Optimisation

2024-10-09

The r-Pareto process

Our rainfall process is $X = \{X_{s,t}, (s,t) \in S \times T\}$, where S is the spatial domain and T the temporal domain.

We have $X_{s,t} \mid r(X_{s,t}) > u \xrightarrow{d} Z_{s,t}$ with a risk function $r(X_{s,t}) = X_{s_0,t_0}$, a threshold $u > 1$ and $Z = \{Z_{s,t}, (s,t) \in S \times T\}$ a r-Pareto process.

With $W = \{W_{s,t}, (s,t) \in S \times T\}$ the Gaussian process of the Brown-Resnick process and its variogram γ, we can define the r-Pareto process as

$$Z_{s,t} = R_{s,t} e^{W_{s,t} - W_{s_0,t_0} - \gamma(s-s_0, t-t_0)}$$

where $R_{s,t}$ is a random variable following a simple Pareto distribution.

The variogram γ is defined as

$$\gamma(ds, dt) = \beta_1 |ds|^{\alpha_1} + \beta_2 dt^{\alpha_2}$$

with $ds = s - s'$, $dt = t - t'$, $\beta_1, \beta_2 > 0$ and $\alpha_1, \alpha_2 \in (0,1)$.

If we add an advection vector $V = (v_x, v_y)$, the variogram becomes

$$\gamma(ds, dt) = \beta_1 |ds - V dt|^{\alpha_1} + \beta_2 dt^{\alpha_2}.$$

The r-Pareto process without advection

Simulation

We simulate the r-Pareto process with the parameters $\beta_1 = 0.4$, $\beta_2 = 0.2$, $\alpha_1 = 1.5$, $\alpha_2 = 1$ and without advection. We simulate the process on a 5×5 grid with 30 time steps and $m = 100$ realizations. We use a conditonal point $s_0 = (1,1)$ at time $t_0 = 1$.

