PPModel

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2.1.1 Three Zone Model

An agent with index i is defined by a position in the two-dimensional space $\vec{r}_i$, an orientation φ_i and the corresponding orientation vector $\vec{v}_i$ as well as a velocity s_i.

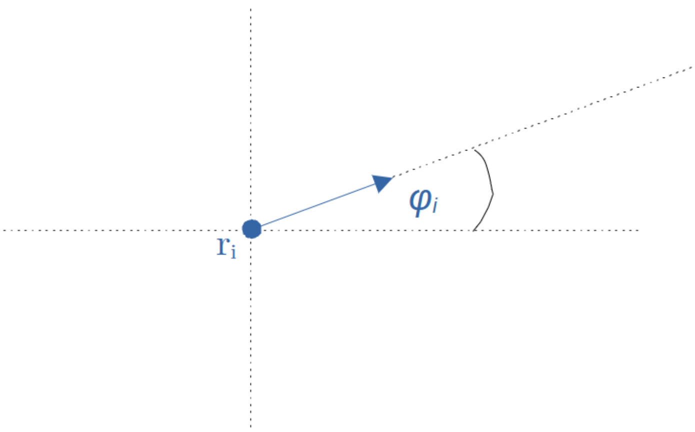

Figure 1: An agent

Interaction with the other agents occurs within 3 zones (see figure 2 on the left). At very short distance (red zone), repulsive forces (**repulsion**) between the agents predominate. At medium distance (green zone), the agents try to align with each other in their orientation (**alignment**). If the agents have a high distance to each other (purple zone), they attract and thus strive towards each other (**Attraction**). A visualization of the zones can be found in figure 2.

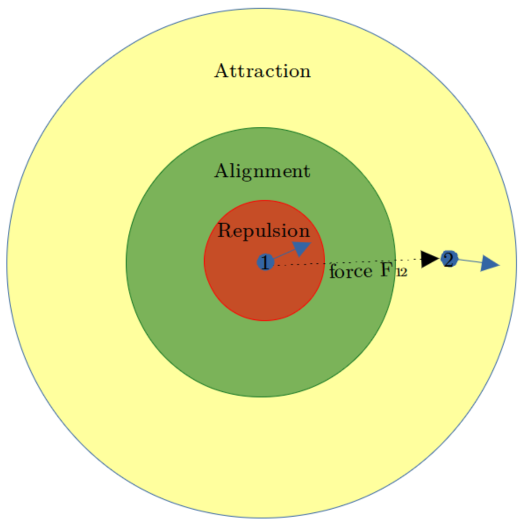

Figure 2: Darstellung der drei Zonen um den Agenten i (Rot : Repulsion, Grün : Alignment, Lila : Attraction). Orientierungsvektoren $\vec{v}_i$ und $\vec{v}_j$, sowie Abstand zwischen beiden Agenten r_{ij}. Quelle : PyGame Modelling Workshop 2022 [**mezey2022threezone**].

The movement of the agents is described by the following differential equations:

$$\frac{d\varphi_i}{dt} = F_{soc,\varphi_i}(t) + \eta_i(t) \tag{1}$$

$$\frac{d\vec{r_i}}{dt} = s_i \cdot \vec{v_i} = \begin{pmatrix} s_i \cdot \cos(\varphi_i(t)) \\ s_i \cdot \sin(\varphi_i(t)) \end{pmatrix} \tag{2}$$

$$\frac{ds}{dt} = F_{soc,s_i}(t) + f(s_i, s_{0,i}, A_i) + \eta_i(t) \tag{3}$$

Here, the variable $\eta_i(t)$ describes a time-dependent function that adds an undirected noise to the process. The social force $F_{soc}(t)$ is composed of all the individual forces acting on an agent. The total force vector is converted to an angle or velocity by a scalar product with the conversion factor u_{φ_i} or u_{s_i}.

$$F_{soc,\varphi_i}(t) = \vec{F}_{soc,i}(t) \cdot u_{\varphi,i}(t) = \vec{F}_{soc,i}(t) \cdot \begin{pmatrix} -\sin(\varphi_i(t)) \\ \cos(\varphi_i(t)) \end{pmatrix} \tag{4}$$

$$F_{soc,s_i}(t) = \vec{F}_{soc,i}(t) \cdot u_{s_i}(t) = \vec{F}_{soc,i}(t) \cdot \begin{pmatrix} \cos(\varphi_i(t)) \\ \sin(\varphi_i(t)) \end{pmatrix} \tag{5}$$

$$\vec{F}_{soc,i} = \vec{F}_{att,i} + \vec{F}_{rep,i} + \vec{F}_{alg,i} \tag{6}$$

