- Simulation et Monte Carlo, ENSAE 2019-2020

Dynamics of transmission of COVID-19

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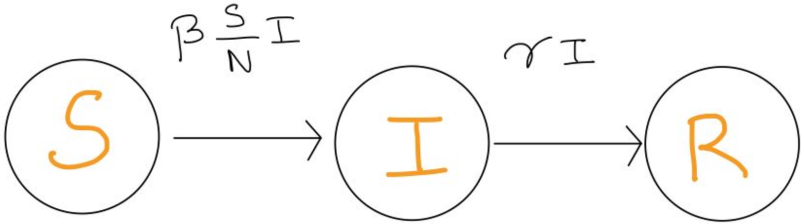
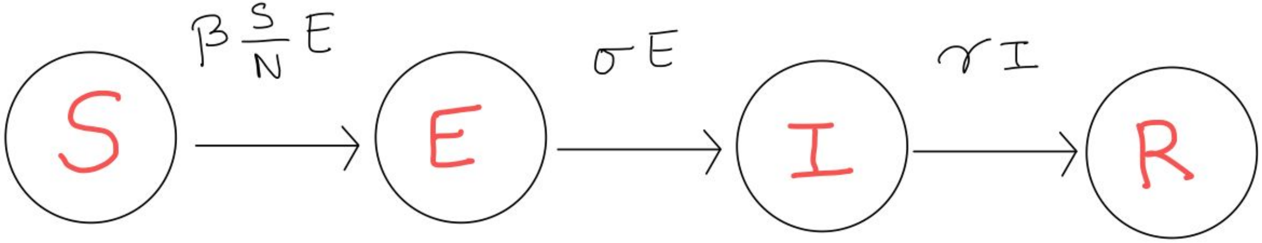
Quick introduction to compartmental models in epidemiology



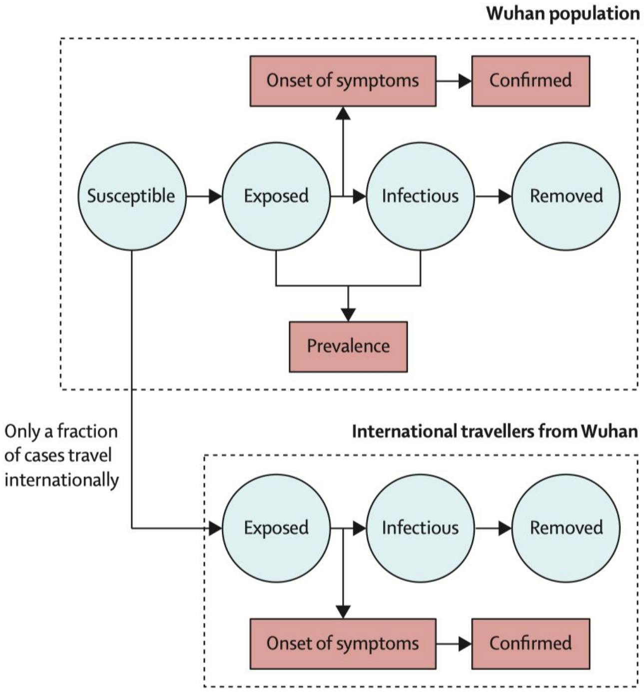
Closed population:

N = S + I + R
or
N = S + E + I + R



A mathematical modelling study in epidemiology

- An extended version of a SEIR model on Wuhan's population

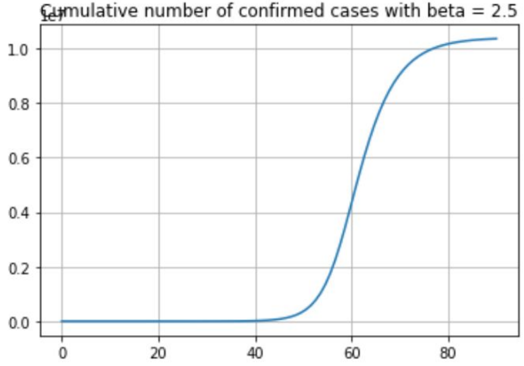


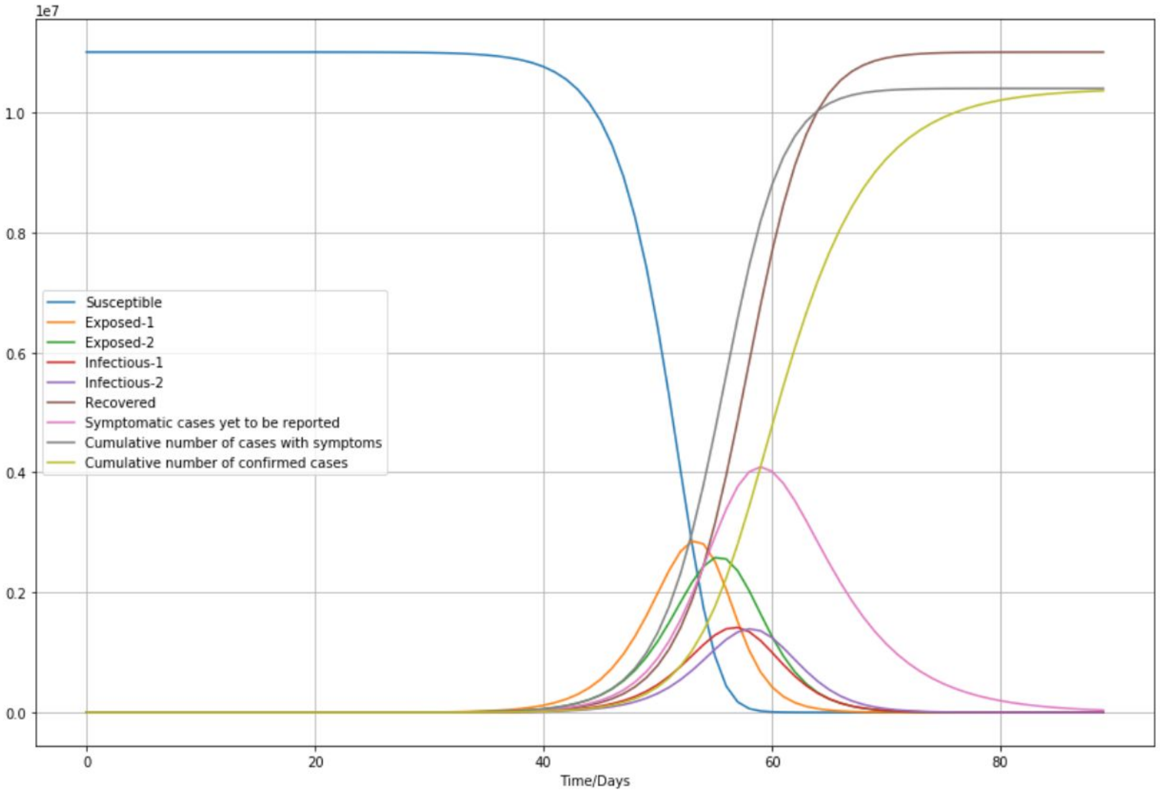
$$
\begin{aligned}
S(t+1) &= S(t) - \beta(t)S(t)[I_{1w}(t) + I_{2w}(t)]/N \\
E_{1w}(t+1) &= E_{1w}(t) + (1-f)\beta(t)S(t)[I_{1w}(t) + I_{2w}(t)]/N - 2\sigma E_{1w}(t) \\
E_{2w}(t+1) &= E_{2w}(t) + 2\sigma E_{1w}(t) - 2\sigma E_{2w}(t) \\
I_{1w}(t+1) &= I_{1w}(t) + 2\sigma E_{2w}(t) - 2\gamma I_{1w}(t) \\
I_{2w}(t+1) &= I_{2w}(t) + 2\gamma I_{1w}(t) - 2\gamma I_{2w}(t) \\
Q_w(t+1) &= Q_w(t) + 2\sigma E_{2w}(t)e^{-\gamma\kappa} - \kappa Q_w(t) \\
D_w(t+1) &= D_w(t) + 2\sigma E_{2w}(t)e^{-\gamma x} \\
C_w(t+1) &= C_w(t) + \kappa Q_w(t) \\
R_w(t+1) &= R_w(t) + \gamma(I_{1w} + I_{2w})
\end{aligned}
$$