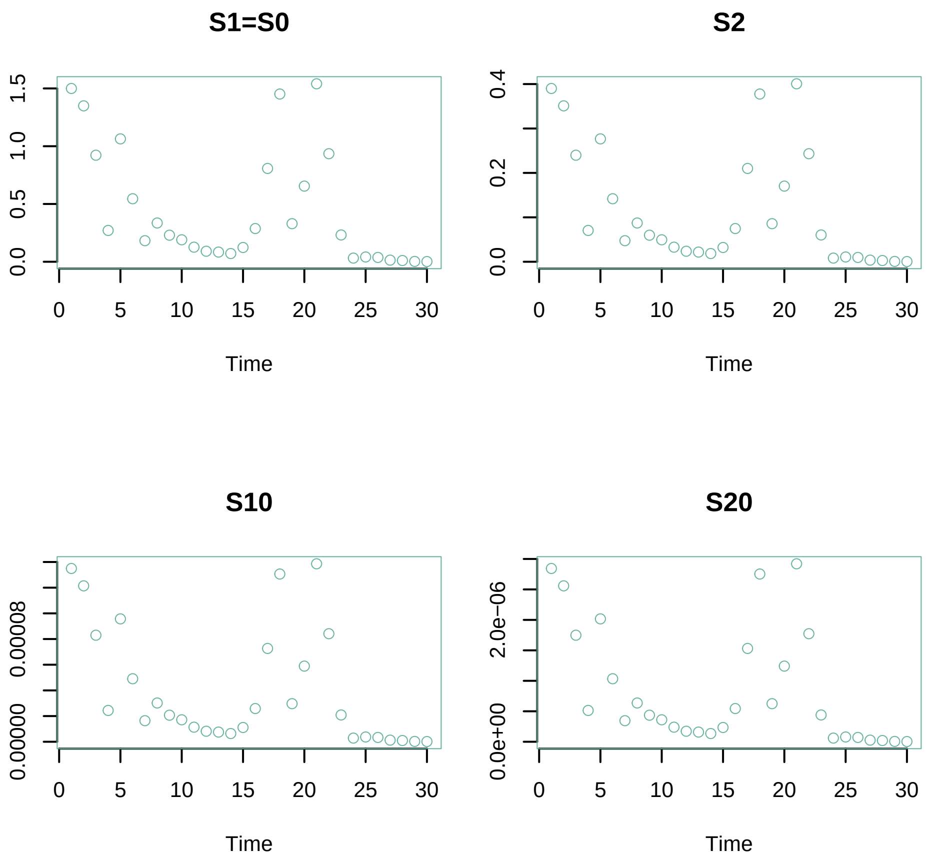

Figure 1: Time series for 4 sites of the first realization

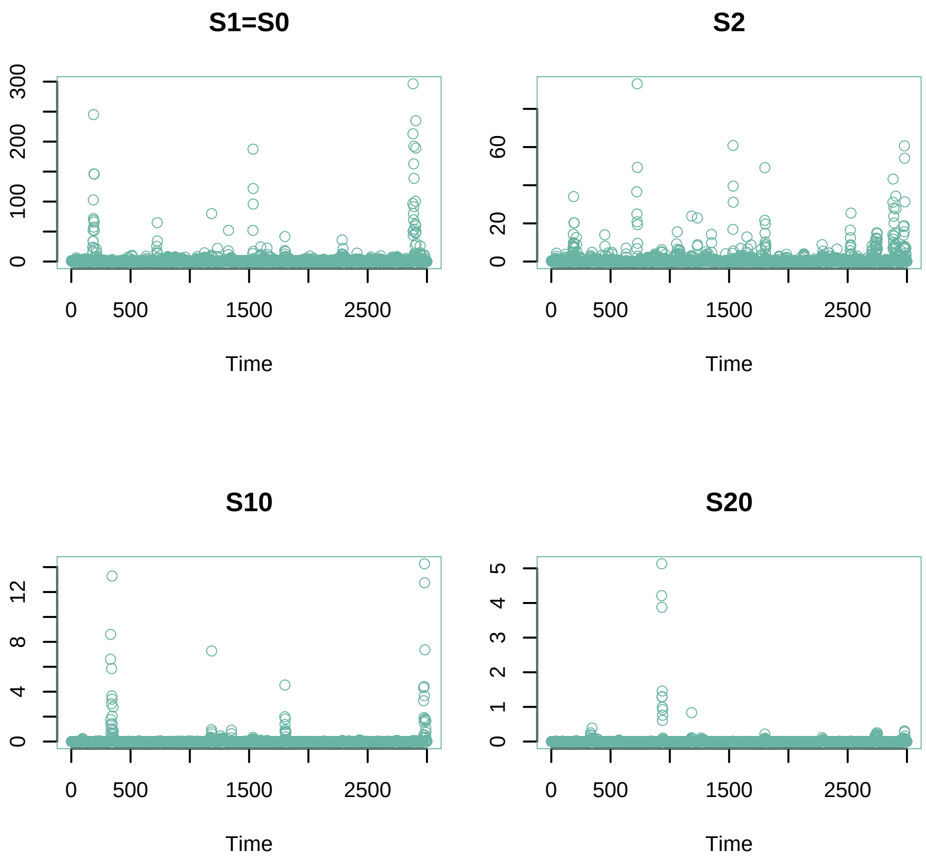

Figure 2: Time series for 4 sites of the all replicates together

Probability of exceedances for the r-Pareto process without advection

For (s_0, t_0)

Verification that $P(X_{s_0,t_0} > u) = 1$ because of the r-Pareto process construction.

```
## [1] "X_s0_t0 = 1.518 > u = 1"
```

```
## [1] "P(X_s0,t > u) = 0.16"
```

For all replicates we can see that the value of the process at the conditional point:

```
## [1] "Minimum value of the process at the conditional point: 1.014 > u = 1"
```

```
## [1] "Maximum value of the process at the conditional point: 96.75"
```

