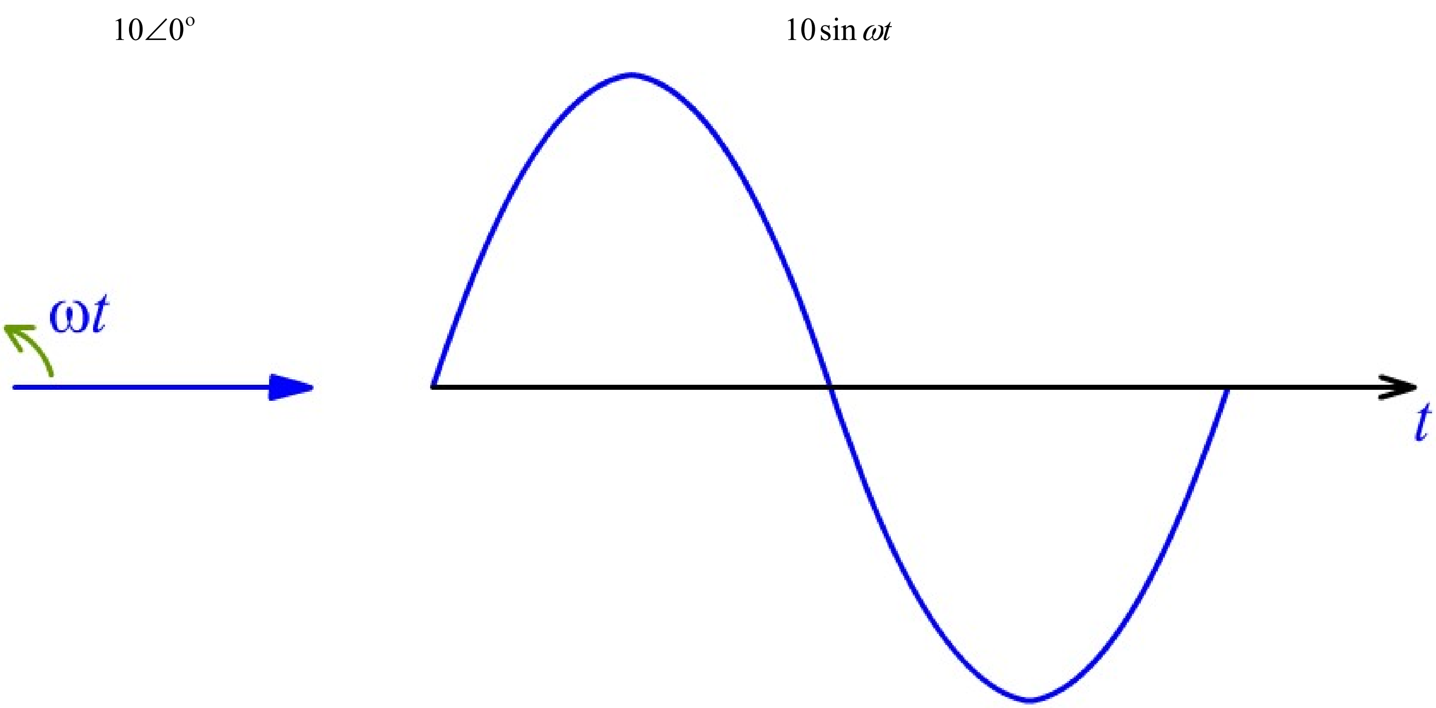
EE2902S

AC Circuits

(Phasor Technique)

Phasor



From *t*-domain to phasor-domain

- Using Euler's identities,

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

- Assume θ = ω*t* + φ, we can find the response of a circuit in both cos and sin at the same time.
- Another interesting point is that ω*t* is always there. We can assume *t* = 0 and solve the problem with φ only. After that just rewrite the solution with (ω*t* + φ).

From *t*-domain to phasor-domain

- Using Euler's identities,

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

- For a sine wave, $10\sin(\omega t)$

  we can use phasor, $10e^{j0^\circ} = 10\cos 0^\circ + j10\sin 0^\circ$

  After solving the problem, write down the answer with either cos or sin.

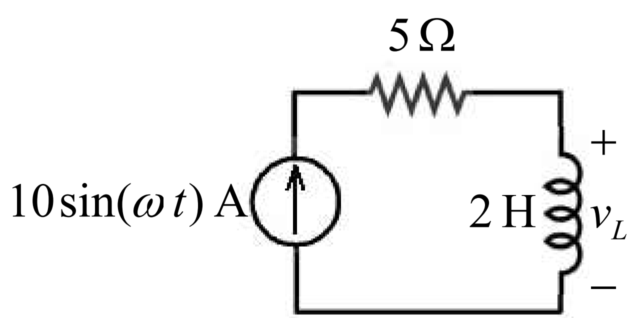
- Note: polar form expression: $A_1 e^{j\theta} = A_1 \angle\theta$ in rad for calculator

  from polar form to rectangular form:

$$A_1\angle\theta = A_1\cos\theta + jA_1\sin\theta$$

Time-domain and phasor technique

Time-domain



$$v_L(t) = L\frac{di}{dt}$$

$$= 2\frac{d\left(10\sin(\omega t)\right)}{dt}$$

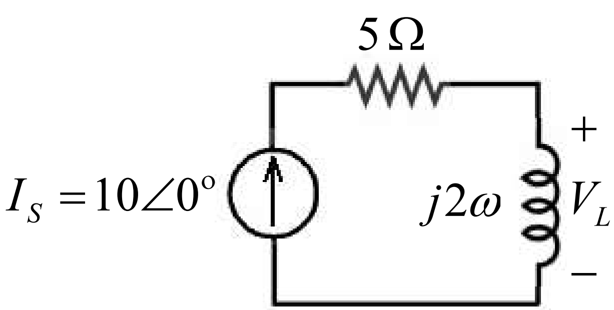
$$= 20\omega\cos(\omega t)$$

$$= 20\omega\sin(\omega t + 90^\circ)$$

Phasor technique

$$I_S = 10\angle 0^\circ \quad \text{ref : sin}$$

$$Z_L = j\omega L = j2\omega$$



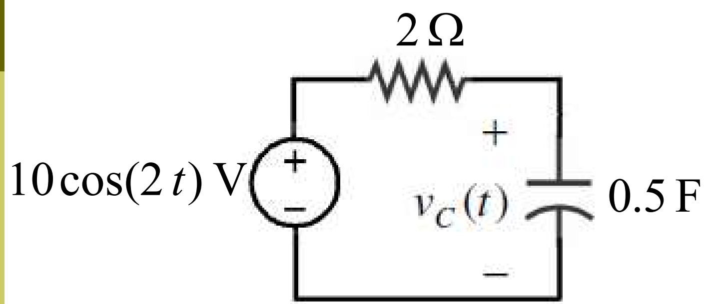
$$V_L = I_S Z_L$$

$$= j20\omega$$

$$v_L(t) = 20\omega\sin(\omega t + 90^\circ)$$

Time-domain and phasor technique

Time-domain



$$i(t) = C\frac{dv_C}{dt}$$

$$\frac{v_S - v_C}{2} = C\frac{dv_C}{dt}$$

$$\frac{10\cos(2t) - v_C}{2} = 0.5\frac{dv_C}{dt}$$

$$10\cos(2t) = \frac{dv_C}{dt} + v_C$$

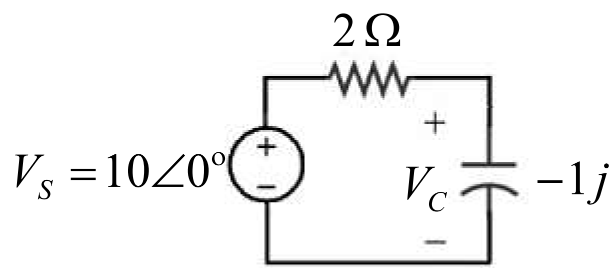
Solving the differential equation:

$$v_C(t) = 4.472\cos(2t - 63.43^\circ)\ \text{V}$$

Phasor technique

$$V_S = 10\angle 0^\circ \quad \text{ref : cos}$$

$$Z_C = \frac{1}{j\omega C} = -1j$$



$$V_C = \frac{Z_C}{R + Z_C}V_S$$

$$= \frac{-1j}{2-1j}10\angle 0^\circ$$

$$= 2 - 4j$$

$$= 4.472\angle -63.43^\circ$$

$$v_C(t) = 4.472\cos(2t - 63.43^\circ)\ \text{V}$$

CASIO fx-50FH

Before using Complex number calculation, set mode to polar form:

[SHIFT] [MODE] [▶] [▶] [▶] [2] $(r\angle\theta)$

$$\mathbf{V_C} = \frac{(-j)}{(2-j)}\left(10\angle 0^\circ\right)$$

Press [MODE].

[2]

←COMP	CMPLX	BASE →
1	2	3

[(] [(−)] [ENG] (i) [)] [×] [(] [1] [0] [SHIFT] [(−)] $(\angle)$ [0] [)] [÷] [(] [2] [−] [ENG] (i) [)] [EXE]

