

MODERATE AND EXTREME URBAN RAINFALL MODELING AT A FINE SPATIO-TEMPORAL RESOLUTION

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STUDY AREA



- ▶ **Geography:**
Verdanson water catchment, tributary of the Lez, located in an urban area
- ▶ **Context:**
Mediterranean events, flood risks

RAIN GAUGES NETWORK

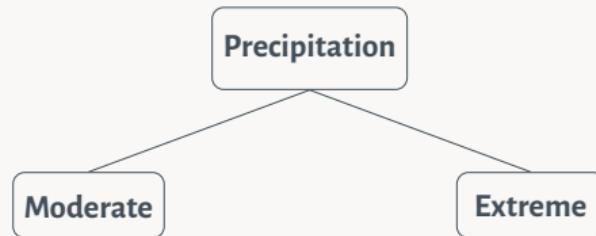


$$\mathcal{S} = \{17 \text{ rain gauges}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$

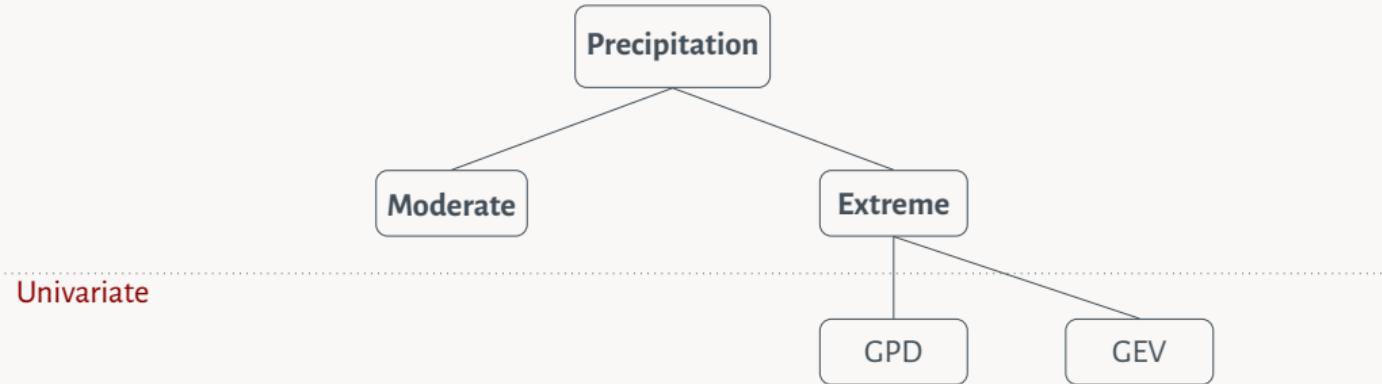
- ▶ **Source:** Urban observatory of HydroScience Montpellier (OHSM)¹
- ▶ **Time period:** [2019, 2022]
- ▶ **High temporal resolution:**
Every minute → 5-minute aggregation
- ▶ **High spatial resolution:**
Interdistance $\in [77, 1531]$ meters

¹FINAUD-GUYOT et al. 2023

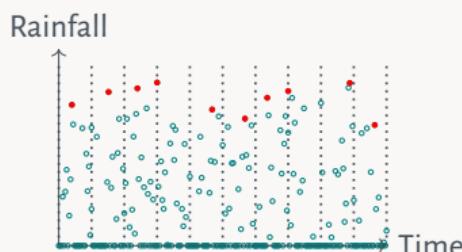
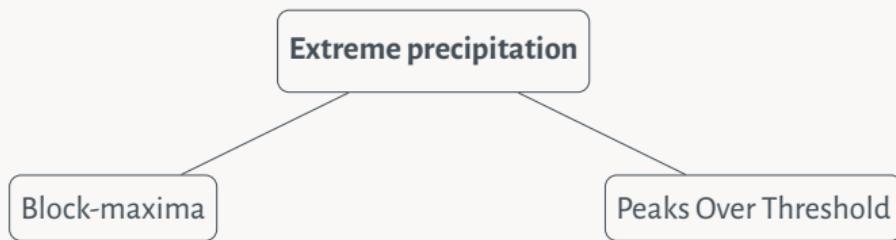
PRECIPITATION MODELING



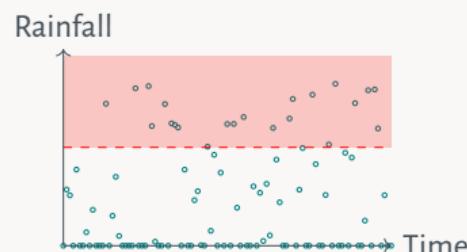
PRECIPITATION MODELING



UNIVARIATE PRECIPITATION MODELING



Generalized Extreme Value
distribution (GEV)



Generalized Pareto Distribution
(GPD)

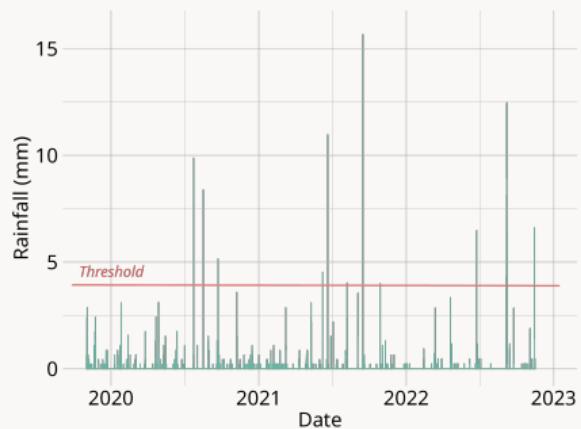
UNIVARIATE PRECIPITATION MODELING

Generalized Pareto Distribution

$$\bar{H}_\xi \left(\frac{x-u}{\sigma} \right) = \begin{cases} \left(1 + \xi \frac{x-u}{\sigma}\right)_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

