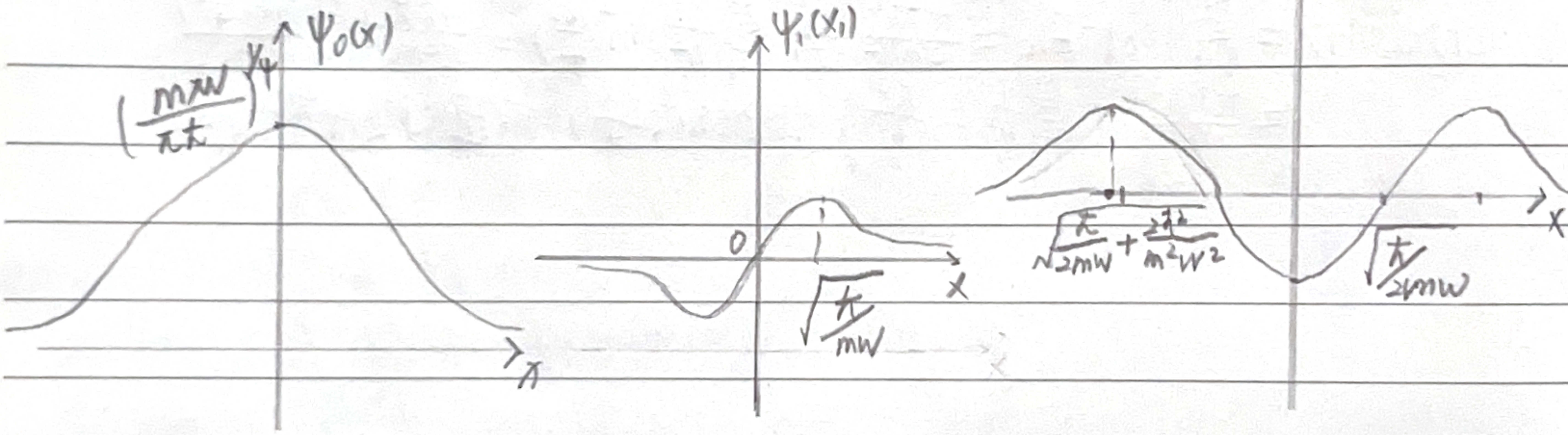
$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\hbar\sqrt{2}} \cdot \left(\frac{4m\omega}{\pi\hbar}\right)^{1/4}\left[-\hbar\left(e^{-\frac{m\omega x^2}{2\hbar}} - \frac{m\omega x^2}{\hbar}x^2 e^{-\frac{m\omega x^2}{2\hbar}}\right) + m\omega x^2 e^{-\frac{m\omega}{2\hbar}x^2}\right]$

$= \left(\frac{m\omega}{4\pi\hbar}\right)^{1/4}\left(\frac{2m\omega x^2}{\hbar} - 1\right)e^{-\frac{m\omega x^2}{2\hbar}}$

(b) $\psi_0(-x) = \psi_0(x)$ 偶

$\psi_1(-x) = -\psi_1(x)$ 奇

$\psi_2(-x) = \psi_2(x)$ 偶



(c) $\psi_0(-x) = \psi_0(x)$ $\psi_1(-x) = -\psi_1(x)$

$\psi_0(-x) \cdot \psi_1(-x) = \psi_0(x) \cdot (-\psi_1(x)) = -(\psi_0(x) \cdot \psi_1(x))$ 奇

故 $\int_{-\infty}^{+\infty}\psi_0(x) \cdot \psi_1(x)\,dx = 0$. $\psi_0(x)$、$\psi_1(x)$ 正交

同理证 $\psi_1(x)$、$\psi_2(x)$ 正交

现证 $\psi_0(x)$ 与 $\psi_2(x)$ 的正交性 令 $\alpha = \sqrt{\frac{m\omega}{\hbar}}$

$\int_{-\infty}^{+\infty}\psi_0(x) \cdot \psi_2(x)\,dx = \left(\frac{\alpha^2}{\pi}\right)^{1/4} \cdot \left(\frac{\alpha^2}{4\pi}\right)^{1/4}\int_{-\infty}^{+\infty}(2\alpha^2x^2 - 1)e^{-\alpha^2x^2}dx$

$= \frac{\alpha}{\sqrt{2\pi}}\left[2\alpha^2\int_{-\infty}^{+\infty}x^2e^{-\alpha^2x^2}dx - \int_{-\infty}^{+\infty}e^{-\alpha^2x^2}dx\right]$ (高斯积分)

$= \frac{\alpha}{\sqrt{2\pi}} \cdot \left[2\alpha^2\frac{\sqrt{\pi}}{2\alpha^3} - \frac{\sqrt{\pi}}{\alpha}\right] = 0$

得证