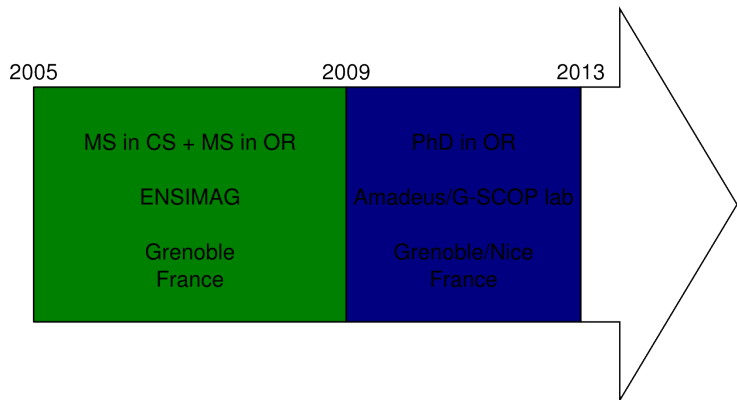


# Market Driven Fleet Assignment

Christophe-Marie Duquesne

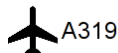
June 17, 2013

# A word about me



Advisors: Denis Naddef, Olivier Briant / Manager: Semi Gabteni

# Fleet Assignment: One simple question



A319



A320



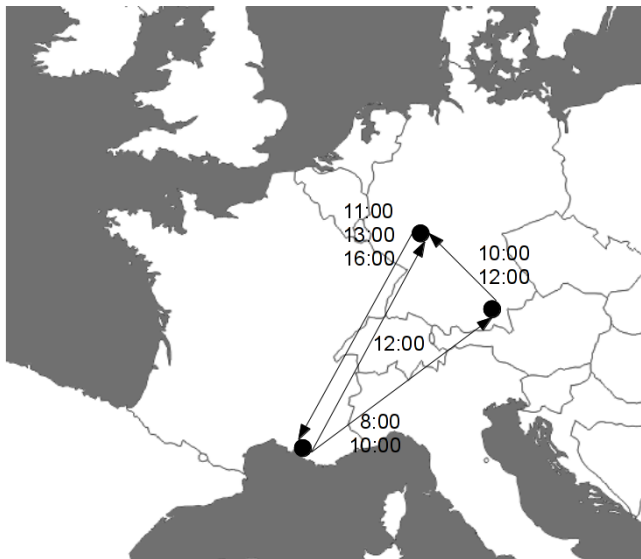
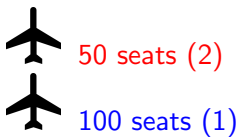
B747

...

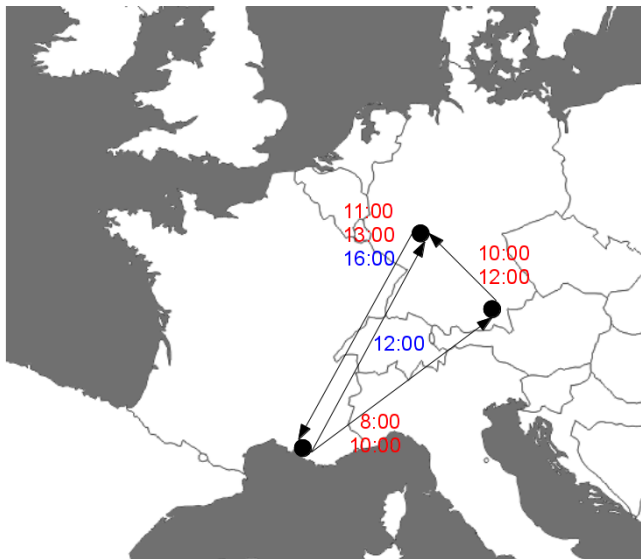
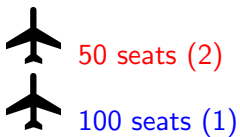


Which **aircraft type** should I put on each **flight leg** ?  
1 take off - 1 landing

# Example



# Example



# What is optimized?



$$\text{Profit} = \text{Revenue} - \text{Costs}$$

Depends on

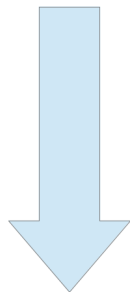
- Fares
- Bookings
- Capacities
- Sales
- Fuel
- Gate rental
- ...

# Context

One step of **Airline Planning**, which decides everything about:

- The flights
- The fleet
- The crew





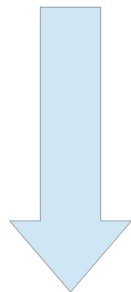
Operations

- ① **Schedule Design**
- ② Fleet Assignment
- ③ Aircraft Routing
- ④ Crew Pairing
- ⑤ Crew Rostering

Decide of the flights to operate.



# Airline Planning



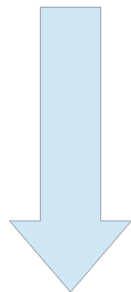
Operations

- ① Schedule Design
- ② **Fleet Assignment**
- ③ Aircraft Routing
- ④ Crew Pairing
- ⑤ Crew Rostering

Decide the type of aircraft to  
operate each flight

(Implies a schedule)

# Airline Planning



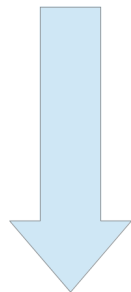
Operations

- ① Schedule Design
- ② Fleet Assignment
- ③ **Aircraft Routing**
- ④ Crew Pairing
- ⑤ Crew Rostering

Decide the route of each aircraft

(Implies a fleet assignment)

# Airline Planning



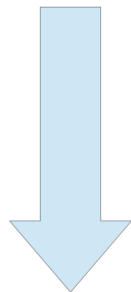
Operations

- ① Schedule Design
- ② Fleet Assignment
- ③ Aircraft Routing
- ④ **Crew Pairing**
- ⑤ Crew Rostering

Decide of the crew types to be assigned on each flight

(Implies an aircraft routing)

# Airline Planning



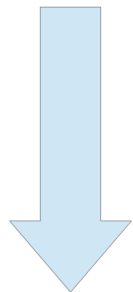
Operations

- ① Schedule Design
- ② Fleet Assignment
- ③ Aircraft Routing
- ④ Crew Pairing
- ⑤ **Crew Rostering**

Decide of the timetables of  
individuals

(Implies a crew pairing)

# Airline Planning



Operations

- ① Schedule Design
- ② **Fleet Assignment**
- ③ Aircraft Routing
- ④ Crew Pairing
- ⑤ Crew Rostering

Focus of my thesis

# Revenue Management



After Airline Planning, next logical step.

