ELEC 4700 Assignment - 4
Circuit Modelling

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1.

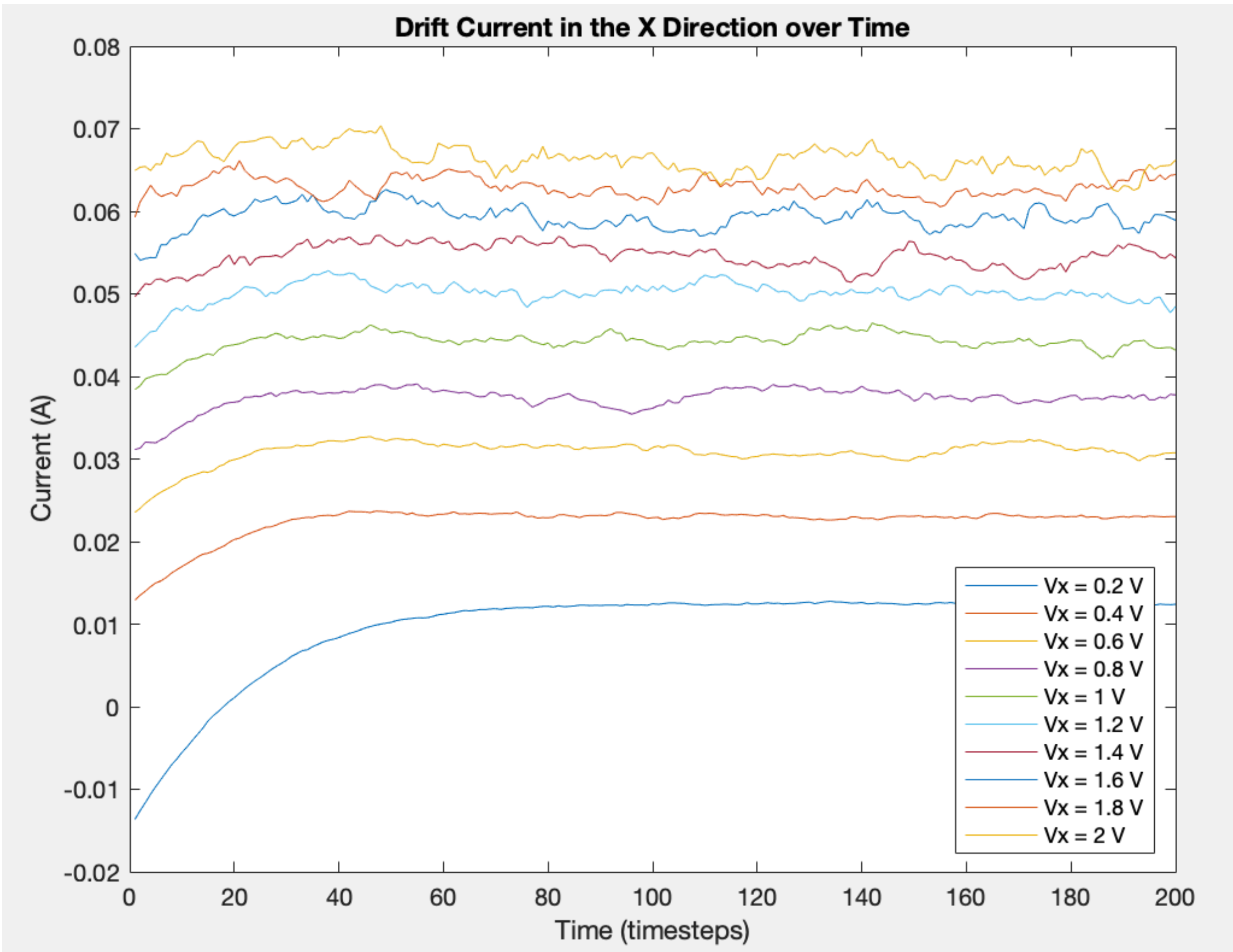

Figure 1: The drift current through the bottleneck in the x-direction over time for a range of voltages.

It should be noted that a range of 0.2 V-2 V was used instead of 0.1-10 V (approved by the prof).

These saturated currents after 200 time steps were used to create the plot below (see Figure 2).

2.

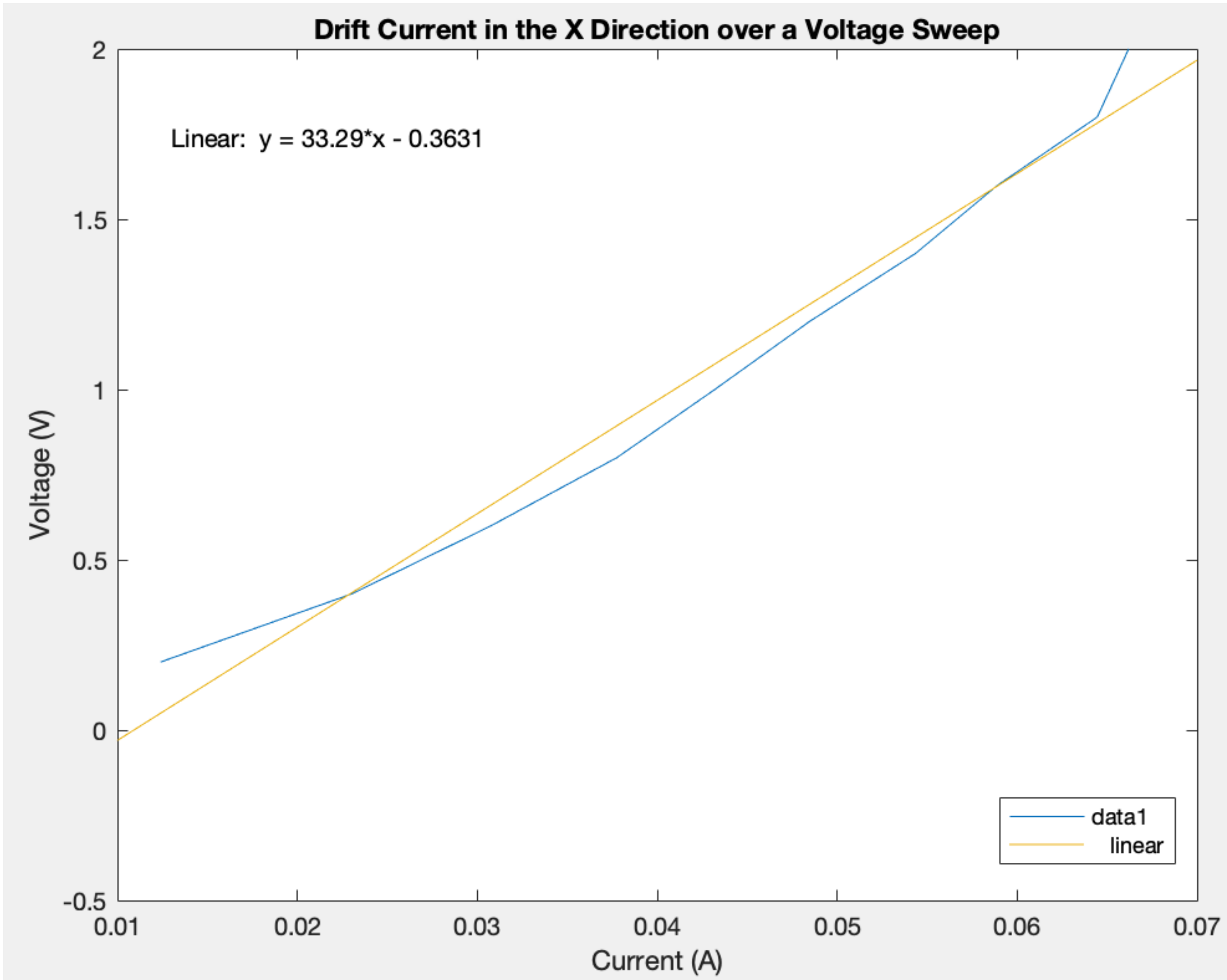

Figure 2: The saturated drift current through the bottleneck in the x-direction for a range of voltages (blue line). The linear fit for the data is also shown (yellow line), with a slope of 33.29.

