

# SPATIO-TEMPORAL MODELING OF URBAN EXTREME RAINFALL EVENTS AT HIGH RESOLUTION

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# GENERAL CONTEXT



Floods in Montpellier, September 2022 and August 2015 (*Midi Libre*)

- ▶ Mediterranean events, localized rainfall
- ▶ Urban area, flood risks



## STUDY AREA



- ▶ North of Montpellier, *Hôpitaux-Facultés* district
- ▶ Verdanson water catchment, tributary of the Lez river

# RAIN GAUGES NETWORK



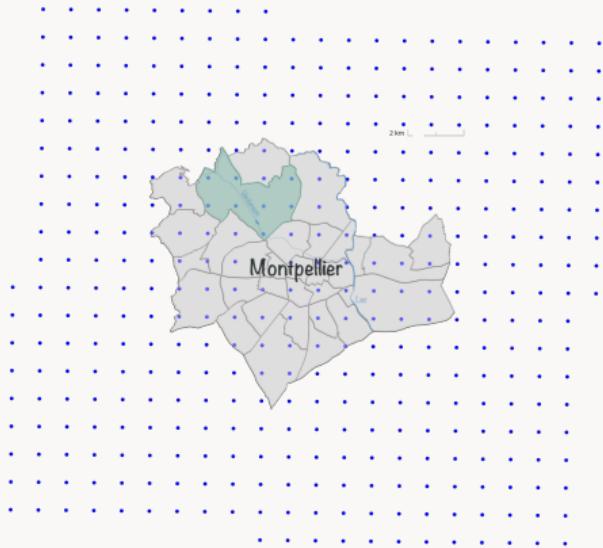
- ▶ **Source:** Urban observatory of HydroScience Montpellier (HSM)<sup>1</sup>
- ▶ **Time period:** [2019, 2023[
- ▶ **Temp. resol.:** 5 minutes
- ▶ **Spatial resol.:** 77 m to 1531 m

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<sup>1</sup>FINAUD-GUYOT et al., 2023

$$\mathcal{S} = \{17 \text{ rain gauges}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$

## ADDITIONAL DATA

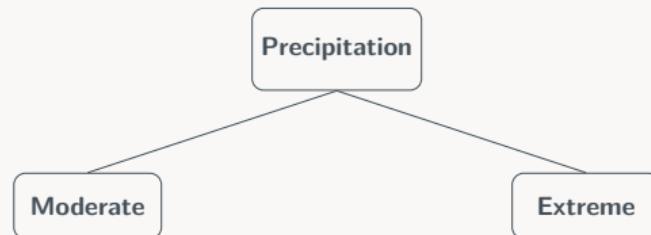


$$\mathcal{S} = \{400 \text{ pixels}\} \subset \mathbb{R}^2 \text{ and } \mathcal{T} \subset \mathbb{R}_+$$

**More consistent data:** HSM + COMEPHORE and Neural Network Downscaling.

- ▶ **Source:** COMEPHORE, Météo France
- ▶ **Time period:** [1997, 2023[
- ▶ **Temporal resolution:** Every hour
- ▶ **Spatial resolution:** 1 km<sup>2</sup>

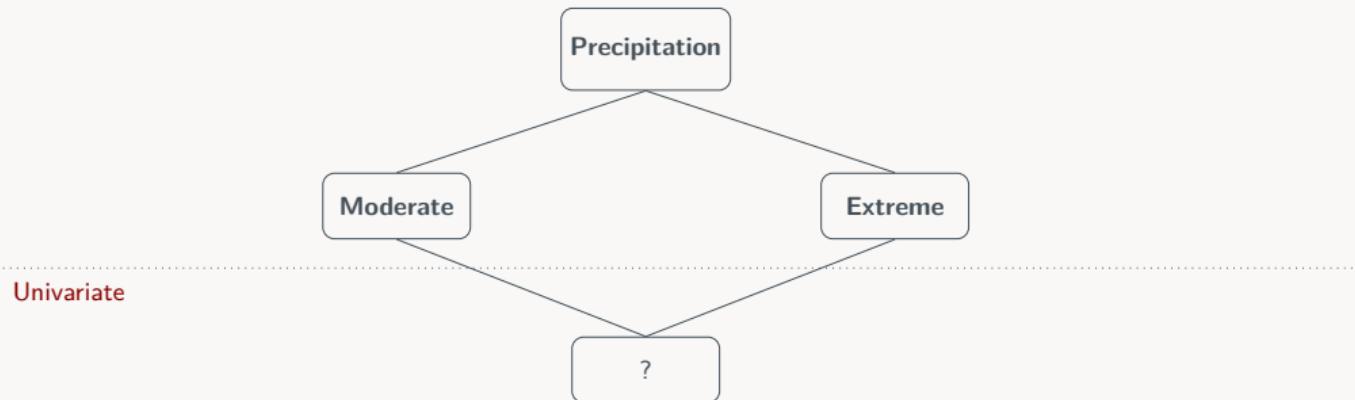
# UNIVARIATE PRECIPITATION MODELING



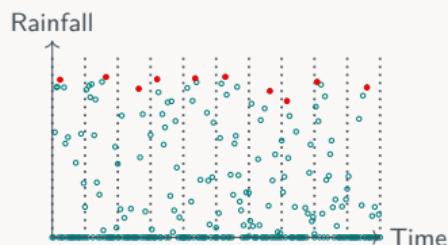
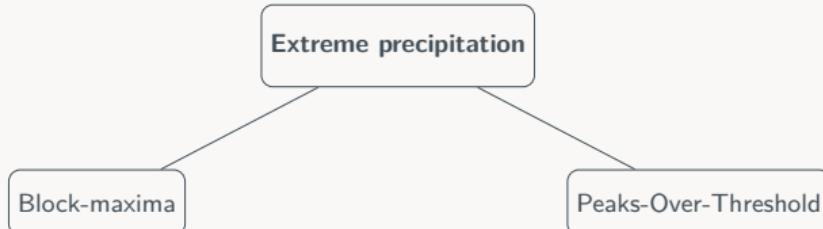
Univariate

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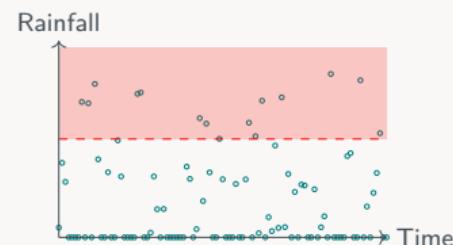
# UNIVARIATE PRECIPITATION MODELING



# EXTREME VALUE THEORY (EVT)



Generalized Extreme Value distribution (GEV)



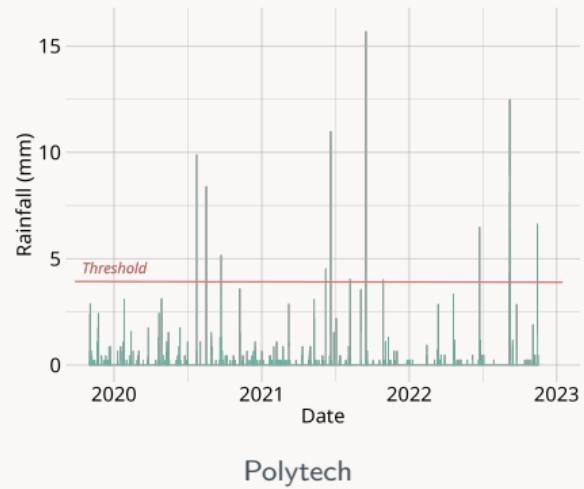
Generalized Pareto Distribution (GPD)

