

Due data: 2/25/2020, end of the day.

For question 1-3, please submit a **PDF file** via Canvas.

For question 4 (programming question), please submit an **.ipynb file** via Canvas.

Please answer the following questions:

- [6 points] Prove Bayes' Theorem. Briefly explain why it is useful for machine learning problems, i.e., by converting posterior probability to likelihood and prior probability.
- [10 points] In Lecture 3-1, we gave the normal equation (i.e., closed-form solution) for linear regression using MSE as the cost function. **Prove that the closed-form solution for Ridge Regression** is  $\mathbf{w} = (\lambda I + X^T \cdot X)^{-1} \cdot X^T \cdot \mathbf{y}$ , where  $I$  is the identity matrix,  $X = (x^{(1)}, x^{(2)}, \dots, x^{(m)})^T$  is the input data matrix,  $x^{(i)} = (1, x_1, x_2, \dots, x_n)$  is the  $i$ th data sample, and  $\mathbf{y} = (y^{(1)}, y^{(2)}, \dots, y^{(m)})$ . Assume the hypothesis function  $h_{\mathbf{w}}(x) = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n$ , and  $y^{(j)}$  is the measurement of  $h_{\mathbf{w}}(x)$  for the  $j$ th training sample. The cost function of the Ridge Regression is  $E(\mathbf{w}) = \text{MSE}(\mathbf{w}) + \frac{\lambda}{2} \sum_{i=1}^m w_i^2$ . [Hint: please refer to the proof of the normal equation of linear regression. [ Note: Please use the following rectified definition of MSE when you prove:  $\text{MSE}(\mathbf{w}) = \sum_{i=1}^m (\mathbf{w}^T \cdot \mathbf{x}^{(i)} - y^{(i)})^2$ . ]
- [10 points] Recall the multi-class Softmax Regression model on page 16 of Lecture 3-3. Assume we have  $K$  different classes. The posterior probability is  $\hat{p}_k = \delta(s_k(x))_k = \frac{\exp(s_k(x))}{\sum_{j=1}^K \exp(s_j(x))}$  for  $k = 1, 2, \dots, K$ , where  $s_k(x) = \theta_k^T \cdot x$ , and input  $x$  is an  $n$ -dimension vector.
  - To learn this Softmax Regression model, how many parameters we need to estimate? What are these parameters?
  - Consider the cross-entropy cost function  $J(\theta)$  (see page 16 of Lecture 3-3) of  $m$  training samples  $\{(x_i, y_i)\}_{i=1,2,\dots,m}$ . Derive the gradient of  $J(\theta)$  regarding to  $\theta_k$  as shown in page 17 of Lecture 3-3

#### Programming Problem:

- [44 points] In this problem, we write a program to find the coefficients for a linear regression model for the dataset provided (data2.txt). Assume a linear model:  $y = w_0 + w_1 \cdot x$ . You need to
  - Plot the data (i.e., x-axis for 1<sup>st</sup> column, y-axis for 2<sup>nd</sup> column), and use Python to implement the following methods to find the coefficients:
    - Normal equation, and
    - Gradient Descent using **batch** AND **stochastic** modes respectively:
      - Determine an appropriate termination condition (e.g., when cost function is less than a threshold, and/or after a given number of iterations).
      - Print the cost function vs. iterations for each mode; compare and discuss batch and stochastic modes in terms of the accuracy and the speed of convergence.
      - Choose a best learning rate. For example, you can plot cost function vs. learning rate to determine the best learning rate.

Please implement the algorithms by yourself and **do NOT use the fit() function** of the library.