A linear fit of the voltage vs saturated current data was performed in order to extract the approximate resistance via the slope of the linear fit. This allowed for a value of 33.29 Ohms to be determined for the resistance of the bottleneck.

It should also be noted that the bottleneck was widened from 20 nm (used in A1, A3) to 60 nm in order to achieve a resistance in the correct range (less than ~50 ohms).

3.

a) Formulation

Equations:

```
i) Time Domain
1 V1 = Vin;
2 (d(V1-V2)/dt)*C + (V1-V2)*G1 + Iin = 0
3 (d(V2-V1)/dt)*C + (V2-V1)*G1 + V2*G2 + IL = 0
4 V2-V3 = L*(d(IL)/dt)  % V2-V3 - 1i*omega*(L)*IL = 0
5 I3 = V3*G3
6 IL+I3 = 0
7 V4 = a*I3
8 (V4-Vo)*G4 + I4 = 0
9 (Vo-V4)*G4 + Vo*Go = 0
```

```
ii) Frequency Domain
1 V1 = Vin;
2 (V1-V2)*1i*omega*C + (V1-V2)*G1 + Iin = 0
3 (V2-V1)*1i*omega*C + (V2-V1)*G1 + V2*G2 + IL = 0
4 V2-V3 - 1i*omega*(L)*IL = 0
5 V3*G3 - I3 = 0
6 IL+I3 = 0
7 V4 - a*I3 = 0
8 (V4-Vo)*G4 + I4 = 0
9 (Vo-V4)*G4 + Vo*Go = 0
```

Code:

DC

```
for i = 1:21
    Vin = i-11;

    F(1) = Vin;

    V = G\F;
    V3(i) = V(n3);
    Vo(i) = V(no);
end
```

AC

```
for i = 1:101
    omega = i-1;

    CFinal = C.*(1i*omega);

    H = G + CFinal;
    V = H\F;

    V3(i) = V(n3);
    Vo(i) = V(no);
end
```

Perturbations on C

```
for i = 1:1000

    C(2,1) = C1(i);
    C(2,2) = -C1(i);

    C(3,1) = -C1(i);
    C(3,2) = C1(i);

    CFinal = C.*(1i*omega);

    H = G + CFinal;
    V = H\F;

    V3(i) = V(n3);
    Vo(i) = V(no);

end
```

G & C Matrices, F Vector

$$\hat{X} = [V_1 \quad V_2 \quad V_3 \quad V_4 \quad V_o \quad I_{in} \quad I_L \quad I_3 \quad I_4]$$

$$\hat{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -G_1 & G_1+G_2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & G_4 & -G_4 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -G_4 & G_4+G_o & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_1 & -C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -C_1 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{F} = \begin{bmatrix} V_{in} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

b) Programming

i)

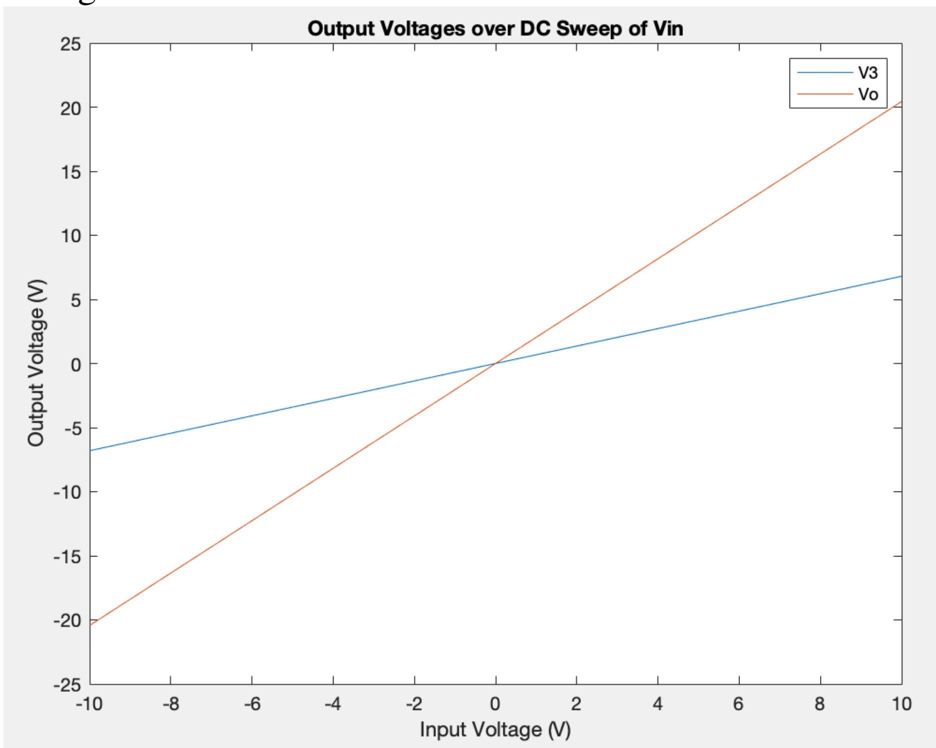

Figure 3: The output voltages (V3, Vo) for a DC voltage sweep of Vin from -10 V to 10 V.

ii)

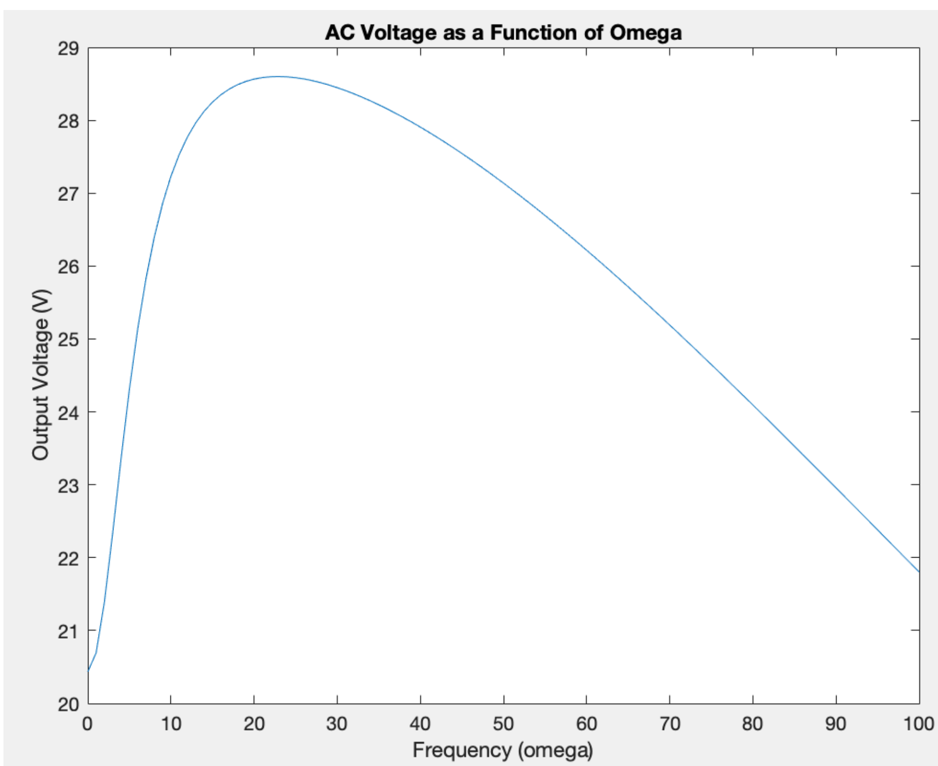

Figure 4: The AC output voltage (Vo) as a function of the frequency (omega).