Implementation of a deterministic model

Simulation up to 90 days (~3 months) with the following parameters (close to the paper ones):

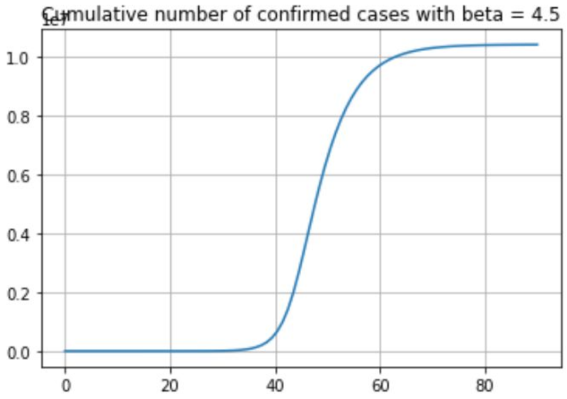
N = 11 000 000
init_vals = [N-1, 1, 0, 0, 0, 0, 0, 0, 0]
beta = 2.5
sigma= 1/5.2
gamma = 1/2.9
kappa = 1/6.1

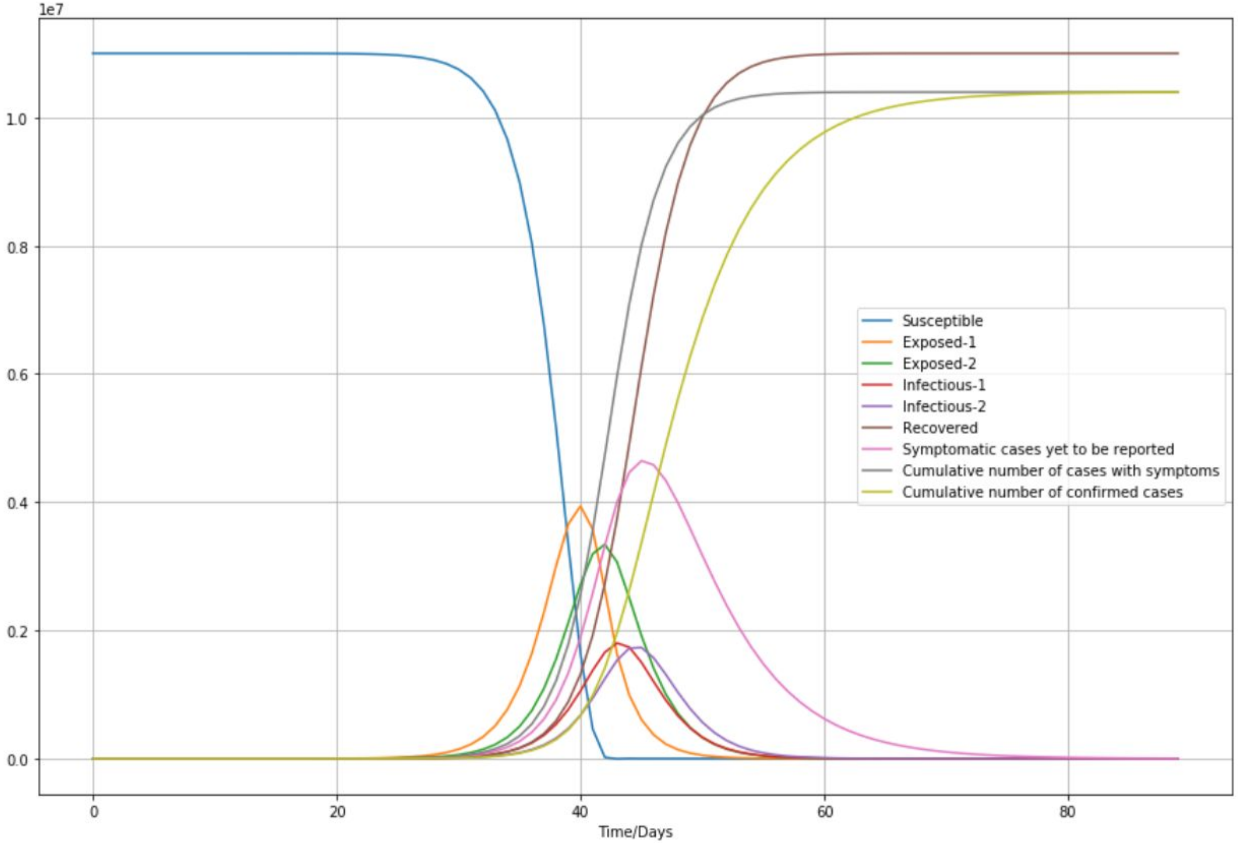