The individual force vectors are always a sum over all individual forces acting between agent i and its neighbors j. The result is:

$$\vec{F}_{att,i} = \sum_{j \neq i}^{N} +\mu_{att} S_{att} \frac{\vec{r_{ij}}}{|\vec{r_{ij}}|} \tag{7}$$

$$\vec{F}_{rep,i} = \sum_{j \neq i}^{N} -\mu_{rep} S_{rep} \frac{\vec{r_{ij}}}{|\vec{r_{ij}}|} \tag{8}$$

$$\vec{F}_{alg,i} = \sum_{j \neq i}^{N} +\mu_{alg} S_{alg}(\vec{v_{ij}}) \tag{9}$$

Here r_{ij} corresponds to the difference between the two position vectors and v_{ij} to the difference of the two direction vectors of the respective agents i and j. The variables S_X describe sigmoid functions by which the 3 zones are realized.

$$S_X = \frac{1}{2}(\tanh(-a(|\vec{r_{ij}}| - r_X)) + 1) \tag{10}$$

This is done by the function taking the value 1 between 0 and r_X and the value 0 between r_X and ∞. Thus, the force acts only in the zone up to the value r_X. The factor a determines the "softness" of the transition between the zone with force and the zone without force. At $a \to 0$ the function becomes linear, whereas at $a \to \infty$ it is a step function.

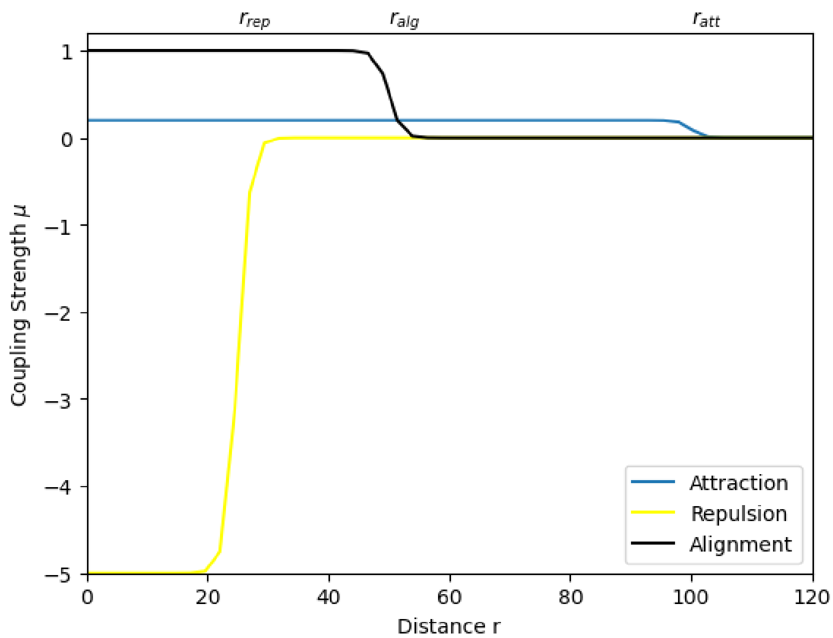

Figure 3: Darstellung der drei sigmoiden Funktionen aus Gleichung 10, welche die Zonen im Modell repräsentieren. Es wurden die folgenden Parameter verwendet : $a = -0.5$, $r_{att} = 100$,$r_{rep} = 25$, $r_{alg} = 50$, $\mu_{att} = 0.2$, $\mu_{rep} = 5$, $\mu_{alg} = 1$.

A timeseries can be computed through discretization of the differential equations.

$$\varphi_i(t+1) = \varphi_i(t) + F_{soc,\varphi_i}(t) \cdot \Delta t + \sigma \cdot \xi \cdot \sqrt{\Delta t} \tag{11}$$

$$s_i(t+1) = s_i(t) + F_{soc,s_i}(t) \cdot \Delta t + A_i(s_{0,i} - s_i(t)) + \sigma \cdot \xi \cdot \sqrt{\Delta t} \tag{12}$$

$$\vec{r}_i(t+1) = \vec{r}_i(t) + s_i(t+1) \cdot \vec{v}_i(t+1) \cdot \Delta t \tag{13}$$

The timestep Δt resembles the accuracy of the representation of the model, a lower timestep corresponds to a closer resemblances to the real formalism but also increases the computation time for the same timespan. The noise function in the equations for angle and velocity is composed of the noise factor σ, a gaussian random number ξ (centered around 0, standard deviation of 1) and the square-root of the timestep. The equation for the velocity is also expanded by a term with the stubborness A_i of the agent (how strongly it will relax into it's prefered velocity $s_{0,i}$).