## Generalized Pareto Distribution

$$\bar{H}_\xi \left( \frac{x-u}{\sigma} \right) = \begin{cases} \left( 1 + \xi \frac{x-u}{\sigma} \right)_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

where  $a_+ = \max(a, 0)$ ,  $\sigma > 0$ ,  $x - u > 0$

- ▶ Models extreme precipitation
- ▶ Depends on a threshold choice



Generalized Pareto Distribution



Extended GPD<sup>2</sup>

$$\bar{H}_\xi \left( \frac{x-u}{\sigma} \right) = \begin{cases} \left( 1 + \xi \frac{x-u}{\sigma} \right)_+^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{-\frac{x-u}{\sigma}} & \text{if } \xi = 0, \end{cases}$$

where  $a_+ = \max(a, 0)$ ,  $\sigma > 0$ ,  $x - u > 0$

$$F(x) = G \left( H_\xi \left( \frac{x}{\sigma} \right) \right),$$

where  $G(x) = x^\kappa$ ,  $\kappa > 0$

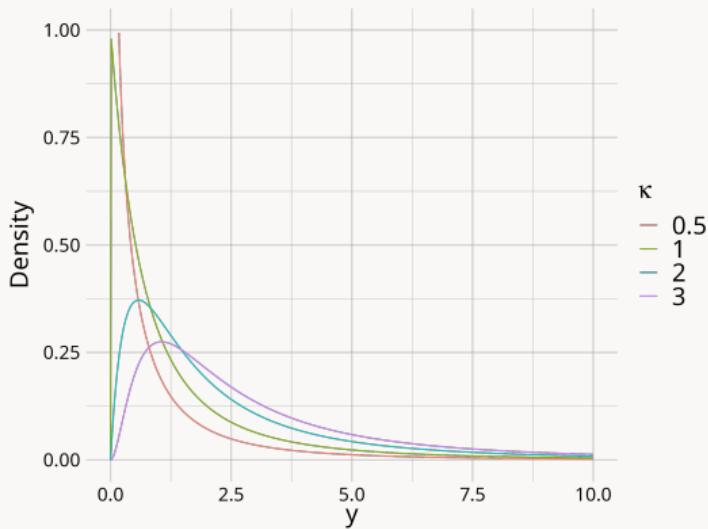
- ▶ Models extreme precipitation
- ▶ Depends on a threshold choice

- ▶ Models **moderate and extreme** precipitation
- ▶ Avoids a threshold choice

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<sup>2</sup>NAVEAU et al., 2016

# MODELING BOTH MODERATE AND EXTREME PRECIPITATION



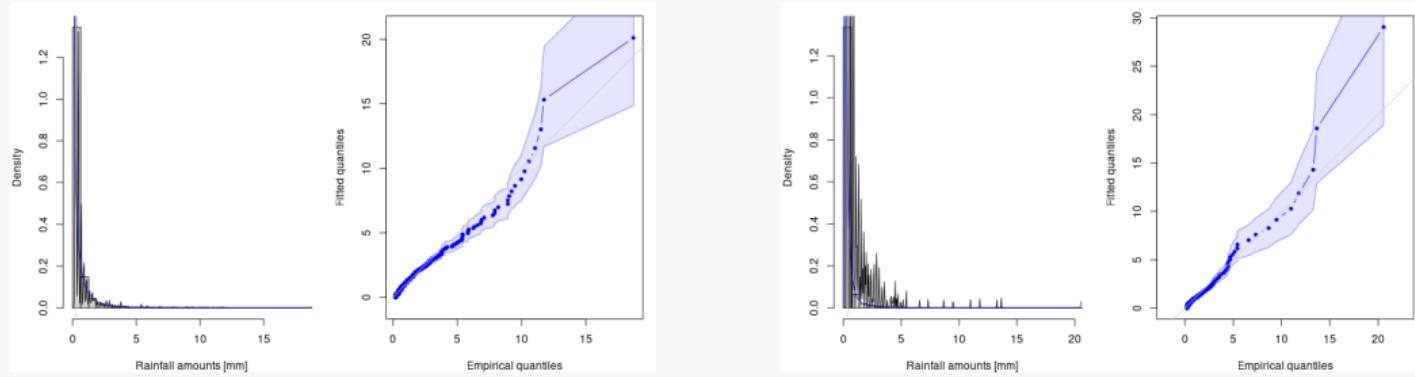
## Extended GPD

$$F(x) = G\left(H_\xi\left(\frac{x}{\sigma}\right)\right),$$

where  $G(x) = x^\kappa$ ,  $\kappa > 0$

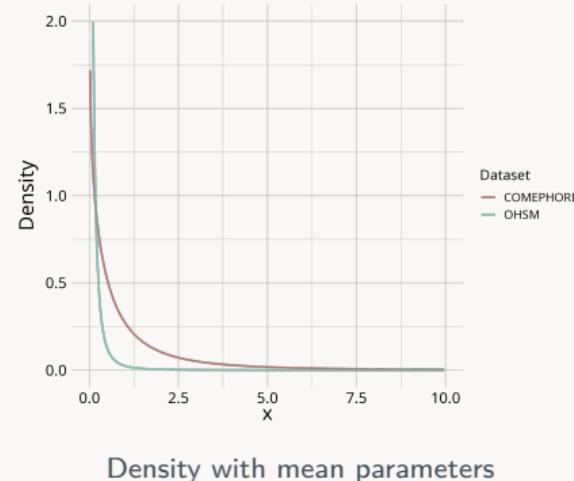
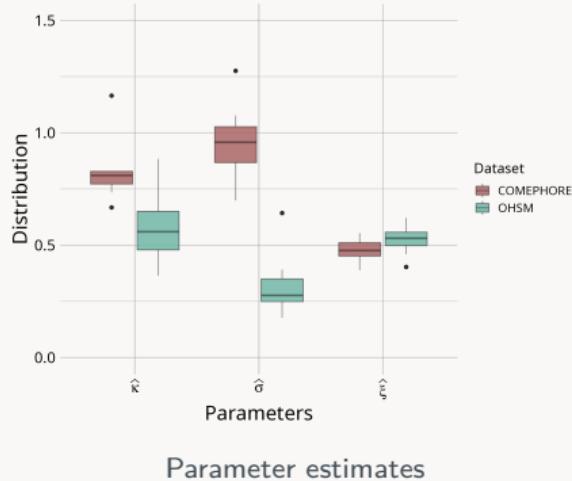
- ▶ Models **moderate and extreme** precipitation
- ▶ Avoids a threshold choice

# EGPD FITTING



EGPD fitting for two rain gauges, CRBM (left) and CNRS (right) with left-censoring and 95% CI

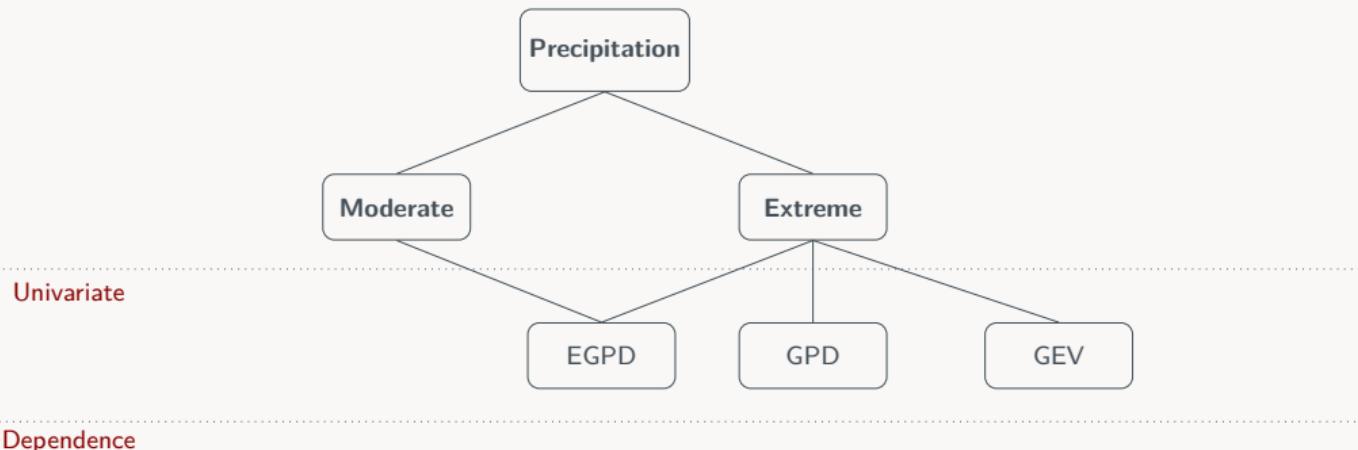
**Left-censoring:** selected according to the RMSE criterion for each site individually<sup>3</sup>



