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CPE 695 WS
HW 4
I pledge my honor that I have abided by the Stevens Honor System - Chloe Quinto

Question 1
Consider again the example application of Bayes rule in Section 6.2.1 of Tom Mitchell's textbook. Suppose the doctor decides to order a second laboratory test for the same patient, and suppose the second test returns a positive result as well. What are the posterior probabilities of cancer and $\neg cancer$ following these two tests. Assume that the two tests are independent.

From the previous application

$P(cancer) = 0.008$	$P(\neg cancer) = 0.992$
$P(+\|cancer) = 0.98$	$P(-\|cancer) = 1 - P(+\|cancer) = 0.02$
$P(+\|\neg cancer) = 1 - P(-\|\neg cancer) = 0.03$	$P(-\|\neg cancer) = 0.97$

We want to calculate the probability of cancer given two positive results
$$P(cancer|++) = \frac{p(+|caner)\,p(cancer|+)}{p(+|cancer)P(cancer|+) + p(+|\neg cancer)p(\neg cancer|+)}$$

Finding
$$P(cancer|+) = \frac{p(+|cancer)\,p(cancer)}{P(+|cancer)P(cancer) + p(+|\neg cancer)p(\neg cancer)}$$

$$P(cancer|+) = \frac{0.98 * 0.008}{0.98 * 0.008 + 0.03 * 0.992} = 0.21$$

Therefore

$$P(\neg cancer|+) = 1 - P(cancer|+) = 0.79$$

$$P(cancer|++) = \frac{p(+|cancer)\,p(cancer|+)}{p(+|cancer)P(cancer|+) + p(+|\neg cancer)p(\neg cancer|+)}$$
$$P(cancer|++) = \frac{0.98 * 0.21}{0.98 * 0.21 + 0.03 * 0.79} = 0.8967$$
$$P(\neg cancer|++) = 1 - P(cancer|++) = 1 - 0.8967 = 0.103$$
***rounding may cause different results**

Question 2

Consider a learned hypothesis, h, for some Boolean concept. When h is tested on a set of 100 examples, it classifies 80 correctly. What is the 95% confidence interval for the true error rate for $Error_D(h)$

95% Confidence Interval for $Error_D(h)$

$Error_D(h) \pm 1.96 * \sqrt{\frac{Error_D(h) * (1 - Error_D(h))}{n}}$

$0.20 \pm 1.96 * \sqrt{\frac{0.20 * 0.80}{100}}$

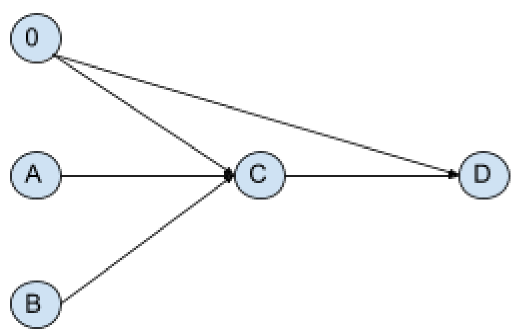
0.20 ± 0.0784

Question 3

Consider a two-layer feedforward ANN with two inputs and a and b, one hidden unit c and out output unit d. This network has five weights w_{ca}, w_{cb}, w_{c0}, w_{dc}, w_{d0} where w_{x0} represents the threshold weight for unit x. Initialize these weights to have values of 0.1. then give their values after each of the first two training iterations of the BACKPROPAGATION algorithm. Assume learning rate is $\eta = 0.3$ and momentum $\alpha = 0.9$ incremental updates and the following training examples:

a	b	d
1	0	1
0	1	0

With sigmoid activation function



Forward Pass 1

$c = w_{ca} * x_a + w_{cb} * x_b + w_{c0} = (0.1 * 1) + (0.1 * 0) + 0.1 = 0.2$

$o_c = \sigma(0.02) = \frac{1}{1+e^{-0.02}} = 0.549834$

$d = w_{dc} * x_c + w_{d0} = (0.1 * 0.54983) + 0.1 = 0.154983$

$o_d = \sigma(0.154983) = \frac{1}{1+e^{-0.154983}} = 0.53866$

Computing Error

$\delta_d = o_d(1 - o_d)(d - o_d) = (0.53866)(1 - 0.53866)(1 - 0.53866) = 0.114645$

$\delta_c = o_c(1 - o_c)(w_{dc} * \delta_d) = (0.549)(1 - 0.549)(0.1 * 0.114645) = 0.002839$

Correction

$\Delta w_{dc} = 0.3 * 0.114645 * 0.549834 = 0.018911$

$\Delta w_{ca} = 0.3 * 0.002839 * 1 + 0.9 * 0 = 0.000852$

$\Delta w_{cb} = 0.3 * 0.002839 * 0 + 0.9 * 0 = 0$

$\Delta w_{c0} = 0.3 * 0.00283 * 1 + 0.9 * 0 = 0.000852$

$\Delta w_{d0} = 0.3 * 0.114645 * 1 + 0.9 * 0 = 0.034394$

New Weights

$w_{dc} = 0.018911 + 0.1 = 0.118911$

$w_{ca} = 0.000852 + 0.1 = 0.100852$

$w_{cb} = 0 + 0.1 = 0.1$

$w_{c0} = 0.000852 + 0.1 = 0.100852$

$w_{d0} = 0.034394 + 0.1 = 0.134394$

Forward Pass 2

$c = w_{ca} * x_a + w_{cb} * x_b + w_{c0} = (0.100852 * 0) + (0.1 * 1) + 0.100852 = 0.200852$

$o_c = \sigma(0.200852) = \frac{1}{1+e^{-0.200852}} = 0.550045$

$d = w_{dc} * x_c + w_{d0} = (0.118911 * 0.54983) + 0.134394 = 0.199775$

$o_d = \sigma(0.199775) = \frac{1}{1+e^{-0.199775}} = 0.549778$

Computing Error

$\delta_d = (y_d)(1 - y_d)(e) = (0.549778)(1 - 0.549778)(0 - 0.549778) = -0.136082$

$\delta_c = (y_c)(1 - y_c)(\delta_d)(w_{dc}) = (0.550045)(1 - 0.550045)(-0.136082)(0.118911) = -0.004005$

Correction

$\Delta w_{dc} = 0.3 \ -0.136082 * 0.550045 + 0.9 * 0.01891 = -0.00543$

$\Delta w_{ca} = 0.3 * -0.004 * 0 + 0.9 * 0.00085 * 1 = 0.0076$

$\Delta w_{cb} = 0.3 * -0.004 * 1 + 0.9 * 0 * 1 = -0.0012$

$\Delta w_{c0} = 0.3 * -0.004 * 1 + 0.9 * 0.00085 * 1 = -0.00043$

$\Delta w_{d0} = 0.3 * -0.136082 * 0 + 0.9 * 0.03439 * 1 = -0.00987$

New Weights

$w_{dc} = 0.018911 + -0.00543 = 0.11348$

$w_{ca} = 0.100852 + 0.0076 = 0.10161$

$w_{cb} = 0.1 + -0.0012 = 0.0988$

$w_{c0} = 0.10085 + -0.00043 = 0.100683$

$w_{d0} = 0.134394 + -0.00987 = 0.12452$