2.1.2 Rules for Interactions

Rules for interaction need to be implemented to represent a realistic swarm, in which not every agent can detect the movement of every other agent in the school. It also saves a lot of computation time for high number of agents, as the forces for every agent only need to be calculated for the respective interaction group.
There are multiple ways of defining the interaction partners of an agent. One relatively simple way would be, to define a range around the agent in which it can potentialy detect other agents and only compute the forces for that agents (metric interactions). Another simple way would be to define a maximum number of neighbors N that the agent can interact with, and then only compute the forces for the N nearest agents.
A more complicated geometrical way of calculating adjacencies is to create a voronoi diagram with the position of every agent as a point and then let every agent only

interact with the first shell of agents around him. In this model both metric and voronoi iteractions are used an compared, as they lead to the best result, and resemble a more real scenario.

2.1.3 Creation of a evenly distributed swarm

To create the initial positions for an evenly distributed swarm, that ressembles random positions, the agents are placed on multiple circumferences of evenly spaced circles around a fixed point. The randomness gets induced adding a noise to every position.

2.1.4 Group measurements

To characterize aspects of the group behavior of the prey agents this work used several measurements. The arguebly most important is **Polarization**, which describes the degree with which the movement of the group is aligned. A Polarization near 1 means, that the group is strongly aligned. A low Polarization near 0 could mean no alignment (random orientation) or some kind of ordered alignment, where the agents have opposite orientations on average (see fountain behavior in ch. 3.1). It is computed as followed:

$$P(t) = \frac{1}{N} |\sum_{i=1}^{N} \vec{v_i}(t)| \tag{14}$$

To calculate how strongly the individuals are clinging to each other, the average inter-individual distance can be computed as:

$$IID(t) = \frac{N(N-1)}{2} \sum_{i=2}^{N} \sum_{j=1}^{i-1} |\vec{r_i}(t) - \vec{r_j}(t)| \tag{15}$$

Alternatively average nearest neighbor distance can be computed as:

$$NND(t) = \frac{1}{N} \sum_{i=1}^{N} d_{i,NN(i)}(t) \tag{16}$$

For further characterization of the group the center of mass is calculated as the average position vector and the average orientation vector of all prey agents ($\vec{r}_{com}(t)$ and $\vec{v}_{com}(t)$). With this information several conclusions can eb drawn about the spatial orientation of the group. The parameter $Q_i(t)$ states the spatial position of every agents with reagrds to the orientation of the school. Individuals at the back will have a value near -1 and at the front near 1. It is computed as:

$$Q_i(t) = \frac{\vec{v}_{com}(t) \cdot \vec{r}_{com,i}(t)}{|\vec{v}_{com}(t)| \cdot |\vec{r}_{com,i}(t)|} \tag{17}$$

The vector $\vec{r}_{com,i}(t)$ describes the distance vector between the position of agent i and the center of mass ($\vec{r_i} - \vec{r}_{com}$).

2.2 Extensions for Predator Interactions

2.2.1 Definition of Preadtor agent

To simulate the movement and interactions of predator and preys another type of agent was added to the model. The predator agent j (or shortly pred) also has a position $\vec{r}_{head,j}$, an orientation φ_j and a velocity like the other agents, but now additional a tail position $\vec{r}_{tail,j}$ and a corresponding length L ($|\vec{r}_{head,j} - \vec{r}_{tail,j}|$).