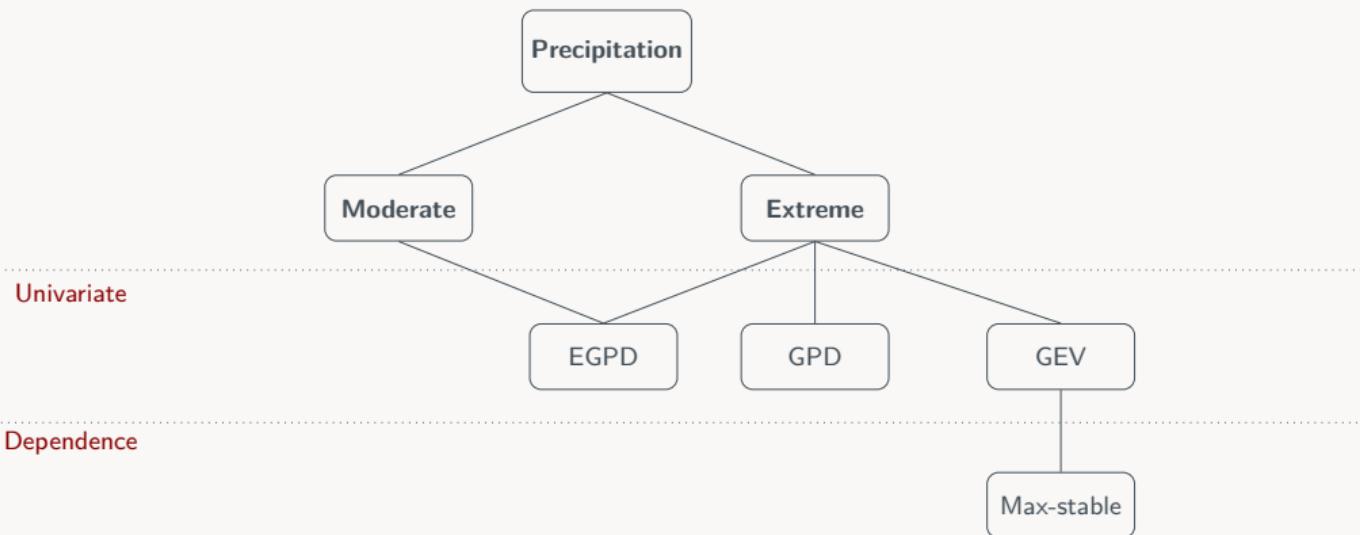
EGPD fitting for two rain gauges, with parameter estimates and density with mean parameters

<sup>3</sup>HARUNA et al., 2023

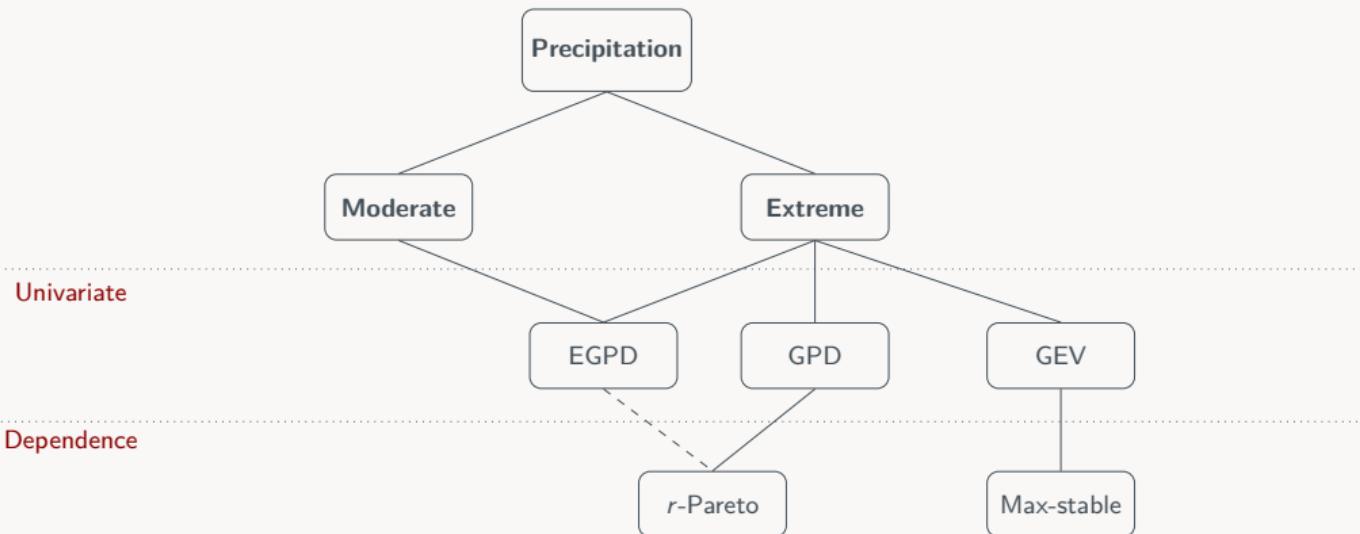
# SPATIO-TEMPORAL DEPENDENCE MODELING

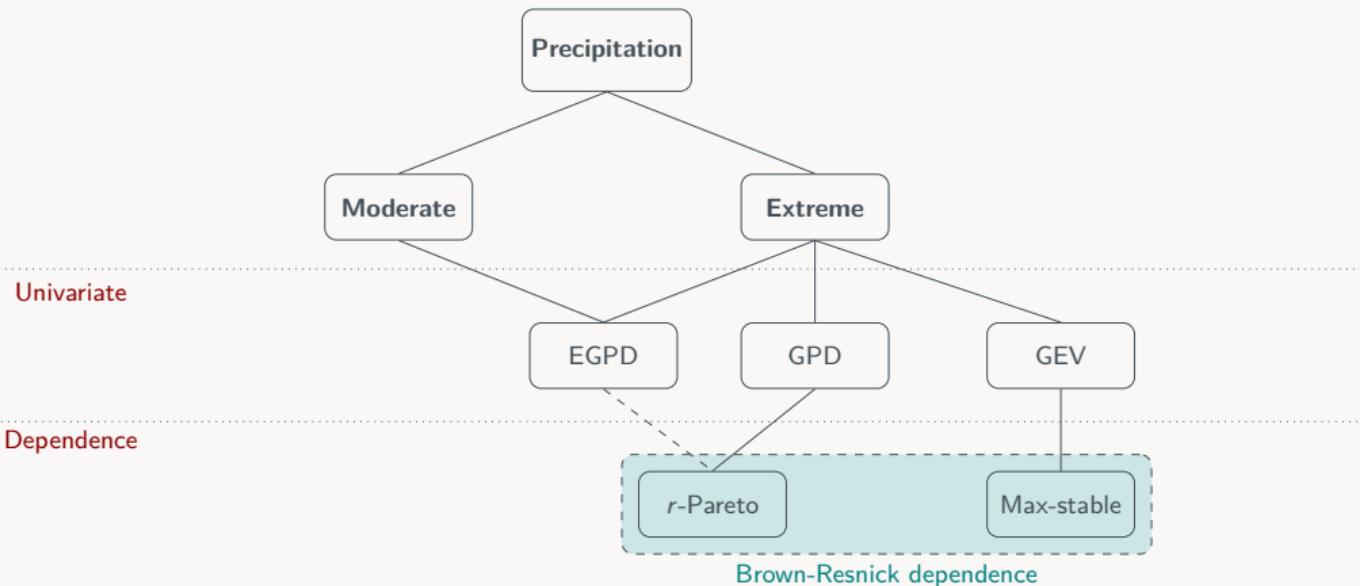


# SPATIO-TEMPORAL DEPENDENCE MODELING



# SPATIO-TEMPORAL DEPENDENCE MODELING





## DEPENDENCE MEASURE

**Rainfall random field:**  $\mathbf{X} = \{X_{s,t}, (s, t) \in \mathcal{S} \times \mathcal{T}\}$  stationary and isotropic

Let  $\Lambda_{\mathcal{S}} \subset \mathbb{R}_+^2$  and  $\Lambda_{\mathcal{T}} \subset \mathbb{R}_+$  be sets of spatial and temporal lags respectively.

### Variogram

The spatio-temporal variogram of  $\mathbf{X}$  is defined as

$$\gamma(\mathbf{h}, \tau) = \frac{1}{2} \text{Var}(X_{s,t} - X_{s+\mathbf{h}, t+\tau}), \mathbf{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$$

- ▶ Describes the spatio-temporal variability
- ▶ Small  $\gamma(\mathbf{h}, \tau)$ : Strong dependence
- ▶ Large  $\gamma(\mathbf{h}, \tau)$ : Weaker dependence

## EXTREME DEPENDENCE MEASURE

Let  $\Lambda_S \subset \mathbb{R}_+^2$  and  $\Lambda_T \subset \mathbb{R}_+$  be sets of spatial and temporal lags respectively.