- Control seat availability **dynamically**.
- **Capacities** are essential.
- Makes the revenue **hard** to accurately estimate!

# Summary



## Fleet Assignment:

- Determines major **costs** (easy)
- Sets capacities for the **revenue** (hard)

# Summary



## Fleet Assignment:

- Determines major **costs** (easy)
- Sets capacities for the **revenue** (hard)

## Goals of this thesis:

- Generally improve the existing models
- Focus on estimating the revenue better

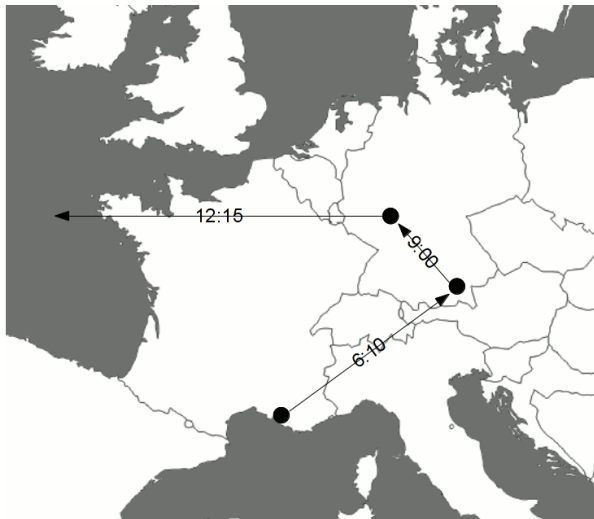


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  - Yaposib
  - lazympsolverlibs
- 4 Conclusion

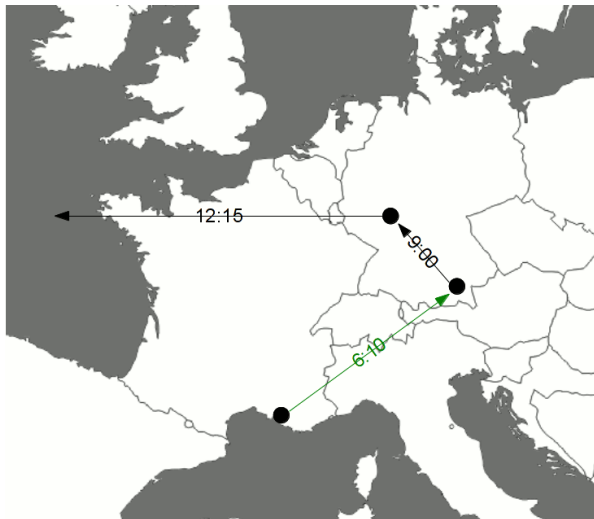
# Some definitions

- legs
- itineraries
- spill
- recapture



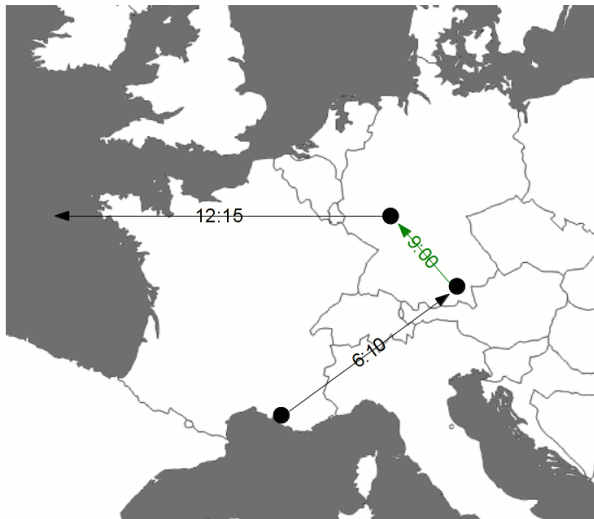
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- legs
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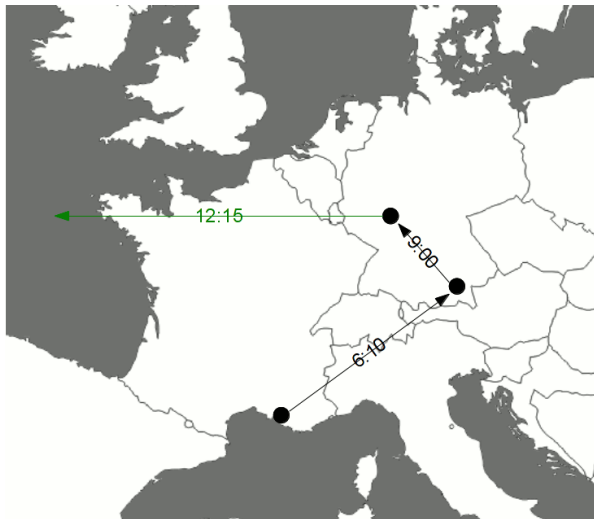
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- legs
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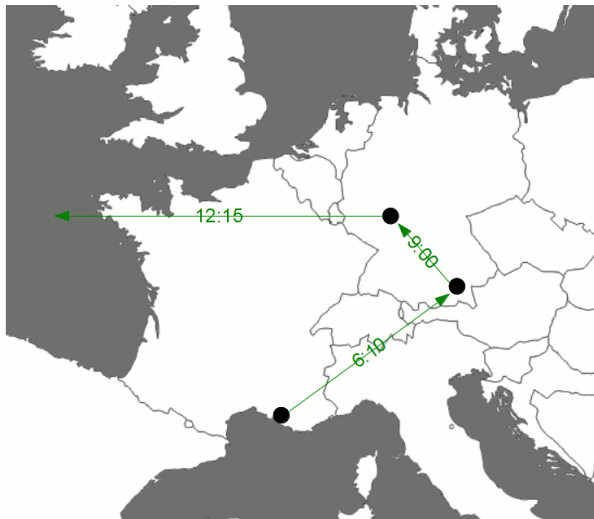
# Some definitions