Implementation of a deterministic model

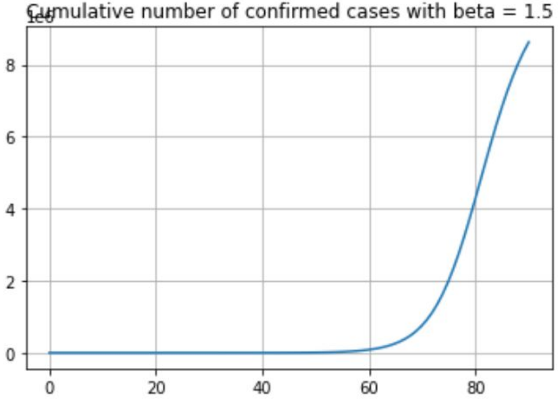
If we change beta to 4.5

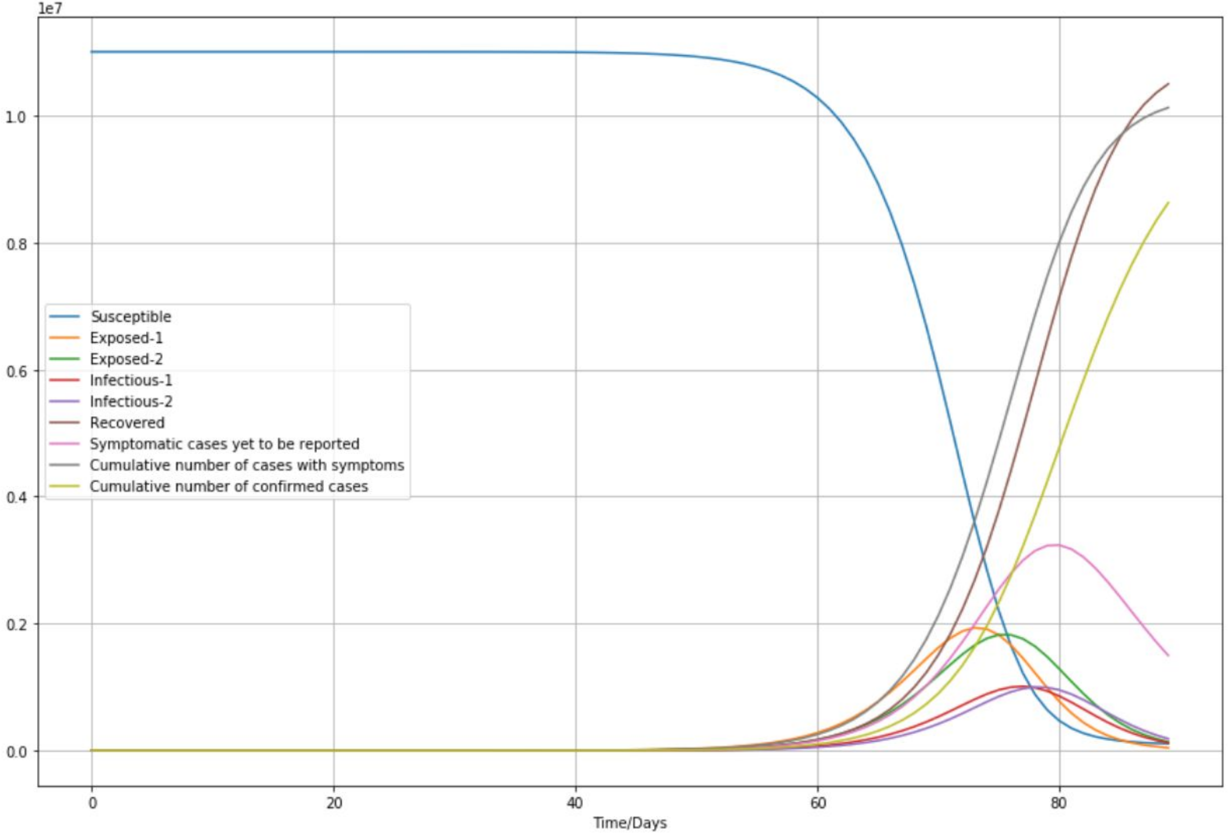




Implementation of a deterministic model

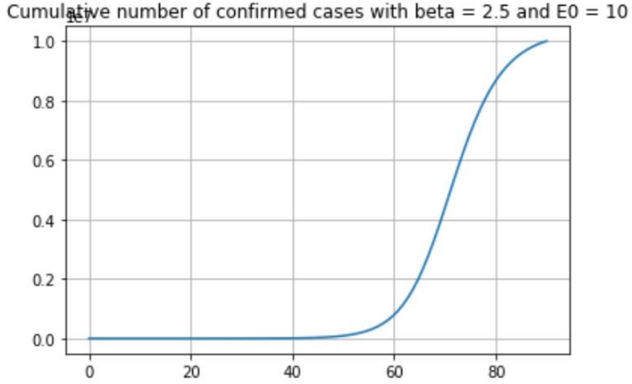
If we change beta to 1.5

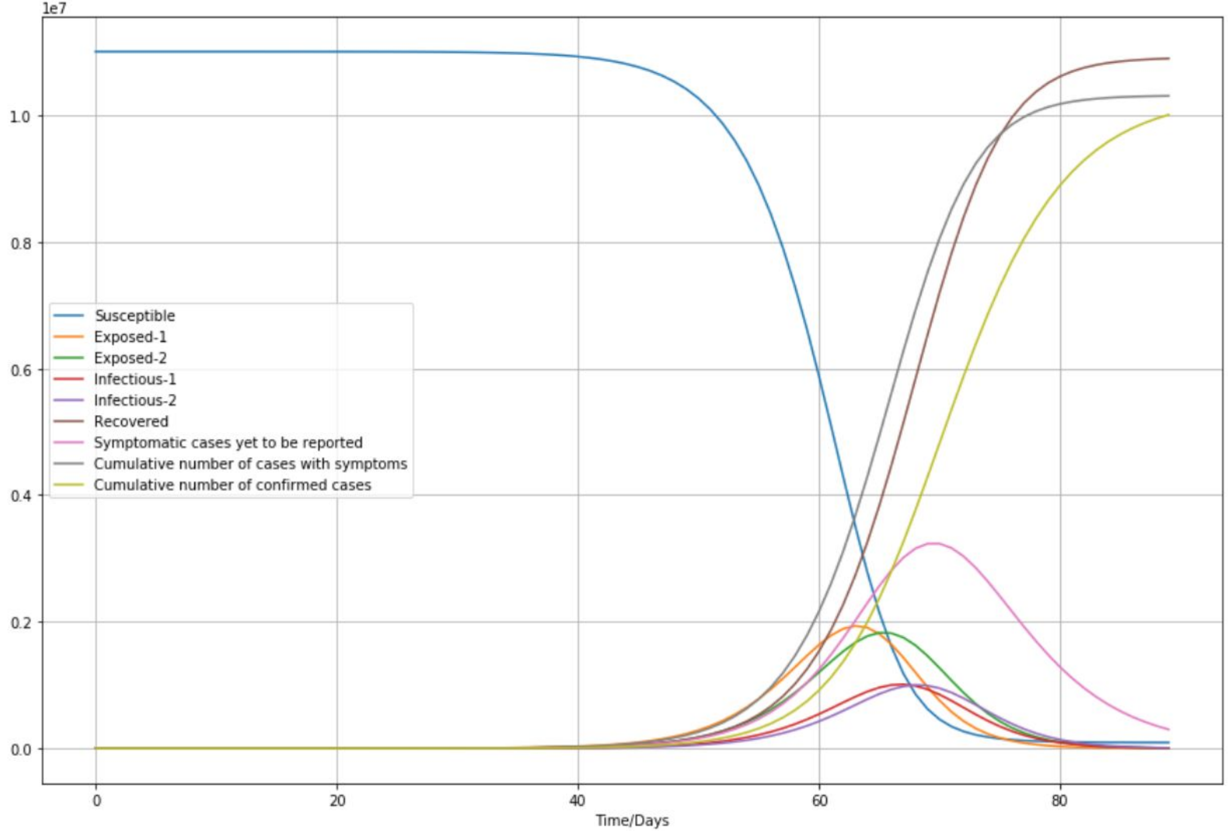




Implementation of a deterministic model

