Numerical Methods Problem Set 10

Due 5/2/2011

1. Solve the differential equation for the Van der Pol oscillator

$$\frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} + x = 0$$

given the initial conditions

$$x(0) = -2$$

$$\dot{x}(0) = 0$$

Take the value of μ to be $\{0.1, 1, 2, 4, 10, 50\}$ respectively and plot for all of them

(1) x as a function of t

(2) the phase portrait of the oscillation, ie., the trajectory of the motion in the phase space $(x, \dot{x})$.

To make the plot, you can generate a data file with descrite values of x and $\dot{x}$, then use tools like Mathematica or MatLab to make the plot.

2. Solve the Lane-Emden equation

$$\frac{1}{s^2}\frac{d}{ds}(s^2\frac{d\psi}{ds}) = \begin{cases} -3\psi^n & \psi > 0 \\ 0 & \psi > 0 \end{cases}$$

with the boundary conditions

$$\psi(0) = 1, \frac{d\psi}{ds} |_0 = 0$$

Where n is an integer. Try different values of n as 0,1,2,3,4,5.

3. (Extra Credit) Solve the following ODE numerically as a two-point boundary value problem

$$\frac{1}{r}\frac{d}{dr}[r\frac{d\Phi(r)}{dr}] + (k^2 - \frac{m^2}{r^2})\Phi(r) = 0$$

with boundary conditions $\Phi(0) = finite$ and $\Phi(1) = 0$.

Hint: $m = 1, 2, 3...$ is an integer and k is the eigen value which need to be determined by the boundary condition. They analytical solution is just the Bessel functions $J_m(r)$. A simple numerical method may not exist, which is what you need to explore.