- ▶ Models extreme precipitation
- ▶ Depends on a threshold choice



UNIVARIATE PRECIPITATION MODELING

Generalized Pareto Distribution



Extended GPD²

$$\bar{H}_\xi \left(\frac{x-u}{\sigma} \right) = \begin{cases} \left(1 + \xi \frac{x-u}{\sigma} \right)_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

$$F(x) = G \left(H_\xi \left(\frac{x}{\sigma} \right) \right),$$

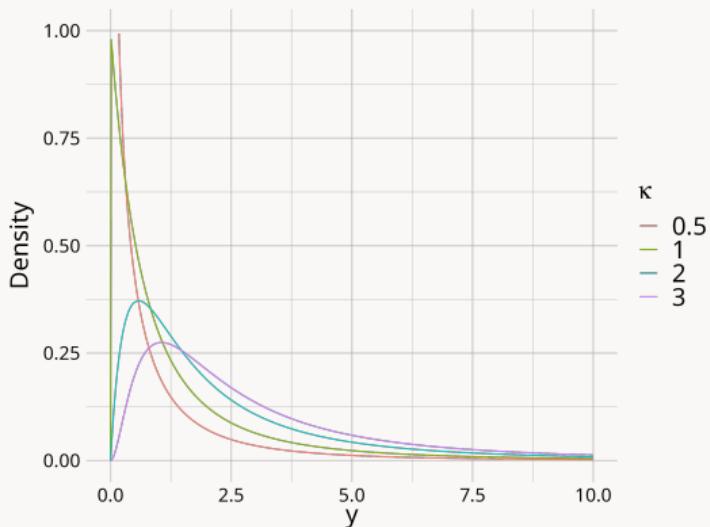
where $G(x) = x^\kappa$, $\kappa > 0$

where $a_+ = \max(a, 0)$, $\sigma > 0$, $x - u > 0$

- ▶ Models extreme precipitation
- ▶ Depends on a threshold choice
- ▶ Models **moderate and extreme** precipitation
- ▶ Avoids a threshold choice

²NAVEAU et al. 2016

UNIVARIATE PRECIPITATION MODELING



$$\sigma = 1, \xi = 0.5$$

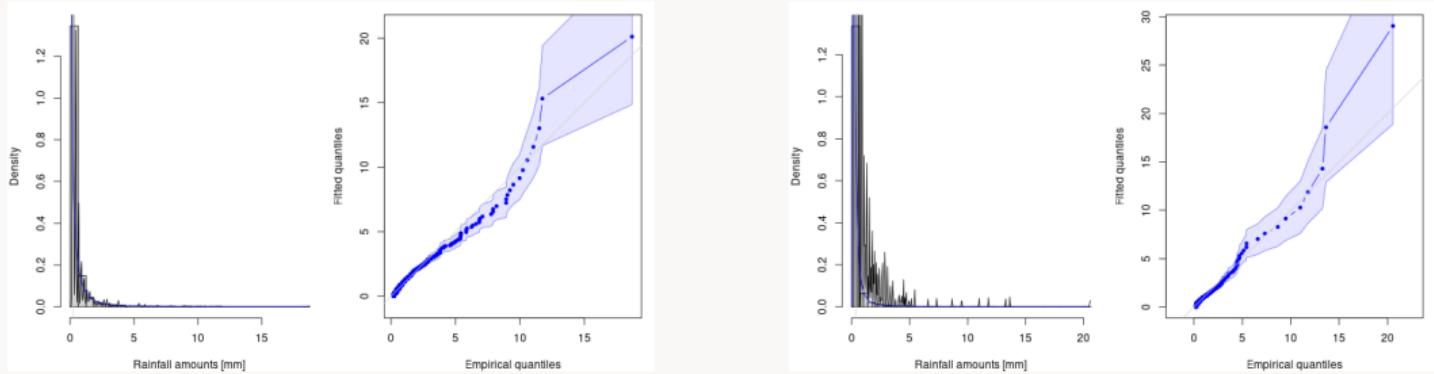
Extended GPD

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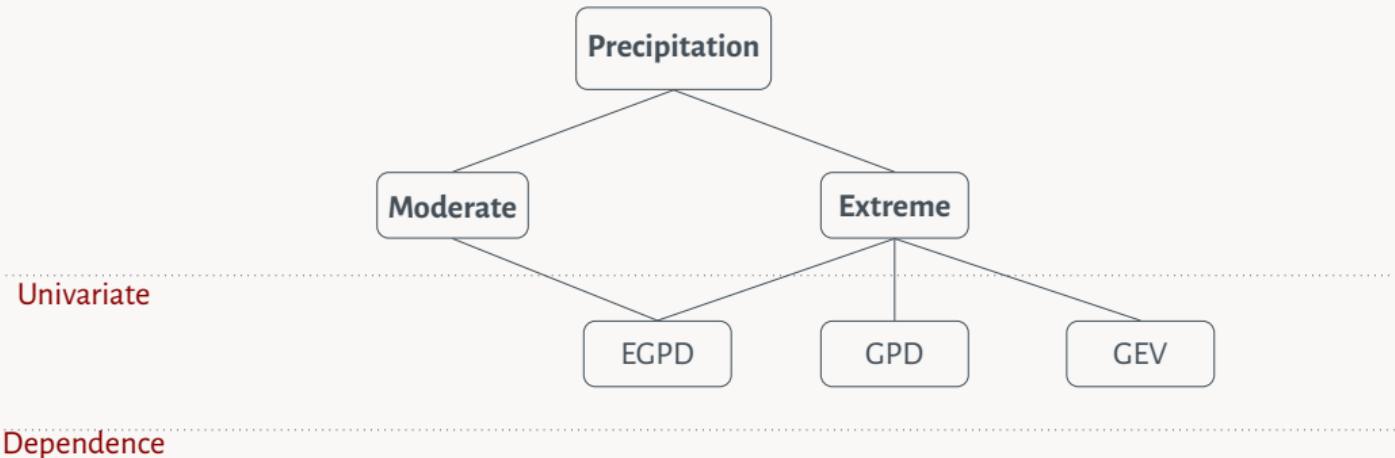
- ▶ Models **moderate and extreme** precipitation
- ▶ Avoids a threshold choice