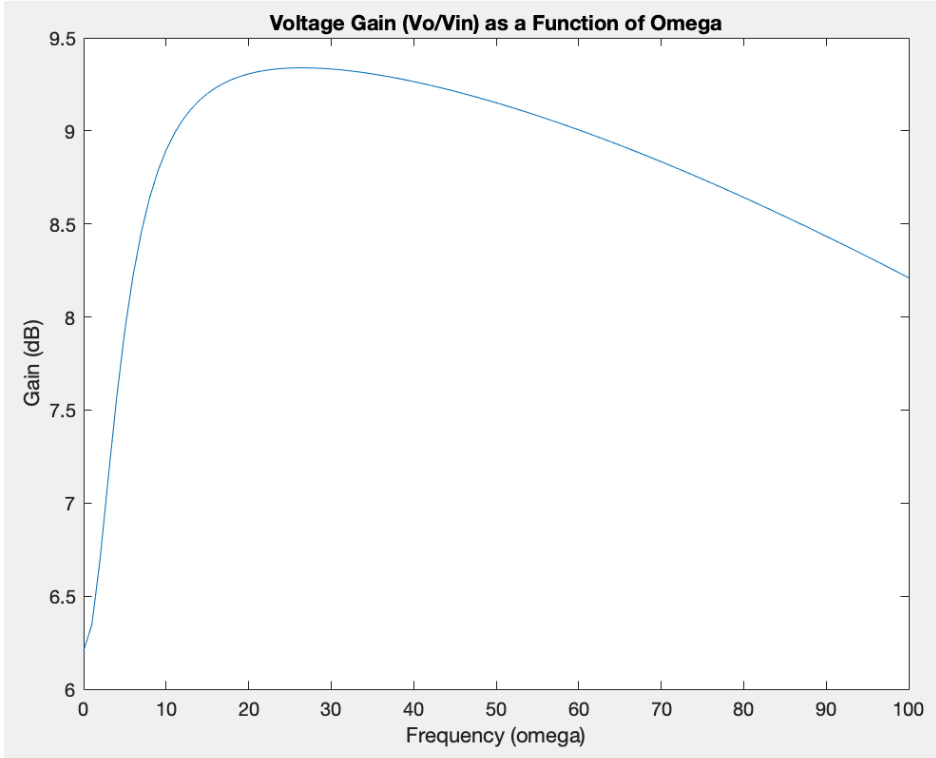

Figure 5: The output voltage gain (Vo/Vin) in dB as a function of the frequency (omega).

iii)

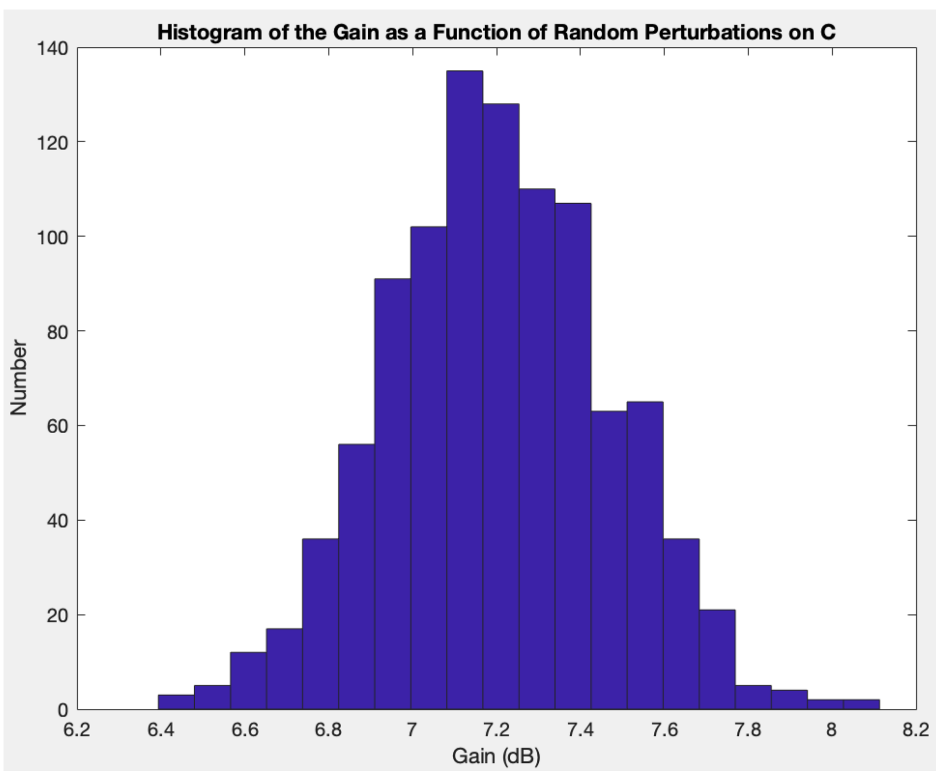

Figure 6: A histogram showing the number frequency of different gains (in dB) for the AC case with random perturbation on C.

4. Transient Circuit Simulation

a) By inspection, we can see that this circuit is an amplifier. The second voltage source having a value that is alpha (100) times the output current (I3) gives a final output value (Vo) that is amplified compared to both the input (Vin) and intermediate (V3) voltages.

b) I would expect this circuit to have an amplified frequency response in the output signal compared to the input.

c) Formulation

```
for i = 1:1000
    t(i) = i*tstep;

    if t(i) < 0.03 && inputA == 1
        Vin(i) = 0;
    elseif t(i) >= 0.03 && inputA == 1
        Vin(i) = 1;

    elseif inputB ==1
        Vin(i) = sin(2*pi*f*t(i));

    elseif inputC == 1
        Vin(i) = gaus(i,mu,sig,amp,vo);
    end

    F(1) = Vin(i);

    H = C/tstep + G;
    V = H\(F + ((C/tstep)*Vp));

    Vo(i) = V(no);
    Vp = V;
end
```

Conditions for the input signals:

-Step

-Sinusoid

-Gaussian

Updated differential equations solving for the V vector, based on the time domain equation:

$$\mathbf{C}\frac{d\mathbf{V}}{dt} + \mathbf{GV} = \mathbf{F}$$

d)

A) A step that transitions from 0 to 1 at 0.03s

iii)

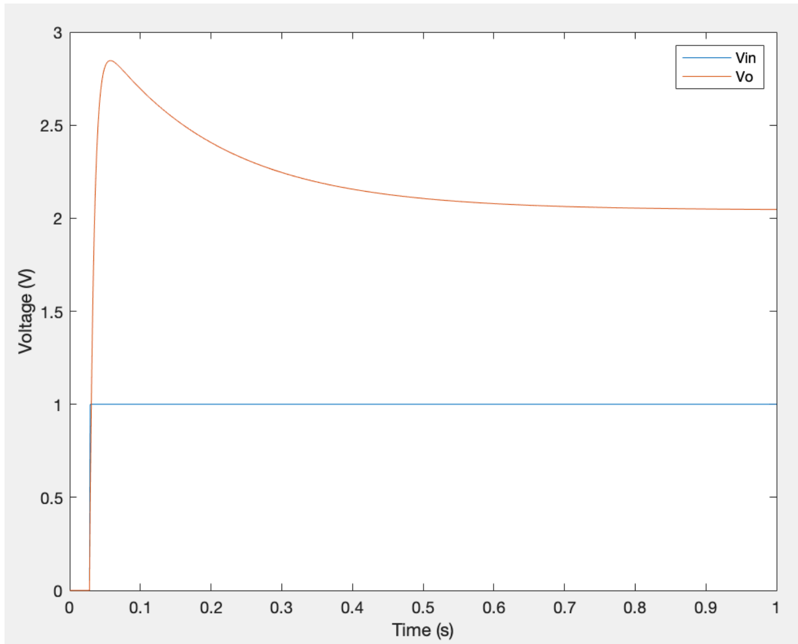

Figure 7: Plot of input and output voltages over the simulation (1000 time steps) for the step.

iv)