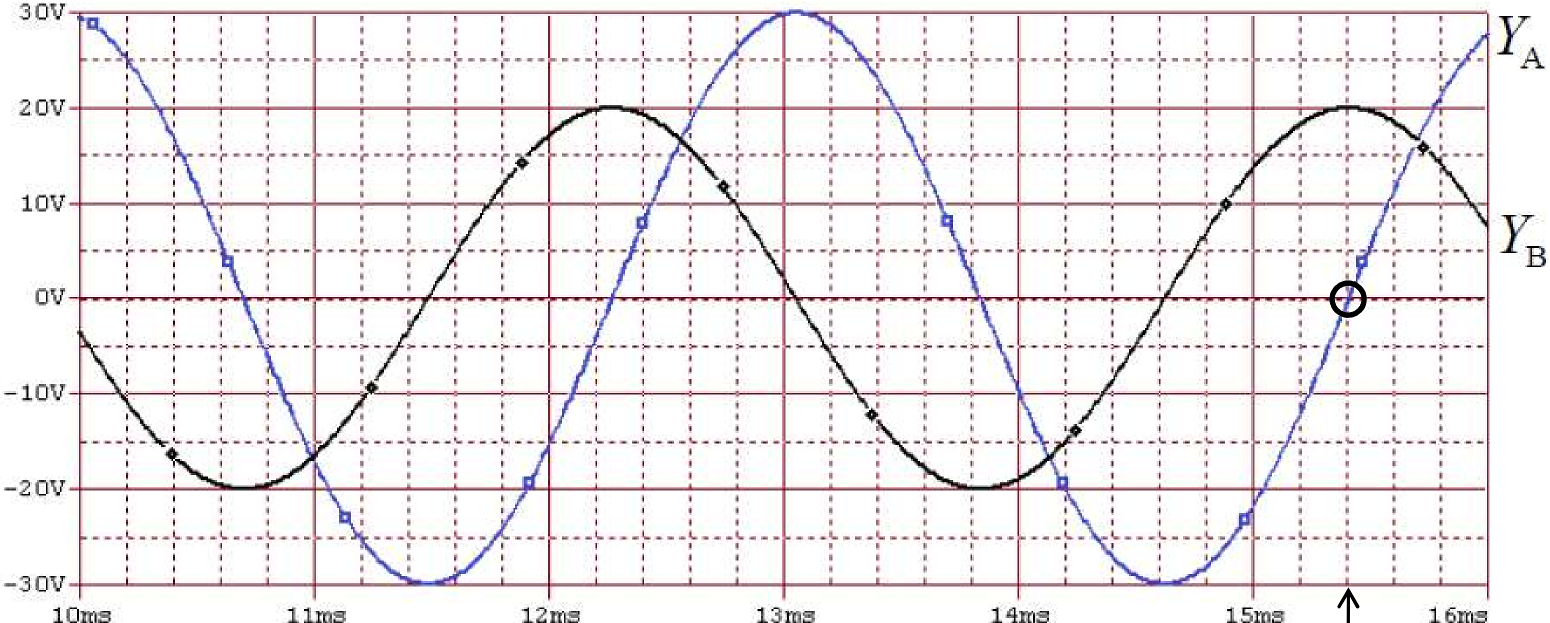
4.472135955

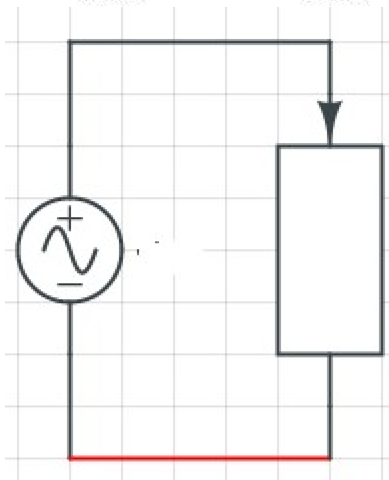
[SHIFT] [EXE] (Re⇔Im)

∠ -63.43494882

$$\mathbf{V_C} = \underline{\underline{4.472\angle(-63.43^\circ)\ \text{V}}} \qquad v_C(t) = 4.472\cos(2t - 63.43^\circ)\ \text{V}$$

If Y_A is voltage and Y_B is current with period = 3.14 ms, determine the impedance.





$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{3.14\times 10^{-3}}$$

$$v_s(t) = v_s(t+nT) = v_s(t-nT)$$

$$t = 15.4\,\text{m} - 4\times 3.14\,\text{m} = 2.84\,\text{ms}$$

$$v_s(t) = 30\sin(\omega t + \theta)\ \text{V}$$

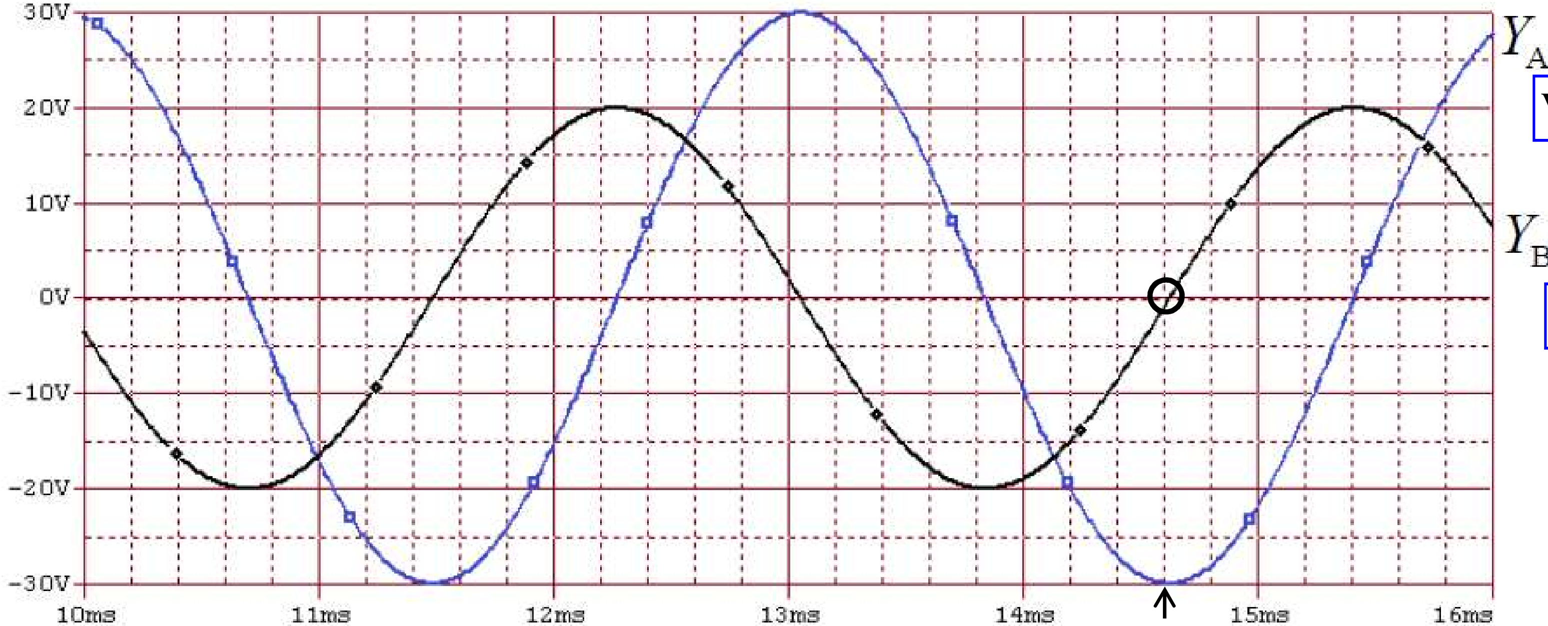
$$0 = 30\sin\left(\left(\frac{2\pi}{3.14\,\text{m}}\right)(2.84\,\text{m}) + \theta\right)$$

$$\left(\frac{2\pi}{3.14\,\text{m}}\right)(2.84\,\text{m}) + \theta = 0$$

$$\theta = -\frac{2\pi\times 2.84}{3.14} = -325.6^\circ = 34.4^\circ$$

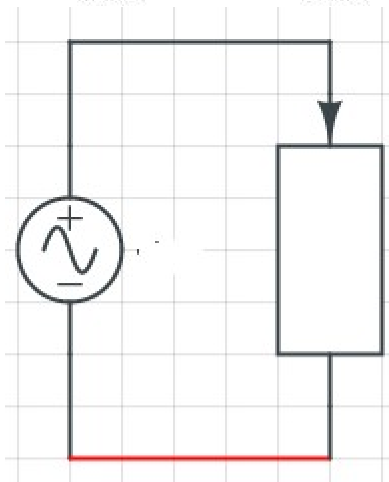
$$\boxed{\mathbf{V}_s = 30\angle 34.4^\circ}$$

If Y_A is voltage and Y_B is current with period = 3.14 ms, determine the impedance.



$\boxed{\mathbf{V}_s = 30\angle 34.4^{\circ}}$

$\boxed{\mathbf{I}_s = 20\angle 126.1^{\circ}}$



$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{3.14\times 10^{-3}}$$

$$i_s(t) = i_s(t+nT) = i_s(t-nT)$$

$$t = 14.6\,\text{m} - 4\times 3.14\,\text{m} = 2.04\,\text{ms}$$

$$i_s(t) = 20\sin(\omega t+\phi)\ \text{A}$$

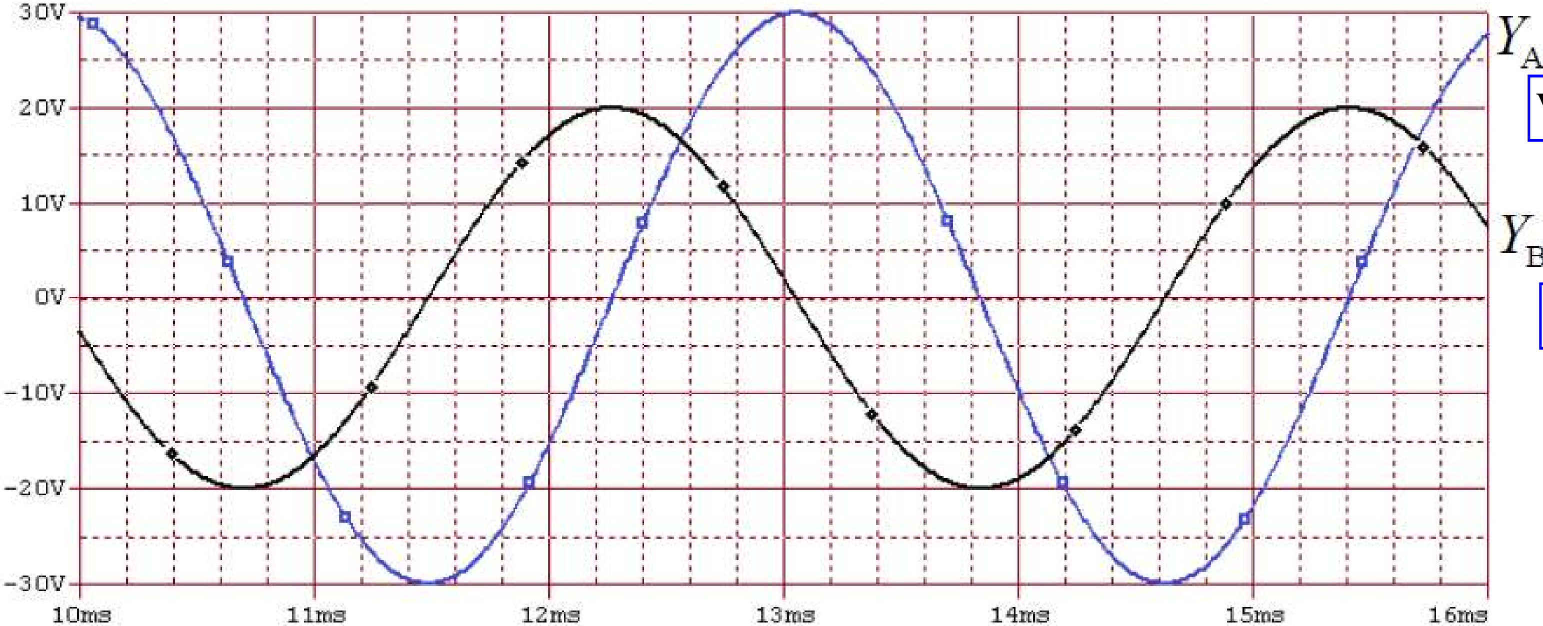
$$0 = 20\sin\left(\left(\frac{2\pi}{3.14\,\text{m}}\right)(2.04\,\text{m}) + \phi\right)$$