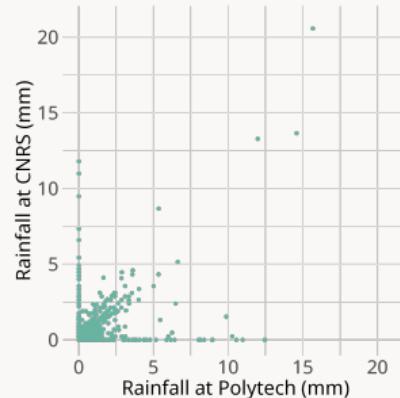
### Spatio-temporal extremogram (COLES et al., 1999)

For all  $\mathbf{h} \in \Lambda_S, \tau \in \Lambda_T$ ,

$$\chi(\mathbf{h}, \tau) = \lim_{q \rightarrow 1} \mathbb{P}(X_{s,t}^* > q \mid X_{s+\mathbf{h}, t+\tau}^* > q),$$

with  $q \in [0, 1[$  and  $X_{s,t}^*$  the uniform margins.

- ▶  $\chi(\mathbf{h}, \tau) \in [0, 1]$
- ▶  $\chi(\mathbf{h}, \tau) > 0$ : asymptotic dependence
- ▶  $\chi(\mathbf{h}, \tau) = 0$ : asymptotic independence



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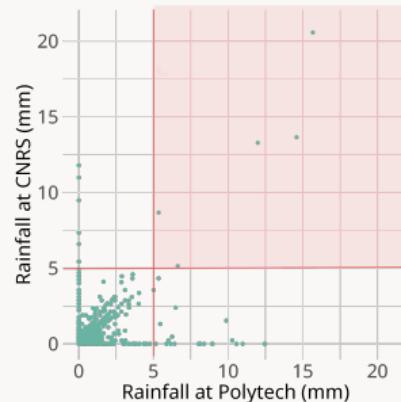
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# BROWN-RESNICK MAX-STABLE PROCESS

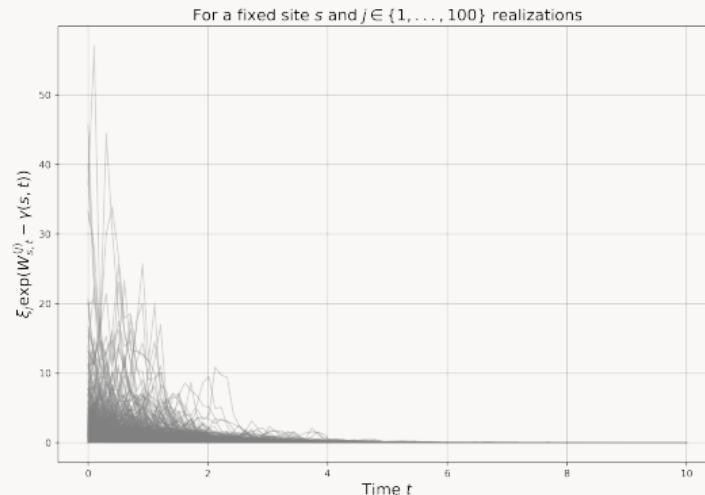
**Rainfall random field:**  $\mathbf{X} = \{X_{s,t}, (s, t) \in \mathcal{S} \times \mathcal{T}\}$  stationary and isotropic

Spectral representation of a Brown-Resnick process (BROWN and RESNICK, 1977)

For all  $s \in \mathcal{S}$  and  $t \in \mathcal{T}$ ,

$$Z_{s,t} = \sqrt{\xi} e^{W_{s,t}^{(j)} - \gamma(s,t)}$$

- ▶  $(\xi_j)_{j \geq 1}$ : points of a Poisson process with intensity  $\xi^{-2} d\xi$
- ▶  $(W^{(j)})_{j \geq 1}$ : independent replicates of an intrinsic stationary and isotropic Gaussian random field  $\mathbf{W}$
- ▶  $\gamma$ : spatio-temporal variogram of  $\mathbf{W}$



# BROWN-RESNICK MAX-STABLE PROCESS

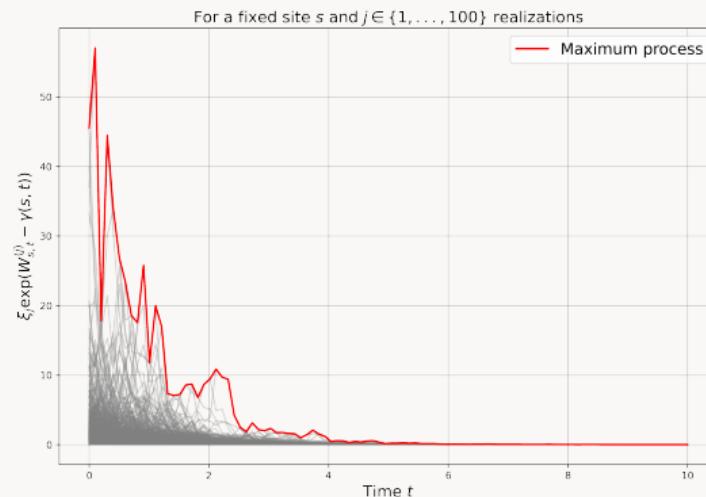
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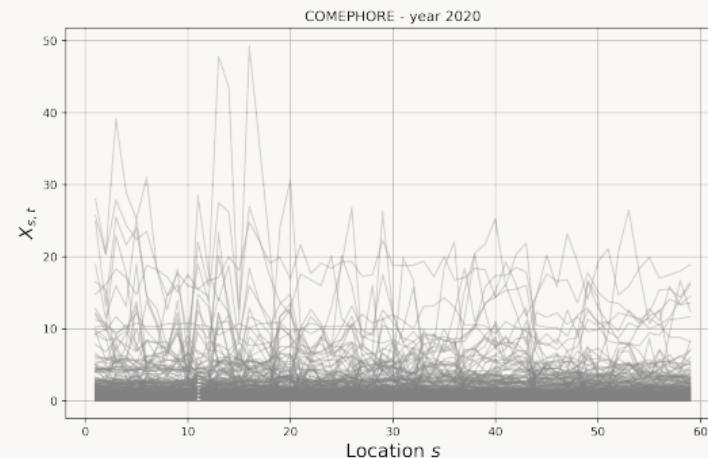


## Definition (de FONDEVILLE and DAVISON, 2018)

For all  $s \in \mathcal{S}$  and  $t \in \mathcal{T}$ , a risk function  $r(\mathbf{X}) = X_{s_0, t_0}$ ,

$$X_{s,t} \mid r(\mathbf{X}) > u \xrightarrow{d} Y_{s,t} \quad \text{with} \quad Y_{s,t} = u R_{s,t} e^{W_{s,t} - W_{s_0,t_0} - \gamma(s-s_0, t-t_0)},$$

with  $(s_0, t_0)$  a given space-time location,  $u$  a high threshold and  $R_{s,t} \sim \text{Pareto}(1)$ .