Back to beta = 2.5 but the number of initial cases is now 10

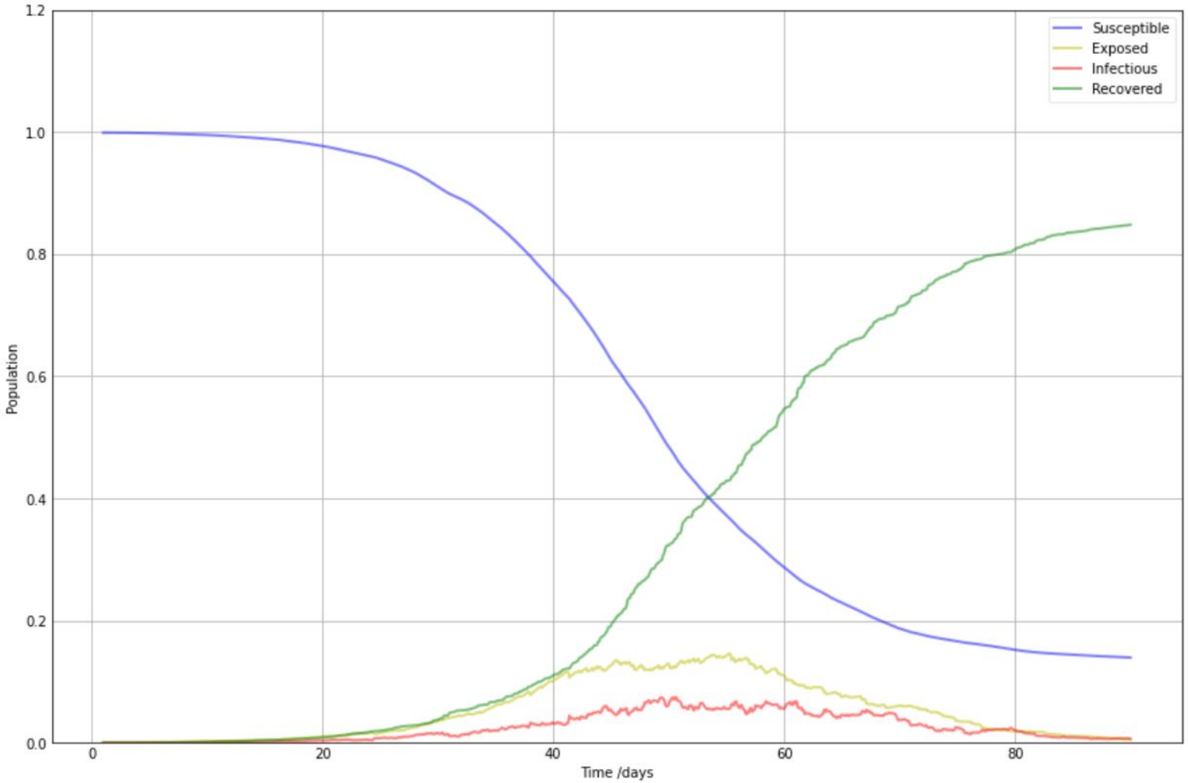




Stochastic implementation using Monte Carlo’s methods

Stochastic modelling on SEIR

Here both sigma and gamma have exponential distribution



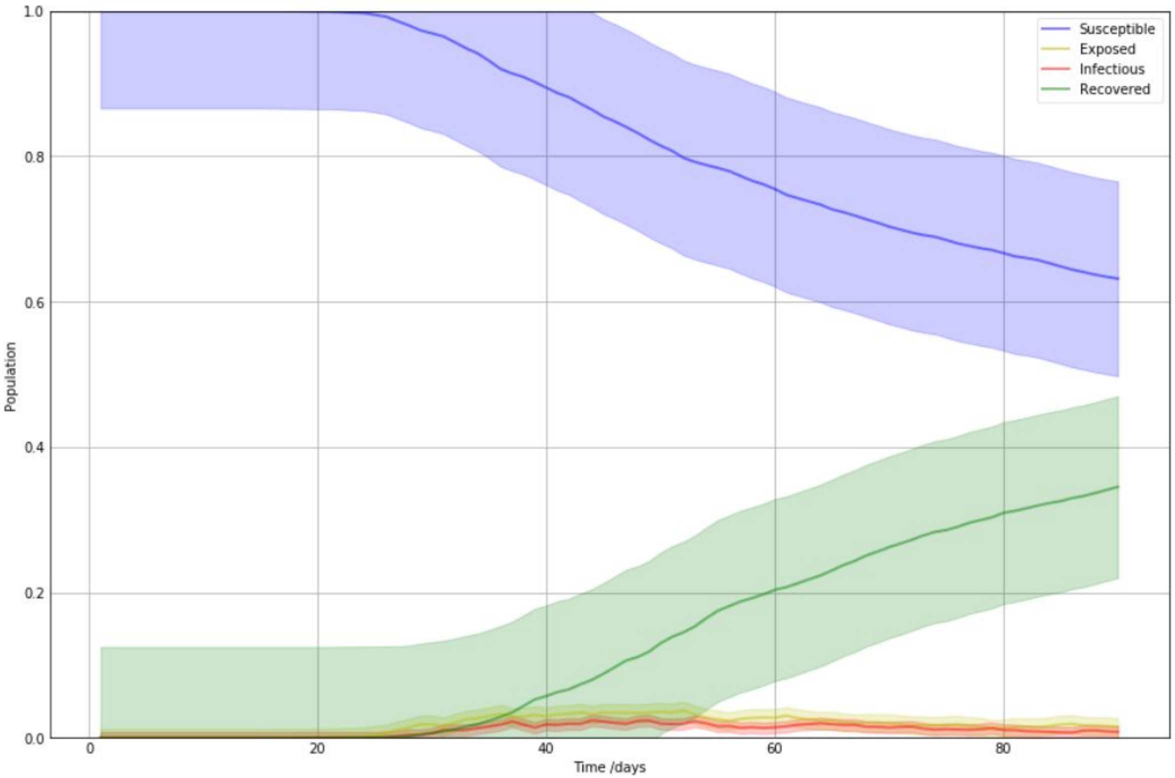
Stochastic modelling: transmission rate a Geometric Brownian Motion