$$\left(\frac{2\pi}{3.14\,\text{m}}\right)(2.04\,\text{m}) + \phi = 0$$

$$\phi = -\frac{2\pi\times 2.04}{3.14} = -233.9^{\circ} = 126.1^{\circ}$$

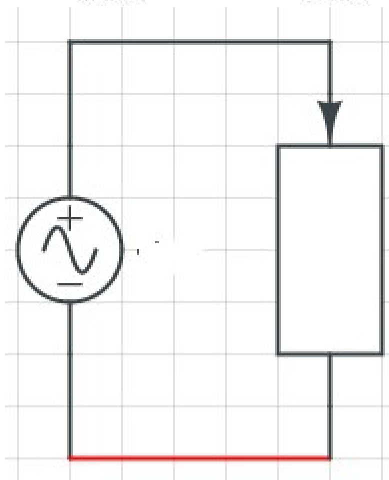
$$\boxed{\mathbf{I}_s = 20\angle 126.1^{\circ}}$$

If Y_A is voltage and Y_B is current with period = 3.14 ms, determine the impedance.



$\boxed{\mathbf{V}_s = 30\angle 34.4^\circ}$

$\boxed{\mathbf{I}_s = 20\angle 126.1^\circ}$



$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{3.14\times 10^{-3}} = 2001.01\,\text{(rad/s)}$$

$$\boxed{Z = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{30\angle 34.4^\circ}{20\angle 126.1^\circ} = 1.5\angle -91.7^\circ = -0.044 - j1.499 = R + jX}$$

Since –ve R is not possible, this R = -0.044 Ω should be round off error.

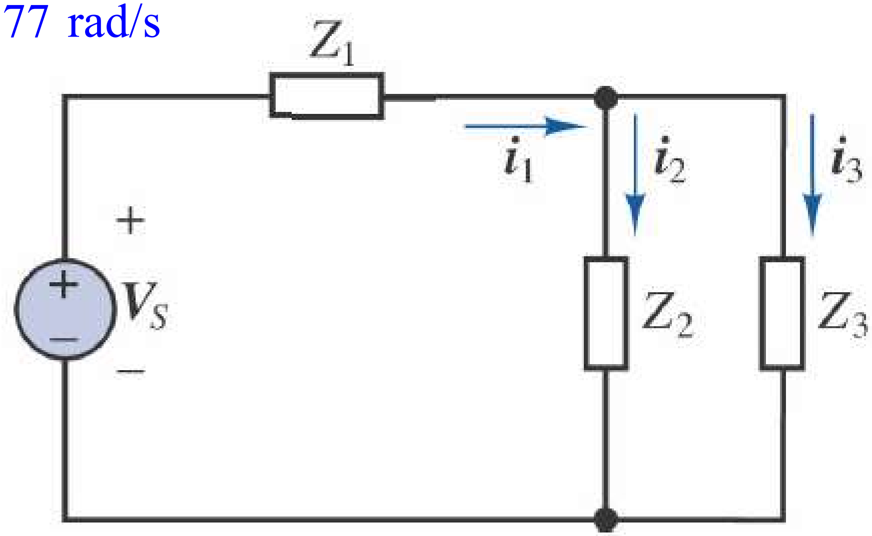
$$\boxed{-j1.499 = \frac{-j}{\omega C} \quad \Rightarrow \quad C = \frac{1}{1.499\omega} = 0.33\,\text{mF}}$$

Q.9 AC Steady State (Phasor)

- Determine $i_3(t)$ if
$$\begin{aligned} i_1(t) &= 141.4cos\,(377t + 2.356)\,\mathrm{mA} \\ i_2(t) &= 50sin\,(377t - 0.927)\,\mathrm{mA} \end{aligned}$$

Using cosine as reference with ω = 377 rad/s

Express $i_1(t)$ and $i_2(t)$ in phasor form,



$$\mathbf{I}_1 = 141.4\angle 2.356 = 141.4\angle 134.99°$$

$$\begin{aligned} \mathbf{I}_2 &= 50\angle(-0.927 - \pi/2) \\ &= 50\angle\,(-2.4978) \\ &= 50\angle\,(-143.11°) \end{aligned}$$

$$\begin{aligned} \mathbf{I}_3 &= \mathbf{I}_1 - \mathbf{I}_2 \\ &= 141.4\angle 134.99° - 50\angle\,(-143.11°) \\ &= 143.1839\angle 114.76° \end{aligned}$$

Convert to time domain, $i_3(t) = 143.1839cos\,(377t + 114.76°)$ mA

Q.10 AC Steady State

$$\frac{A\angle\theta}{B\angle\phi} = (A/B)\angle(\theta - \phi)$$

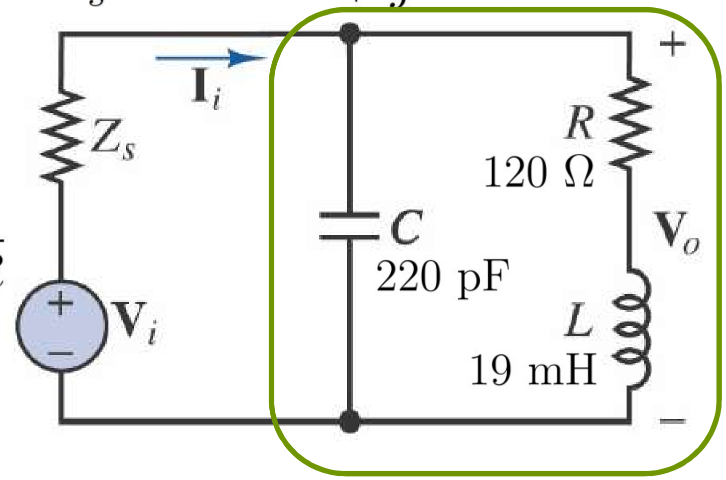
- Determine the frequency so that the current $\mathbf{I}_i$ and the voltage $\mathbf{V}_o$ are in phase.

Only pure resistance with current and voltage in phase,

$$Z_{eq} = \frac{1}{j\omega C} \Big\| (R + j\omega L) = \frac{\left(\frac{1}{j\omega C}\right)(R + j\omega L)}{\frac{1}{j\omega C} + R + j\omega L}$$

$$= \frac{R + j\omega L}{1 + j\omega CR - \omega^2 CL} = \frac{R + j\omega L}{(1 - \omega^2 CL) + j\omega CR}$$

$Z_s = 13000 + j\omega 3\ \Omega$



Make this part becomes pure resistance.

The phase angle of $Z_{eq} = 0$

$$\frac{\omega L}{R} = \frac{\omega CR}{1 - \omega^2 CL}$$

$$1 - \omega^2 CL = \frac{CR^2}{L}$$

$$\omega = \sqrt{\frac{1}{CL} - \frac{R^2}{L^2}}$$

$$= \sqrt{\frac{1}{(220 \times 10^{-12})(0.019)} - \frac{120^2}{0.019^2}} = \underline{\underline{489.075\ (\mathrm{krad/s})}}$$

Q.13 AC Steady State

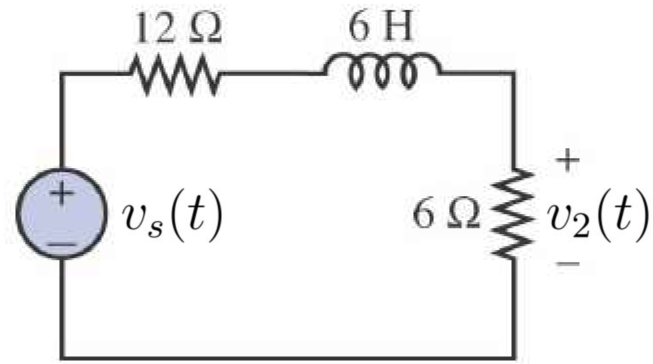
- Solve for $v_2(t)$ and show the answer in time domain. Given that $v_s(t) = 25sin(2t)$ V

From the given voltage source, use $sin\,(\omega t)$ as reference

$\omega = 2$ rad/s

In phasor domain, $\mathbf{V_s} = 25\angle 0^\circ$

Apply voltage divider equation,



$$\mathbf{V_2} = \frac{6}{12 + j\omega 6 + 6} \times \mathbf{V_s}$$

$$= \left(\frac{6}{18 + j12}\right)(25\angle 0^\circ)$$

$$= 6.9337\angle(-33.69^\circ)$$

Convert to time domain, $v_2(t) = \underline{\underline{6.9337sin\,(2t - 33.69^\circ)\ \ (\mathrm{V})}}$

CASIO fx-50FH

Before using Complex number calculation set mode to polar form:

[SHIFT] [MODE] ▶ ▶ ▶ [2] $(r\angle\theta)$

$$\mathbf{V}_2 = \left(\frac{6}{18 + j12}\right)(25\angle 0)$$

Press [MODE].
[2]

←COMP	CMPLX	BASE →
1	2	3

[6] [×] [(] [2] [5] [SHIFT] [(–)] (∠) [0] [)] [÷] [(] [1] [8] [+] [1] [2] [ENG] (i) [)] [EXE]