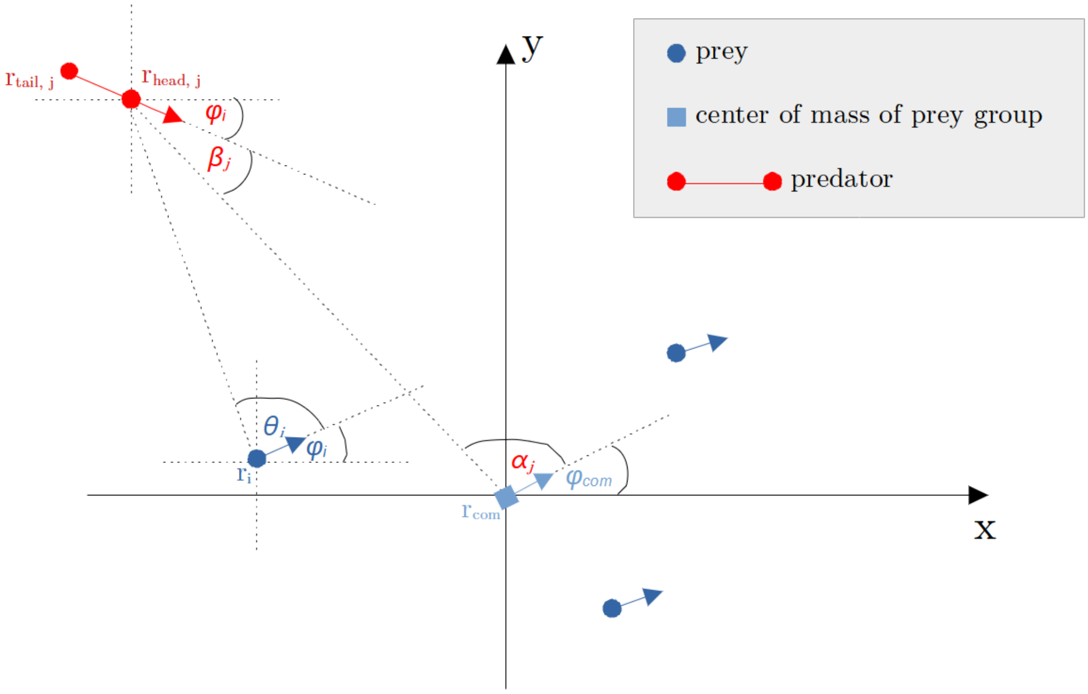

Figure 4: An agent

2.2.2 Prey Interaction with Predator

The prey agent interacts with its conspecifics according to the rules defined in the above mentioned three zone model. The force vector of equation 6 is then expanded with another force vector, that is composed of an attraction to the tail and a repulsion from the head of the predator. Like in the 3 Zone Model both forces correspond to a zone around the head and tail respectively, while the attraction zone is always bigger than the repulsion zone and the attraction strength always smaller than the repulsion strength. This is defined as:

$$\vec{F}_{pred,i} = \vec{F}_{att,pred,i} + \vec{F}_{rep,pred,i} \tag{18}$$

$$\vec{F}_{att,pred,i} = \sum_{j}^{N_{pred}} +\mu_{att,pred} S_{att,j} \frac{\vec{r}_{ij,tail}}{|\vec{r}_{ij,tail}|} \tag{19}$$

$$\vec{F}_{rep,pred,i} = \sum_{j}^{N_{pred}} -\mu_{rep,pred} S_{rep,j} \frac{\vec{r}_{ij,head}}{|\vec{r}_{ij,head}|} \tag{20}$$

The sum of both attraction and repulsion is then added to the social force vector. The movement is updated as in the 3 zone model description with the new force vector. The angle between the connecting vector between the position of agent i and the position of predator j and the orientation vector of the agent is called fleeangle θ_{ij}.

2.2.3 Predator Movement

The movement of the predator is divided into 2 phases. During the **preparation phase** the predator will be attracted to a starting position, where it can start its attack. This starting position is calculated with regards to the center of mass of the school, the attack angle α_j, and a distance from the center of the school $D_{atk,j}$ with the following formula:

$$\vec{r}_{proj,j} = \begin{pmatrix} cos(\alpha_j)cos(\varphi_{com}) - sin(\alpha_j)sin(\varphi_{com}) \\ sin(\alpha_j)cos(\varphi_{com}) + cos(\alpha_j)sin(\varphi_{com}) \end{pmatrix}$$

$$\vec{r}_{start,j} = \vec{r}_{com} + D_{atk,j}\vec{r}_{proj,j} \tag{21}$$

The force is then calculated as:

$$\vec{F}_{att,j} = \mu_j \cdot \frac{\vec{r}_{start,j} - \vec{r}_j}{|\vec{r}_{start,j} - \vec{r}_j|} \tag{22}$$

Addiotionally the predator has a repulsion to from the center of mass of the school, that is computed as:

$$\vec{F}_{rep,j} = \mu_j \cdot \frac{\vec{r}_{com} - \vec{r}_j}{|\vec{r}_{com} - \vec{r}_j|} \tag{23}$$

The sum of both forces yields the total forcevector $\vec{F}_j$ If the predator gets into a distance lower than e threshold X to the starting position, it will get into the **attack phase**. In the attack phase the predator is attracted to the center of mass projected with an offset angle β_j. The force vector is computed as:

$$\vec{r}_{j,com} = \vec{r}_{com} - \vec{r}_j = \begin{pmatrix} x_{j,com} \\ y_{j,com} \end{pmatrix}$$

$$\vec{r}_{off,j} = \begin{pmatrix} x_{j,com}cos(\beta_j) - y_{j,com}sin(\beta_j) \\ x_{j,com}sin(\beta_j) + y_{j,com}cos(\beta_j) \end{pmatrix}$$

$$\vec{F}_{att,j} = \mu_j \cdot \frac{\vec{r}_{off,j}}{|\vec{r}_{off,j}|} \tag{24}$$

If the predator gets near the center of mass with the afforementioned threshold X it will transition again into a preparation phase.