- legs
- itineraries
- spill
- recapture



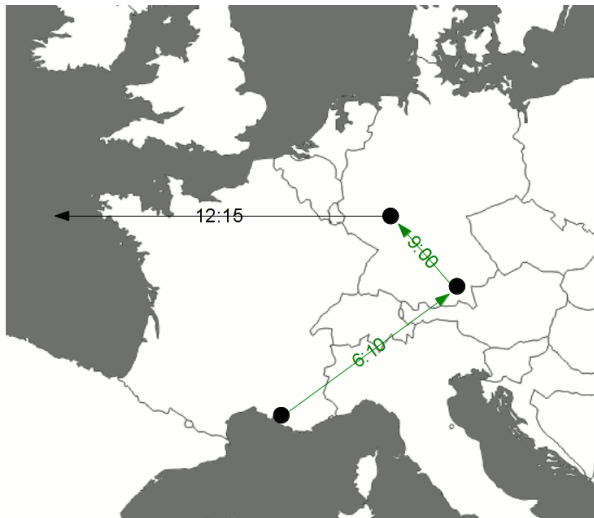
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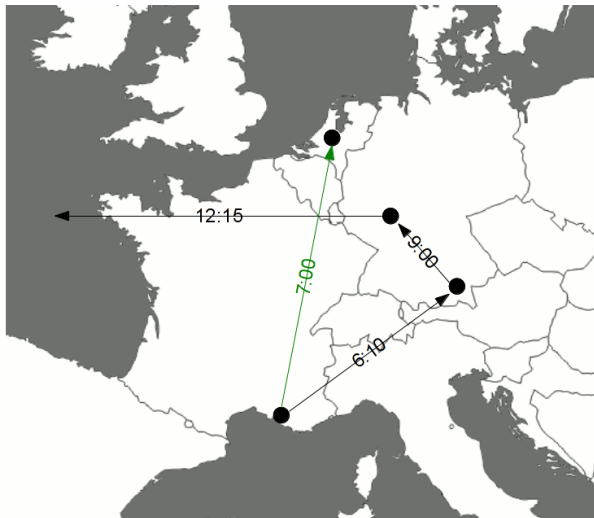
# Some definitions

- legs
- **itineraries**
- spill
- recapture



# Some definitions

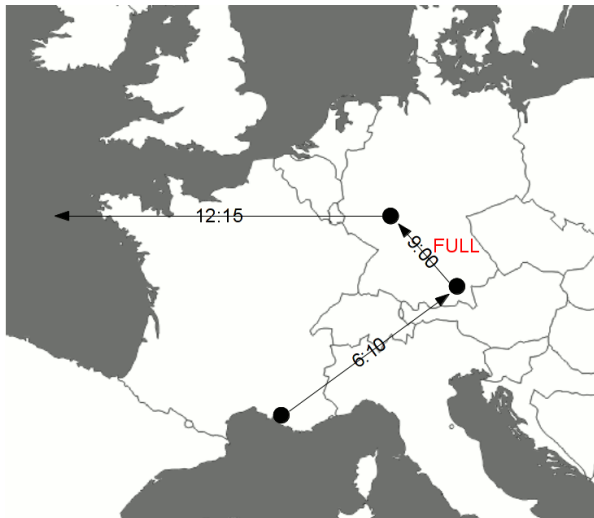
- legs
- **itineraries**
- spill
- recapture





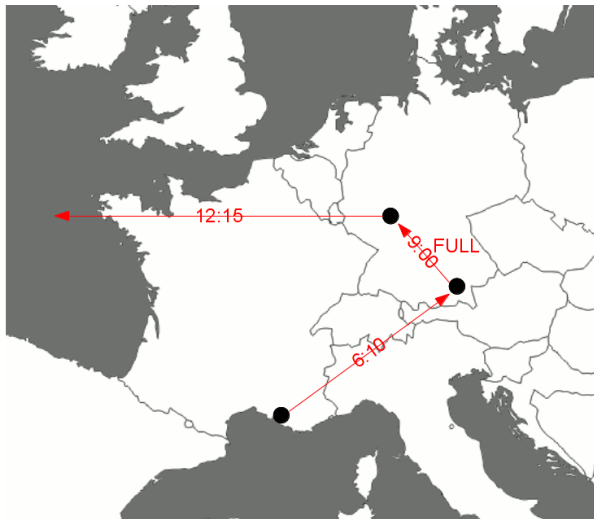
# Some definitions

- legs
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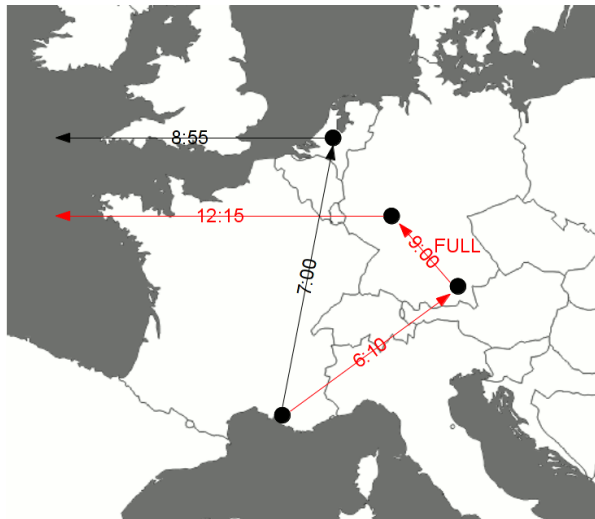
# Some definitions

- legs
- itineraries
- **spill**
- recapture



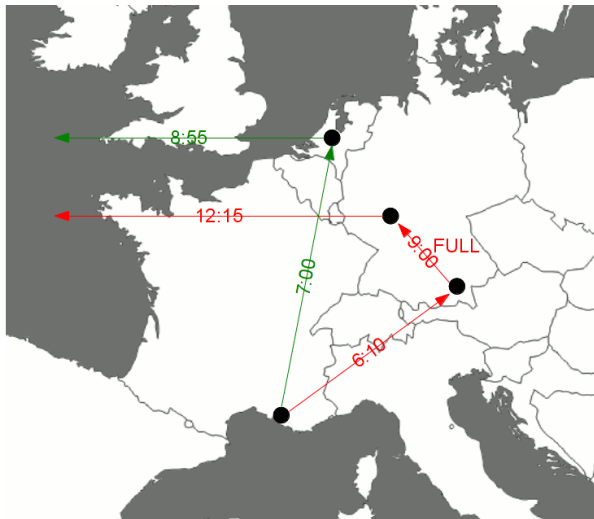
# Some definitions

- legs
- itineraries
- spill
- **recapture**



# Some definitions

- legs
- itineraries
- spill
- **recapture**



# Summary

- **legs:** 1 takeoff - 1 landing
- **itineraries:** sequences of legs
- **spill:** unsatisfied demand
- **recapture:** demand satisfied elsewhere