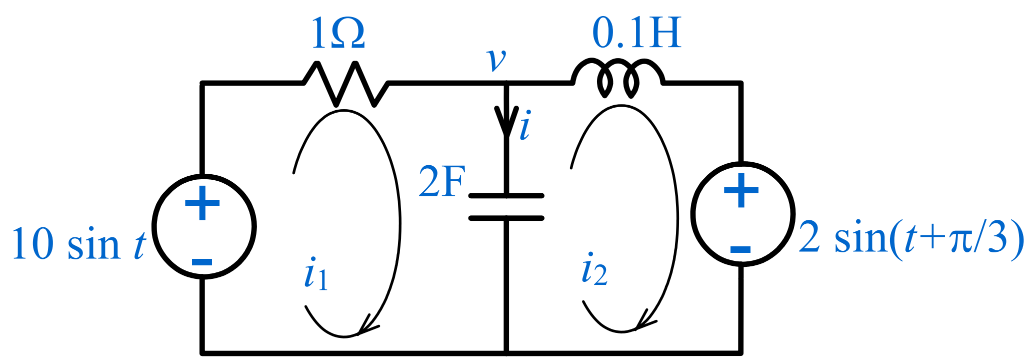
6.933752453

[SHIFT] [EXE] (Re⇔Im) ∠ -33.69006753

$$\mathbf{V}_2 = \underline{\underline{6.9337\angle(-33.69^\circ)\ \mathrm{V}}}$$

AC Steady State

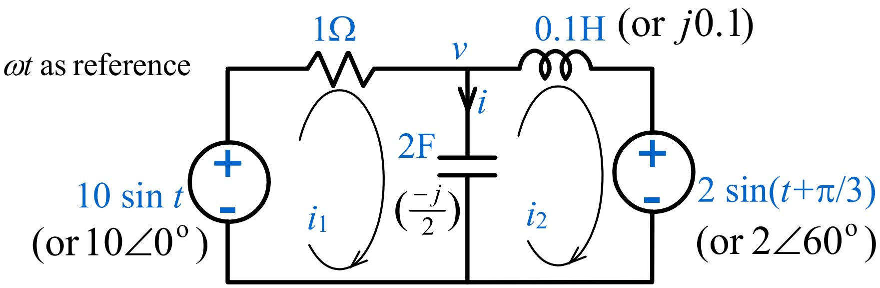
- Use the mesh-current method to solve the circuit. Convert all voltages and currents to the form $X \sin(t+\phi)$. You need only find the steady-state solutions.



AC Steady State (Phasor)

Apply KVL for loop i_1,

consider sin ωt as reference



$$-10\angle 0^{\circ} + 1\times \mathbf{I}_1 + \frac{-j}{2}(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(1 - j0.5)\mathbf{I}_1 + j0.5\mathbf{I}_2 = 10$$

Apply KVL for loop i_2,

$$2\angle 60^{\circ} + \frac{-j}{2}(\mathbf{I}_2 - \mathbf{I}_1) + j0.1\times \mathbf{I}_2 = 0$$

$$-j0.5\mathbf{I}_1 + j0.4\mathbf{I}_2 = 1 + j1.732051$$

$$\begin{cases} (1 - j0.5)\mathbf{I}_1 + j0.5\mathbf{I}_2 + (0)\mathbf{I}_3 = 10 \\ -j0.5\mathbf{I}_1 + j0.4\mathbf{I}_2 + (0)\mathbf{I}_3 = 1 + j1.732051 \\ \mathbf{I}_3 = 1 \end{cases}$$

Using *fx-50FH* calculator enter the following:

Press Prog1 and then press:

1–0.5 i EXE 0.5 i EXE 0 EXE 10 EXE

–0.5 i EXE 0.4 i EXE 0 EXE 1 + 1.732051 i EXE

0 EXE 0 EXE 1 EXE 1 EXE (*disp* $\mathbf{I}_1$) EXE (*disp* $\mathbf{I}_2$) EXE (*disp* 1).

RCL A (*disp* $\mathbf{I}_1$) SHIFT r∠θ EXE (*disp* $\mathbf{I}_1$ in polar form).

RCL B (*disp* $\mathbf{I}_2$) SHIFT r∠θ EXE (*disp* $\mathbf{I}_2$ in polar form).

$$\Delta = \begin{vmatrix} 1 - j0.5 & j0.5 \\ -j0.5 & j0.4 \end{vmatrix} = (1 - j0.5)\times j0.4 + j0.5\times j0.5$$
$$= -0.05 + j0.4$$

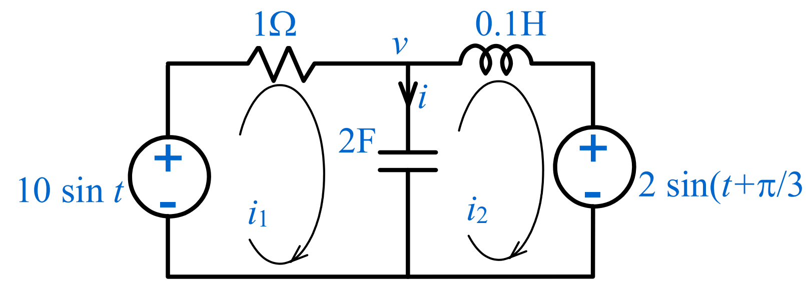
$$\Delta_1 = \begin{vmatrix} 10 & j0.5 \\ 1 + j1.732051 & j0.4 \end{vmatrix} = 10\times j0.4 - (1 + j1.732051)\times j0.5$$
$$= 0.8660255 + j3.5$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{0.8660255 + j3.5}{-0.05 + j0.4} = 8.944271\angle(-21.0229^{\circ})$$

Convert to time-domain notation,

$$i_1(t) = 8.944271\sin(t - 21.0229^{\circ})\,\text{A}$$

Q.4 AC Steady State (Phasor)



$$\Delta = \begin{vmatrix} 1-j0.5 & j0.5 \\ -j0.5 & j0.4 \end{vmatrix} = -0.05 + j0.4$$

$$\Delta_2 = \begin{vmatrix} 1-j0.5 & 10 \\ -j0.5 & 1+j1.732051 \end{vmatrix}$$

$$= (1-j0.5)\times(1+j1.732051) + (j0.5)\times 10$$

$$= 1.866026 + j6.232051$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{1.866026 + j6.232051}{-0.05 + j0.4}$$

$$= 16.137961\angle(-23.794^\circ)$$

$$i_2(t) = 16.137961\sin\left(t - 23.794^\circ\right)\text{A}$$

$$\mathbf{I} = (\mathbf{I}_1 - \mathbf{I}_2)$$

$$= 8.944271\angle -21.0229^\circ - 16.137961\angle -23.794^\circ$$

$$= 7.217114\angle 152.771^\circ$$

$$i(t) = 7.217114\sin\left(t + 152.771^\circ\right)\text{A}$$

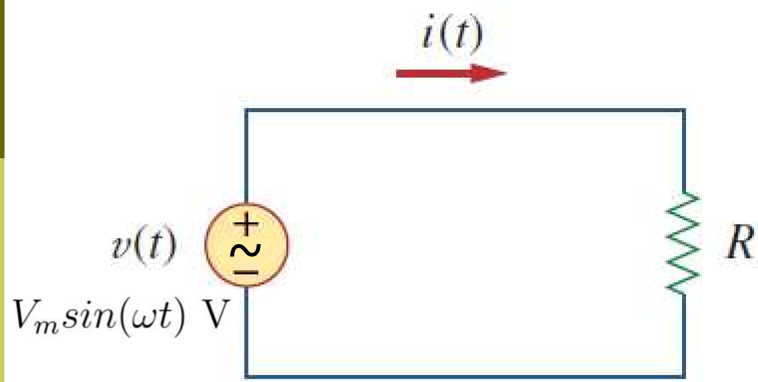
$$\mathbf{V} = (\mathbf{I})\left(\frac{-j}{2}\right)$$

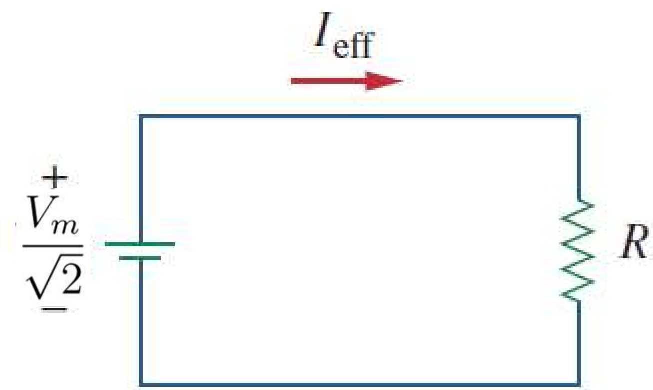
$$= \left(7.217114\angle 152.771^\circ\right)\times\left(0.5\angle -90^\circ\right)$$

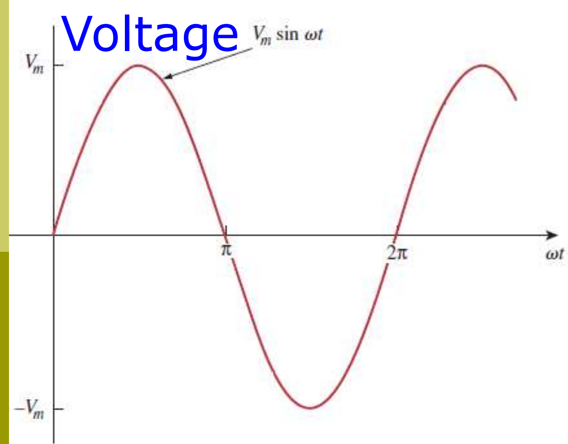
$$= 3.608557\angle 62.771^\circ$$

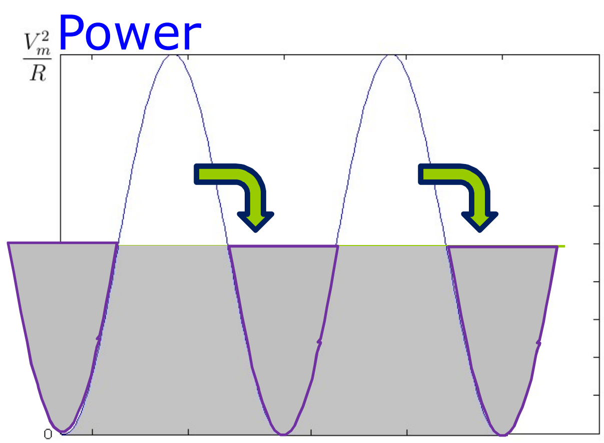
$$v(t) = 3.608557\sin\left(t + 62.771^\circ\right)\text{V}$$

root-mean-square value

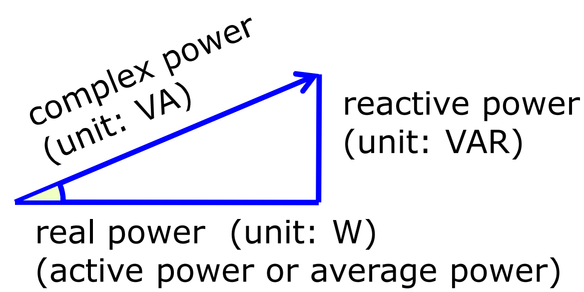








$$V_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T (V_m sin(\omega t))^2 \, dt} = \frac{V_m}{\sqrt{2}}$$