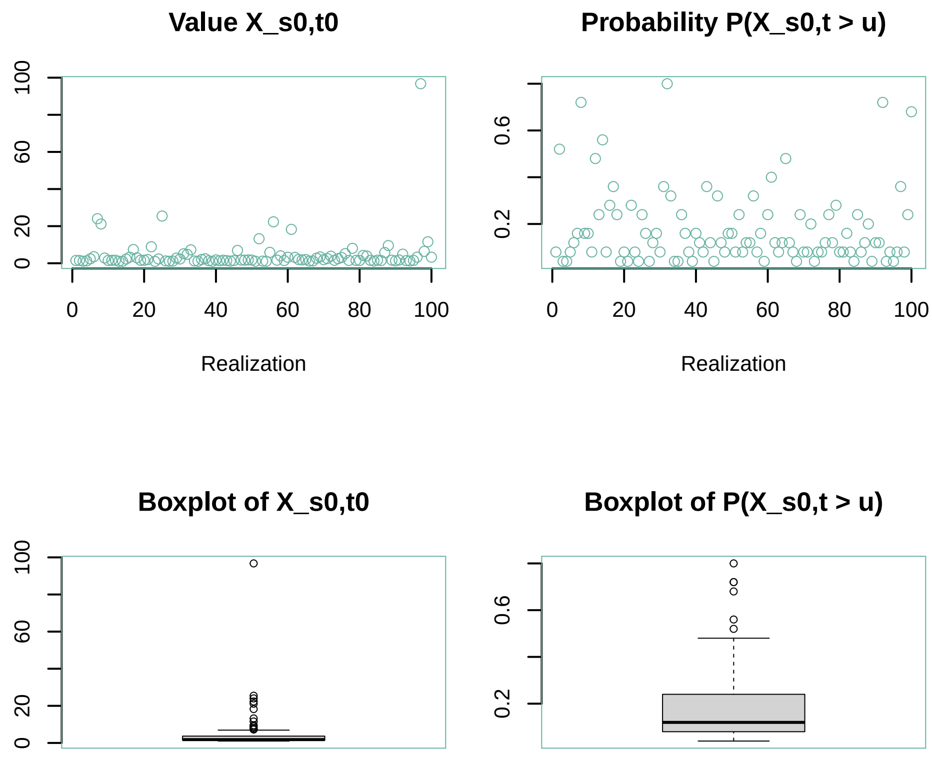

Figure 3: Value and probability at conditional point for all replicates

Marginal exceedance probability

We have different probability of exceedances for each site as we can see in the following plot.