EGPD FITTING



EGPD fitting for two rain gauges, CRBM (left) and CNRS (right) with left-censoring and 95% CI

PRECIPITATION MODELING



SPATIO-TEMPORAL DEPENDENCE MODELING

Rainfall field: $\mathbf{X} = \{X_{s,t}, (s, t) \in \mathcal{S} \times \mathcal{T}\}$

Domain of attraction: Stationary isotropic max-stable Brown-Resnick process

Brown-Resnick process (BROWN and RESNICK 1977)

For all $s \in \mathcal{S}$ and $t \in \mathcal{T}$,

$$X_{s,t} = \bigvee_{j=1}^{\infty} \xi_j e^{W_{s,t}^j - \gamma(s,t)}$$

- ▶ ξ_j : point of a Poisson process with intensity $\xi^{-2} d\xi$
- ▶ W^j : independent replicates of an intrinsic stationary and isotropic Gaussian random field \mathbf{W}
- ▶ γ : spatio-temporal variogram of \mathbf{W}

DEPENDENCE MEASURES

Let $\Lambda_S \subset \mathbb{R}_+^2$ and $\Lambda_T \subset \mathbb{R}_+$ be sets of spatial and temporal lags respectively.

Spatio-temporal extremogram

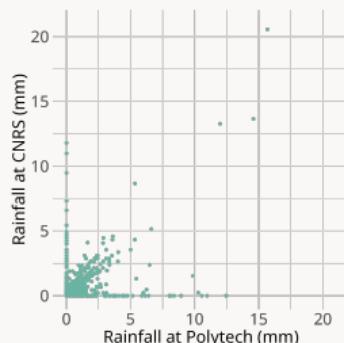
For all $\mathbf{h} \in \Lambda_S, \tau \in \Lambda_T$,

$$\chi(\mathbf{h}, \tau) = \lim_{q \rightarrow 1} \mathbb{P}(X_{s,t}^* > q \mid X_{s+\mathbf{h}, t+\tau}^* > q),$$

with $q \in [0, 1[$ and $X_{s,t}^*$ the uniform margins.

Spatio-temporal variogram γ

$$\gamma(\mathbf{h}, \tau) = \frac{1}{2} \text{Var} (W_{s,t} - W_{s+\mathbf{h}, t+\tau}), \mathbf{h} \in \Lambda_S, \tau \in \Lambda_T$$



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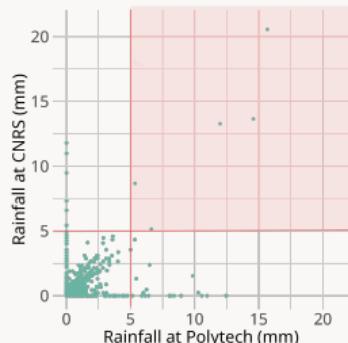
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DEPENDENCE MEASURES

Spatio-temporal extremogram of a Brown-Resnick process

Let $\mathbf{h} \in \Lambda_S$ and $\tau \in \Lambda_T$. We have

$$\chi(\mathbf{h}, \tau) = 2 \left(1 - \phi \left(\sqrt{\frac{1}{2} \gamma(\mathbf{h}, \tau)} \right) \right)$$

with ϕ the std normal c.d.f. and γ the variogram of \mathbf{W} .

Dependence model framework: BUHL et al. 2019

SPATIO-TEMPORAL DEPENDENCE MODELING

Case of additive separability: $\frac{\gamma(h, \tau)}{2} = \beta_1 \|h\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$, $0 < \alpha_1, \alpha_2 \leq 2$, $\beta_1, \beta_2 > 0$

Spatio-temporal

$$\chi(h, \tau) = 2 \left(1 - \phi \left(\sqrt{\frac{1}{2}} \gamma(h, \tau) \right) \right)$$

Transformation:

$$\eta(x) = 2 \log \left(\phi^{-1} \left(1 - \frac{1}{2}x \right) \right)$$

Spatial

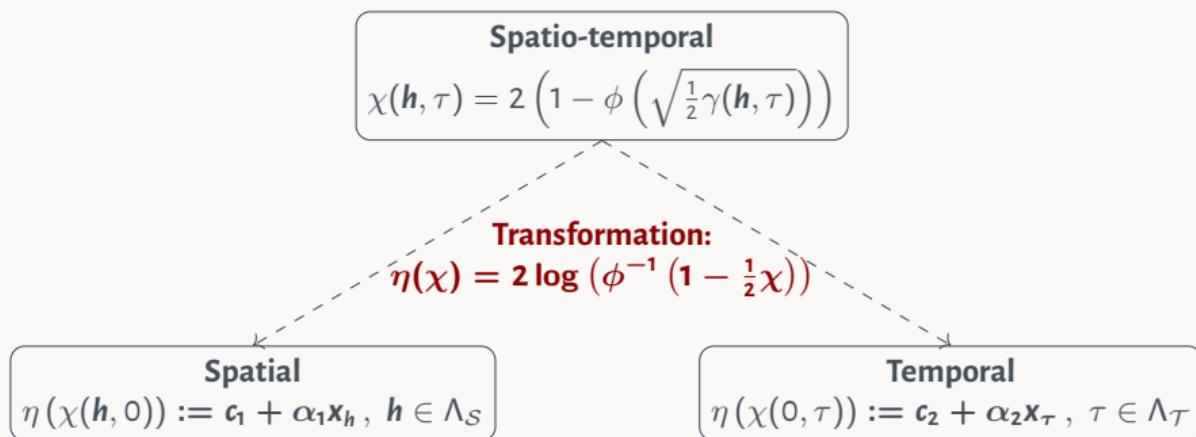
$$\eta(\chi(h, 0)) = \log \beta_1 + \alpha_1 \log \|h\|, \quad h \in \Lambda_S$$