$$\text{At time } t, \text{ power} = \frac{v(t)^2}{R} = \frac{V_m^2 sin^2(\omega t)}{R}$$

$$\text{Average power} = \frac{V_m^2}{2R} = \frac{(V_m/\sqrt{2})^2}{R}$$

$$\mathbf{S} = \mathrm{P} + j\mathrm{Q}$$

$$|\mathbf{S}| = \text{apparent power (unit: VA)}$$

$$\cos\theta = \text{power factor}$$

AC Power #1

SHIFT MODE ▶ ▶ ▶ 2 $(r\angle\theta)$

Press MODE 2

(1 0 0 SHIFT (–) $(\angle)$ 2 5) SHIFT = (▶$a+bi$) EXE

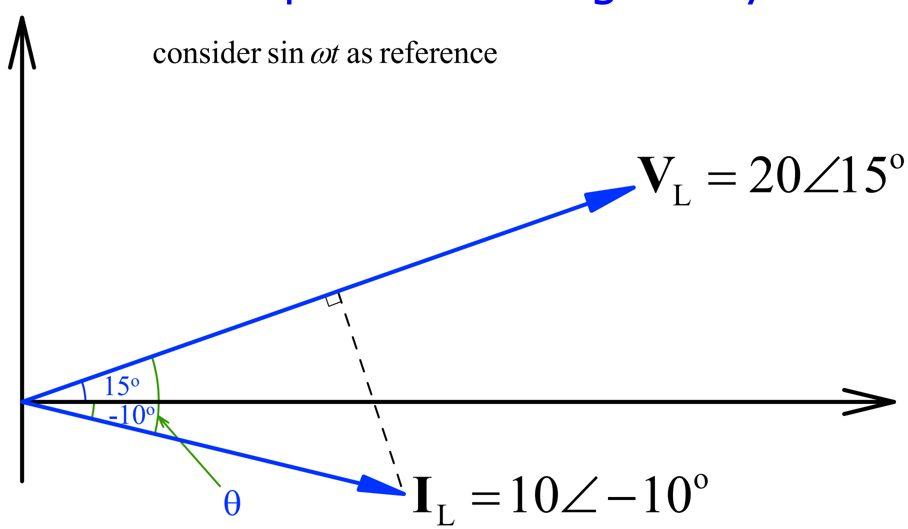
SHIFT EXE (Re⇔Im)

- The current flowing in an impedance, Z_L, is $10\sin(\omega t - 10°)$ A, and the voltage across it is $20\sin(\omega t + 15°)$ V. Calculate the complex power, real power, and reactive power in the impedance. What is the power factor of Z_L? Find Z_L.

Given $i = 10\sin(\omega t - 10^{\circ})$ A

$v = 20\sin(\omega t + 15^{\circ})$ V

Sketch the phasors: I lags V by 25°



Complex power

$$\mathbf{S}_L = \tilde{\mathbf{V}}_L \tilde{\mathbf{I}}_L^*$$

$$= \left(\frac{20\angle 15^{\circ}}{\sqrt{2}}\right) \times \left(\frac{10\angle 10^{\circ}}{\sqrt{2}}\right)$$

$$= 100\angle 25^{\circ} \text{ VA}$$

$$= 90.63 + j42.26 \text{ VA}$$

Real power (or average power)

$$P = \text{Re}\{\mathbf{S}\} = 90.63 \text{ W}$$

Reactive power

$$Q = \text{Im}\{\mathbf{S}\} = 42.26 \text{ VAR}$$

Power factor of Z_L

$$= cos\,(25°) = 0.9063 \text{ (lagging)}$$

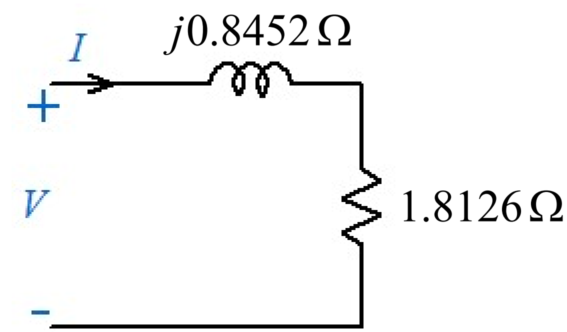
AC Power #1

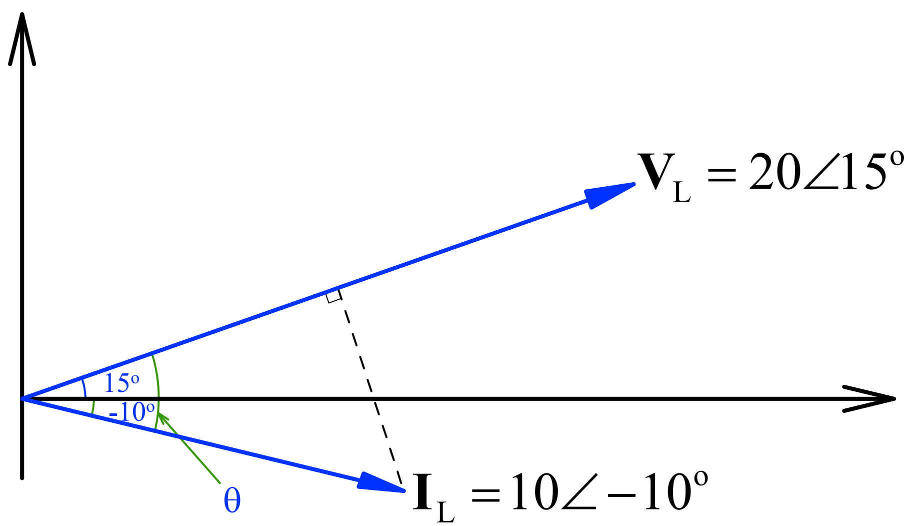
To find Z_L, I lags V $\Rightarrow$ inductive load.

$$Z_L = \frac{\mathbf{V}_L}{\mathbf{I}_L}$$

$$= \frac{20\angle 15^o}{10\angle -10^o}$$

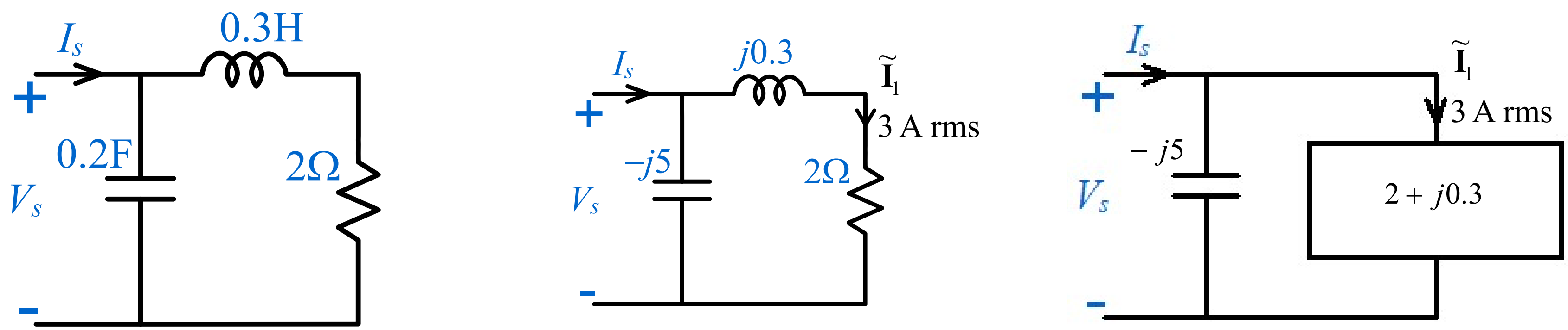
$$= 1.8126 + j0.8452\ \Omega$$





AC Power #2

- The current through the resistor is 3 A rms. What are the input current and voltage, assume the driving angular frequency is 1 rad/s? Calculate the input active, reactive, and apparent powers, and hence the input power factor.



AC Power #2

Apply current divider equation,

$$3 = \left(\frac{-j5}{-j5+2+j0.3}\right)\left(\tilde{\mathbf{I}}_S\right)$$

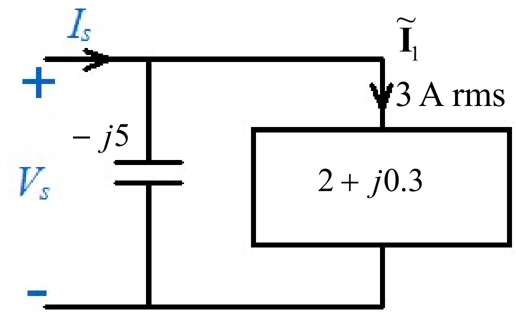
$$= \left(\frac{-j5}{2-j4.7}\right)\left(\tilde{\mathbf{I}}_S\right)$$

$$\tilde{\mathbf{I}}_S = \frac{3(2-j4.7)}{-j5}$$

$$= \frac{6-j14.1}{-j5}$$

$$= \frac{15.3235\angle -66.95^{\circ}}{5\angle -90^{\circ}}$$

$$= 3.0647\angle 23.05^{\circ}$$



By Ohm's law,

$$\tilde{\mathbf{V}}_S = \left(\tilde{\mathbf{I}}_1\right)(2+j0.3)$$

$$= 3(2+j0.3)$$

$$= 6+j0.9$$

$$= 6.0671\angle 8.53^{\circ}$$

Complex power

$$\mathbf{S} = \tilde{\mathbf{V}}_s\tilde{\mathbf{I}}_s^*$$

$$= \left(6.0671\angle 8.53^{\circ}\right)\left(3.0647\angle(-23.05^{\circ})\right)$$

$$= \left(6.0671\times 3.0647\right)\angle(8.53^{\circ}-23.05^{\circ})$$

$$= 18.5939\angle(-14.52^{\circ})\ \text{VA}$$

AC Power #2

Complex power,

$$\mathbf{S} = 18.5939\angle(-14.52^\circ)$$
$$= 18.5939\cos(-14.52^\circ) + j18.5939\sin(-14.52^\circ)$$
$$= 18 - j4.662$$