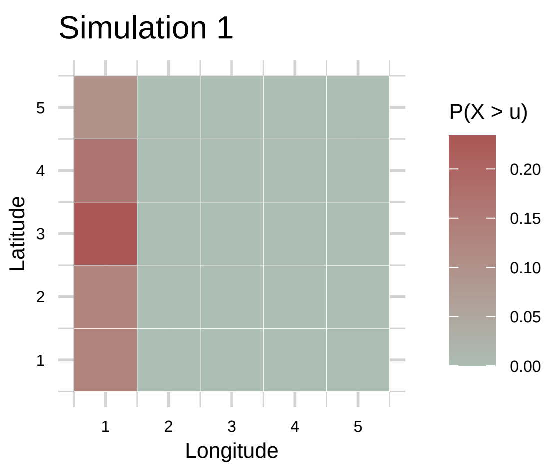

Figure 4: Marginal probability of exceedances for each site for one simulation

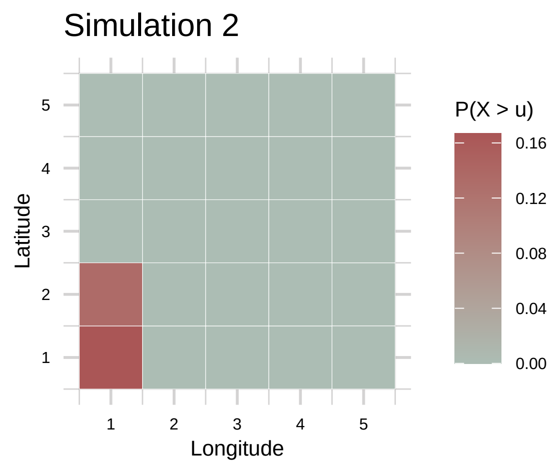

Figure 5: Marginal probability of exceedances for each site for one simulation

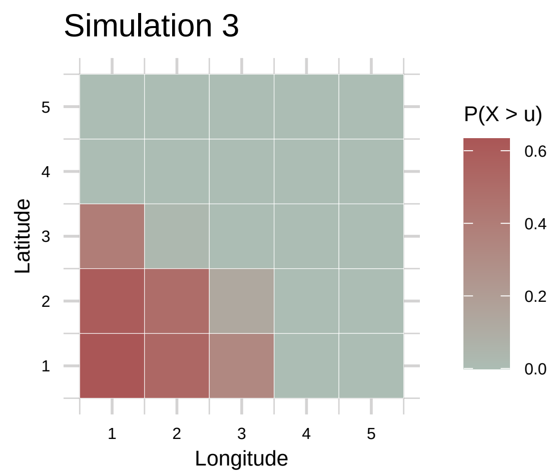

Figure 6: Marginal probability of exceedances for each site for one simulation

Multiple replicates

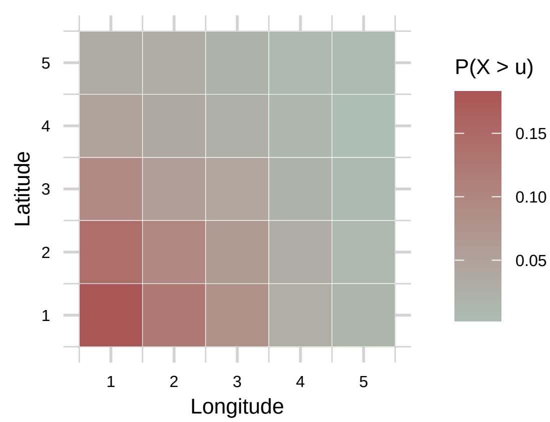

Figure 7: Marginal probability of exceedances for m replicates in a single dataframe

The r-Pareto process with advection

Simulation

We simulate the r-Pareto process with the parameters $\beta_1 = 0.4$, $\beta_2 = 0.2$, $\alpha_1 = 1.5$, $\alpha_2 = 1$ and the advection vector $V = (0.5, 0.3)$. We simulate the process on a 5×5 grid with 30 time steps and 100 realizations. We use a conditonal point $s_0 = (1, 1)$ at time $t_0 = 1$.