# Major models



Fleet Assignment Models (for assigning fleet):

- FAM (Abara, 1989)
- IFAM (Barnhart et al, 2002)

Passenger Mix Model (determining revenue fleeted network):

- PMM (Barnhart et al, 2002)

# Major models



Fleet Assignment Models (for assigning fleet):

- FAM (Abara, 1989)
- IFAM (Barnhart et al, 2002)

Passenger Mix Model (determining revenue fleeted network):

- PMM (Barnhart et al, 2002)

# Fleet Assignment Model (FAM)

- MIP to find the best assignment
- **Leg-based** costs
- Revenue included as spill cost



# Fleet Assignment Model (FAM)

Costs:

- $c$ : costs (type, leg)

Constraints:

- network

Variables:

- $x$ : assignment (type, leg)

# Fleet Assignment Model (FAM)

$$\begin{array}{ll} \min & cx \\ \text{s.t.} & x \in X = \left\{ \begin{array}{l} \text{cover} \\ \text{aircraft flow} \\ \text{fleet size} \end{array} \right. \end{array}$$

# Fleet Assignment Model (FAM)

$$\begin{array}{ll} \min & cx \\ \text{s.t.} & x \in X = \left\{ \begin{array}{l} \text{cover} \\ \text{aircraft flow} \\ \text{fleet size} \end{array} \right. \end{array}$$

Leg Costs:

$$C_{l,t} = C_{l,t}^{\text{operating}} + C_{l,t}^{\text{spill}}$$

# Fleet Assignment Model (FAM)

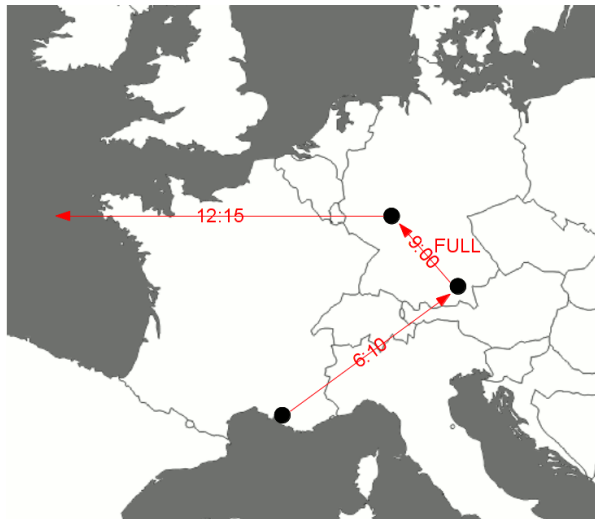
$$\begin{array}{ll} \min & cx \\ \text{s.t.} & x \in X = \left\{ \begin{array}{l} \text{cover} \\ \text{aircraft flow} \\ \text{fleet size} \end{array} \right. \end{array}$$

Leg Costs:

$$C_{l,t} = C_{l,t}^{\text{operating}} + C_{l,t}^{\text{spill}}$$

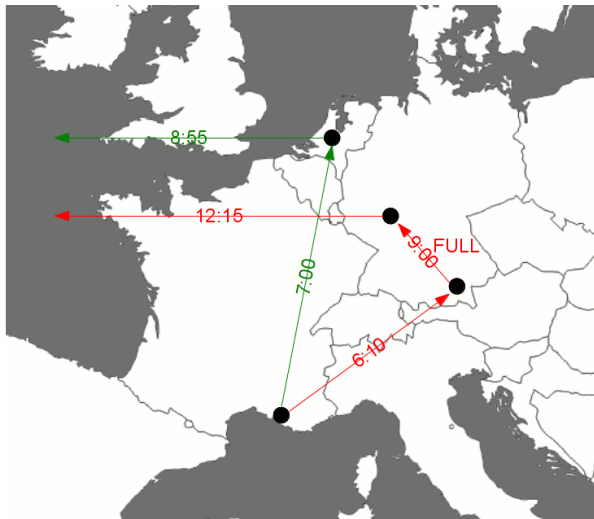
# Problem of FAM

- Leg-based costs, but spill costs are not local



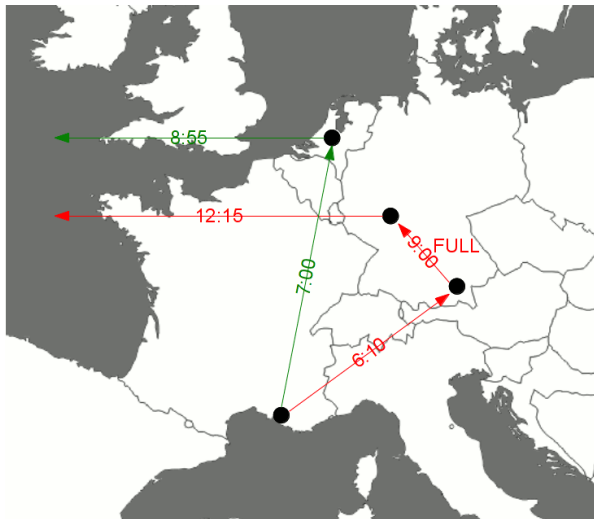
# Problem of FAM

- Leg-based costs, but spill costs are not local
- Recapture is ignored



# Problem of FAM

- Leg-based costs, but spill costs are not local
- Recapture is ignored
- Poor revenue estimation



# Passenger Mix Model (PMM)

- Computes the revenue of a **fleeted network**
- Minimizes the spill cost
- Based on demand forecasts



# Passenger Mix Model (PMM)

Costs:

- $f$ : fares (itinerary)

Constraints:

- $x$ : **fleet assignment**, demand

Variables:

- $s$ : spill (itinerary)

# Passenger Mix Model (PMM)

Costs:

- $f$ : fares (itinerary)