Temporal

$$\eta(\chi(0, \tau)) = \log \beta_2 + \alpha_2 \log \tau, \quad \tau \in \Lambda_T$$

SPATIO-TEMPORAL DEPENDENCE MODELING

Case of additive separability: $\frac{\gamma(h, \tau)}{2} = \beta_1 \|h\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$, $0 < \alpha_1, \alpha_2 \leq 2$, $\beta_1, \beta_2 > 0$



Weighted Least Squares Estimation (WLSE)

$$\begin{pmatrix} \hat{c}_i \\ \hat{\alpha}_i \end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_i} \sum_x w_x (\eta(\hat{\chi}) - (c_i + \alpha_i x))^2$$

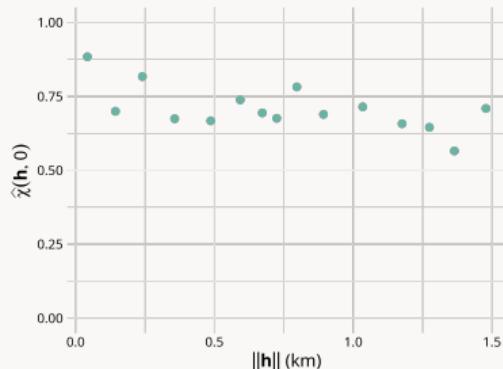
SPATIAL DEPENDENCE ESTIMATION

Empirical spatial extremogram

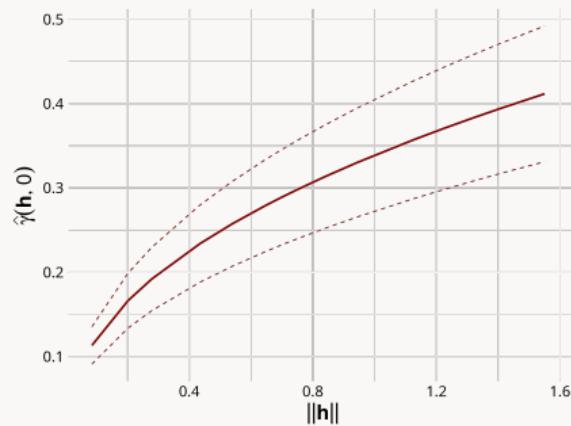
For a fixed $t \in \mathcal{T}$ and q a high quantile,

$$\widehat{\chi}_q^{(t)}(\mathbf{h}, 0) = \frac{\frac{1}{|N_h|} \sum_{i,j \mid (\mathbf{s}_i, \mathbf{s}_j) \in N_h} \mathbb{1}_{\{X_{s_i,t}^* > q, X_{s_j,t}^* > q\}}}{\frac{1}{|\mathcal{S}|} \sum_{i=1}^{|\mathcal{S}|} \mathbb{1}_{\{X_{s_i,t}^* > q\}}},$$

where C_h are equifrequent distance classes and $N_h = \{(\mathbf{s}_i, \mathbf{s}_j) \in \mathcal{S}^2 \mid \|\mathbf{s}_i - \mathbf{s}_j\| \in C_h\}$.



Transformation and WLSE



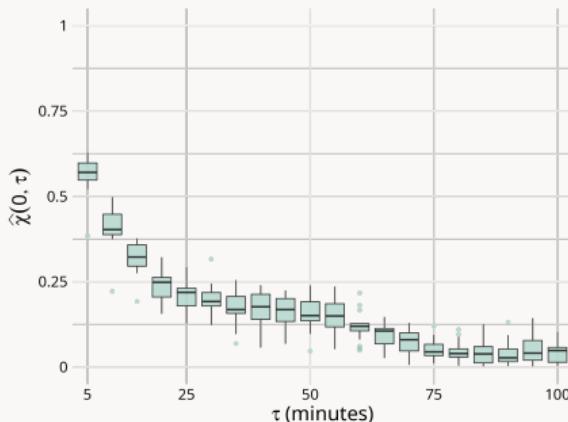
Spatial variogram $\widehat{\gamma}(\mathbf{h}, 0) = 2\widehat{\beta}_1 \|\mathbf{h}\|^{\widehat{\alpha}_1}$

TEMPORAL DEPENDENCE ESTIMATION

Empirical temporal extremogram

For a location $s \in \mathcal{S}$, a high quantile q and $t_k \in \{t_1, \dots, t_T\}$,

$$\hat{\chi}_q^{(s)}(\mathbf{0}, \tau) = \frac{\frac{1}{T-\tau} \sum_{k=1}^{T-\tau} \mathbb{1}_{\{X_{s,t_k}^* > q, X_{s,t_k+\tau}^* > q\}}}{\frac{1}{T} \sum_{k=1}^T \mathbb{1}_{\{X_{s,t_k}^* > q\}}}$$



Transformation and WLSE



Temporal variogram $\hat{\gamma}(\mathbf{0}, \tau) = 2\hat{\beta}_2\tau^{\hat{\alpha}_2}$

CONSIDERING ADVECTION

Advection vector \mathbf{V}

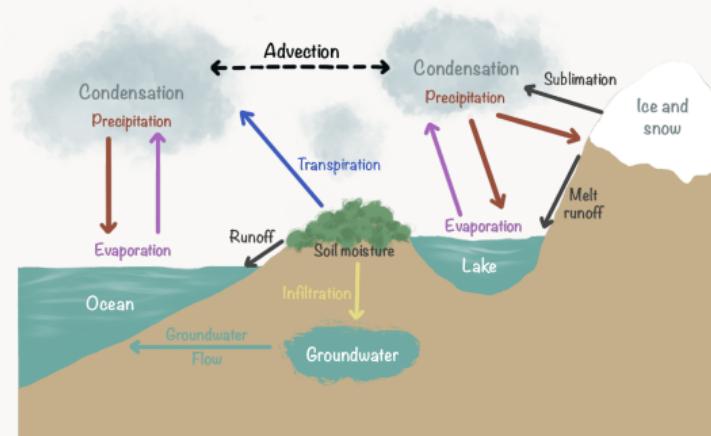
- ▶ Horizontal transport of air masses
- ▶ To relax the separability assumption