Real power,

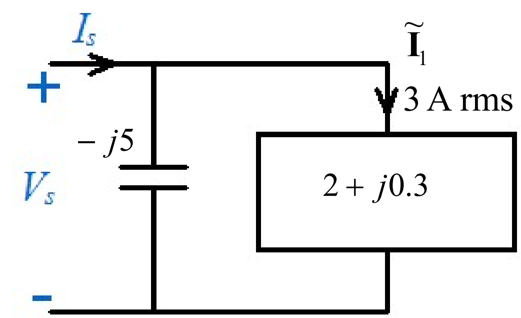
$$P = 18\ \text{W}$$

Reactive power,

$$Q = -4.662\ \text{VAR}$$

Apparent power,

$$|\mathbf{S}| = 18.5939\ (\text{VA})$$



The power factor is

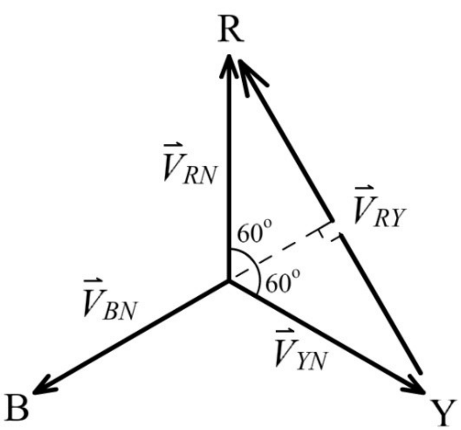
$$\text{p.f.} = \cos(-14.52^\circ)$$
$$= 0.9681\ (\text{leading})$$

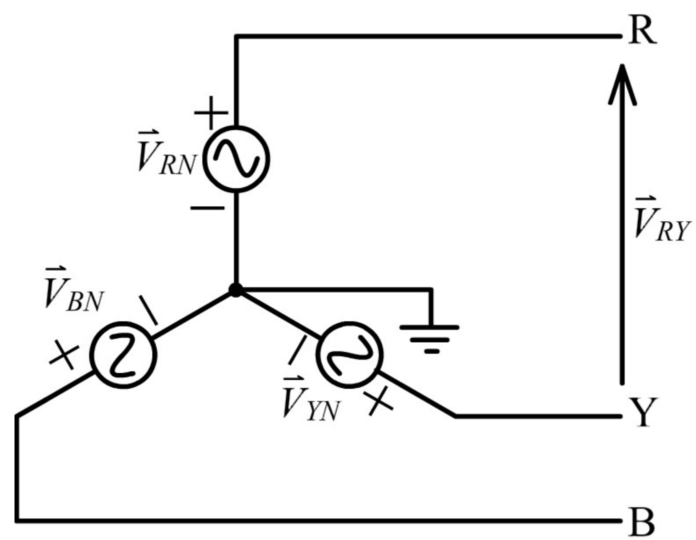
Leading means “I leads V.”

Phase and Line Voltages

- Find the phase voltage V_P in terms of line voltage V_L. $\vec{V}_{RN} = V_P\angle 0°, \vec{V}_{BN} = V_P\angle 120°, and \vec{V}_{YN} = V_P\angle -120°$
- If V_L = 380 V, what is V_P?

Draw the voltage diagram as follows:





$$V_L = |\vec{V}_{RY}| = |\vec{V}_{RN}| \sin 60° + |\vec{V}_{YN}| \sin 60°$$

$$= V_P \sin 60° + V_P \sin 60°$$

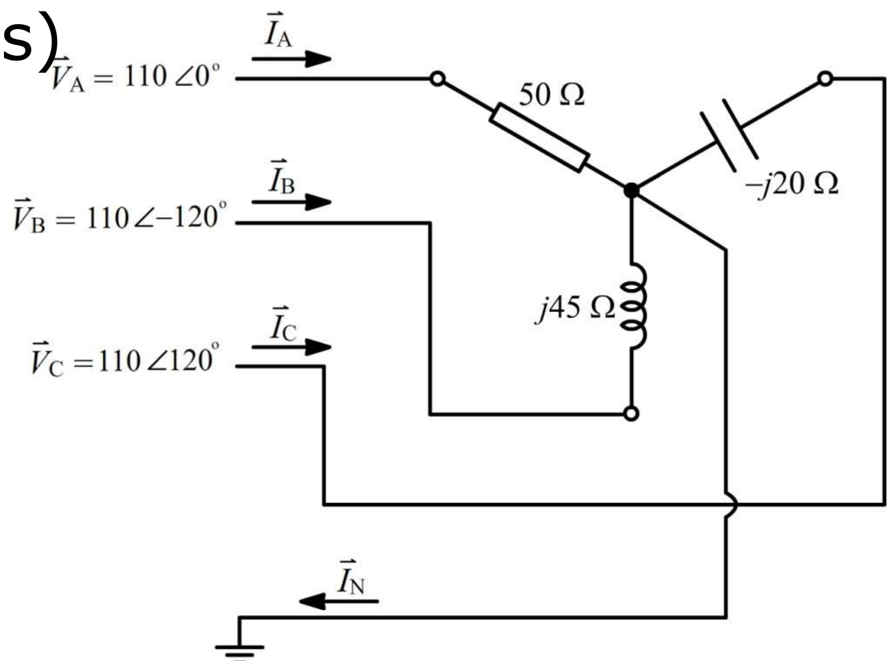
$$= 2V_P \sin 60° = 2V_P \left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_P$$

$$V_P = \frac{V_L}{\sqrt{3}}$$

$$V_P = \frac{380}{\sqrt{3}} = \underline{\underline{219.4\text{ V}}}$$

Three-phase Circuit #1

- For the 3-phase circuit shown in Figure below, find the current in the neutral wire and the real power. (all values in rms)



Solution:

$$\vec{I}_A = \frac{\vec{V}_A}{50} = \frac{110\angle 0^\circ}{50} = 2.2\angle 0^\circ \text{ (A)}$$

$$\vec{I}_B = \frac{\vec{V}_B}{j45} = \frac{110\angle -120^\circ}{45\angle 90^\circ} = 2.44\angle -210^\circ \text{ (A)}$$

$$\text{or } 2.44\angle 150^\circ \text{ (A)}$$

$$\vec{I}_C = \frac{\vec{V}_C}{-j20} = \frac{110\angle 120^\circ}{20\angle -90^\circ} = 5.5\angle 210^\circ \text{ (A)}$$

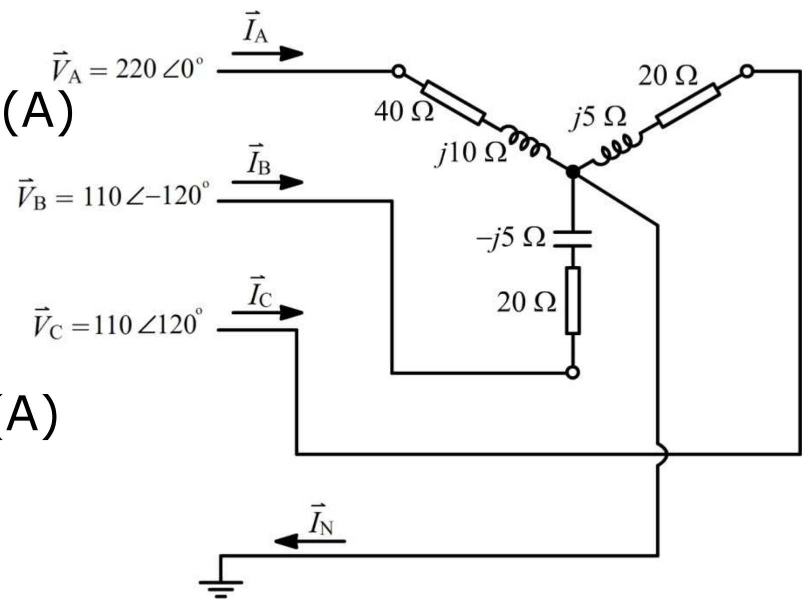
$$\vec{I}_N = \vec{I}_A + \vec{I}_B + \vec{I}_C$$
$$= 2.2\angle 0^\circ + 2.44\angle 150^\circ + 5.5\angle 210^\circ = 4.92\angle -161.9^\circ \text{ (A)}$$

$$Real\ Power = |\vec{V}_A||\vec{I}_A| = 110 \times 2.2 = 242 \text{ (W)}$$

Three-phase Circuit #2

- Find the current in the neutral wire and the real power. (all values are in rms)

Solution:



$$\vec{I}_A = \frac{\vec{V}_A}{40+j10} = \frac{22\ \ \angle 0^\circ}{41.23\angle 14.04^\circ} = 5.3358\angle -14.04^\circ \text{ (A)}$$

$$\vec{I}_B = \frac{\vec{V}_B}{20-} = \frac{110\angle -120^\circ}{20.6155\angle -1\ \ .04^\circ}$$

$$= 5.338\angle -105.96^\circ \text{ (A)}$$

$$\vec{I}_C = \frac{\vec{V}_C}{20+j} = \frac{110\angle 120^\circ}{20.6155\angle 14.04^\circ} = 5.3358\angle 105.96^\circ \text{ (A)}$$

$$\vec{I}_N = \vec{I}_A + \vec{I}_B + \vec{I}_C$$

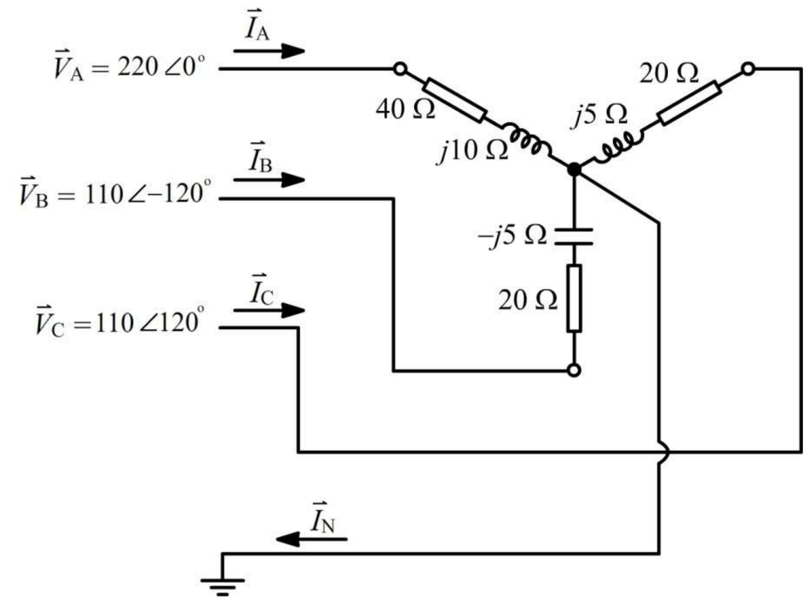
$$= 5.3358\angle -14.04^\circ + 5.3358\angle -105.96^\circ + 5.3358\angle 105.96^\circ$$

$$= 2.59\angle -30^\circ \text{ (A)}$$

$$P_C = \left|\overset{\varpi}{I}_C\right|^2 R_C = (5.3358)^2(20) = 569.41\ \text{W}$$

Three-phase Circuit #2

Find the real power.