Constraints:

- $x$ : **fleet assignment**, demand[, recapture rates]

Variables:

- $s$ : spill (itinerary[ $\times$ itinerary])

# Passenger Mix Model (PMM)

$$\begin{array}{ll} \min & fs \\ \text{s.t.} & s \in S(x) = \begin{cases} \text{capacity}(x) \\ \text{population size} \end{cases} \end{array}$$

# Passenger Mix Model (PMM)

$$\begin{array}{ll} \min & fs \\ \text{s.t.} & s \in S(x) = \begin{cases} \text{capacity}(x) \\ \text{population size} \end{cases} \end{array}$$

- Can be embedded in a Fleet Assignment Model (IFAM)

# Itinerary-based Fleet Assignment Model (IFAM)

- Combines FAM and PMM
- Find the best assignment **and** the spill solution
- Embeds a simulation of the passenger flow

# Itinerary-based Fleet Assignment Model (IFAM)

## Costs:

- $c$ : operating costs (type, leg)
- $f$ : fares (itinerary)

## Constraints:

- network, demand [,recapture rates]

## Variables:

- $x$ : assignment (type, leg)
- $s$ : spill (itinerary[  $\times$  itinerary])

# Itinerary-based Fleet Assignment Model (IFAM)

$$\begin{array}{ll}
 \min & \underbrace{\text{operating cost}}_{cx} + \underbrace{\text{spill cost}}_{fs} \\
 \text{s.t.} & x \in X = \left\{ \begin{array}{l} \text{cover} \\ \text{aircraft flow} \\ \text{fleet size} \end{array} \right. \\
 & s \in S(x) = \left\{ \begin{array}{l} \text{capacity}(x) \\ \text{population size} \end{array} \right.
 \end{array}$$

# Itinerary-based Fleet Assignment Model (IFAM)

$$\begin{array}{ll}
 \min & \underbrace{\text{operating cost}}_{cx} + \underbrace{\text{spill cost}}_{fs} \\
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 & s \in S(x) = \left\{ \begin{array}{l} \text{capacity}(x) \\ \text{population size} \end{array} \right.
 \end{array}$$

- Better models the revenue
- But forecasting demands (and recapture rates) is delicate



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- 4 Conclusion

# Idea

- Itinerary demands are highly variable
- Grouped demands are easier to forecast (law of large numbers)

We thus include the demands as decision variables, and we constrain them

# MDFAM

## Costs:

- $c$ : operating costs (type, leg)
- $f$ : fares (itinerary)

## Constraints:

- network, **constraints on demand**

## Variables:

- $x$ : assignment (type, leg)
- $s$ : spill (itinerary)
- **$d$ : demand (itinerary)**

# MDFAM

$$\begin{array}{ll}
 \min & \overbrace{cx}^{\text{operating cost}} + \overbrace{f(s-d)}^{\text{revenue loss}} \\
 \text{s.t.} & x \in X \\
 & s, d \in SD(x) = \left\{ \begin{array}{l} \text{cover} \\ \text{aircraft flow} \\ \text{fleet size} \\ \text{capacity}(x) \\ \text{population size} \\ \text{market constraints} \end{array} \right.
 \end{array}$$

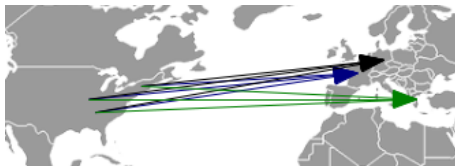
# MDFAM

$$\begin{array}{ll}
 \min & \overbrace{cx}^{\text{operating cost}} + \overbrace{f(s-d)}^{\text{revenue loss}} \\
 \text{s.t.} & x \in X \\
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 \end{array}$$

- With fixed demands, this is exactly IFAM (without recapture)
- Demand is chosen as a best case scenario.

# Examples of market constraints

## Matching a geographical criterion



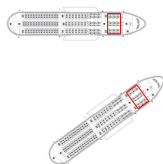
$$D_{\text{France}}^{\min} \leq \sum_{i \in I_{\text{France}}} d_i \leq D_{\text{France}}^{\max}$$

$$D_{\text{Germany}}^{\min} \leq \sum_{i \in I_{\text{Germany}}} d_i \leq D_{\text{Germany}}^{\max}$$

$$D_{\text{Greece}}^{\min} \leq \sum_{i \in I_{\text{Greece}}} d_i \leq D_{\text{Greece}}^{\max}$$

# Examples of market constraints

Matching based on the fare class



$$D_{\text{First class}}^{\min} \leq \sum_{i \in I_{\text{First class}}} d_i \leq D_{\text{First class}}^{\max}$$

# Examples of market constraints

Enforcing correlations between demands

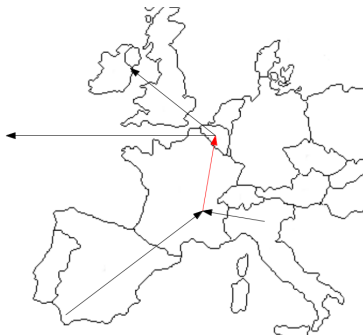


$$-\epsilon \leq d_i - d'_i \leq \epsilon$$



# Examples of market constraints

Leg based



$$D_l^{\min} \leq \sum_{i \in I_l} d_i \leq D_l^{\max}$$

# Many possibilities

Flexible: any linear expression can be used

$$D^{\min} \leq \sum_i \alpha_i d_i \leq D^{\max}$$

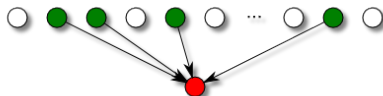
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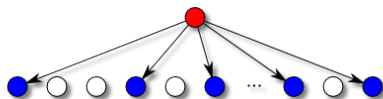
# Comparing the models



sample (demand scenarios)



forecast the best fleet assignment based on a subsample



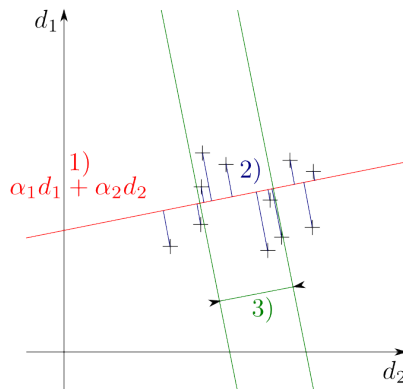
test the assignment on another subsample

# Forecasting the best Assignments

FAM and IFAM: solve according to average demands

MDFAM requires ranged constraints. How to proceed?