$$d \log(\beta(t)) = a dB_t$$

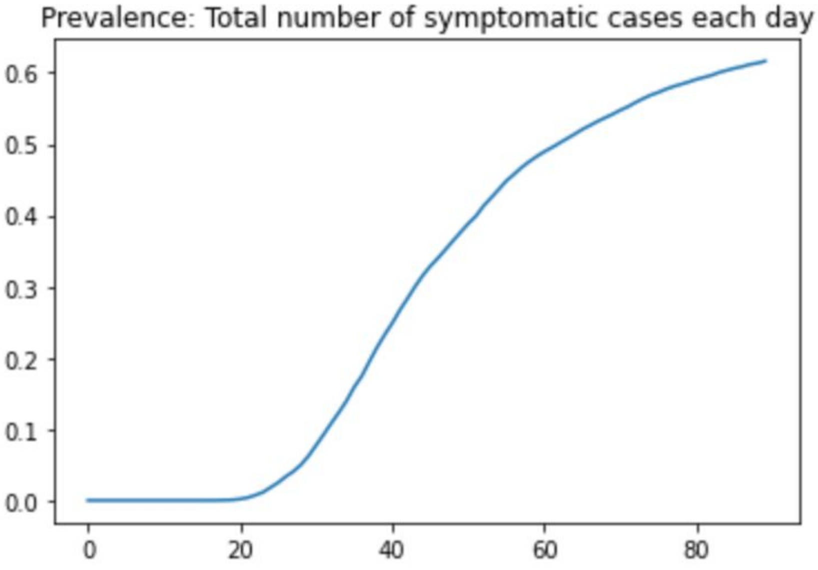
where a is the volatility of transmission over time and Bt is Brownian motion.
In the paper, they estimate a = 0.395

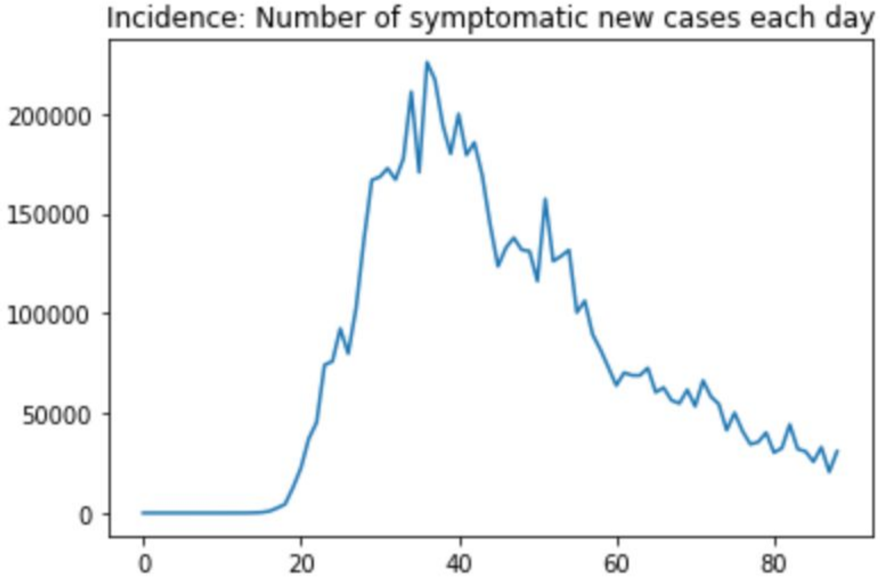
$$\beta(t+1) = \beta(t)e^{X}$$

where X → N(0,a)



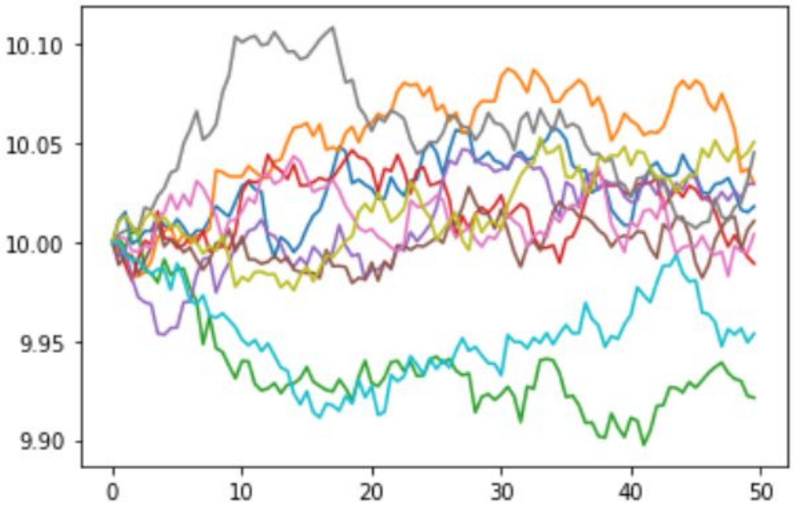
Stochastic modelling: simulation in order to estimate