## $r$ -PARETO PROCESS

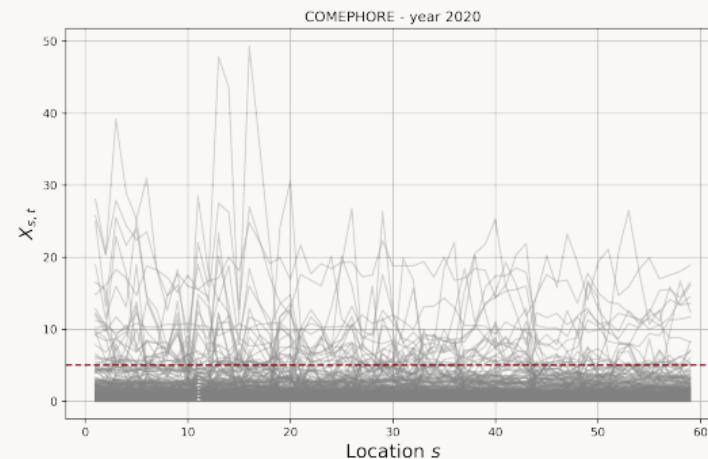
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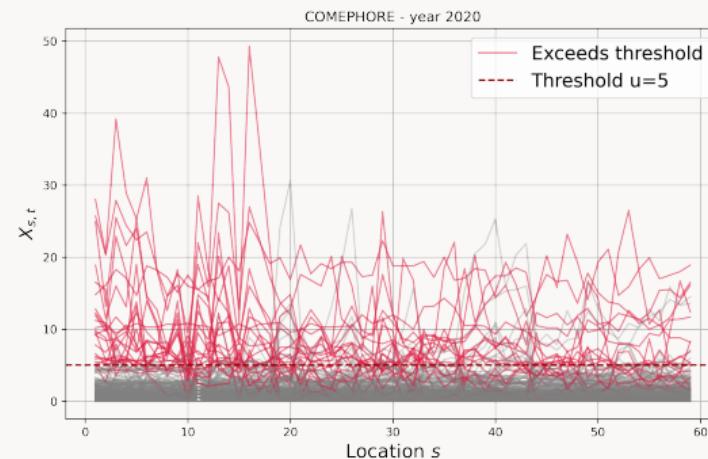
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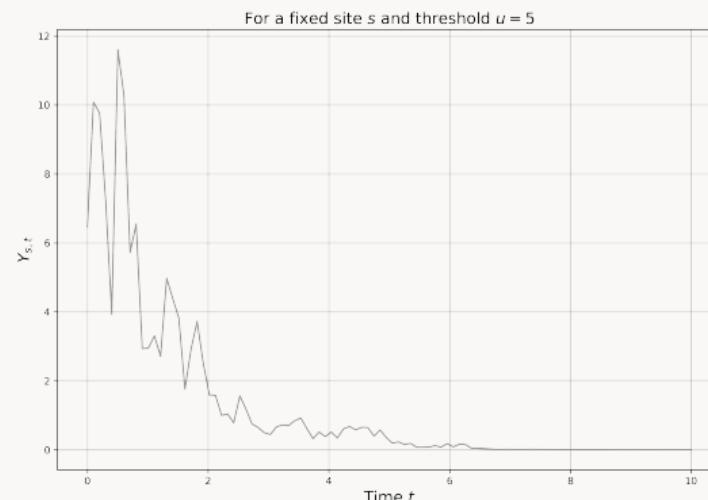
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with  $(s_0, t_0)$  a given space-time location,  $u$  a high threshold and  $R_{s,t} \sim \text{Pareto}(1)$ .



# DEPENDENCE MEASURE FOR AN $r$ -PARETO PROCESS

Let  $\{X_{s,t} \mid r(\mathbf{X}) > u, s \in \mathcal{S}, t \in \mathcal{T}\}$  converges in distribution to a  $r$ -Pareto process with  $r(\mathbf{X}) = X_{s_0, t_0}$ .

## Spatio-temporal $r$ -extremogram

For all  $\mathbf{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$ ,

$$\begin{aligned}\chi_r(\mathbf{h}, \tau) &= \lim_{q \rightarrow 1} \mathbb{P}(X_{s_0+\mathbf{h}, t_0+\tau}^* > q \mid X_{s_0, t_0}^* > q) \\ &= \lim_{q \rightarrow 1} \mathbb{P}(X_{s_0+\mathbf{h}, t_0+\tau}^* > q),\end{aligned}$$

with  $q \in [0, 1[$  corresponding to threshold  $u$  and  $X_{s,t}^*$  the uniform margins.

## Spatio-temporal $r$ -variogram of $\mathbf{W}$

For all  $\mathbf{h} \in \Lambda_{\mathcal{S}}, \tau \in \Lambda_{\mathcal{T}}$ ,

$$\gamma_r(\mathbf{h}, \tau) = \frac{1}{2} \text{Var}(W_{s_0, t_0} - W_{s_0+\mathbf{h}, t_0+\tau}).$$

Let  $\Lambda_S \subset \mathbb{R}_+^2$  and  $\Lambda_T \subset \mathbb{R}_+$  be sets of spatial and temporal lags respectively.

## Spatio-temporal extremogram with a Brown-Resnick dependence

Let  $\mathbf{h} \in \Lambda_S$  and  $\tau \in \Lambda_T$ . We have

$$\chi(\mathbf{h}, \tau) = 2 \left( 1 - \phi \left( \sqrt{\frac{1}{2} \gamma(\mathbf{h}, \tau)} \right) \right)$$

with  $\phi$  the std normal c.d.f. and  $\gamma$  the variogram of  $\mathbf{W}$ .

**Dependence framework:** BUHL et al., 2019

- ▶ Fractional Brownian motion
- ▶ Additive separability

$$\frac{\gamma(\mathbf{h}, \tau)}{2} = \beta_1 \|\mathbf{h}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2},$$

with  $0 < \alpha_1, \alpha_2 \leq 2$ ,  $\beta_1, \beta_2 > 0$ .

# ESTIMATION OF THE SEPARABLE VARIOGRAM PARAMETERS

**Case of additive separability:**  $\frac{\gamma(\mathbf{h}, \tau)}{2} = \beta_1 \|\mathbf{h}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}, \quad 0 < \alpha_1, \alpha_2 \leq 2, \quad \beta_1, \beta_2 > 0$

## Spatio-temporal

$$\chi(\mathbf{h}, \tau) = 2 \left( 1 - \phi \left( \sqrt{\frac{1}{2}} \gamma(\mathbf{h}, \tau) \right) \right)$$

## Transformation:

$$\eta(\chi) = 2 \log \left( \phi^{-1} \left( 1 - \frac{1}{2} \chi \right) \right)$$

## Spatial

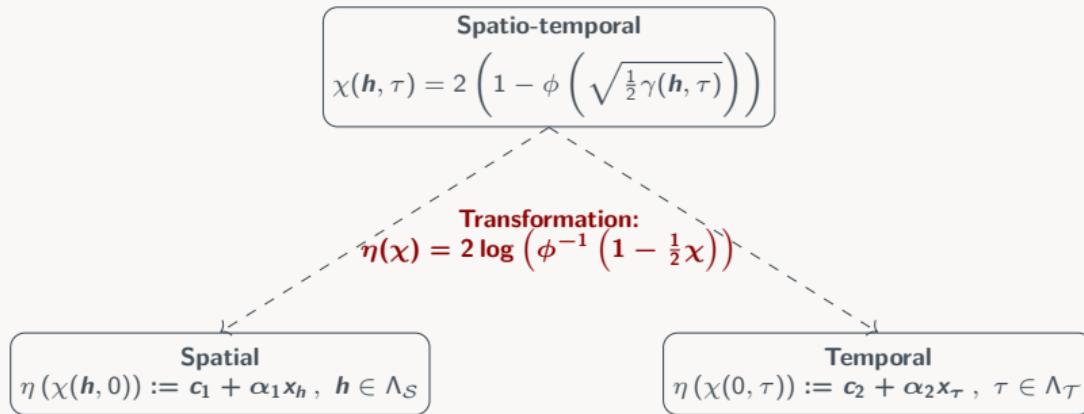
$$\eta(\chi(\mathbf{h}, 0)) = \log \beta_1 + \alpha_1 \log \|\mathbf{h}\|, \quad \mathbf{h} \in \Lambda_S$$

## Temporal

$$\eta(\chi(0, \tau)) = \log \beta_2 + \alpha_2 \log \tau, \quad \tau \in \Lambda_T$$

# ESTIMATION OF THE SEPARABLE VARIOGRAM PARAMETERS

**Case of additive separability:**  $\frac{\gamma(\mathbf{h}, \tau)}{2} = \beta_1 \|\mathbf{h}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}, \quad 0 < \alpha_1, \alpha_2 \leq 2, \quad \beta_1, \beta_2 > 0$



**Weighted Least Squares Estimation (WLSE)**

$$\begin{pmatrix} \widehat{c}_i \\ \widehat{\alpha}_i \end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_i} \sum_x w_x \left( \eta \left( \widehat{\chi} \right) - (c_i + \alpha_i x) \right)^2$$