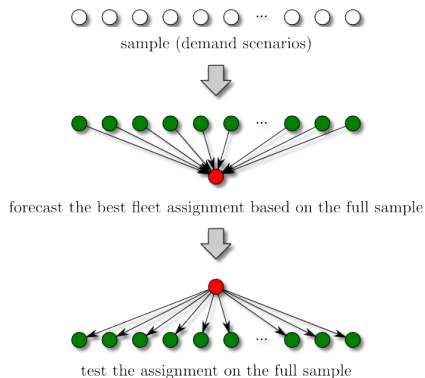
# Market Constraints Forecast



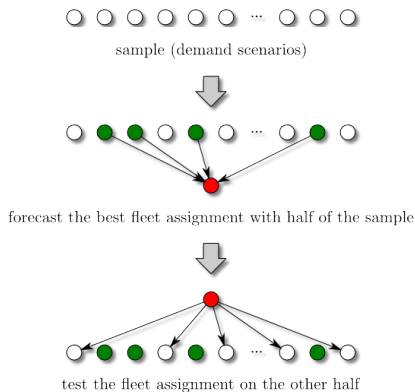
Bounds are determined from the occurrences of  $\sum_i \alpha_i d_i$

# Tests

## Whole season



## 2-fold cross-validation



# Results

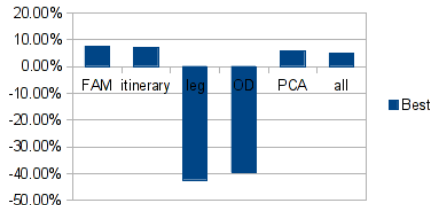
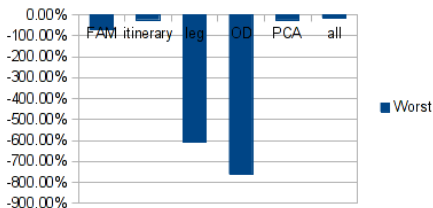
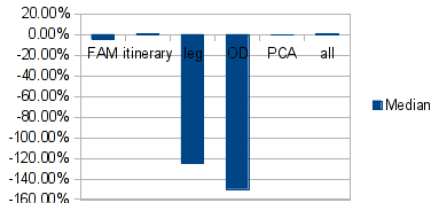
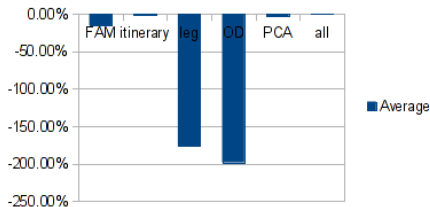
Compared to IFAM:

Whole season	+4%
2-fold cross validation	-0.67%

- With perfect forecasts, MDFAM would perform very well
- With less information, it needs improvement (but not too far)



# Effects of the market constraints (Profit%IFAM)



# About this work

MDFAM will be improved: as a generalisation, it can be IFAM.

- Part of my thesis, defended in January
- Positive comments from Barnhart: It simplifies the whole recapture concept.
- Amadeus has 2 master students working on it.

# About this work

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- Amadeus has 2 master students working on it.

Speaking about code...

# Related Achievements

I will now present 3 open source projects related to this thesis.

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  - Yaposib
  - lazympsolverlibs
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# Story

- Needed to implement FAM, IFAM, PMM...
- Wanted to benchmark different solvers
- Already knew Coin-OSI

# PuLP-OR

- Solver-agnostic Python Linear Programming library
- COIN-OR project, actively maintained, Growing user base
- Support 5 solvers
- Interpreted, but 90% time = solver

# What I got out of it

A solver that could do

```
cat FAM.json | assignfleet --solver=cplex -
```

Contributions:

- support for python-glpk
- support for cplex runtime licenses



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  - **Yaposib**
  - lazylp solverlibs
- 4 Conclusion

# Story

- OSI is ugly, but supports many solvers
- Let's make a python binding

# Yaposib

- Integrated in PuLP-OR

```
problem.Solve(YAPOSIB("Clp"))
```

- Can be used standalone (startup **Prediction Appliance**)
- Extended documentation, examples, and a test suite
- Should become COIN-OR when they support git

# Yaposib

**Welcome to yaposib's documentation!**

Yaposib is a python binding to CGL, the Open Solver Interface from CON-OR. It intends to give access to various solvers through python. Yaposib was created to be integrated with pulp, and plays nicely with it.

**Manual**

- Getting Started
  - Installing
  - Checking your installation
  - Code samples
- Reference API
  - Problem
  - Helper
  - FAQ

**Various Infos**

The repository is hosted on [code.google.com](https://code.google.com/p/yaposib/)

```
git clone https://code.google.com/p/yaposib/
```

A mirror is also maintained on github, and can be useful in a number of situations, like when you only have set

```
git clone https://github.com/yaposib/yaposib
```

[Read the Docs](#)

**Getting Started**

What follows is a guide for installing yaposib very quickly and solve your first linear program using it.

**Installing**

**Recommended method: pip**

- Install pip, python dev and local python. On Ubuntu:
 

```
sudo apt-get install python-pip python-dev libboost-python-dev
```
- Install os. If you want support for commercial solvers, relying on your distribo is not recommended. Otherwise, using a package from your distribution is fine. Note that if you modify the Cgl installed on your machine, you will have to recompile yaposib.
- Use pip to install yaposib:
 

```
sudo pip install yaposib
```

**Alternative: development version**

- Follow 1. and 2 from the previous method
- Clone the repository
 

```
git clone https://code.google.com/p/yaposib/
```

[Read the Docs](#)

**Reference API**

**Problem**

class **Problem**

Models an LP problem

Main methods

Problem **.markHotStart()**

Makes an internal optimization snapshot of the problem (an internal warmstart object is built)

Problem **.unmarkHotStart()**

Deletes the internal snapshot of the problem (if existing)

Problem **.solve(Timeout=False)**

Solves the internal problem

- If an internal snapshot exists, use it.