Lagrangian/Eulerian variogram

$$\gamma_L(\mathbf{h}, \tau) = \gamma(\mathbf{h} - \tau\mathbf{V}, \tau)$$

Dependence model

$$\frac{1}{2}\gamma_L(\mathbf{h}, \tau) = \beta_1 \|\mathbf{h} - \tau\mathbf{V}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$$



Hydrologic cycle

CONSIDERING ADVECTION - ESTIMATION

Parameter optimization of $\Theta = (\beta_1, \beta_2, \alpha_1, \alpha_2, V)$

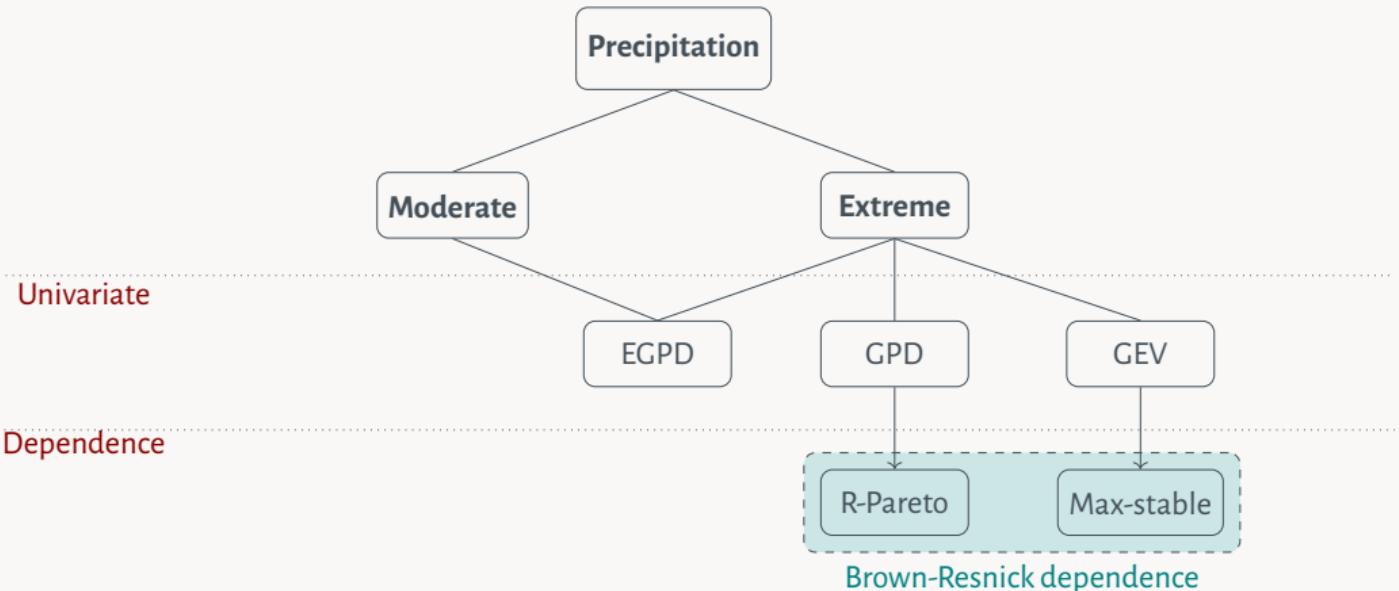
Excesses: for all spatial pairs (s_i, s_j) ,

$$k_{ij} = \sum_{t=1}^n \mathbb{1}_{\{X_{s_i,t} > q, X_{s_j,t} > q\}} \mid n_j \sim \mathcal{B}(n_j, \chi_{ij,\Theta})$$

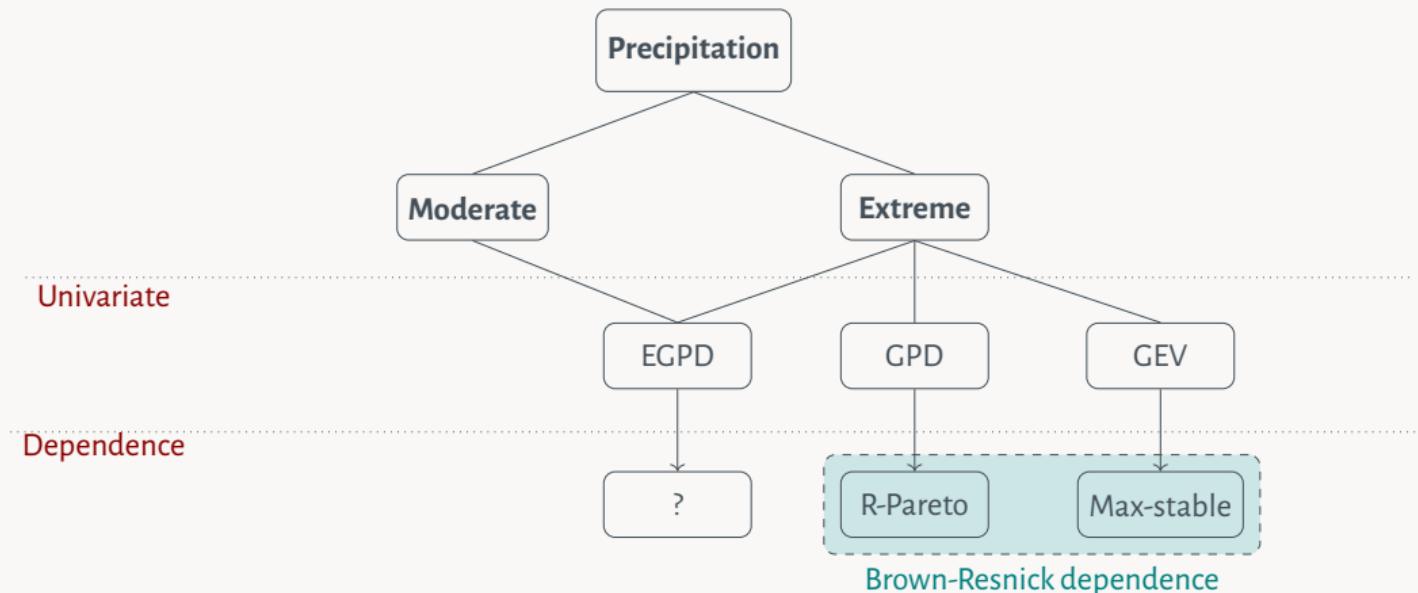
Composite log-likelihood:

$$l_C(\Theta) \approx \sum_{ij} k_{ij} \log \chi_{ij,\Theta} + (n_j - k_{ij}) \log(1 - \chi_{ij,\Theta})$$

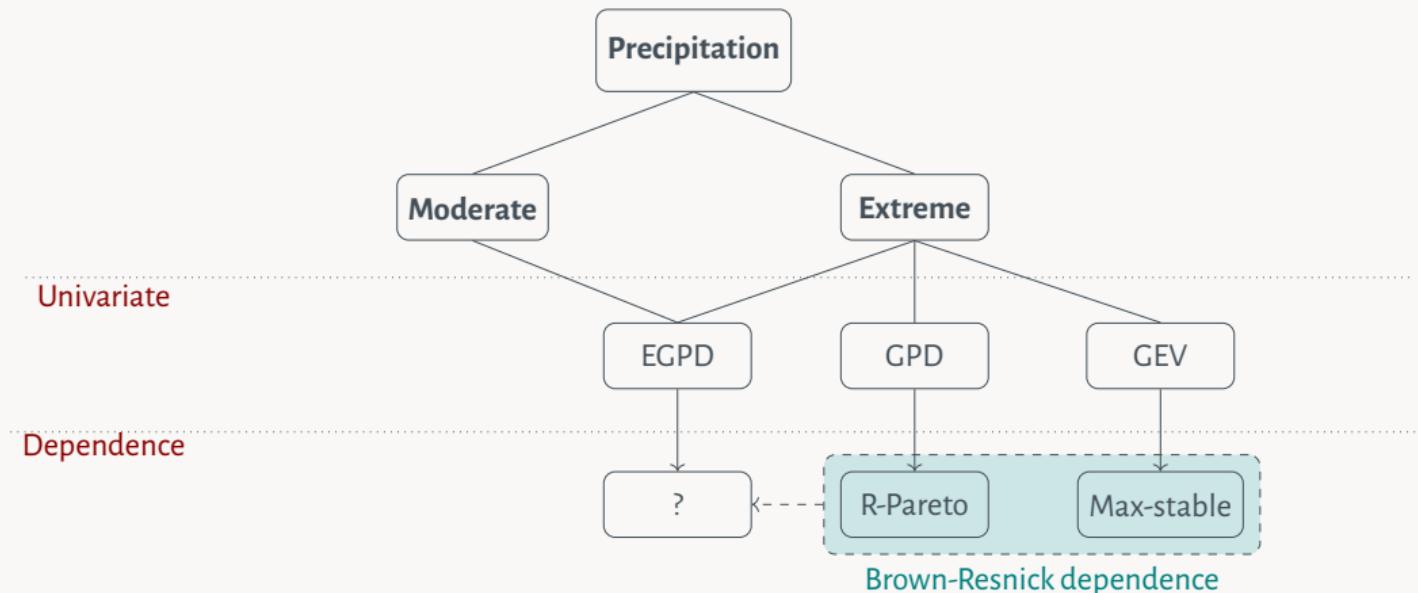
CONCLUSION



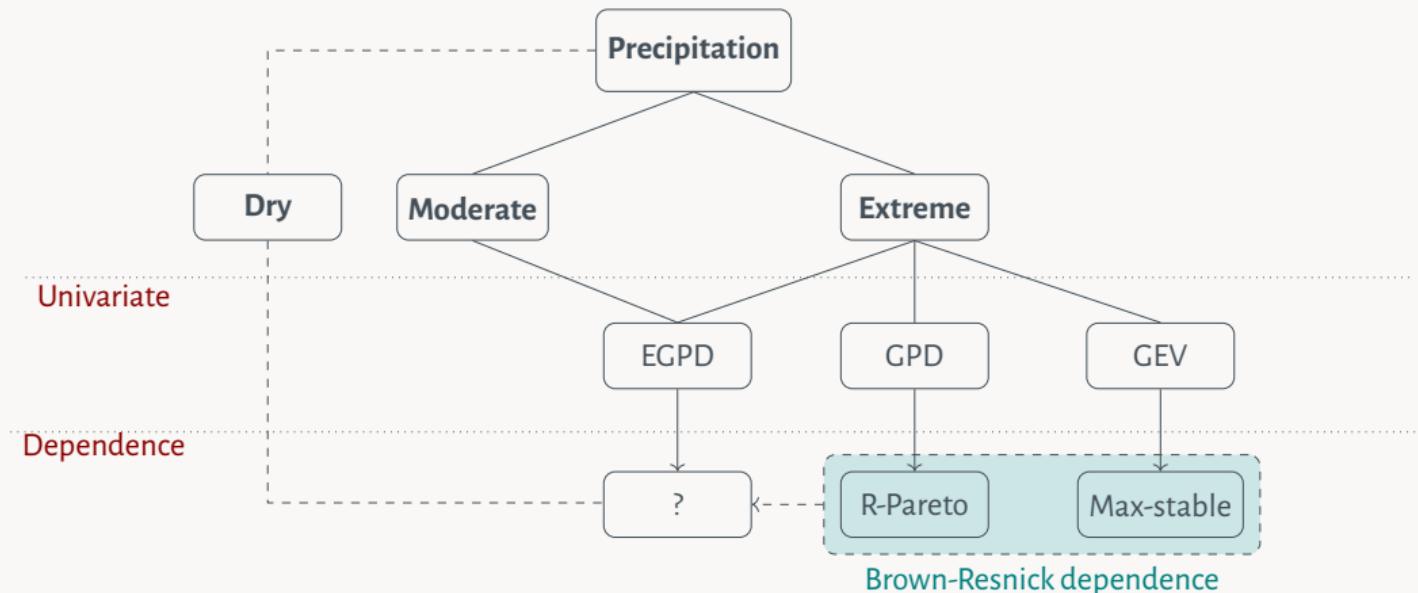
CONCLUSION



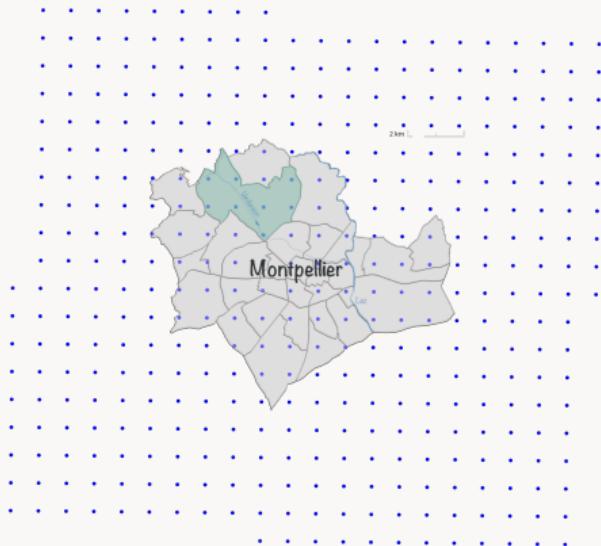
CONCLUSION



CONCLUSION



ADDITIONAL DATA



- ▶ **Source:** COMEPHORE, Météo France
- ▶ **Time period:** [1997, 2023]
- ▶ **Temporal resolution:** Every hour
- ▶ **Spatial resolution:** 1 km^2

$$\mathcal{S} = \{\text{400 pixels}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$

REFERENCES

- BROWN, Bruce M. and Sidney I. RESNICK (1977). "Extreme values of independent stochastic processes". In: *Journal of Applied Probability*. DOI: [10.2307/3213346](https://doi.org/10.2307/3213346).
- BUHL, Sven et al. (2019). "Semiparametric estimation for isotropic max-stable space-time processes". In: *Bernoulli*.
- FINAUD-GUYOT, Pascal et al. (2023). *Rainfall data collected by the HSM urban observatory (OMSEV)*. DOI: [10.23708/67LC36](https://doi.org/10.23708/67LC36).