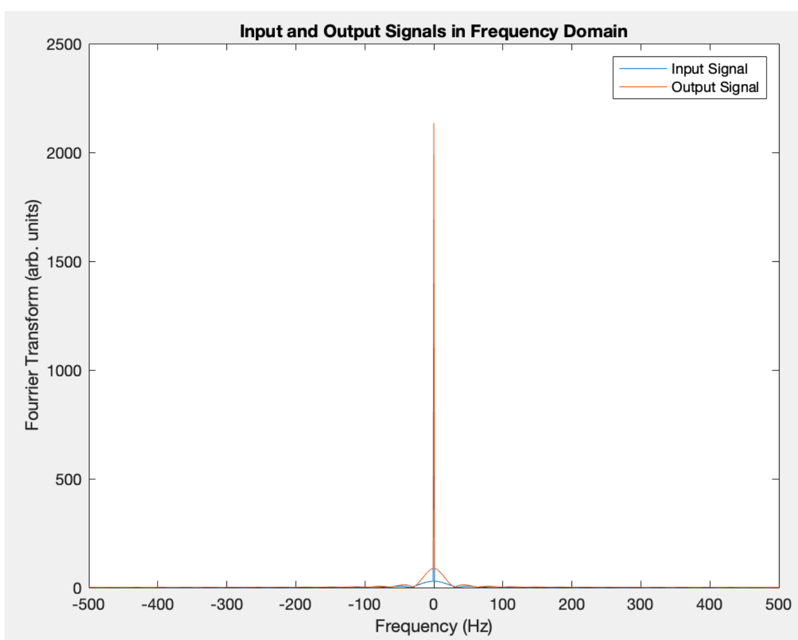

Figure 8: Fast Fourier Transform for the input and output signals (1000 time steps) for the step.

v) Increasing the Time Step

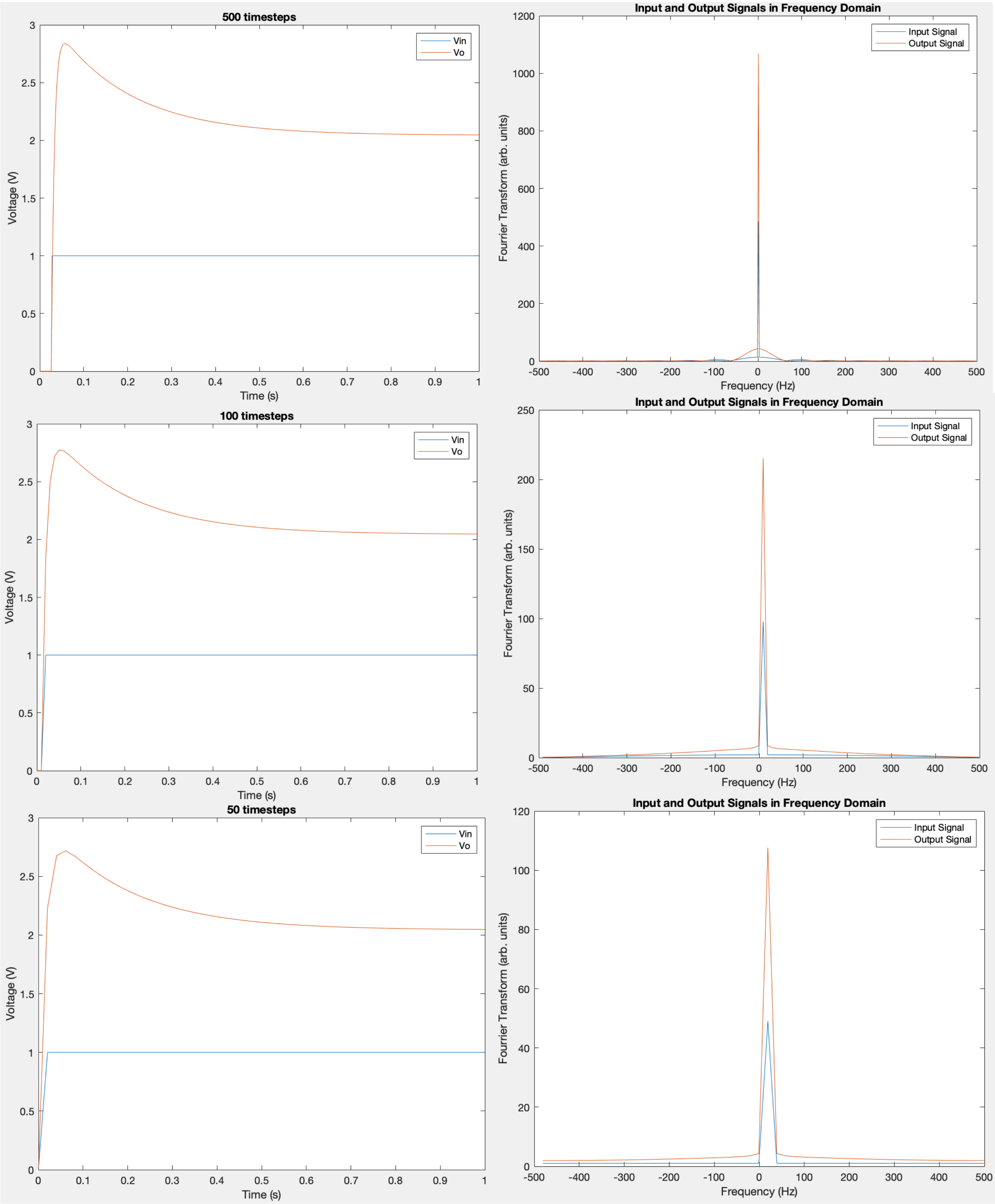

Figure 9: Plot of the input and output voltages (left) and the Fast Fourier Transforms (right) over the simulation for the step for 3 different time steps (500, 100, 50).

B) A sin(2πf t) function with f = 1/(0.03) Hz

iii)

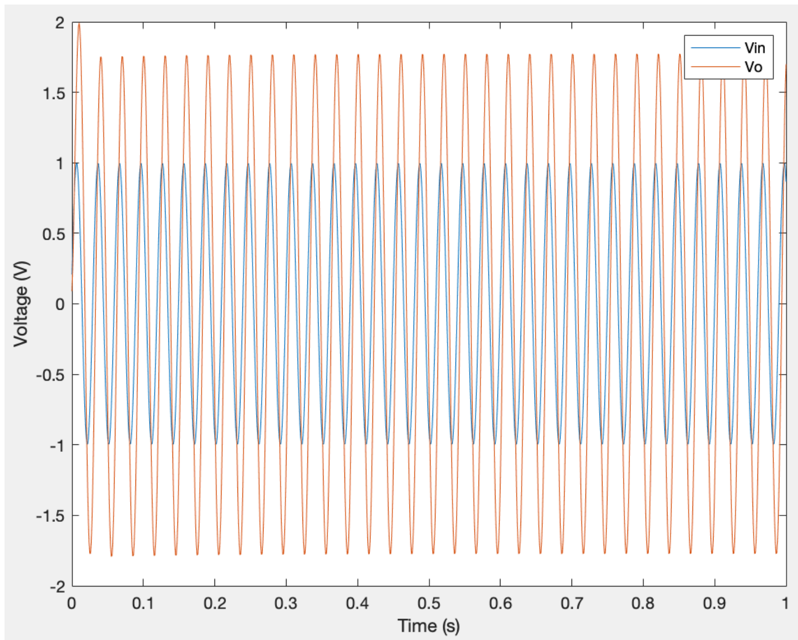

Figure 10: Plot of input and output voltages over the simulation (1000 time steps) for the sinusoid.

iv)

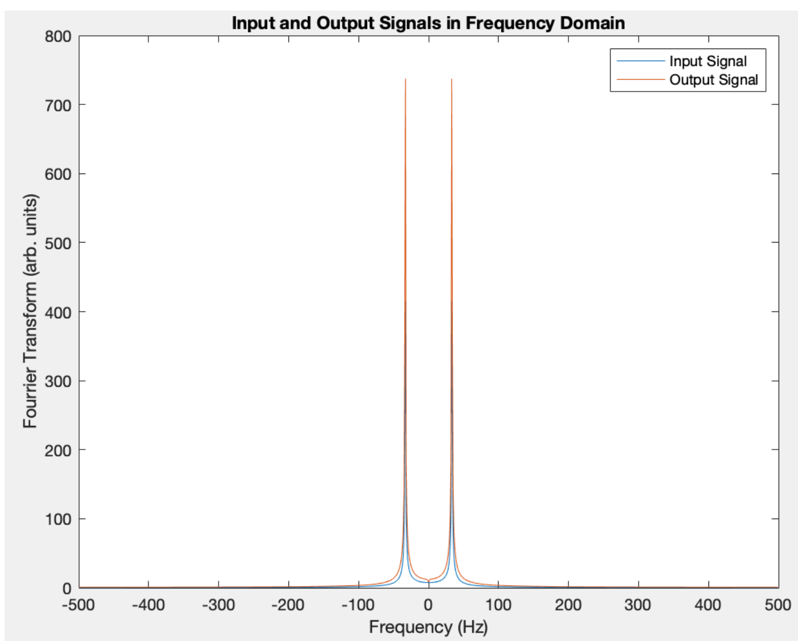

Figure 11: Fast Fourier Transform for the input and output signals (1000 time steps) for the sinusoid.