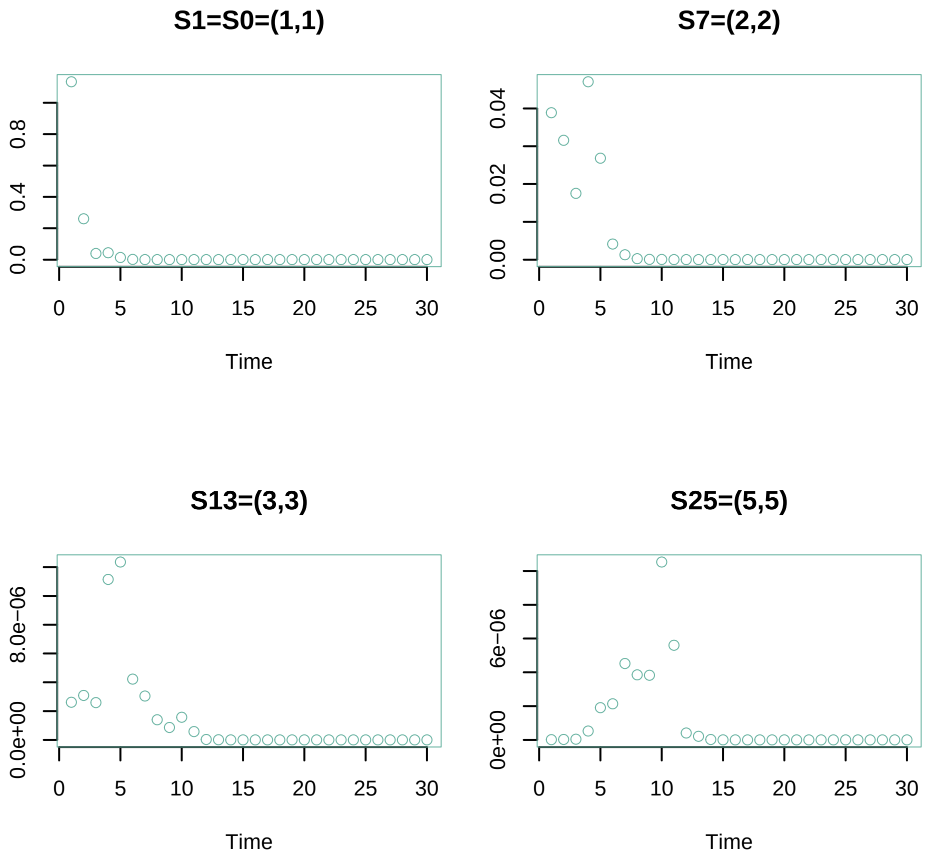

Figure 8: Time series for 4 sites of the first realization in the advection direction

r-Pareto process with advection

Probability of exceedances for the r-Pareto process

For (s_0, t_0)

Verification that $P(X_{s_0,t_0} > u) = 1$ because of the r-Pareto process construction.

```
## [1] "X_s0_t0 = 2.188 > u = 1"
```

```
## [1] "P(X_s0,t > u) = 0.04"
```

For all replicates we can see that the value of the process at the conditional point:

```
## [1] "Minimum value of the process at the conditional point: 1.011 > u = 1"

## [1] "Maximum value of the process at the conditional point: 660.524"
```

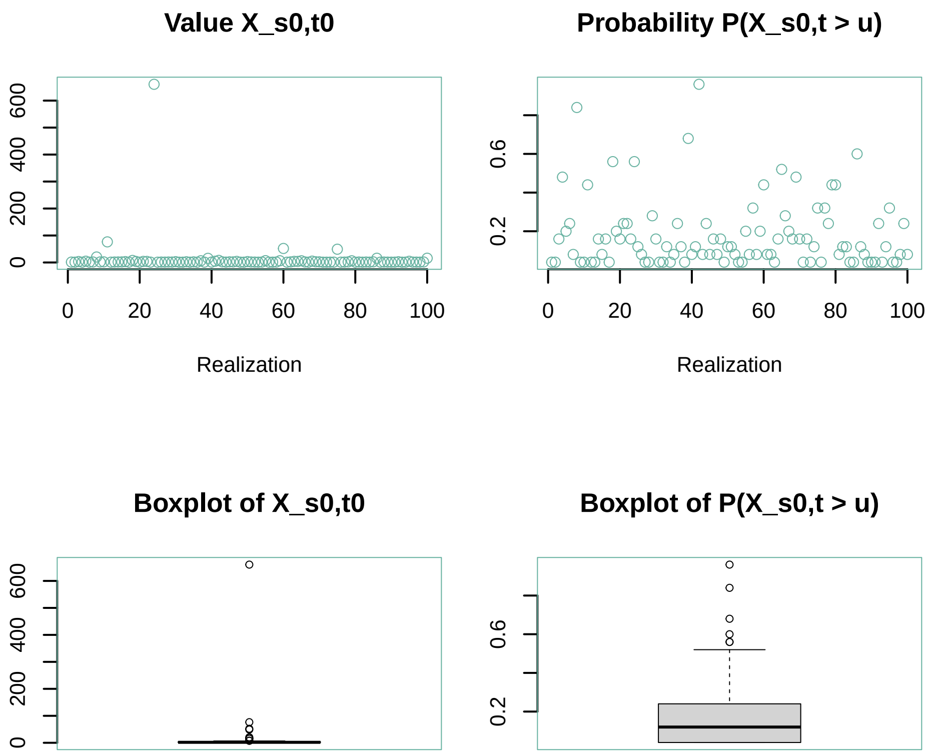

Figure 9: Value and probability at conditional point for all replicates

Marginal exceedance probability

We have different probability of exceedances for each site as we can see in the following plot.