- HARUNA, Abubakar, Juliette BLANCHET, and Anne-Catherine FAVRE (2023). *Modeling Areal Precipitation Hazard: A Data-driven Approach to Model Intensity-Duration-Area-Frequency Relationships using the Full Range of Non-Zero Precipitation in Switzerland*. preprint. Preprints. DOI: [10.22541/essnar.169111775.53035997/v1](https://doi.org/10.22541/essnar.169111775.53035997/v1).
- NAVEAU, Philippe et al. (2016). "Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection". In: *Water Resources Research*. DOI: [10.1002/2015WR018552](https://doi.org/10.1002/2015WR018552).

MODEL VALIDATION - SIMULATIONS

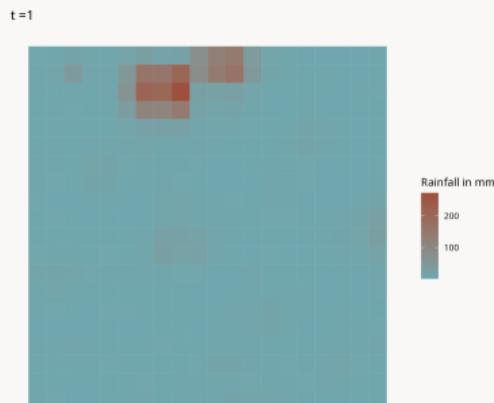
Brown-Resnick simulations

- Spatial: $\mathcal{S} = \{400\text{ sites}\}$, $\mathcal{T} = \{1, \dots, 50\}$, $|\Lambda_{\mathcal{S}}| = 10$ and $|\Lambda_{\mathcal{T}}| = 10$
- Temporal: $\mathcal{S} = \{25\text{ sites}\}$, $\mathcal{T} = \{1, \dots, 300\}$, $|\Lambda_{\mathcal{S}}| = 10$ and $|\Lambda_{\mathcal{T}}| = 10$

	True	Mean	RMSE	MAE
$\hat{\beta}_1$	0.4	0.524	0.138	0.126
$\hat{\alpha}_1$	1.5	1.507	0.120	0.088
$\hat{\beta}_2$	0.2	0.259	0.093	0.074
$\hat{\alpha}_2$	1	0.873	0.149	0.128

Parameter estimates for 100 realisations

True variogram: $\frac{1}{2}\gamma(\mathbf{h}, \tau) = 0.4\|\mathbf{h}\|^{3/2} + 0.2\tau$