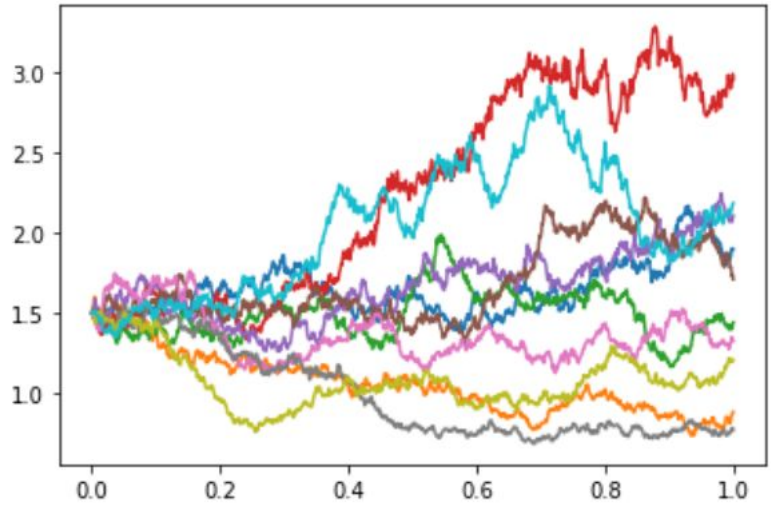


Stochastic modelling: Geometric Brownian Motion

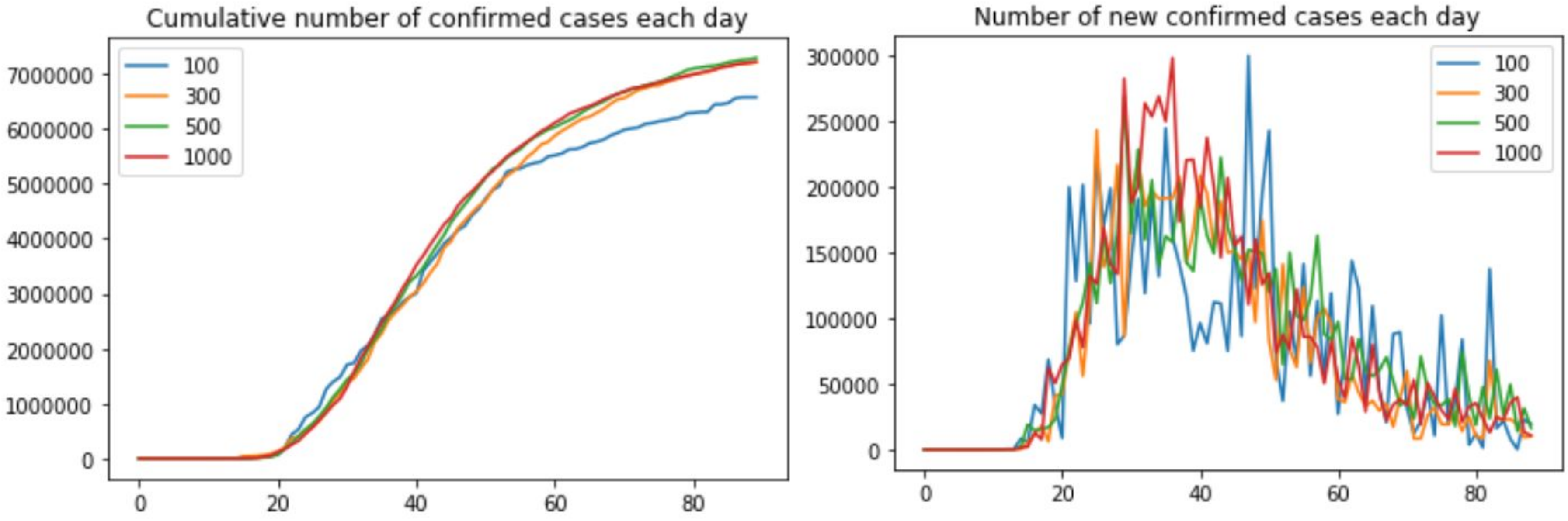
First technique: using a random walk



Second technique: RQMC with Sobol sequence



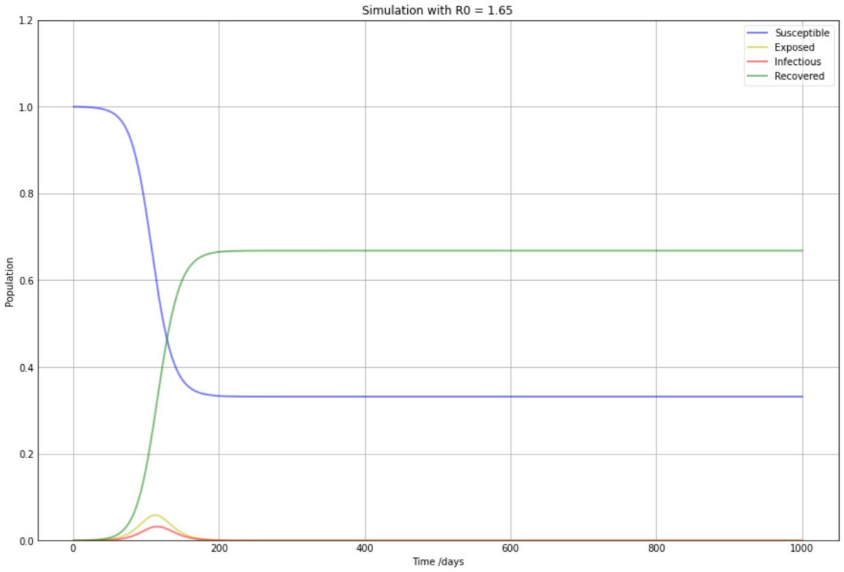
Stochastic modelling: RQMC has better convergence rate and smaller approximation error than MC