v) Increasing the Time Step

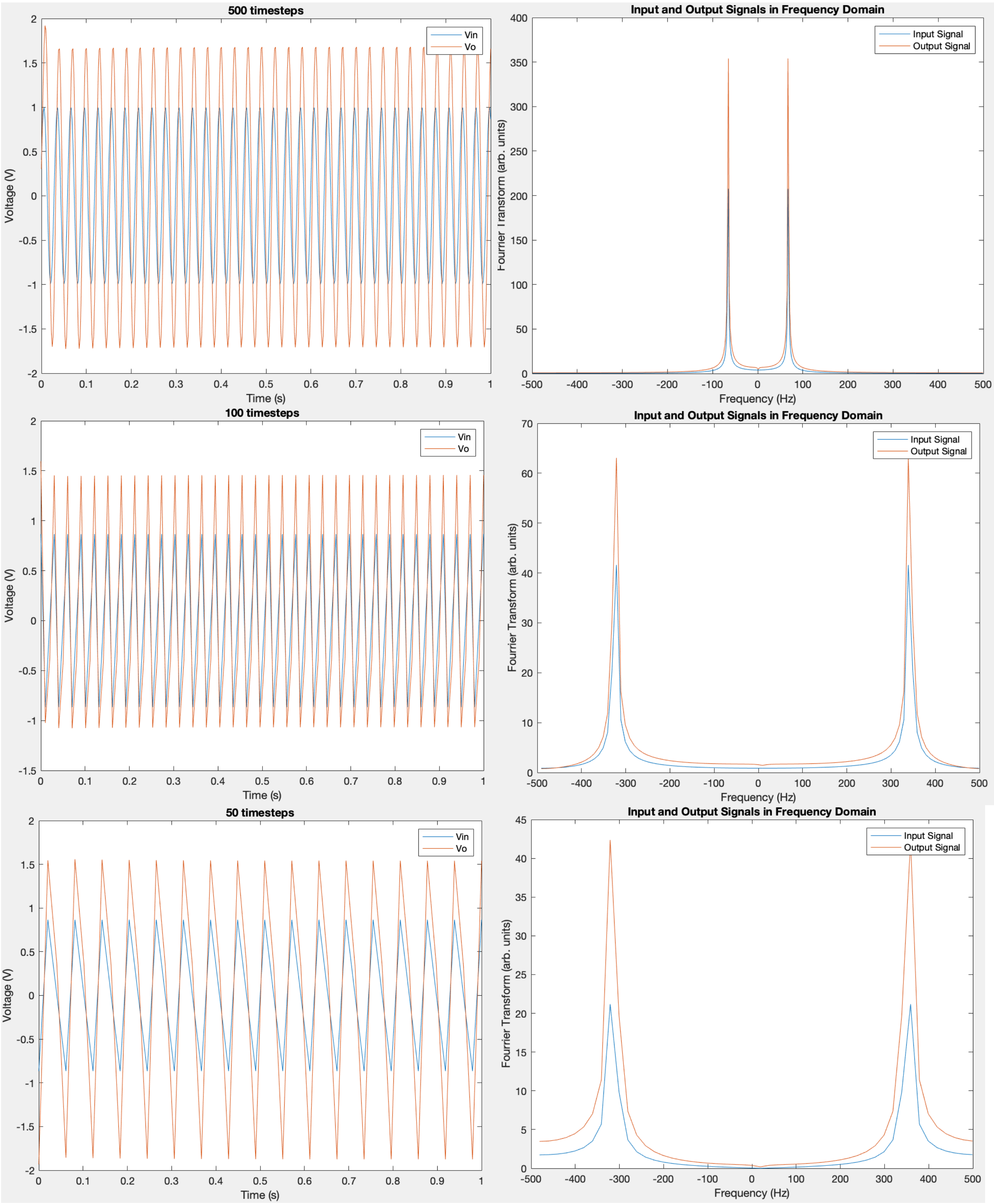

Figure 12: Plot of the input and output voltages (left) and the Fast Fourier Transforms (right) over the simulation for the sinusoid for 3 different time steps (500, 100, 50).

vi) Other frequencies

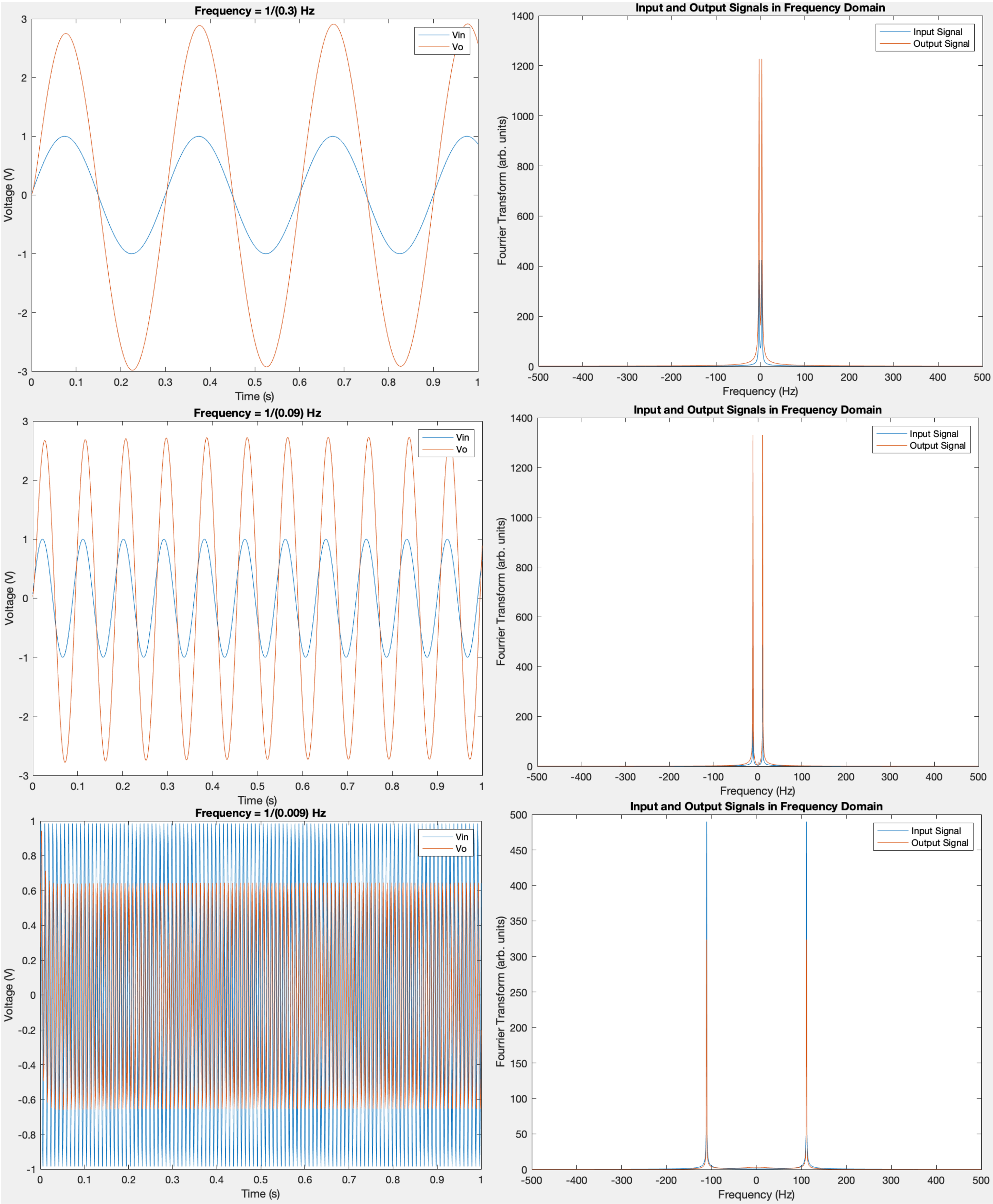

Figure 13: Plot of the input and output voltages (left) and the Fast Fourier Transforms (right) over the simulation for the sinusoid for 3 frequencies (1/0.3 Hz, 1/0.09 Hz, 1/0.009 Hz).