CONSIDERING ADVECTION

Advection vector \mathbf{V}

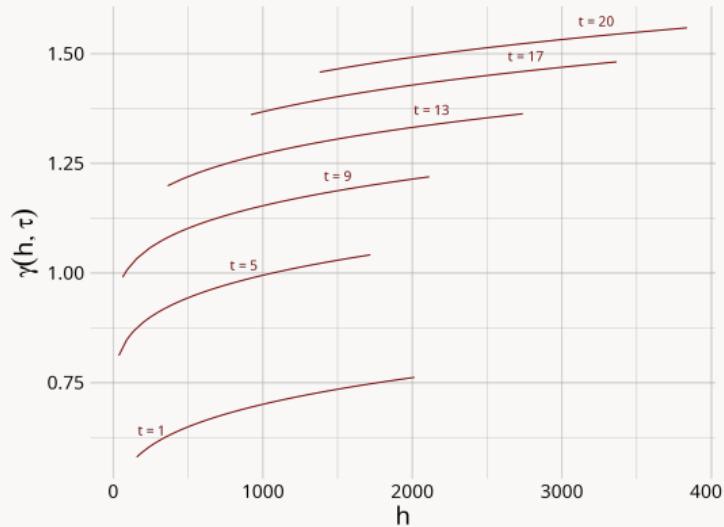
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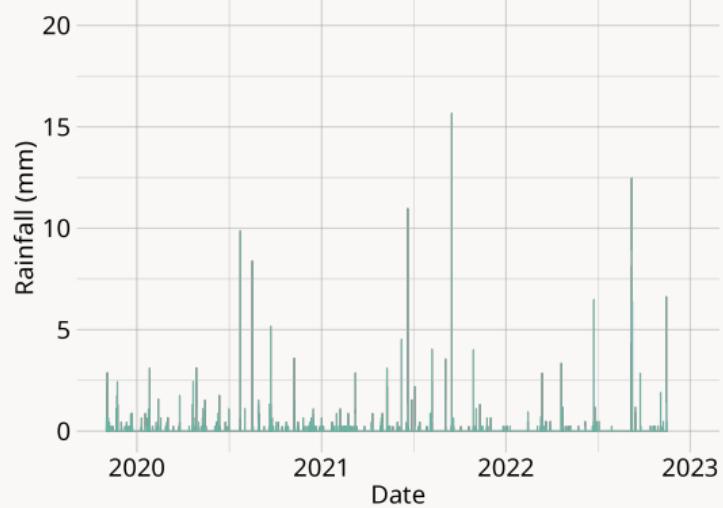
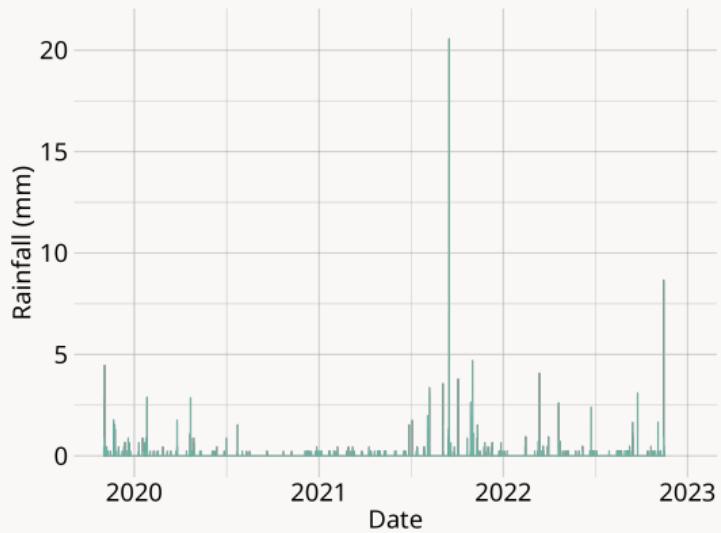
Dependence model

$$\frac{1}{2}\gamma_L(\mathbf{h}, \tau) = \beta_1 \|\mathbf{h} - \tau\mathbf{V}\|^{\alpha_1} + \beta_2 \tau^{\alpha_2}$$



Spatial variogram with a constant advection
 $\mathbf{V} = (0.001, 45)^T$ on OHSM data

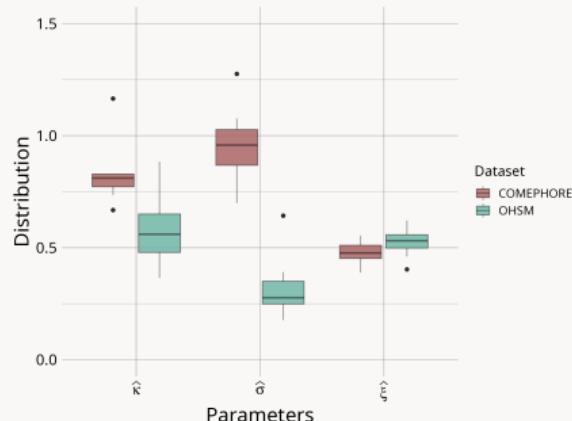
RAINFALL DATA - OHSM



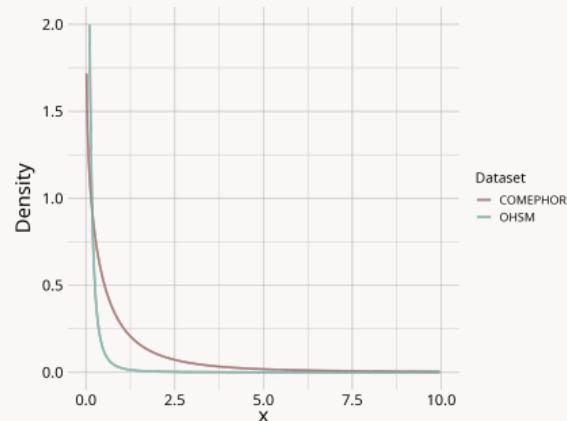
Rainfall amounts on CNRS and Polytech rain gauges

EGPD FITTING

Left-censoring: selected according to the NRMSE criterion for each site individually³



Parameter estimates



Density with mean parameters

³HARUNA, BLANCHET, and FAVRE 2023