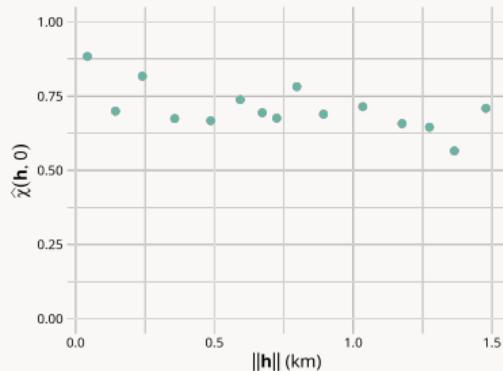
## Empirical spatial extremogram

For a fixed  $t \in \mathcal{T}$  and  $q$  a high quantile,

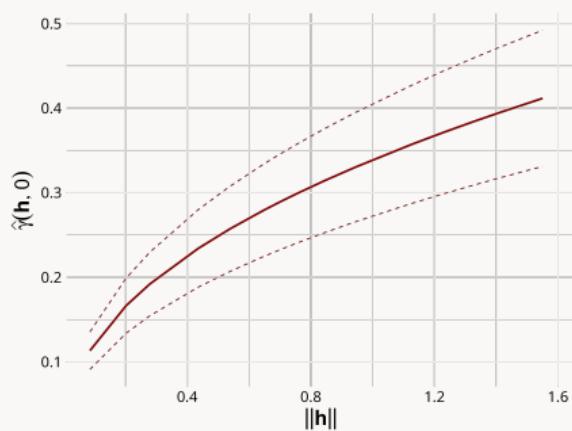
$$\hat{\chi}_q^{(t)}(\mathbf{h}, 0) = \frac{\frac{1}{|N_{\mathbf{h}}|} \sum_{i,j \mid (\mathbf{s}_i, \mathbf{s}_j) \in N_{\mathbf{h}}} \mathbb{1}_{\{X_{\mathbf{s}_i, t}^* > q, X_{\mathbf{s}_j, t}^* > q\}}}{\frac{1}{|\mathcal{S}|} \sum_{i=1}^{|\mathcal{S}|} \mathbb{1}_{\{X_{\mathbf{s}_i, t}^* > q\}}},$$

where  $C_{\mathbf{h}}$  are equifrequent distance classes and

$$N_{\mathbf{h}} = \left\{ (\mathbf{s}_i, \mathbf{s}_j) \in \mathcal{S}^2 \mid \|\mathbf{s}_i - \mathbf{s}_j\| \in C_{\mathbf{h}} \right\}.$$



Transformation and WLSE



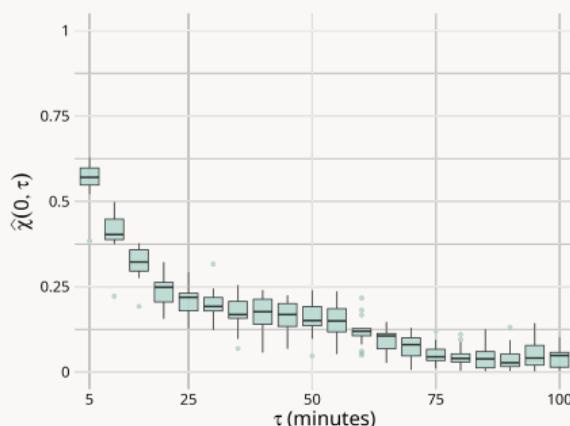
$$\text{Spatial variogram } \hat{\gamma}(\mathbf{h}, 0) = 2\hat{\beta}_1 \|\mathbf{h}\|^{\hat{\alpha}_1}$$

# TEMPORAL DEPENDENCE ESTIMATION

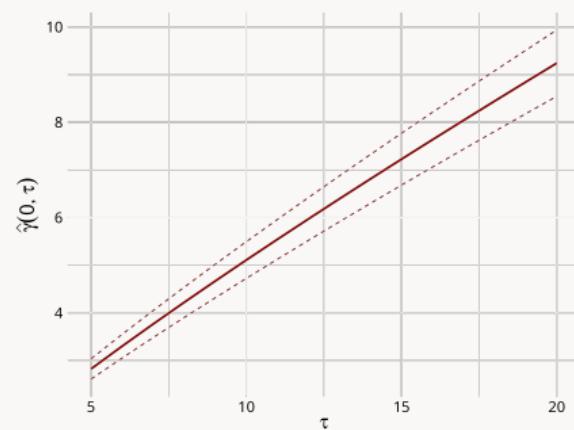
## Empirical temporal extremogram

For a location  $s \in \mathcal{S}$ , a high quantile  $q$  and  $t_k \in \{t_1, \dots, t_T\}$ ,

$$\hat{\chi}_q^{(s)}(\mathbf{0}, \tau) = \frac{\frac{1}{T-\tau} \sum_{k=1}^{T-\tau} \mathbb{1}_{\{X_{s,t_k}^* > q, X_{s,t_k+\tau}^* > q\}}}{\frac{1}{T} \sum_{k=1}^T \mathbb{1}_{\{X_{s,t_k}^* > q\}}}$$



## Transformation and WLSE



Temporal variogram  $\hat{\gamma}(\mathbf{0}, \tau) = 2\hat{\beta}_2|\tau|^{\alpha_2}$

## Advection vector $\mathbf{V}$

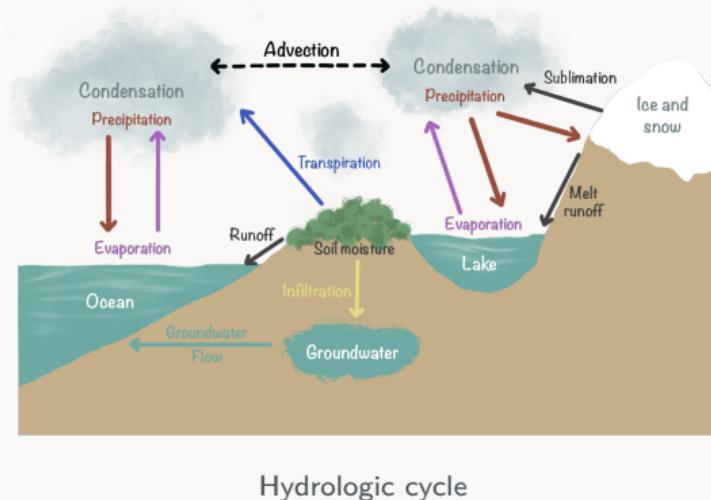
- ▶ Horizontal transport of air masses
- ▶ To relax the separability assumption

## Lagrangian/Eulerian variogram

$$\gamma_L(\mathbf{h}, \tau) = \gamma(\mathbf{h} - \tau \mathbf{V}, \tau)$$

## Dependence model

$$\frac{1}{2}\gamma_L(\mathbf{h}, \tau) = \beta_1 \|\mathbf{h} - \tau \mathbf{V}\|^{\alpha_1} + \beta_2 |\tau|^{\alpha_2}$$



**Parameter optimization of  $\Theta = (\beta_1, \beta_2, \alpha_1, \alpha_2, \mathbf{V})$**

**Joint excesses for  $r$ -Pareto processes:**

$$\begin{aligned} K_{\mathbf{h}, \tau}(u) &= \sum_{s \in \mathcal{S}: (\mathbf{s}, s_0) \in \mathcal{N}(\mathbf{h})} \sum_{t \in \mathcal{T}: t - t_0 = \tau} \mathbb{1}_{\{X_{s,t} > u, X_{s_0 + \mathbf{h}, t_0 + \tau} > u\}} \\ &\sim \mathcal{B} \left( \sum_{(s_0, t_0) \in \mathcal{R}} \#\{s \in \mathcal{S} : (\mathbf{s}, s_0) \in \mathcal{N}(\mathbf{h})\}, \chi_{r, \Theta}(\mathbf{h}, \tau) \right), \text{ for a large } u \end{aligned}$$

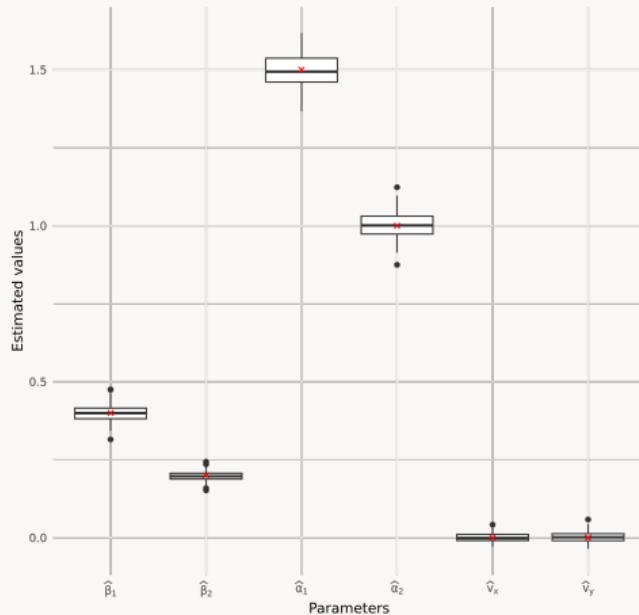
with  $\mathcal{R}$  a specific set of  $r$ -conditioning spatio-temporal locations.

