Numerical Methods Problem Set

Due 3/20/2011

1. Please comment in detail the Householder matrix program in Numerical Recipes. Show in your comment explicitly what is going on at each step. (If you know C or C++, commenting in these codes is encouraged. The code you comment in this question does not need to be the same one that is used in question 2.)

2. Transfrom the matrix

$$A = \begin{pmatrix} 7 & 2 & 3 & -1 \\ 2 & 8 & 5 & 1 \\ 3 & 5 & 12 & 9 \\ -1 & 1 & 9 & 7 \end{pmatrix}$$

to tridiagonal form using Householder reduction. Determine the transformation matrix P. Calculate the associated eigenvectors and eigenvalues of A.

3. (This is an analytical problem instead of a numerical one.)

Show that if $A = a_{ik}$ is Hermitian, then for every diagonal element a_{ii}, there exists an eigenvalue $\lambda(A)$ of A such that

$$|\lambda(A) - a_{ii}| \leq \sqrt{\sum_{j \neq i} |a_{ij}|^2}$$