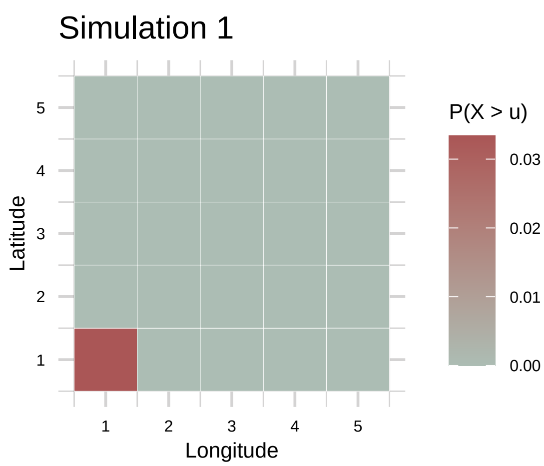

Figure 10: Marginal probability of exceedances for each site for one simulation

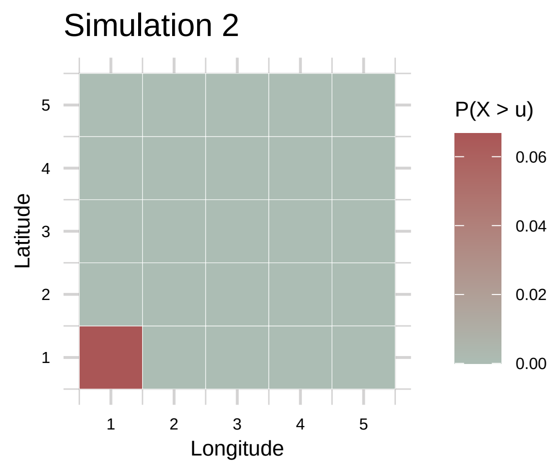

Figure 11: Marginal probability of exceedances for each site for one simulation

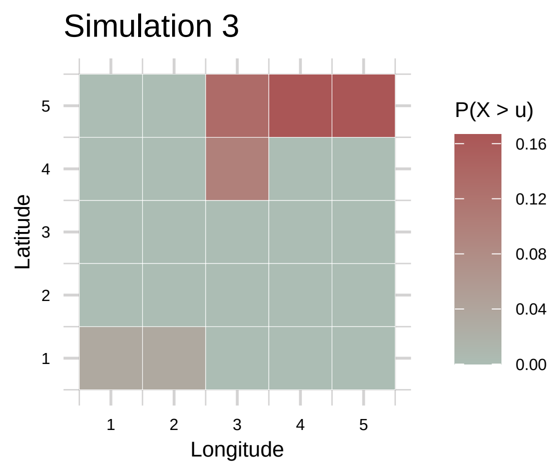

Figure 12: Marginal probability of exceedances for each site for one simulation

TODO : verifier la forme de ma grille dans mes gifs, j'ai un doute

For multiple replicates of the r-Pareto process

We concate the data of $m = 30$ replicates to have a single simulation of the r-Pareto process.