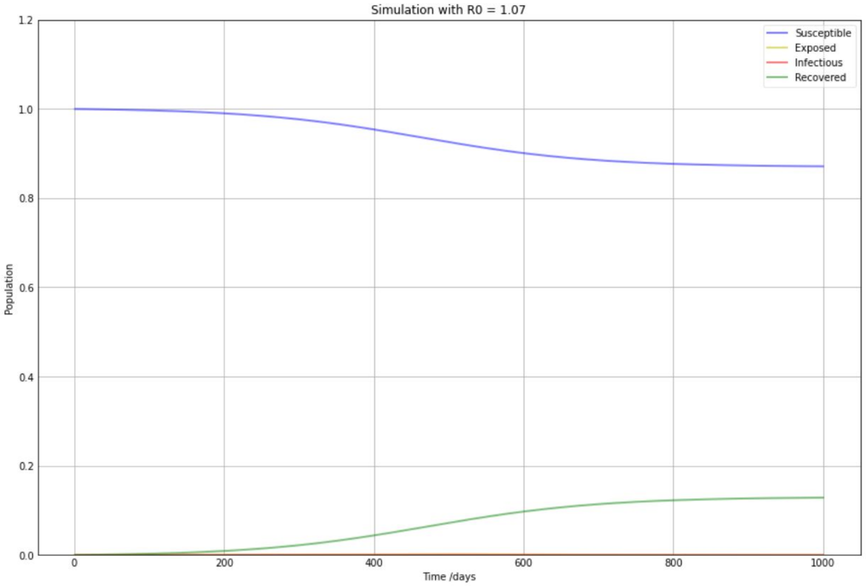


Application of the model on France: estimation of R0

Linear estimation of R0 between 25/03/2020 and 26/02/2020: 1.65

Linear estimation of R0 after 18/03/2020 (after containment measures): 1.07



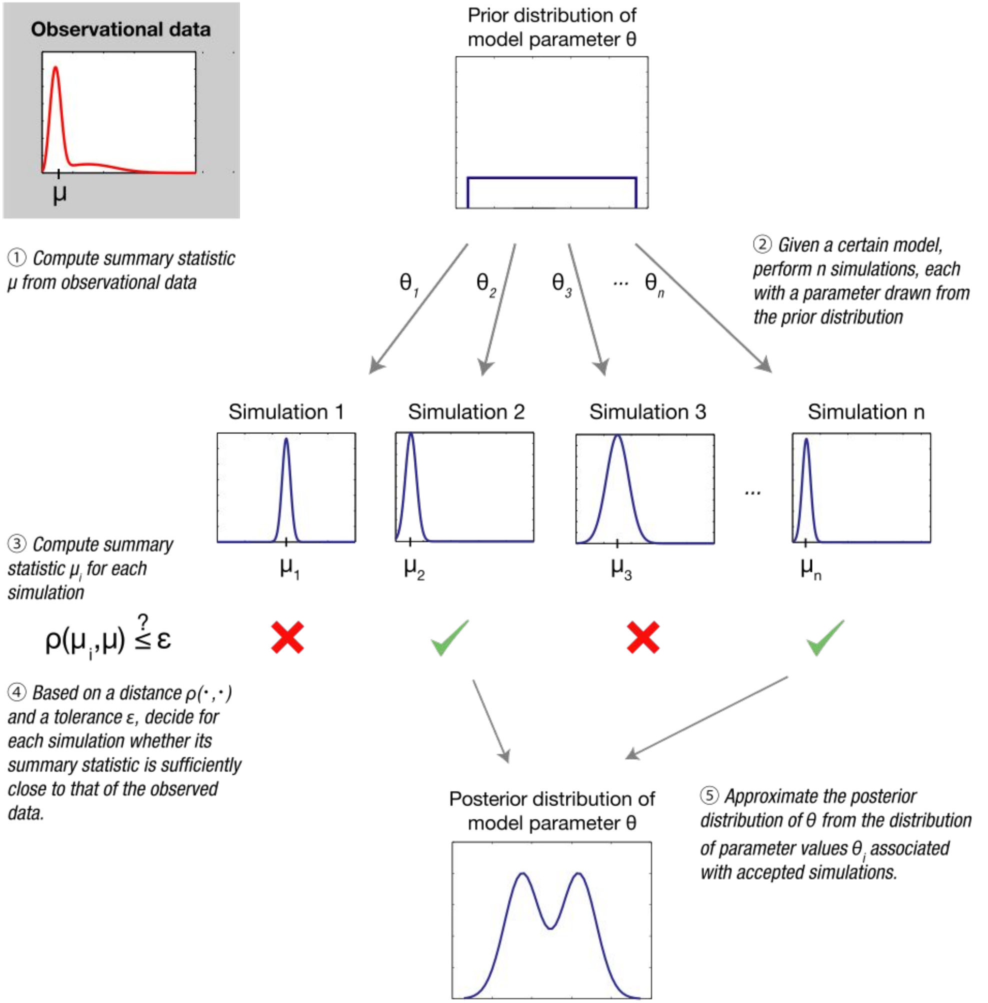


Estimate the posterior distribution without the likelihood function

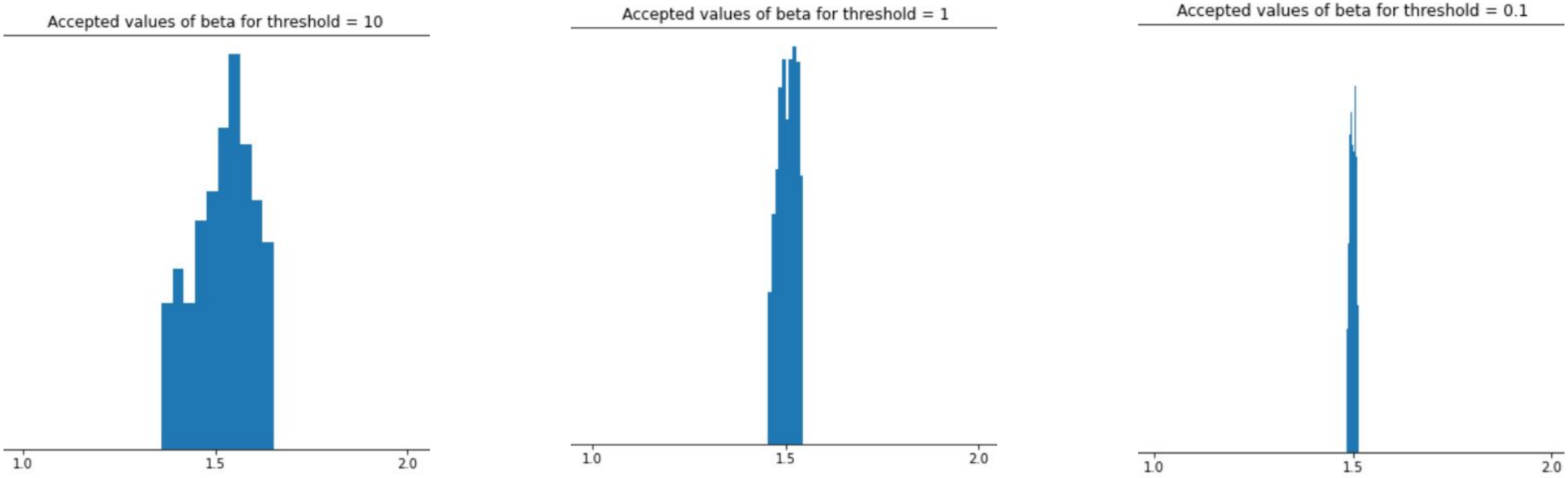
Approximate Bayesian Computation (ABC)

ABC rejection algorithm :

- sample many particles from a prior distribution (e.g. uniform)
- generate synthetic datasets from these particles and compare them to observed data
- accept particles with a given tolerance rate
- when N particles are accepted, estimate the posterior from the accepted particles (histogram)


Observational data
μ
① Compute summary statistic μ from observational data
Prior distribution of model parameter θ
θ1 θ2 θ3 … θn
② Given a certain model, perform n simulations, each with a parameter drawn from the prior distribution
Simulation 1
Simulation 2
Simulation 3
Simulation n
…
μ1 μ2 μ3 μn
③ Compute summary statistic μi for each simulation
ρ(μi,μ) ≤? ε
④ Based on a distance ρ(·,·) and a tolerance ε, decide for each simulation whether its summary statistic is sufficiently close to that of the observed data.
Posterior distribution of model parameter θ
⑤ Approximate the posterior distribution of θ from the distribution of parameter values θi associated with accepted simulations.