C) A gaussian pulse with a magnitude of 1, std dev. of 0.03s and a delay of 0.06s

iii)

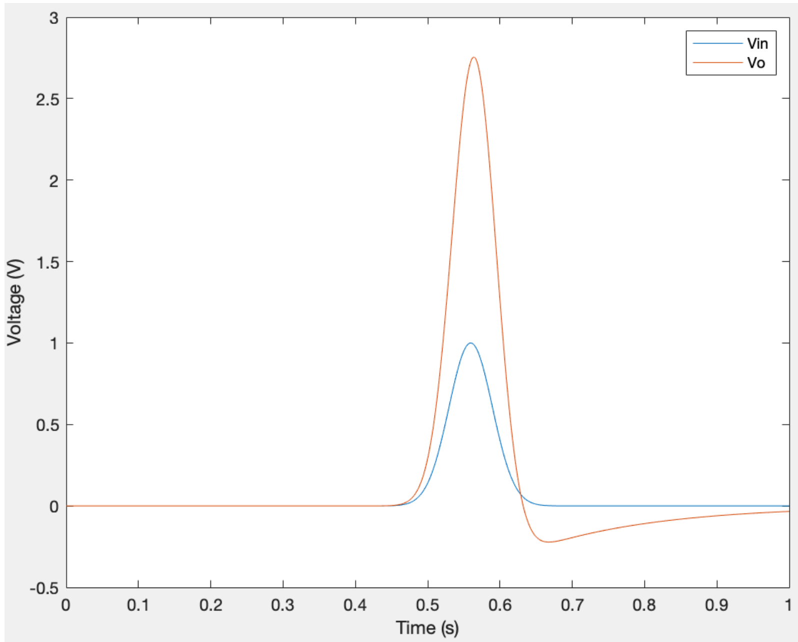

Figure 14: Plot of input and output voltages over the simulation (1000 time steps) for the Gaussian.

iv)

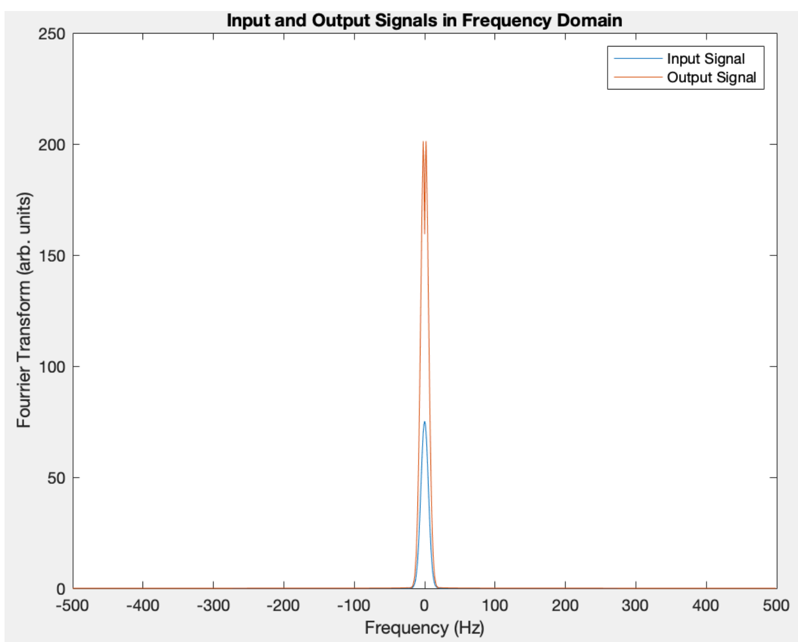

Figure 15: Fast Fourier Transform for the input and output signals (1000 time steps) for the Gaussian.

v) Increasing the Time Step

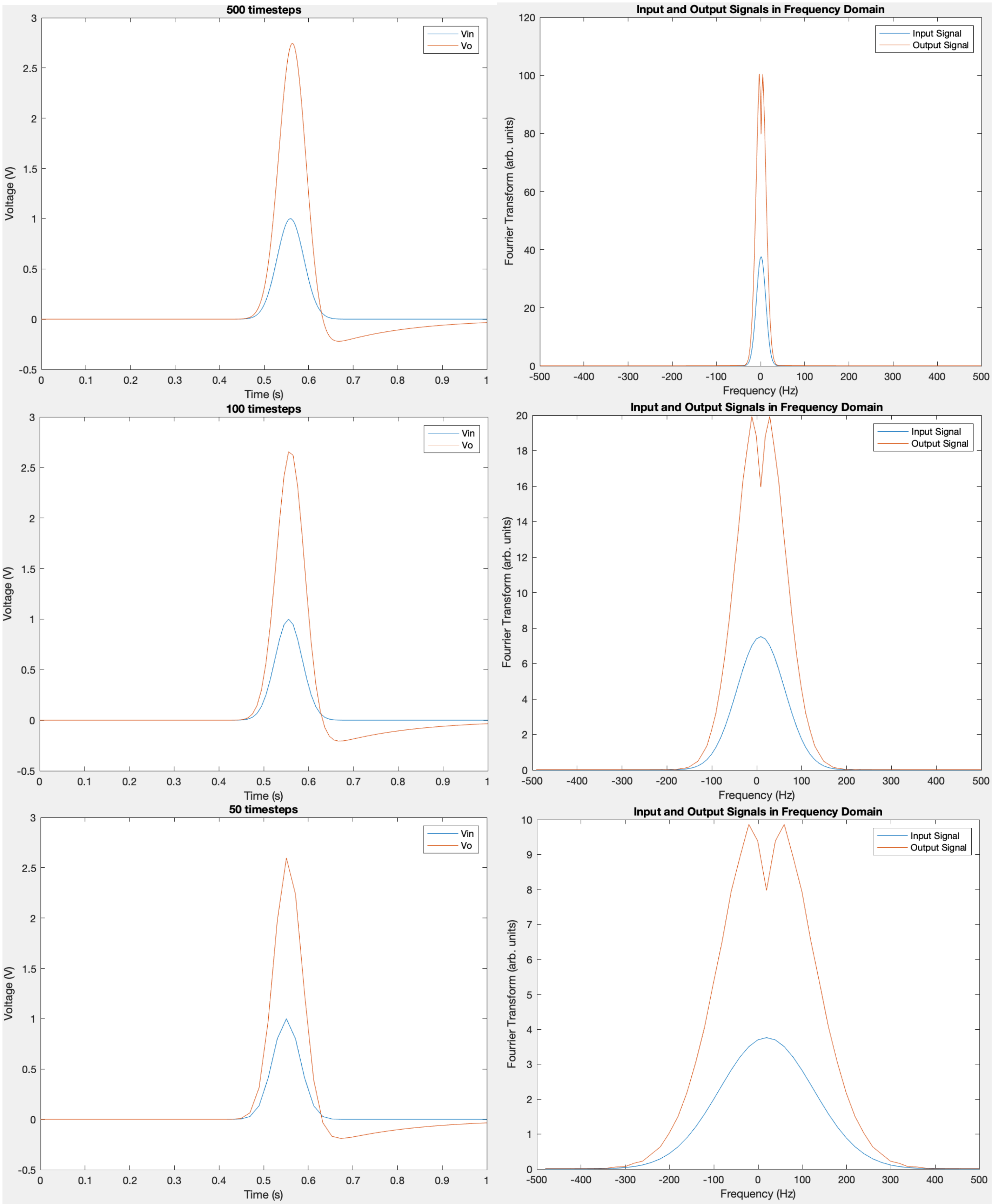

Figure 16: Plot of the input and output voltages (left) and the Fast Fourier Transforms (right) over the simulation for the Gaussian for 3 different time steps (500, 100, 50).