[Read the Docs](#)

```
import yaposib
import sys

def main():
    """Extra simple command line mps solver"""
    if len(sys.argv) == 1:
        print("Usage: yaposib-solve <file.mps> [<file.mps> ...]&")
        sys.exit(0)
    solver = yaposib.available_solvers[0]
    for filename in sys.argv[1:]:
        problem = yaposib.Problem(solver)
        print("Will now solve %s" % filename)
        err = problem.readmps(filename)
        if not err:
            problem.solve()
            if problem.status == 'optimal':
                print("Optimal value: %s" % problem.obj.value)
            for var in problem.columns:
                print("%s = %s" % (var.name, var.solution))
            print("No optimal solution could be found.")
        else:
            print("No optimal solution could be found.")

if __name__ == "__main__":
    main()
```

Other examples are available in the examples directory

[Read the Docs](#)

# Outline

- 1 Introduction
  - Basic Concepts
  - Literature Review
- 2 Market Driven Fleet Assignment
  - Idea
  - Model
  - Experiments
- 3 Open Source Projects
  - PuLP-OR
  - Yaposib
  - lazympsolverlibs
- 4 Conclusion

# Rationale

- Yaposib builds on OSI: let's package both
- Solver support: compile time
- Users do not have every solvers (e.g. cplex)

# lazympsolverlibs

- Fake libraries proxying to cplex, gurobi, xpress:

```
-lcplex -I$INCLUDE_DIR -L$LINK_DIR
```

```
# versus #
```

```
$(pkg-config --libs --cflags lazycplex)
```

```
export LAZLPSEVERLIBS_CPLEX_LIB=$LINK_DIR/libcplex.so
```

- Man page, solver availability test tool
- Debian package on the way

# Outline

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# Conclusion

- 3 active years
- Very happy to talk about them
- I am looking forward to doing new projects!

Thank You

# Bibliography I



J. Abara.

Applying integer linear programming to the fleet assignment problem.  
[Interfaces](#), page 20–28, 1989.



Y. Ageeva and J. P. Clarke.

Approaches to incorporating robustness into airline scheduling.  
2000.



R. Ahuja, J. Goodstein, A. Mukherjee, J. Orlin, and D. Sharma.

A very large-scale neighborhood search algorithm for the combined  
through and fleet assignment model.  
[MIT Sloan Working Paper No. 4388-01](#), 2001.

# Bibliography II



R.K. Ahuja, J. Liu, J. Goodstein, A. Mukherjee, J.B. Orlin, and D. Sharma.

Solving multi-criteria through-fleet assignment models.  
Applied Optimization, 79:233–256, 2003.



R. K Ahuja, T. L Magnanti, and J. B Orlin.

Network Flows: Theory, Algorithms, and Applications.  
Prentice-Hall, 1993.



C. Barnhart, N.L. Boland, L.W. Clarke, E.L. Johnson, G.L. Nemhauser, and R.G. Shenoi.

Flight string models for aircraft fleetting and routing.  
Transportation science, 32(3):208–220, 1998.

# Bibliography III



C. Barnhart, P. Belobaba, and A.R. Odoni.

Applications of operations research in the air transport industry.  
[Transportation science](#), 37(4):368–391, 2003.



JF Bard and IG Cunningham.

Improving through-flight schedules.  
[IIE transactions](#), 19(3):242–251, 1987.



N. Bélanger, G. Desaulniers, F. Soumis, J. Desrosiers, HEC Montréal,  
and G. Jacques.

Fleet assignment with time windows, spacing constraints and time  
dependent profits.

# Bibliography IV



P. P. Belobaba and A. Farkas.

The influence of network effects and yield management on airline fleet assignment decisions.

PhD thesis, Massachusetts Institute of Technology, 1996.



C. Barnhart, A. Farahat, and M. Lohatepanont.

Airline fleet assignment with enhanced revenue modeling.

Operations research, 57(1):231–244, 2009.



M.E. Berge and C.A. Hopperstad.

Demand driven dispatch: A method for dynamic aircraft capacity assignment, models and algorithms.

Operations Research, page 153–168, 1993.

# Bibliography V



C. Barnhart, E.L. Johnson, G.L. Nemhauser, M.W.P. Savelsbergh, and P.H. Vance.

Branch-and-price: Column generation for solving huge integer programs.

[Operations Research](#), page 316–329, 1998.



C. Barnhart, T.S. Kniker, and M. Lohatepanont.

Itinerary-based airline fleet assignment.

[Transportation Science](#), 36(2):199–217, 2002.



O. Briant, C. Lemaréchal, P. Meurdesoif, S. Michel, N. Perrot, and F. Vanderbeck.

Comparison of bundle and classical column generation.

[Mathematical Programming](#), 113(2):299–344, 2008.

# Bibliography VI



LW Clarke, CA Hane, E.L. Johnson, and G.L. Nemhauser.  
Maintenance and crew considerations in fleet assignment.  
[Transportation Science](#), 30(3):249–260, 1996.



R.S. Chandler, S.S. Ja, and T.L. Jacobs.  
Advantages of real-time revenue management.  
[Journal of Revenue and Pricing Management](#), 3(3):254–264, 2004.



L. Clarke, E. Johnson, G. Nemhauser, and Z. Zhu.  
The aircraft rotation problem.  
[Annals of Operations Research](#), 69:33–46, 1997.



L. Cadarso and Á Marín.  
Robust passenger oriented airline scheduling.



# Bibliography VII



J.F. Cordeau, G. Stojković, F. Soumis, and J. Desrosiers.  
Benders decomposition for simultaneous aircraft routing and crew scheduling.  
[Transportation science](#), 35(4):375–388, 2001.



J-C Culioli.  
Modèles et problèmes de revenue management pour une compagnie aérienne.  
Toulouse, 2006.



J. Dumas, F. Aithnard, and F. Soumis.  
Improving the objective function of the fleet assignment problem.  
[Transportation Research Part B: Methodological](#), 43(4):466–475, 2009.

## Bibliography VIII



G. Desaulniers, J. Desrosiers, Y. Dumas, M.M. Solomon, and F. Soumis.

Daily aircraft routing and scheduling.

[Management Science](#), page 841–855, 1997.



G. Dobson and P. J Lederer.

Airline scheduling and routing in a hub-and-spoke system.

[Transportation Science](#), 27(3):281–297, 1993.



M.S. Daskin and N.D. Panayotopoulos.

A lagrangian relaxation approach to assigning aircraft to routes in hub and spoke networks.