$\vec{I}_A = 5.3358\angle -14.04^\circ$ (A)

$\vec{I}_B = 5.3358\angle -105.96^\circ$ (A)

$\vec{I}_C = 5.3358\angle 105.96^\circ$ (A)

$$P_A = \left|\vec{I}_A\right|^2 R_A = (5.3358)^2(40) = 1138.8\ \text{(W)}$$

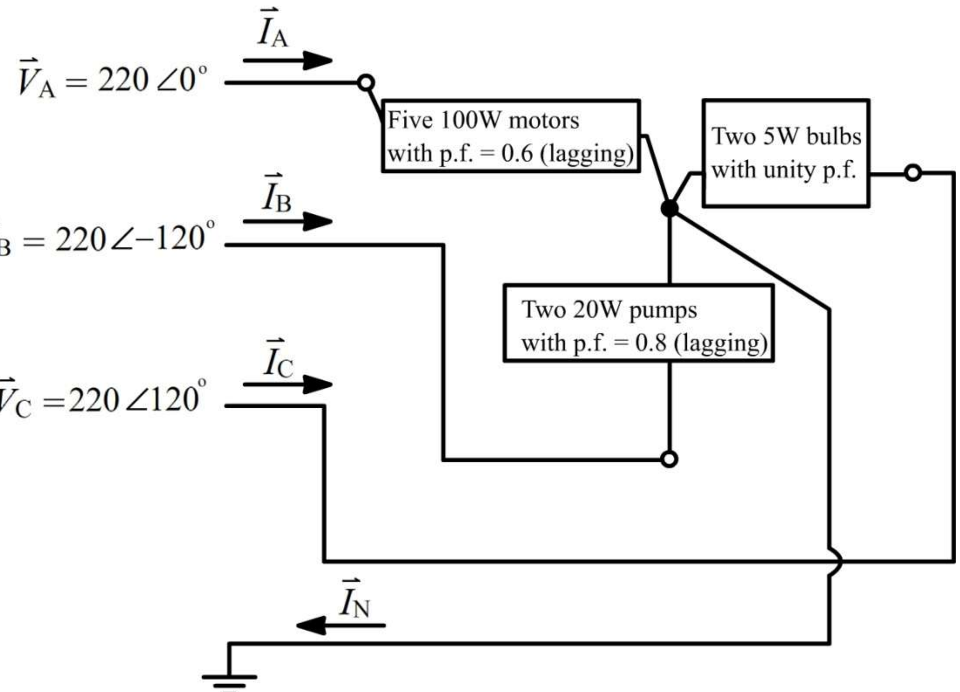
$$P_B = \left|\vec{I}_B\right|^2 R_B = (5.3358)^2(20) = 569.41\ \text{(W)}$$

$$P_C = \left|\vec{I}_C\right|^2 R_C = (5.3358)^2(20) = 569.41\ \text{(W)}$$

$$\begin{aligned}\text{Total real power} &= P_A + P_B + P_C \\ &= 1138.8 + 569.41 + 569.41 \\ &= \underline{\underline{2277.62\ \text{W}}}\end{aligned}$$

Three-phase Power

- Find the current in the neutral wire. (all values are in rms)



$$|S_A|\cos\theta_A = P_A = 5\times 100$$

$$|S_A| = \frac{500}{p.f.} = \frac{500}{0.6} = 833.33$$

$$S_A = 833.33\angle(\cos^{-1}0.6) = \vec{V}_A\vec{I}_A^*$$

$$\vec{I}_A^* = \frac{833.33\angle 53.13^\circ}{220\angle 0^\circ} = 3.7879\angle 53.13^\circ$$

$$\vec{I}_A = 3.7879\angle -53.13^\circ$$

$$|S_B| = \frac{2\times 20}{p.f.} = \frac{40}{0.8} = 50$$

$$S_B = 50\angle(\cos^{-1}0.8) = \vec{V}_B\vec{I}_B^*$$

$$\vec{I}_B^* = \frac{50\angle 36.87^\circ}{220\angle -120^\circ} = 0.2273\angle 156.87^\circ$$

$$\vec{I}_B = 0.2273\angle -156.87^\circ$$

$$|S_C| = \frac{2\times 5}{p.f.} = \frac{10}{1} = 10 \Rightarrow S_C = 10\angle 0^\circ = \vec{V}_C\vec{I}_C^*$$

$$\vec{I}_C^* = \frac{10\angle 0^\circ}{220\angle 120^\circ} = 0.04545\angle -120^\circ$$

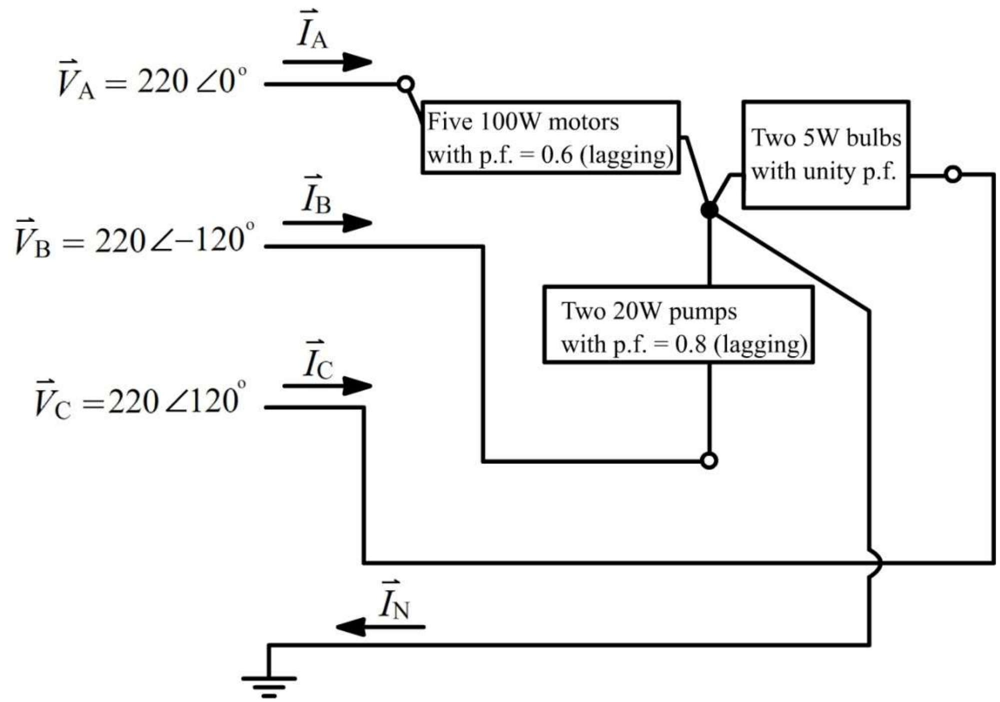
$$\vec{I}_C = 0.04545\angle 120^\circ$$

Three-phase Power

$\vec{I}_A = 3.7879\angle -53.13°$

$\vec{I}_B = 0.2273\angle -156.87°$

$\vec{I}_C = 0.04545\angle 120°$



$$\vec{I}_N = \vec{I}_A + \vec{I}_B + \vec{I}_C$$

$$= 3.7879\angle -53.13° + 0.2273\angle -156.78° + 0.04545\angle 120°$$

$$= 3.695\angle -56.47° \text{ (A)}$$

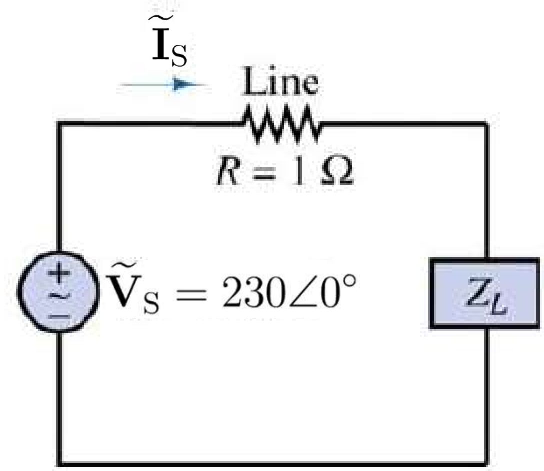
Q22

The load Z_L consists of a 25 Ω resistor in series with a 0.1 mF capacitor and a 70.35 mH inductor. Assuming f = 60 Hz, calculate the following

a) The apparent power delivered to the load.

b) The real (or active) power supplied by the source.

c) The power factor of the load.



$$\omega = 2\pi f \ = 120\pi \ \text{(rad/sec)}$$

$$Z_L = 25 + j\omega L + \frac{1}{j\omega C} = 25 - j0.0045 = 25\angle -0.01^\circ$$

$$\tilde{\mathbf{I}}_S = \frac{\tilde{\mathbf{V}}_S}{R + Z_L} = \frac{230\angle 0^\circ}{1 + 25 - j0.0045} = 8.846\angle 0.01^\circ$$

$$\tilde{\mathbf{V}}_L = \tilde{\mathbf{I}}_S Z_L = (8.846\angle 0.01^\circ)(25 - j0.0045) \approx 221.15\angle 0^\circ \ \text{V}$$

$$\mathbf{S}_L = \tilde{\mathbf{V}}_L \tilde{\mathbf{I}}_S^* = (221.15\angle 0^\circ)(8.846\angle -0.01^\circ) = 1956.36\angle -0.01^\circ \ \text{VA}$$

$$\boxed{|\mathbf{S}_L| = 1956.36 \ \text{VA}}$$

$$\mathbf{S}_S = \tilde{\mathbf{V}}_S \tilde{\mathbf{I}}_S^* = (230\angle 0^\circ)(8.846\angle -0.01^\circ) = 2034.6\angle -0.01^\circ$$
$$= \boxed{2034.615} - j0.352 \ \text{VA}$$

Real power = 2034.615 W

Power factor of the load

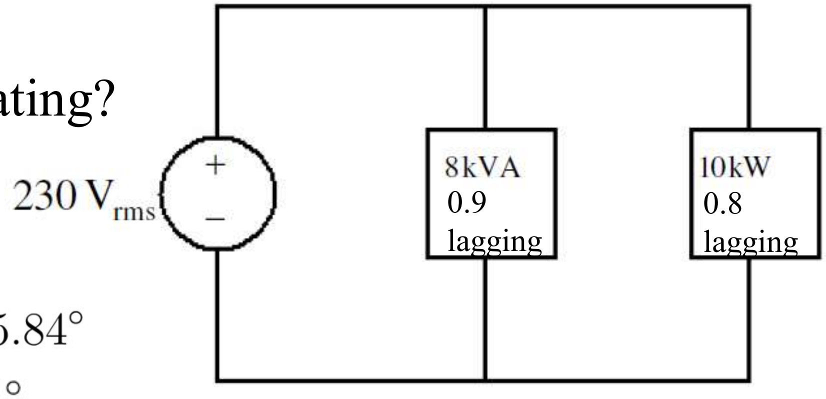
$$= \cos(-0.01^\circ) \approx 1.0$$

Q23

A 230 V_{rms} source has two loads connected to it in parallel. One draws 8 kVA at lagging power of 0.9, and the other draws 10 kW at a lagging power factor of 0.8.

a) Find the rms source current amplitude.

b) At what power factor is the source operating?