2.2.4 Varible Turning Velocity and Speed

Realistically length has impact on turning velocity and speed. For example predators with a high length can normally turn slower. To simulate this, one could vary the interaction strength and speed with regards to length. Because there is no clear data on which this variation can be based upon, the following analysis disregards this factor for now.

2.3 Genetic Algorithm to train parameters of Predator

The fitness function needs to incoporate both low movement of the school and compactness. For the low movement one could look at the center of mass deviation from the average center of mass of the whole timeseries or during attack phase.

$$\bar{d}_{com} = \frac{1}{T}\sum_{t}^{T} |\bar{r}_{com} - \vec{r}_{com,t}| \tag{25}$$

An obvious way to compute the compactness of the school would be the average inter-individual distance over the timeseries. This however is computationally really expensive and thus the training woudl take a lot of time. A simpler way of looking at the spatial compactness would be to look at the distance between minimum and maximum x and y of the school.

3 Results

3.1 Trajectories

Different interactions strength of the predator agent, have different impacts on the succes of the attack. At low interaction strength, the turn velocity of the prey is much stronger than that of the predator and thus their attack radii differ substantially, which leads to the predator not reaching the prey. The higher the interaction strength of the predator, the higher its turn velocity and thus their attack radius. This can be examplified in a trajectory plot for different interactions tregth with only one prey and predator.

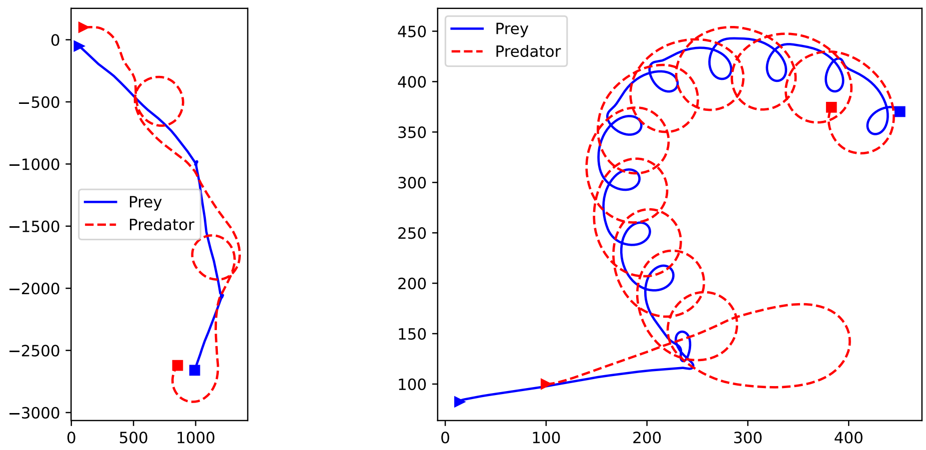

Figure 5: Trajectories of Predator and Prey for different attack strengths (0.1 and 0.5). Example of Predator trying to catch Prey, but not reaching it.

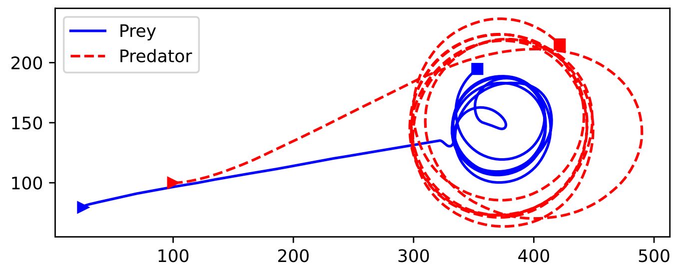

Figure 6: Trajectories of Predator and Prey for attack strength = 0,25. Example of Predator and Prey reaching stable orbit

Currently I don't know when those stable orbits occur and on which parameters they most often occur. Repeated simulations show stable orbits and the "catch-me-if-you-can„cases for all tested parameters. One could look quantitatievly at how often this occurs at different parameters. Regarding the multidemnsionality of the model parameters this might be pointless, as a small change in another parameter could totally change this behavior.