[Transportation Science](#), 23(2):91–99, 1989.

# Bibliography IX



J. Dumas and F. Soumis.

Passenger flow model for airline networks.

Transportation Science, 42(2):197–207, 2008.



T. A Feo and J. F Bard.

Flight scheduling and maintenance base planning.

Management Science, pages 1415–1432, 1989.



T. A Feo and J. F Bard.

Flight scheduling and maintenance base planning.

Management Science, pages 1415–1432, 1989.



G. Gagnon.

A model for flowing passengers over airline networks.

Transportation science, 1(3):232–248, 1967.

# Bibliography X



F. Glover, R. Glover, J. Lorenzo, and C. McMillan.  
The passenger-mix problem in the scheduled airlines.  
[Interfaces](#), pages 73–80, 1982.



Z. Gu, E.L. Johnson, G.L. Nemhauser, and Y. Wang.  
Some properties of the fleet assignment problem.  
[Operations Research Letters](#), 15(2):59–71, 1994.



R. Gopalan and K. T Talluri.  
The aircraft maintenance routing problem.  
[Operations Research](#), pages 260–271, 1998.



R. Gopalan and K.T. Talluri.  
Mathematical models in airline schedule planning: A survey.  
[Annals of Operations Research](#), 76:155–185, 1998.

# Bibliography XI



C.A. Hane, C. Barnhart, E.L. Johnson, R.E. Marsten, G.L. Nemhauser, and G. Sigismondi.

The fleet assignment problem: solving a large-scale integer program.  
[Mathematical Programming](#), 70(1):211–232, 1995.



D. C. Hoaglin, F. Mosteller, and J. W. Tukey.

[Exploring data tables, trends, and shapes](#), volume 101.  
Wiley, 2011.



C. Hopperstad.

PFLOW: a passenger flow allocation model.  
[Boeing Internal Report](#), 1983.



O. Holst and B. Srensen.

Combined scheduling and maintenance planning for an aircraft fleet.  
In [Operational Research](#), volume 84, 1984.

## Bibliography XII



I. Ioachim, J. Desrosiers, F. Soumis, and N. Bélanger.

Fleet assignment and routing with schedule synchronization constraints.

[European Journal of Operational Research](#), 119(1):75–90, 1999.



H. Jiang and C. Barnhart.

Dynamic airline scheduling.

[Transportation Science](#), 43(3):336–354, 2009.



Hai Jiang and Cynthia Barnhart.

Robust airline schedule design in a dynamic scheduling environment.

[Computers & Operations Research](#), 2011.



I. Jolliffe.

[Principal component analysis](#).

Wiley Online Library, 2005.

# Bibliography XIII



T. L. Jacobs, B. C. Smith, and E. L. Johnson.

Incorporating network flow effects into the airline fleet assignment process.

[Transportation Science](#), 42(4):514–529, 2008.



Thimothy S. Kniker and Cynthia Banhart.

Shortcomings of the conventional airline fleet assignment model. Puerto Rico, 1998.



M. Lohatepanont and C. Barnhart.

Airline schedule planning: Integrated models and algorithms for schedule design and fleet assignment.

[Transportation Science](#), 38(1):19–32, 2004.

## Bibliography XIV



C. Lemaréchal.

Lagrangian relaxation.

[Computational Combinatorial Optimization](#), pages 112–156, 2001.



J.I. McGill and G.J. Van Ryzin.

Revenue management: Research overview and prospects.

[Transportation science](#), 33(2):233, 1999.



C. C. Queenan, M. Ferguson, J. Higbie, and R. Kapoor.

A comparison of unconstraining methods to improve revenue management systems.

[Production and Operations Management](#), 16(6):729–746, 2007.



B. Rexing.

[Airline fleet assignment with time windows](#).

PhD thesis, Massachusetts Institute of Technology, 1997.



# Bibliography XV



J. M Rosenberger, E. L Johnson, and G. L Nemhauser.

A robust fleet-assignment model with hub isolation and short cycles.  
[Transportation Science](#), 38(3):357–368, 2004.



R.A. Rushmeier and S.A. Kontogiorgis.

Advances in the optimization of airline fleet assignment.  
[Transportation science](#), 31(2):159–169, 1997.



H. D Sherali, K. H Bae, and M. Haouari.

A benders decomposition approach for an integrated airline schedule design and fleet assignment problem with flight retiming, schedule balance, and demand recapture.  
[Annals of Operations Research](#), pages 1–32, 2010.

# Bibliography XVI



H. D Sherali, E. K Bish, and X. Zhu.

Airline fleet assignment concepts, models, and algorithms.

[European Journal of Operational Research](#), 172(1):1–30, 2006.



F. Soumis, J. A Ferland, and J. M Rousseau.

A model for large-scale aircraft routing and scheduling problems.

[Transportation Research Part B: Methodological](#), 14(1-2):191–201, 1980.



F. Soumis, J. A Ferland, and J. M Rousseau.

MAPUM: a model for assigning passengers to a flight schedule.

[Transportation Research Part A: General](#), 15(2):155–162, 1981.



S. Shebalov.

Practical overview of demand-driven dispatch.

[Journal of Revenue & Pricing Management](#), 8(2):166–173, 2009.

# Bibliography XVII



B. C Smith and E. L Johnson.

Robust airline fleet assignment: Imposing station purity using station decomposition.

[Transportation Science](#), 40(4):497–516, 2006.



B. C Smith.

Robust airline fleet assignment.  
2004.



H. D Sherali and X. Zhu.

Two-stage fleet assignment model considering stochastic passenger demands.

[Operations research](#), 56(2):383–399, 2008.

## Bibliography XVIII



K. T Talluri.

Swapping applications in a daily airline fleet assignment.  
[Transportation Science](#), 30(3):237–248, 1996.



K.T. Talluri and G. Van Ryzin.

[The theory and practice of revenue management](#), volume 68.  
Springer Verlag, 2005.



X. Wang and A. Regan.

Dynamic yield management when aircraft assignments are subject to swap.  
[Transportation Research Part B: Methodological](#), 40(7):563–576,  
2006.

# Bibliography XIX



S. Yan and C. H Tseng.

A passenger demand model for airline flight scheduling and fleet routing.

[Computers & Operations Research](#), 29(11):1559–1581, 2002.