Part 4 Comments:

The results were mostly as expected; the FFT plots well fit what was expected for each given input signal. For example, the FFT of the step function was found to be the sinc function (see Figure 8) and the FFT of the Gaussian function was also found to be a Gaussian (see Figure 15). The output voltage signals and output FFTs were consistently amplified compared to the input signals, which also aligns with what was predicted.

For all three input signals, increasing time step was found to decrease the precision of both the voltage signals and FFT. This can be explained by the lower temporal resolution due to the increase in time between taking samples of the simulated data. Increasing the time step also appeared to increase the FFT frequency bandwidth.

For the sinusoidal input signal, changing frequency had a fairly direct impact on the voltage signals. The higher frequencies had shorter wavelengths for the sinusoid, and the lower frequencies had longer wavelengths for the sinusoid. Additionally, increasing the frequency resulted in a greater separation between the two characteristic peaks in the FFT.

5. Circuit with Noise

a)/b)

<u>Updated Equations:</u>

```
i) Time Domain
1 V1 = Vin;
2 (d(V1-V2)/dt)*C1 + (V1-V2)*G1 + Iin = 0
3 (d(V2-V1)/dt)*C1 + (V2-V1)*G1 + V2*G2 + IL = 0
4 V2-V3 = L*(d(IL)/dt) --> V2-V3 - 1i*omega*(L)*IL = 0
5 I3 = V3*G3
6 IL + I3 + In + d(V3-0)/dt)*Cn= 0
7 V4 = a*I3
8 (V4-Vo)*G4 + I4 = 0
9 (Vo-V4)*G4 + Vo*Go = 0
```

<u>Updated C matrix & F Vector:</u>

```
C(6,3) = Cn;
```

```
F(3) = 0.1*randn();
```

The only changes to the C matrix and F vector compared to those from Part 3 are the ones above; the 6th row & 3rd column of the C matrix has a value of Cn, and the 3rd row of the F vector has a random Gaussian distribution to simulate the thermal noise.

In addition, it should be noted that a value of In = 0.1 A was used so that noise was visible in the plots.

c) Simulation

iv) Vo with Noise

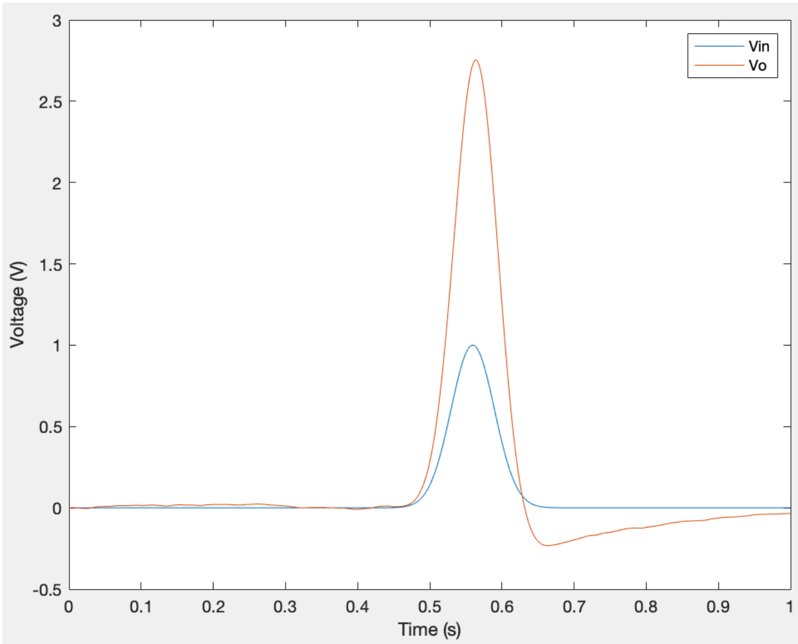

Figure 17: Plot of input and output voltages over the simulation (1000 time steps) for the Gaussian with noise (Cn = 0.00001 F).

v) Fast Fourier Transform

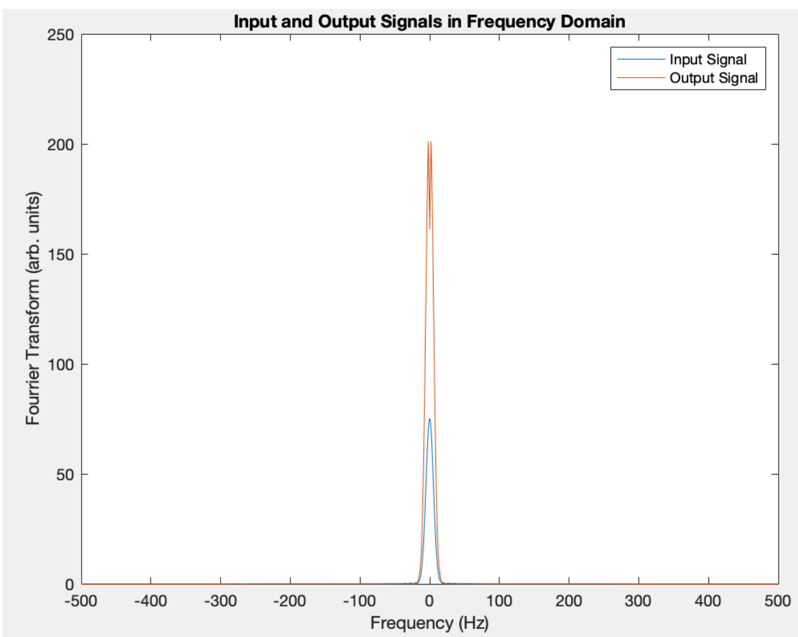

Figure 18: Fast Fourier Transform for the input and output signals (1000 time steps) for the Gaussian with noise (Cn = 0.00001 F).