3.2 Fountain Behavior

On a broad range of parameters fountain behavior could be replicated.

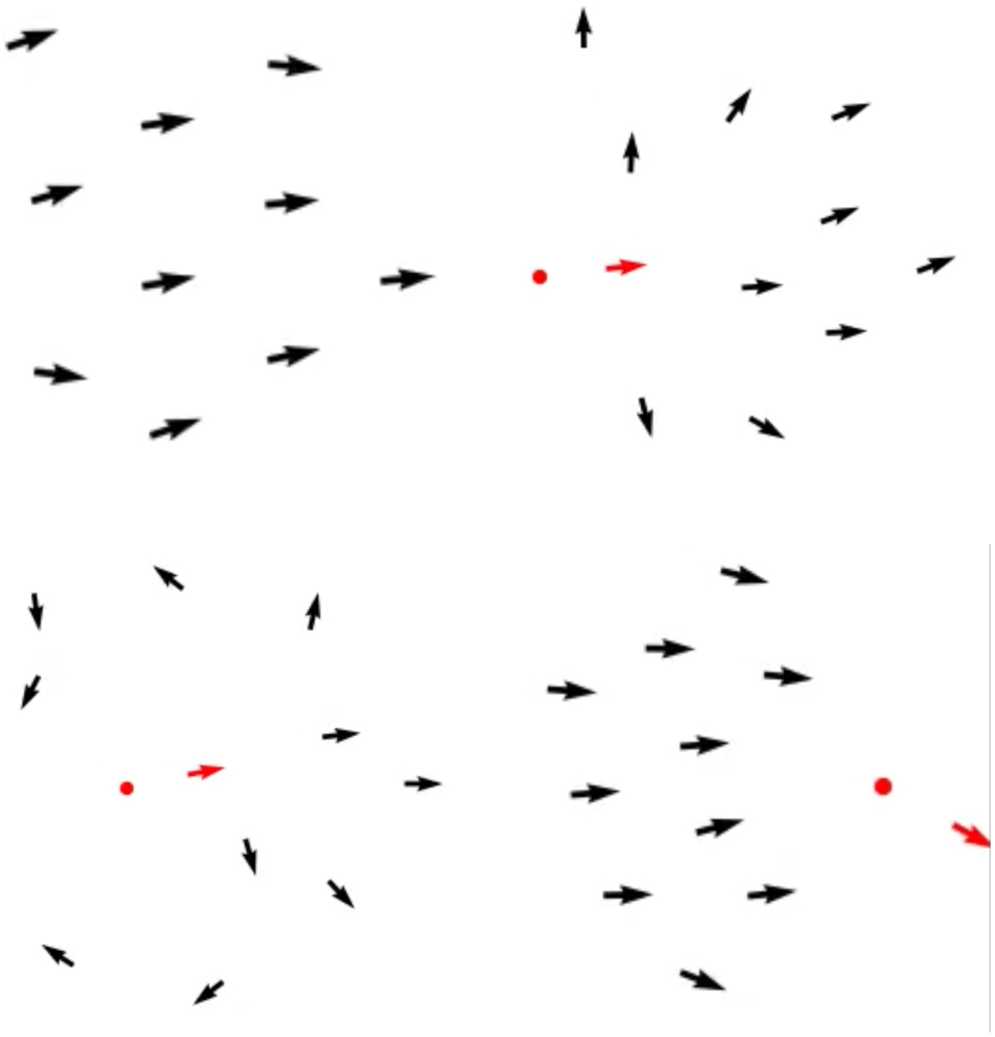

Figure 7: Fountain behavior

3.3 Genetic Algorithm

For the first try i did a genetic algorithm for only one parameter, in this case β, and tried to find the optimal value that maximizises the low movement of the school, disregarding the conmpactness for now. The resuöts of the fitness optimization can be seen here:

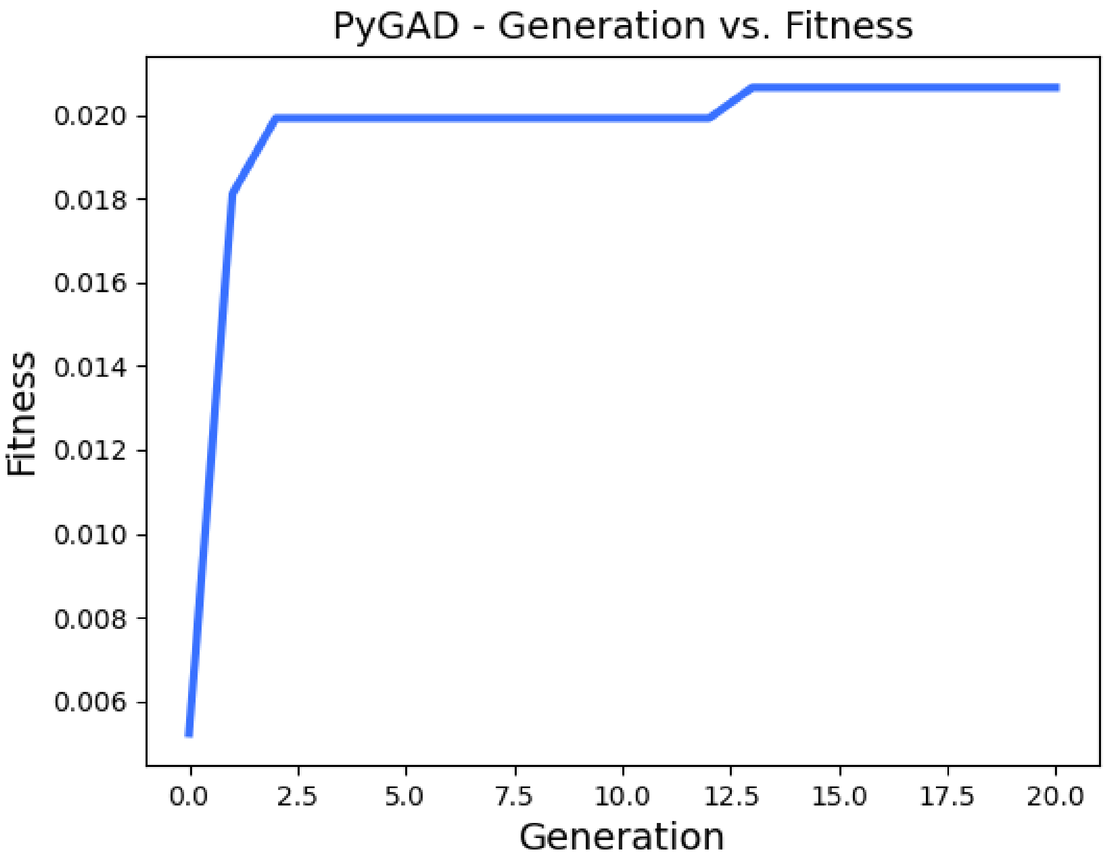

Figure 8: Genetic Algorithm for 10 Prey and 1 Predator on β. 20 Generations

The optimal value, that the algorithm found was $\beta = 6.92265909$.
Secondly i tried optimizing for β and μ, while still only looking at the low movement of the school.

3.4 Heatmaps length and attack angle

Polarization .

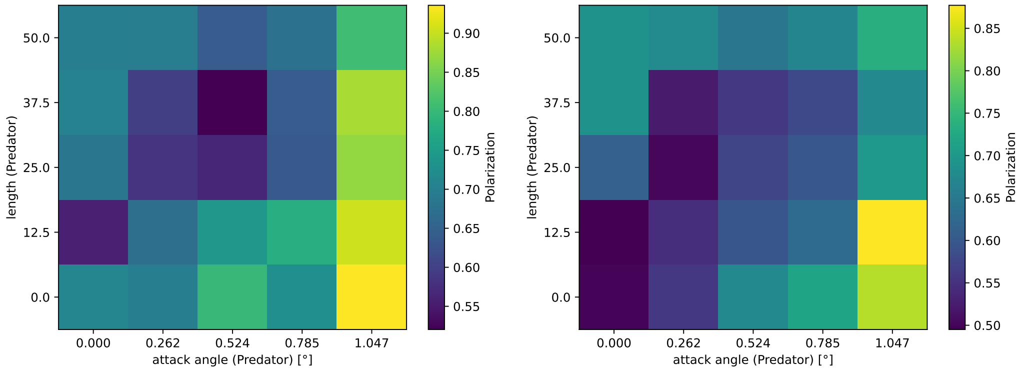

Figure 9: Polarizations of groups of 20 agents with 1 attacking predator. length of predator and attackangle is varied. Average over 50 Runs per value pair. Comparison of voronoi Interaction and Metric Interaction

Interindividual Distance .

