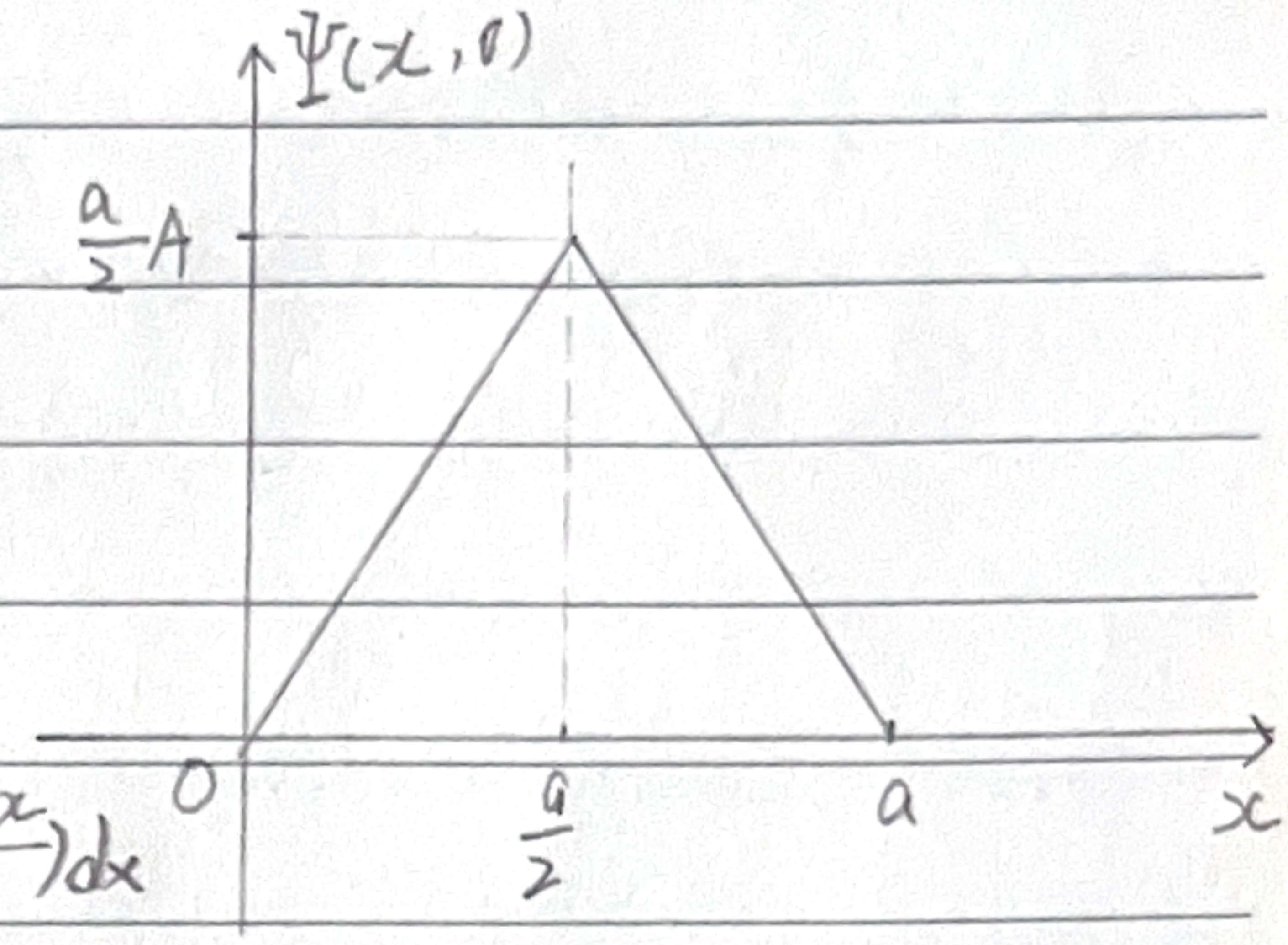
Ex.2.7 | Ch.2 | P31 | Chloe | 2025.7.16

解：(a) 求常数A $\int_{-\infty}^{+\infty} |\Psi(x,0)|^2 dx$



$= \int_0^{a/2} (Ax)^2 dx + \int_{a/2}^{a} [A(a-x)]^2 dx = \frac{A^2a^3}{24} + \frac{A^2a^3}{24} = \frac{A^2}{12}a^3 = 1$

故 $A = (12/a^3)^{\frac{1}{2}} = \sqrt{12/a^3}$

(b) 求 $\Psi(x,t) = \sum_{n=1}^{\infty} C_n \psi_n(x) e^{-iE_n t/\hbar}$，其中

$C_n = \int_{-\infty}^{+\infty} \Psi(x,0)\psi_n^*(x)dx$，$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

$C_n = \int_0^{a/2} Ax \cdot \sqrt{\frac{2}{a}} \sin(\frac{n\pi x}{a})dx + \int_{a/2}^{a} A(a-x) \cdot \sqrt{\frac{2}{a}} \sin(\frac{n\pi x}{a})dx$

$= \frac{4\sqrt{6}}{n^2\pi^2} \cdot \sin(\frac{n\pi}{4})$（书后积分公式）

故 $\Psi(x,t) = \sum_{n=1}^{\infty} \frac{4\sqrt{6}}{n^2\pi^2} \sin(\frac{n\pi}{4}) \cdot \sqrt{\frac{2}{a}} \sin(\frac{n\pi x}{a}) e^{-i\frac{n^2\pi^2\hbar}{2ma^2}t}$

(c) 求 E_1 的几率为 $|C_1|^2 = (\frac{4\sqrt{6}}{\pi^2} \cdot \sin\frac{\pi}{4})^2 = (\frac{4\sqrt{3}}{\pi^2})^2 = \frac{48}{\pi^4}$

(d) 求 $\langle H \rangle = \sum_{n=1}^{\infty} |C_n|^2 E_n = \frac{\pi^2\hbar^2}{2ma^2} \cdot \frac{96}{\pi^4} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin^2(\frac{n\pi}{4}) = \frac{48\hbar^2}{\pi^2 ma^2} \sum_{n=1}^{\infty} \frac{\sin^2(\frac{n\pi}{4})}{n^2}$

$= \frac{\pi^2\hbar^2}{ma^2}$（$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$，结合 $\sin^2(\frac{n\pi}{4})$ 的取值分段计算求和）