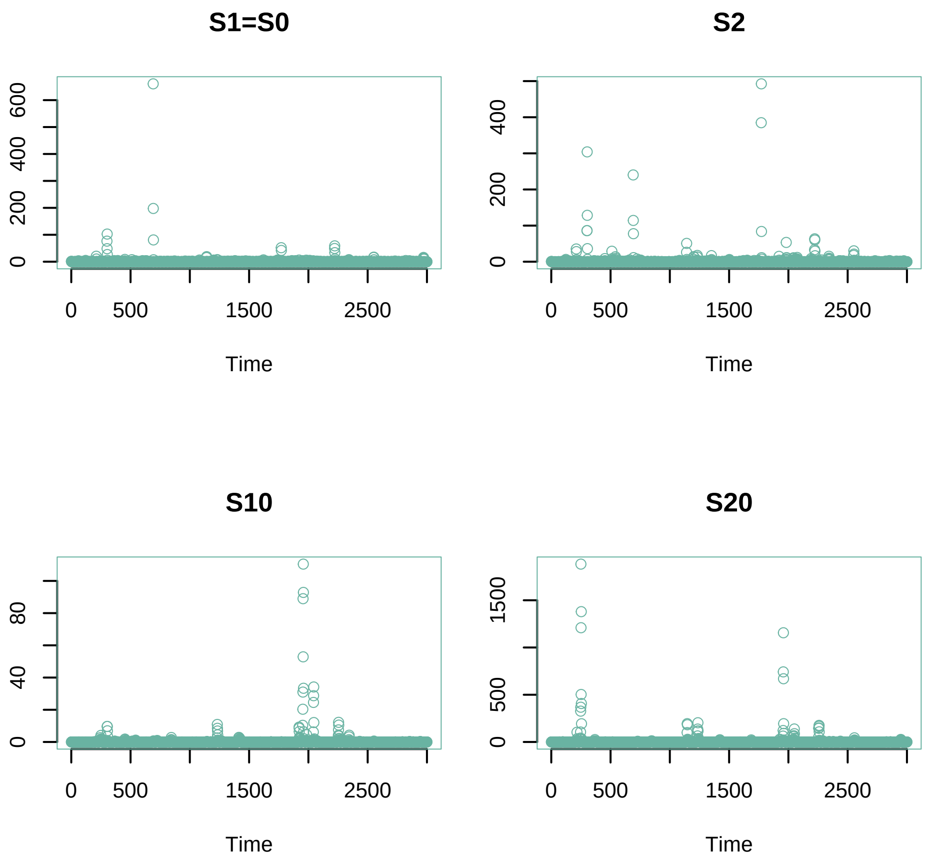

Figure 13: Time series for 4 sites of the replicates together

The probability of exceedances for each site is computed for all replicates.

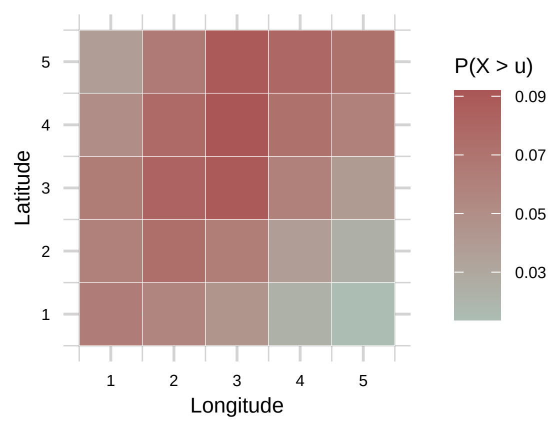

Figure 14: Marginal probability of exceedances for m replicates in a single dataframe

Optimisation

We want to estimate the parameters β_1, β_2, α_1 and α_2 of the r-Pareto process. We use the maximum likelihood estimation with the composite likelihood method.

We compute the number of joint excesses for each replicate i, $k_{s,t}^{(i)} = \sum_{t=1}^{T} \mathbb{1}_{\{X_{s,t}>u, X_{s_0,t_0}>u\}}$ and $k_{s-s_0,t-t_0}^{(i)} \sim Bin(T-t-t_0, \chi(s-s_0, t-t_0))$ with T the number of observations within a replicate (same for all replicates).

Si je bouge pas c'est bon, si je bouge un peu trouve tout sauf l'advection estimée à 21, 21 (bcp trop)

Avec les vrais parametres en valeurs initiales c(0.4, 0.2, 1.5, 1, 0.5, 0.3) et 30 simus on a comme estimation: 0.4 0.2 1.5 1 0.5 0.3

Avec les vrais parametres en valeurs initiales c(0.4, 0.2, 1.5, 1, 0.5, 0.3) et 20 simus on a comme estimation: 0.3999390 0.1999695 1.4997711 0.9998579 0.4922282 0.2880588

Pour valeurs initiales c(0.45, 0.2, 1.55, 1, 0.5, 0.3) et 20 simus on a comme estimation: 0.3540622 0.4971539 1.4470578 1.1838994 0.5037758 47.9497756

Plot by fixing params