**Composite log-likelihood:**

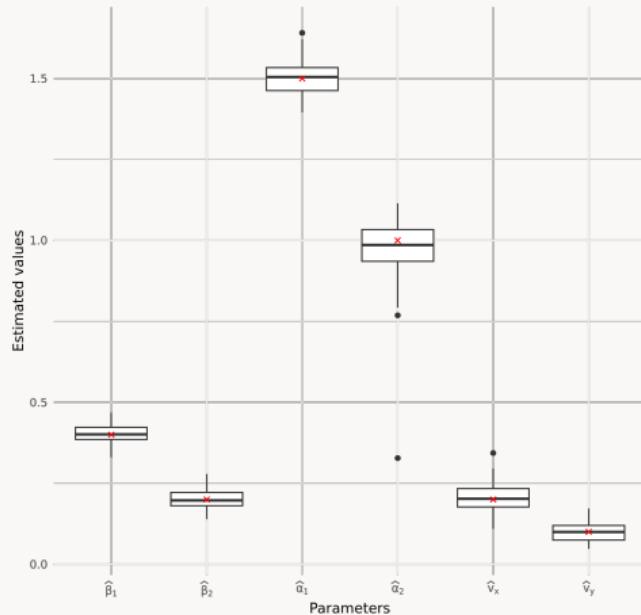
$$l_C(\Theta) \propto \sum_{(\mathbf{s}_0, t_0) \in \mathcal{R}} \sum_{(\mathbf{s}, s_0) \in \mathcal{N}(\mathbf{h})} \sum_{t \in \mathcal{T}: t - t_0 = \tau} k_{s,t} \log \chi_{r, \Theta}(\mathbf{h}, \tau) + (1 - k_{s,t}) \log (1 - \chi_{r, \Theta}(\mathbf{h}, \tau)).$$

where  $\chi_{r, \Theta}(\mathbf{h}, \tau)$  is the  $r$ -extremogram.

# VALIDATION ON SIMULATIONS



Without advection



With advection

Parameter estimation on 100 simulations of 1000 replicates of  $r$ -Pareto processes with 25 sites and 30 time observations

# USING THE OPTIMIZATION MODEL

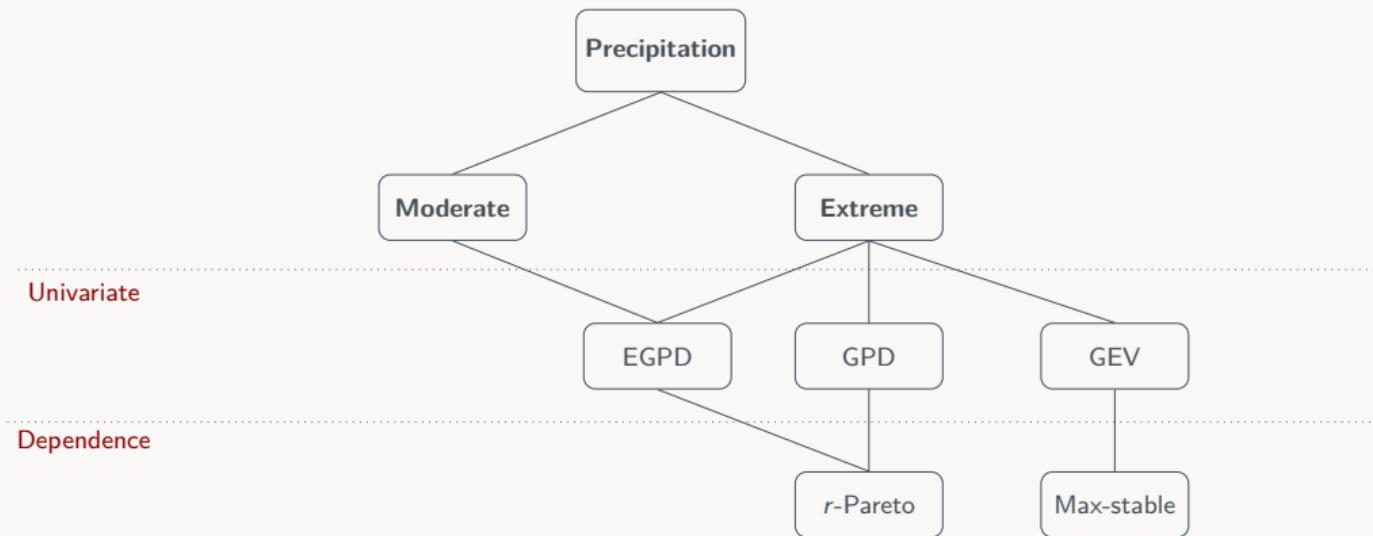
Estimate constant advection  $\hat{V}$

- ▶ Use COMEPHORE data.
- ▶ Initial parameters  $\beta_1, \beta_2, \alpha_1, \alpha_2$ : WLSE
- ▶ Initial advection: wind data

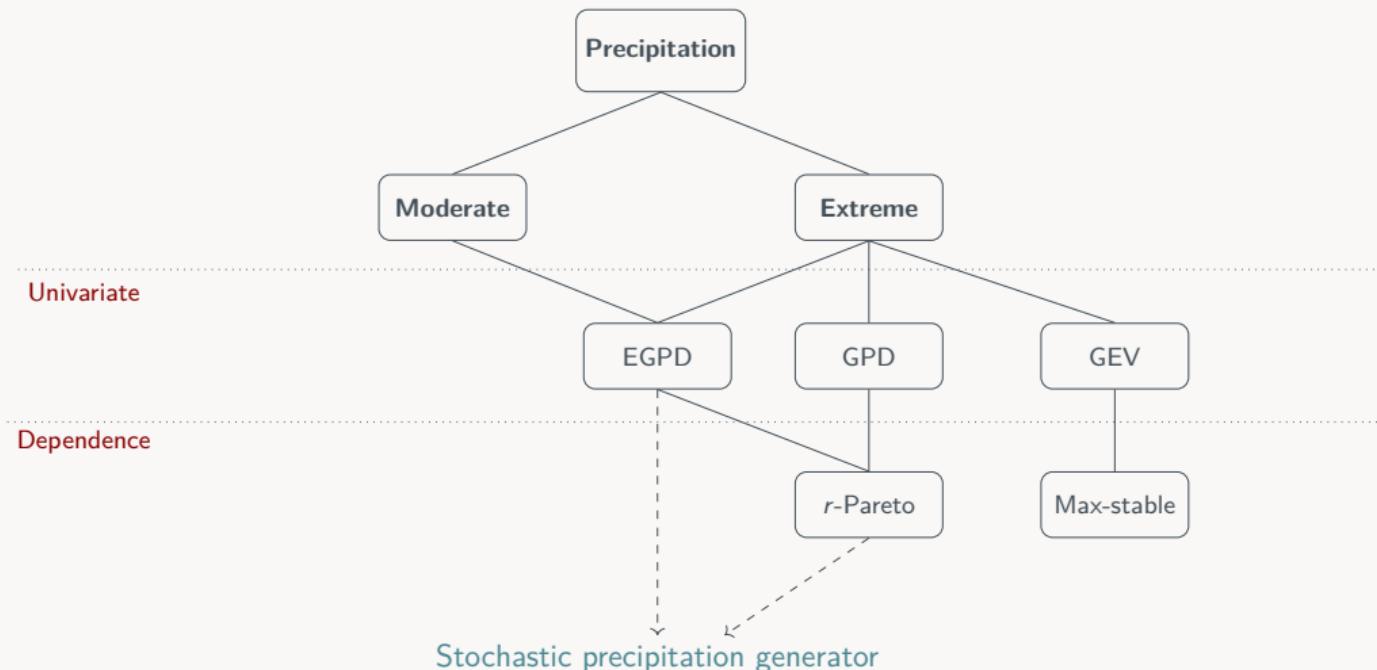
Estimate variogram parameters  
on HSM data