Approximate Bayesian Computation (ABC)



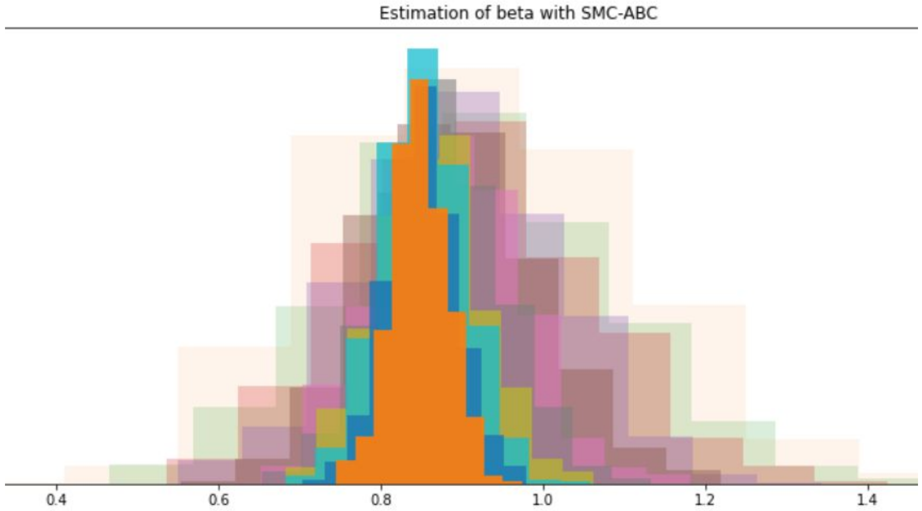
trade-off between the <u>precision</u> of the estimated posterior and the <u>computation time</u>

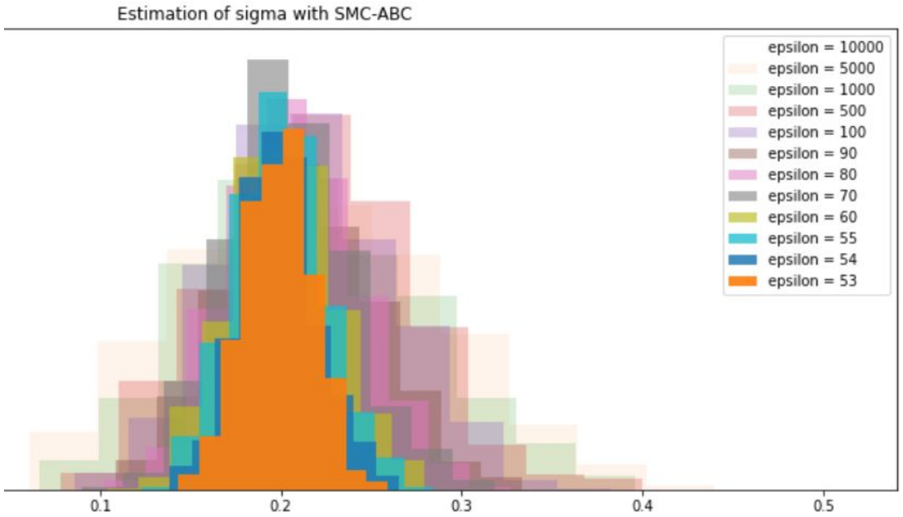
ABC-SMC (Sequential Monte Carlo)

ABC-SMC rejection algorithm :

- define a list of tolerance rates : eps1 > eps2 > … > epsT
- at first, run ABC algorithm with tolerance eps1 and particles from prior distribution
- estimate the posterior
- then, run ABC with successive tolerance rates
- sample particles from intermediate posterior distribution
- refine the estimated posterior distribution
- repeat

ABC-SMC (Sequential Monte Carlo)



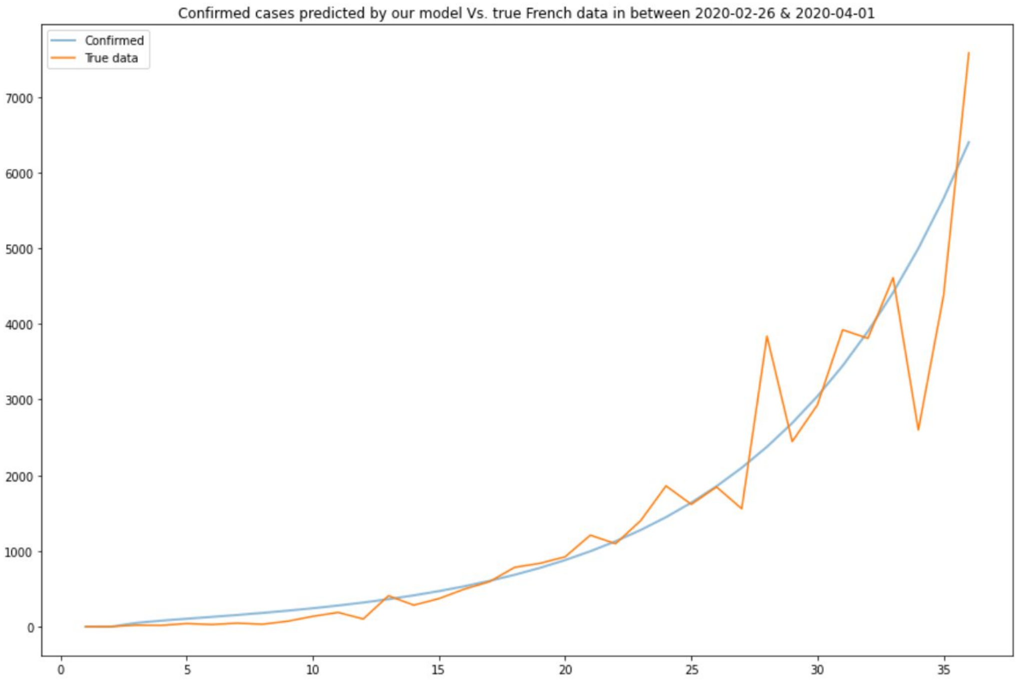


much faster and more precise, a lot of possible improvements (how to sample from intermediate posterior? how to choose tolerance rate? how to decide whether to accept a particle?)

Application on French covid-19 data

Setting:

- SEIR model with delay between symptomatic and confirmed cases
- from 26/02 to 31/03, before lockdown impact
- parameters to estimate: infection rate, incubation period, report delay, initial number of exposed/infected
- observed data: nb of new confirmed cases each day



Conclusion / Limitations

- SIR model : basic model of epidemics, good at predicting trends (not precise numbers), can be complexified

- Infection rate much more deterministic than expected

- ABC algorithm is very powerful for complex model with no obvious likelihood and latent variable model with partially observed data