Since we only need to find amplitude of $\widetilde{\mathbf{I}}_S$,
we may assume source voltage having angle of 0°.

Load 1: $\text{p.f. of load1} = cos\,(\theta_1) = 0.9 \Rightarrow \theta_1 = 25.84°$

$$\mathbf{S_1} = \widetilde{\mathbf{V}}_S \widetilde{\mathbf{I}}_1^* \Rightarrow \widetilde{\mathbf{I}}_1^* = \frac{\mathbf{S_1}}{\widetilde{\mathbf{V}}_S} = \frac{8000\angle 25.84°}{230\angle 0°} = 34.7826\angle 25.84°$$

Load 2: $\text{p.f. of load2} = cos\,(\theta_2) = 0.8 \Rightarrow \theta_2 = 36.87°$

$$\mathbf{S}_2 = \widetilde{\mathbf{V}}_S \widetilde{\mathbf{I}}_2^* \Rightarrow \widetilde{\mathbf{I}}_2^* = \frac{\mathbf{S_2}}{\widetilde{\mathbf{V}}_S} = \frac{\left(\frac{10000}{0.8}\right)\angle 36.87°}{230\angle 0°} = 54.3478\angle 36.87°$$

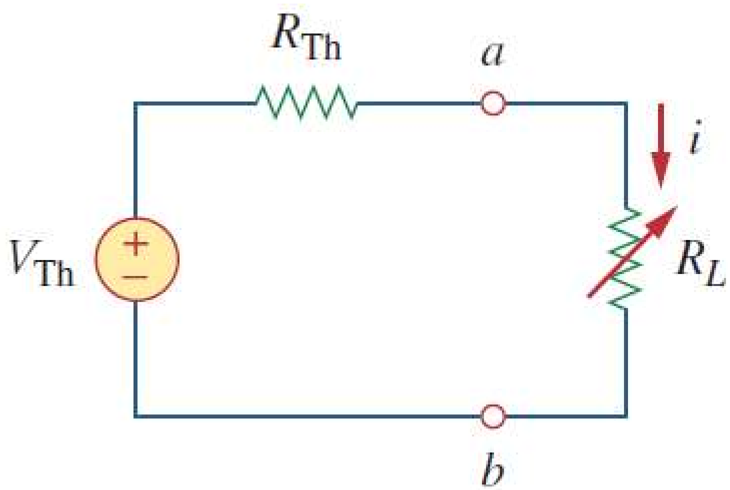
$$\widetilde{\mathbf{I}}_S = \widetilde{\mathbf{I}}_1 + \widetilde{\mathbf{I}}_2 = (34.7826\angle -25.84°) + (54.3478\angle -36.87°) = 88.7377\angle -32.57°$$

$$\text{The rms source current amplitude} = 88.7377\ \text{A}_{\text{rms}}$$

$$\mathbf{S}_S = \widetilde{\mathbf{V}}_S \widetilde{\mathbf{I}}_S^* \Rightarrow \text{p.f. of the source} = \cos\,(32.57°) = 0.8427 \text{ lagging}$$

AC Maximum Real Power Transfer

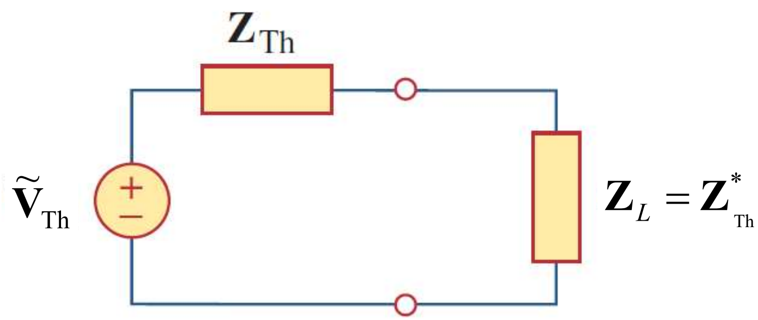
- For DC, max. power transfer occurs when $R_L = R_{Th}$:



$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

$$\boxed{p_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

- For AC, max. power transfer occurs when $Z_L = Z_{Th}^*$, Z_{Th}^* is complex conjugate of Z_{Th}



If $Z_{\mathrm{Th}} = 5 + j4$ and $Z_L = 5 - j4$, max. power transfer occurs.

Complex power from source is real.

$$\mathbf{S}_{\text{source}} = \tilde{\mathbf{V}}_{\mathrm{Th}}\tilde{\mathbf{I}}^* = \left(\tilde{\mathbf{V}}_{\mathrm{Th}}\right)\left(\frac{\tilde{\mathbf{V}}_{\mathrm{Th}}}{Z_{\mathrm{Th}} + Z_L}\right)^* = \left(\tilde{\mathbf{V}}_{\mathrm{Th}}\right)\left(\frac{\tilde{\mathbf{V}}_{\mathrm{Th}}}{5+5}\right)^* = \frac{\tilde{\mathbf{V}}_{\mathrm{Th}}\tilde{\mathbf{V}}_{\mathrm{Th}}^*}{10}$$

As an example, when $\tilde{\mathbf{V}}_{\mathrm{Th}} = 220\angle 25^\circ$ (Vrms)

$$\mathbf{S}_{\text{source}} = \frac{\tilde{\mathbf{V}}_{\mathrm{Th}}\tilde{\mathbf{V}}_{\mathrm{Th}}^*}{10} = \frac{(220\angle 25^\circ)(220\angle -25^\circ)}{10} = 4840\angle 0^\circ$$

Voltage source supplies real power = 4840 W

AC Maximum Real Power Transfer

Voltage source supplies real power = 4840 W

Since only resistors absorbs real power, the two 5 Ω resistors share 4840 W:

Max. real power absorbed by $\mathbf{Z}_L = \underline{\underline{2420 \text{ W}}}$

Check this result by $\mathbf{S}_L = \tilde{\mathbf{V}}_L \tilde{\mathbf{I}}_L^*$

$$\mathbf{S}_L = \left(\frac{Z_L}{Z_{\text{Th}} + Z_L}\tilde{\mathbf{V}}_{\text{Th}}\right)\left(\frac{\tilde{\mathbf{V}}_{\text{Th}}}{Z_{\text{Th}} + Z_L}\right)^* = \frac{(5 - j4)\tilde{\mathbf{V}}_{\text{Th}}\tilde{\mathbf{V}}_{\text{Th}}^*}{100}$$

$$\mathbf{S}_L = \frac{(5 - j4)(220\angle 25^\circ)(220\angle -25^\circ)}{100} = 2420 - j1936$$

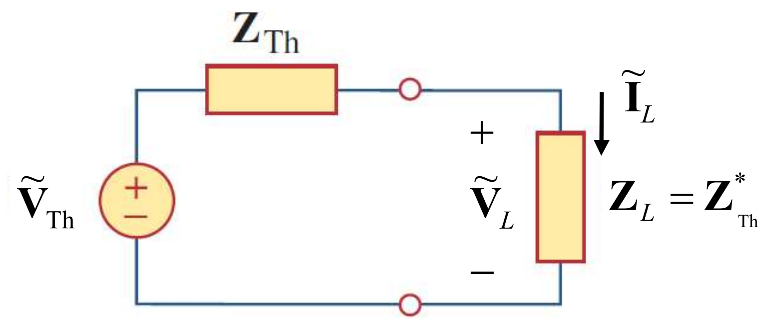
Max. real power absorbed by $\mathbf{Z}_L = 2420 \text{ W}$

OR use the DC formula:

$$\text{P}_{L,\max} = \frac{\left|\tilde{\mathbf{V}}_{\text{Th}}\right|^2}{4R_{\text{Th}}} = \frac{220^2}{4 \times 5} = 2420 \text{ W}$$

$$\boxed{p_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

- For AC, max. power transfer occurs when $Z_L = Z_{Th}^*$, Z_{Th}^* is complex conjugate of Z_{Th}



$Z_{\text{Th}} = 5 + j4$

$Z_L = 5 - j4$

$$\mathbf{S}_{\text{source}} = \tilde{\mathbf{V}}_{\text{Th}}\tilde{\mathbf{I}}^* = \left(\tilde{\mathbf{V}}_{\text{Th}}\right)\left(\frac{\tilde{\mathbf{V}}_{\text{Th}}}{Z_{\text{Th}} + Z_L}\right)^* = \left(\tilde{\mathbf{V}}_{\text{Th}}\right)\left(\frac{\tilde{\mathbf{V}}_{\text{Th}}}{5+5}\right)^* = \frac{\tilde{\mathbf{V}}_{\text{Th}}\tilde{\mathbf{V}}_{\text{Th}}^*}{10}$$

As an example, when $\tilde{\mathbf{V}}_{\text{Th}} = 220\angle 25^\circ \text{ (Vrms)}$

$$\mathbf{S}_{\text{source}} = \frac{\tilde{\mathbf{V}}_{\text{Th}}\tilde{\mathbf{V}}_{\text{Th}}^*}{10} = \frac{(220\angle 25^\circ)(220\angle -25^\circ)}{10} = 4840\angle 0^\circ$$