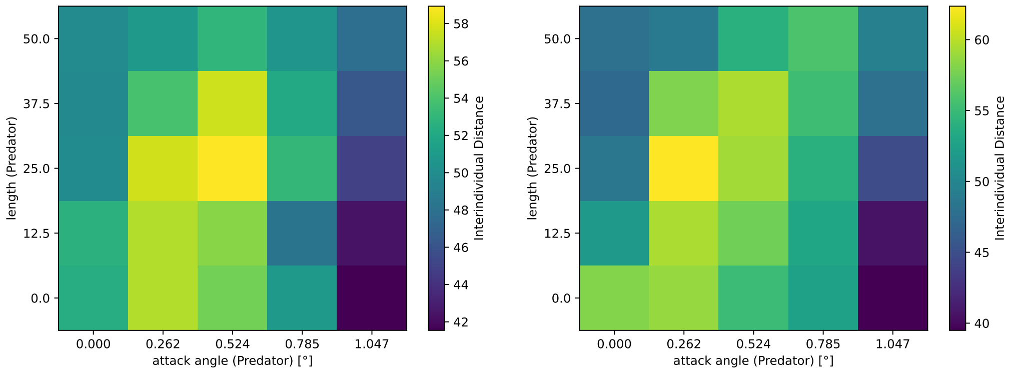

Figure 10: Interindividual Distances of groups of 20 agents with 1 attacking predator. Length of predator and attack-angle is varied. Average over 50 Runs per value pair. Comparison of Voronoi Interaction and Metric Interaction

Distance to Nearest Neighbors .

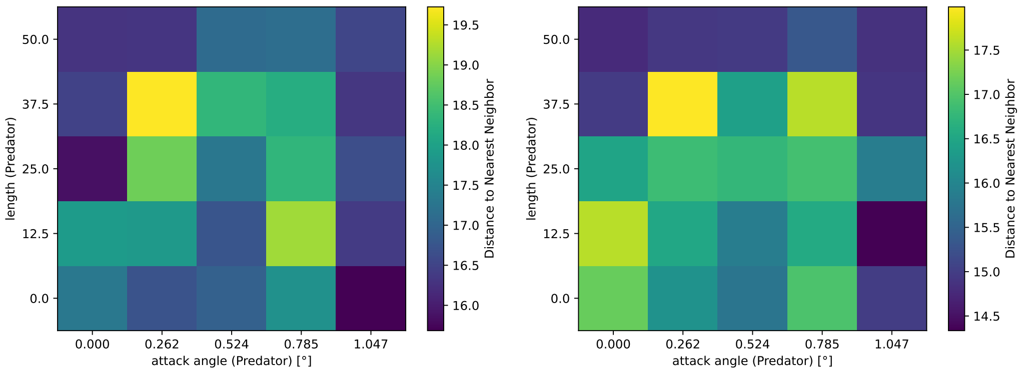

Figure 11: Average Distance to Nearest Neighbor of groups of 20 agents with 1 attacking predator. length of predator and attackangle is varied. Average over 50 Runs per value pair. Comparison of Voronoi Interaction and Metric Interaction

Distance to head of Predator .

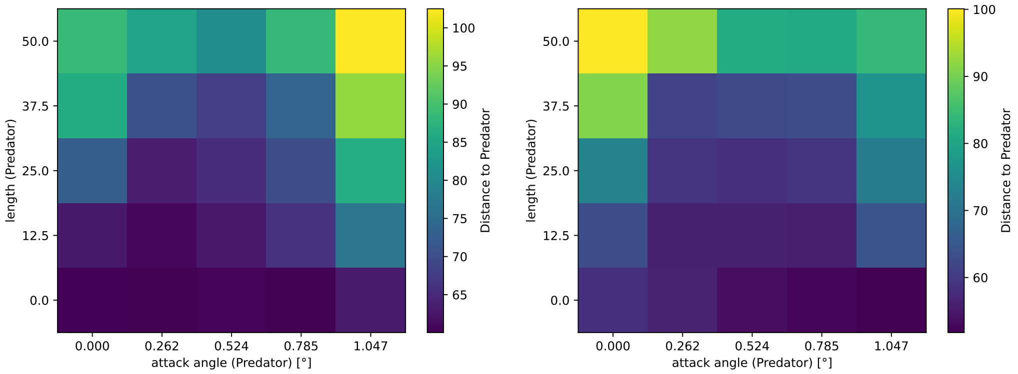

Figure 12: Average Distance to Predator of groups of 20 agents with 1 attacking predator. length of predator and attackangle is varied. Average over 50 Runs per value pair. Comparison of Voronoi Interaction and Metric Interaction