- ▶ Initial parameters  $\beta_1, \beta_2, \alpha_1, \alpha_2$ : WLSE
- ▶ Initial advection:  $\hat{V}$  (from COMEPHORE)

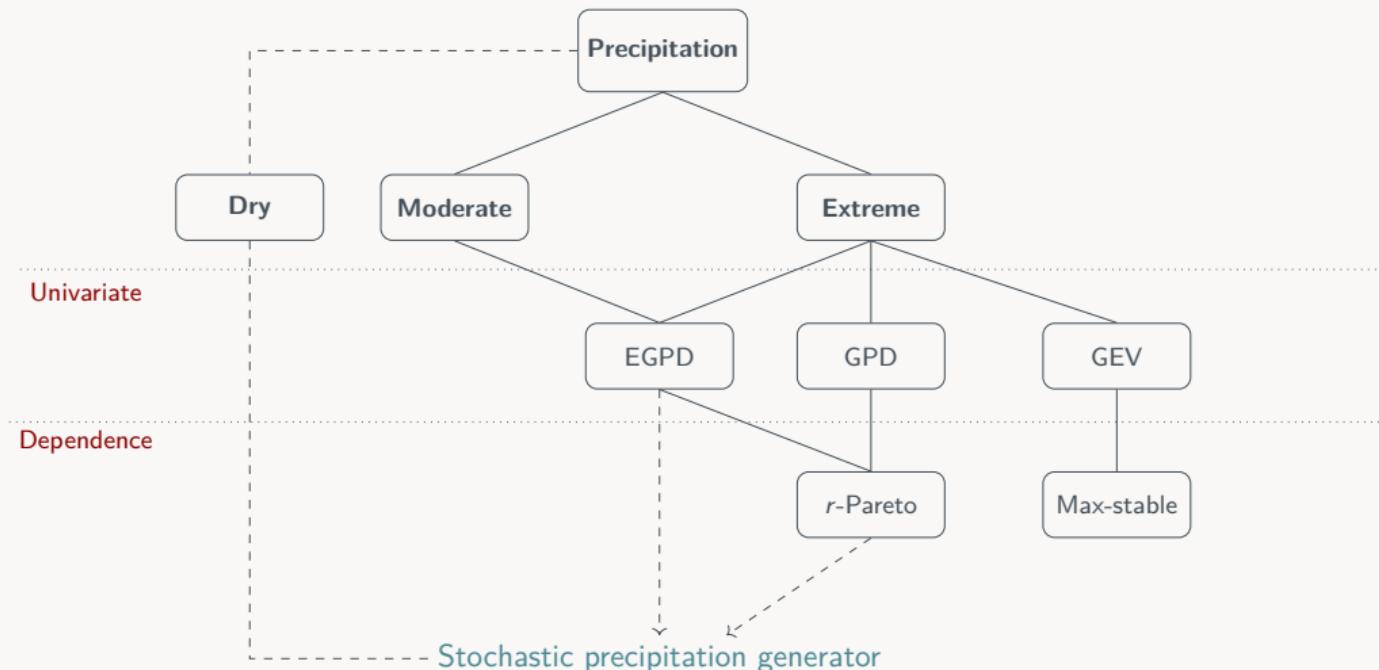
# CONCLUSION



# CONCLUSION



# CONCLUSION

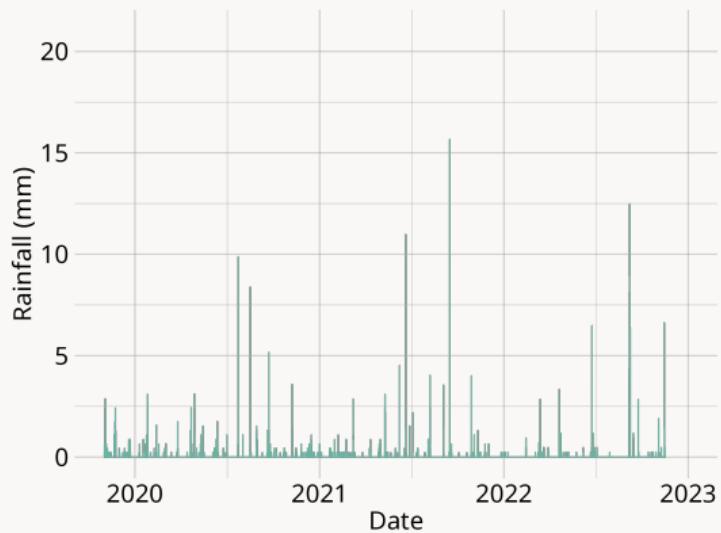
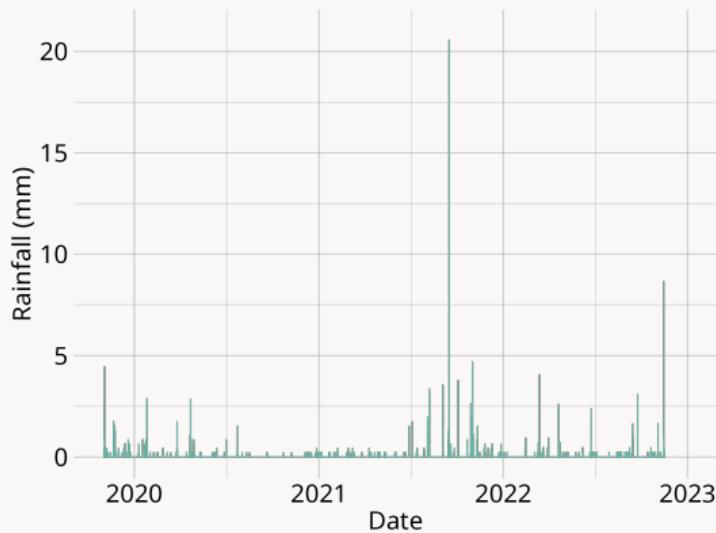


**Thank you for your attention!**

## REFERENCES

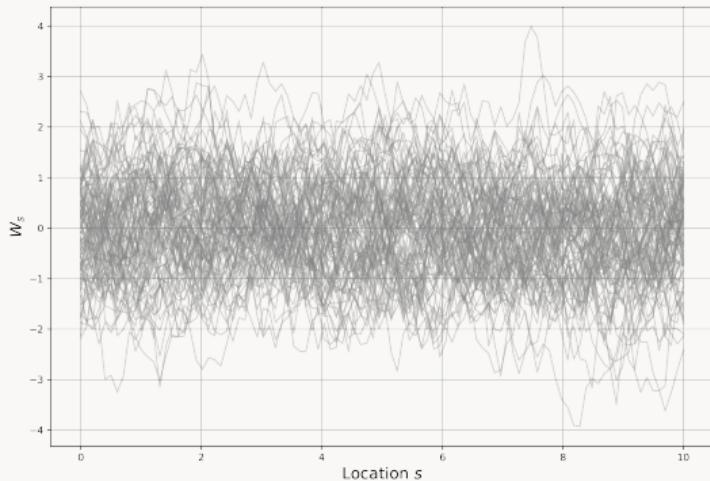
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# RAINFALL DATA - HSM

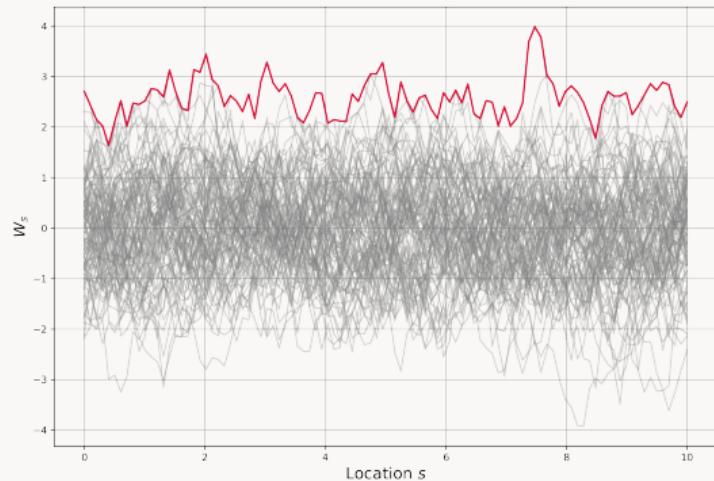


Rainfall amounts on CNRS and Polytech rain gauges

# GAUSSIAN PROCESSES



100 Gaussian processes



Maximum process in red