vi) Changing Cn

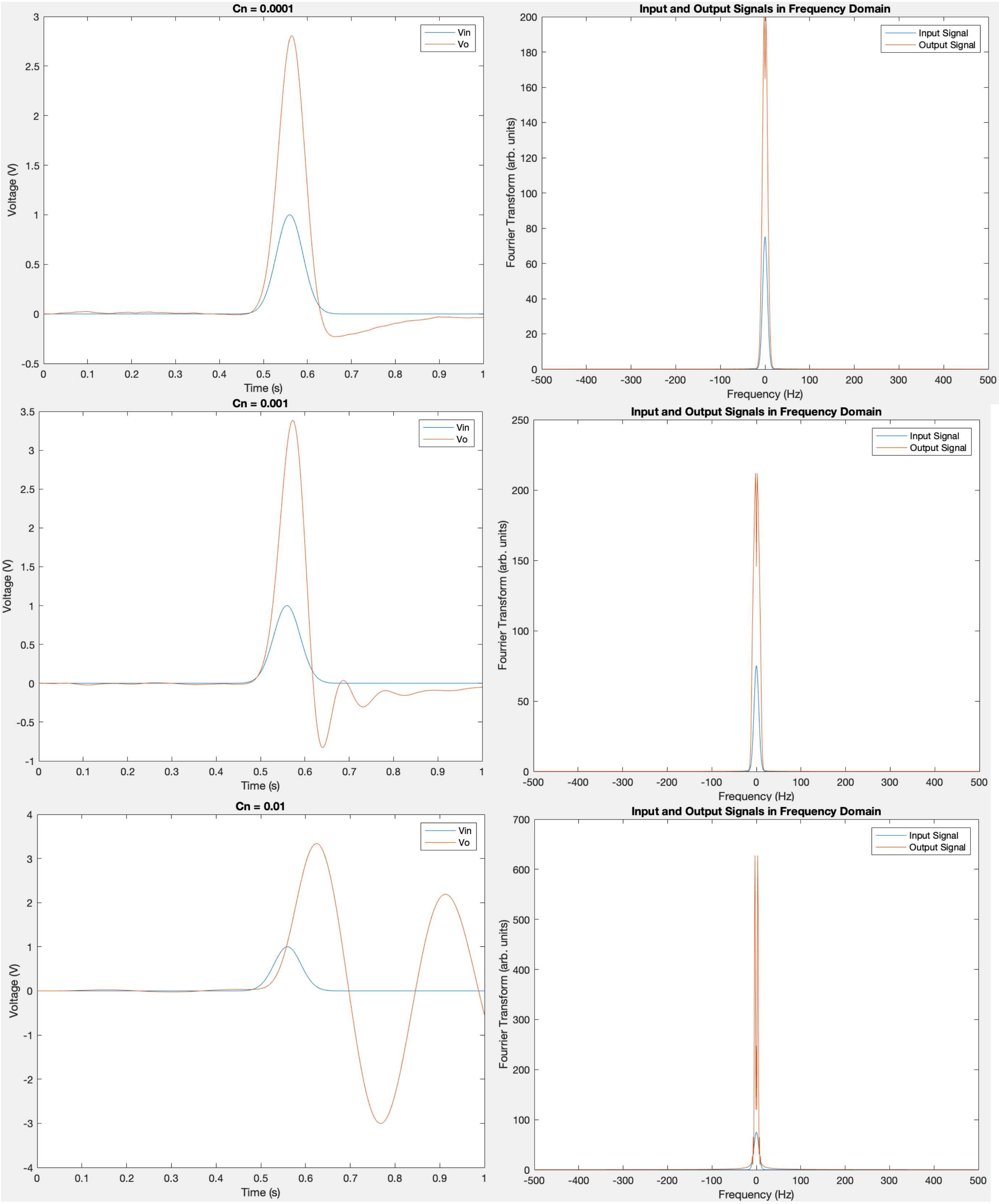

Figure 19: Plot of the input and output voltages (left) and the Fast Fourier Transforms (right) over the simulation for the Gaussian with noise for 3 different Cn values (0.0001 F, 0.001 F, 0.01 F).

vii) Increasing the Time Step

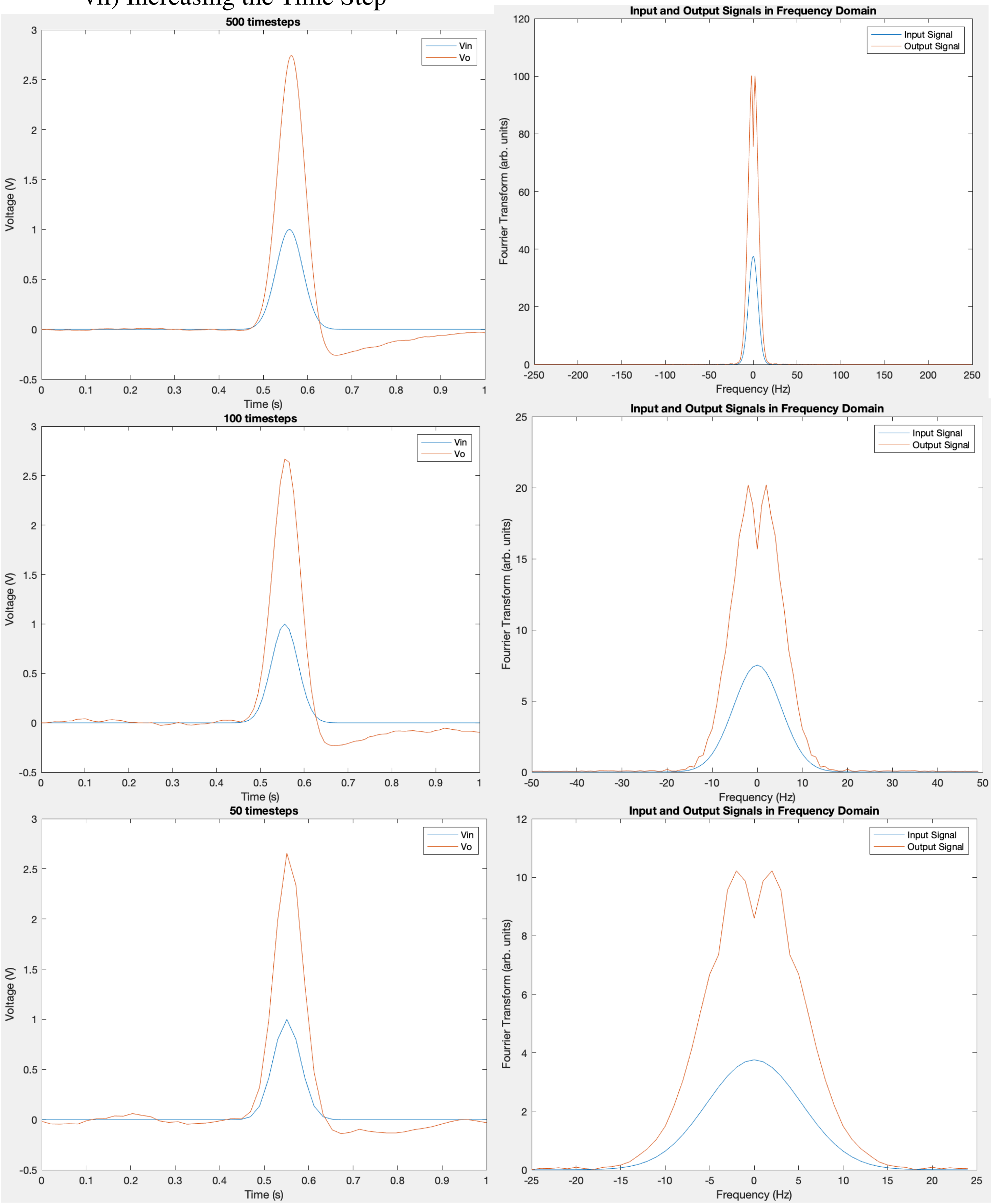

Figure 20: Plot of the input and output voltages (left) and the Fast Fourier Transforms (right) over the simulation for the Gaussian with noise for 3 different time steps (500, 100, 50).

Part 5 Comments:

The results for this section were mostly as expected. All the shown results in part 5 were done using the Gaussian extinction.

At first, there was no visible noise seen on the output voltage plots; however, after increasing the thermal noise current (In) to 0.1 A, the noise on the output signal started to become evident.

It was noticed that changing Cn from 0.00001 F to 0.0001 F had little effect on the output signal or FFT. However, the output signal began to more closely resemble a decaying sinusoidal function than a Gaussian pulse when Cn was decreased to 0.001 F, then 0.01 F. Additionally, the FFT began to more closely resemble the sinusoid's FFTs with 2 characteristic peaks than a Gaussian FFT when Cn was decreased to 0.001 F, then 0.01 F.

As seen in Part 4, increasing time step decreased the resolution of the output voltage signal and FFT. It was also noticed that the amplitude of the FFT was lowered as the time step was increased. Additionally, with increased time step sizes (fewer number of times steps), the Gaussian pulse output signals became much more noisy.

6. Non-Linearity

In order to be able to model a non-linear transconductance equation, the differential equation used to model the simulator would need to include a term for non-linear components. Typically this is represented by the B vector, which can represent things such as inductors, diodes, and non-linear voltage sources (see equation below).

$$\hat{C}\frac{d\hat{X}}{dt} + \hat{G}\hat{X} + \hat{B} = \hat{F}$$

Next, a Jacobian will need to be formed via an expansion on the differential equation. This will act as the multidimensional derivative of the B vector with respect to voltage (in this case).

Once the Jacobian is formed, the problem can be solved using an iterative method known as the Newton-Raphson method. This method requires an initial guess, and then improves the solution during each iteration until V = V + dV (essentially solving for a "root").