PIPES & CISTERNS

Inlet

A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet

A pipe connected with a tank or cistern or reservoir, emptying it, is known as an outlet.

- ✓ If a pipe can fill a tank in x hours, then

Part of tank filled in 1 hour $= \frac{1}{x}$

- ✓ If a pipe can empty a tank in y hours, then

Part of tank emptied in 1 hour $= \frac{1}{y}$

Rules and Tricks:

Rule 1:

Two pipes A and B can fill (or empty) a cistern in X and Y hours respectively, while working alone. If both the pipes are opened together, then the time taken to fill (or empty) the cistern is given by

$$\left\{\frac{XY}{X+Y}\right\} \textbf{ hours.}$$

Example:

Two pipes A and B can fill a cistern in 20 and 30 minutes respectively. If both the pipes are opened together, how long will it take to fill the cistern?

Solution 1:

Let's say x = 20 and y = 30, then

$$\frac{20 * 30}{20 + 30} = 600/50 = 12 \text{minutes.}$$

So it will take 12 minutes for both the pipes to full the cistern.

Solution 2: (UNITARY METHOD)

Let the total work be assumed as LCM(20,30)= 60 Units

Now to complete 60 units A takes 20 mins

To complete 60 units B takes 30 mins

Units done in 1 min by A = 3

Units done in 1 min by B = 2

Units done in 1 min by A & B = 5

To complete 60 units they will take 60/5 = 12 minutes.

Rule 2:

Three pipes can fill (or empty) a cistern in X, Y and Z hours while working alone. If all the three pipes are opened together, the time taken to fill (or empty) the cistern is given by

$$\left\{\frac{XYZ}{XY + YZ + XZ}\right\}$$ **hours.**

Example:

Three pipes A,B and C can fill a tank in 20 minutes, 30 minutes and 40 minutes respectively while working alone. If, all the pipes are opened together, how long will it take to fill the tank full?

Solution 1:

Let's say X = 20 minutes, Y = 30 minutes, Z = 40 minutes, then

$$= \frac{20 * 30 * 40}{(20*30) + (30*40) + (20*40)} = 9.23 \text{ mins}$$

So it will take 9.23 minutes to fill the tank full.

Solution 2:(UNITARY METHOD)

Let the total work be assumed as LCM(20,30,40)= 120 Units

Now to complete 120 units A takes 20 mins

To complete 120 units B takes 30 mins

To complete 120 units C takes 40 mins

Units done in 1 min by A = 6

Units done in 1 min by B = 4

Units done in 1 min by C = 3

Units done in 1 min by A+B+C = 13

To complete 120 units they will take 120/13 = 9.23 minutes.

Rule 3:

If a pipe can fill a cistern in X hours and another can fill the same cistern in Y hours, but a third one can empty the full tank in Y hours, and all of them are opened together, then

Net part filled in 1 hour = $1/X + 1/Y - 1/Z$

Time taken to fill the full cistern =

$$\frac{XYZ}{YZ + XZ - XY} \text{ hours.}$$

Example:

Two pipes can fill a cistern in 20 minutes and 30 minutes respectively. Third pipe can empty the tank in 40 minutes. If all the three pipes are opened together, how long it will take to fill the tank full?

Solution:

Let's say x = 20, y = 30 and z = 40

$$= \frac{20 * 30 * 40}{(30*40) + (20*40) - (20*30)} = 17.14 \text{ min}$$

So it will take 17.14 minutes to fill the tank full.

Question

Pipe A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in :

A $1\frac{13}{17}$ hours

B $2\frac{8}{11}$ hours

C $3\frac{9}{17}$ hours

Answer: Option **C**

Explanation:

Net part filled in 1 hour $\frac{1}{5} + \frac{1}{6} - \frac{1}{12} = \frac{17}{60}$

The tank will be full in $\frac{60}{17}$ hours or $3\frac{9}{17}$ hours

Shortcut

- LCM (5,6,12)= 60=Number of Chocolates
- Efficiency Of A=60/5=12 Chocolates per hour
- Efficiency of B=60/6=10 Chocolates per hour
- Efficiency of C=60/12= - 5 Chocolates per hour
- Why minus 5 ? Because outlet pipe C is not finishing the task it is increasing the task to fill the tank or it is not consuming the chocolates it is putting the 5 chocolates per hour
- **So where ever outlet pipe is used take efficiency with negative sign**
- Now working together A+B+C=12+10-5=17 Chocolates per hour so to complete 60 Chocolates all together will take 60/17 hours

Question

Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?

A. 10 min 20 seconds

B. 11 min 45 seconds

C. 12 min 30 seconds

D. 14 min 40 seconds

Answer: Option D

Explanation:

Part filled in 4 minutes = $4\left(\frac{1}{15}+\frac{1}{20}\right)=\frac{7}{15}$

Remaining Part = $1-\frac{7}{15}=\frac{8}{15}$

Part filled by B in 1 minutes = $\frac{1}{20}$

1/20 th part in 1 min
1 part in 20 min
8/15 part in 20*8/15= 32/3 min=10 min 40 seconds
So total time 10 min 40 seconds+ 4 minutes=14 minutes 40 seconds Ans

Shortcut

- LCM(15,20)=60 =Number of Chocolates
- Efficiency of A= 60/15=4 Chocolates per minute
- Efficiency of B= 60/20=3 Chocolates per minute
- A and B working together can consume 4+3=7 Chocolates per minute so in 4 minutes =7*4=28 Chocolates
- Pending number of Chocolates =60-28=32 Chocolates which must be consumed by B
- since B consume 3 Chocolates per minute so B will take 32/3 minutes=10 minutes 40 seconds
- Hence Total time 10 minutes 40 seconds + 4 minutes=14 minutes 40 SecondsAns

Question

Two pipes can fill a tank in 12 hours and 15 hours respectively while a third pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time the tank will be filled?

A.5 hours

B.10 hours

C.15 hours

D. 20 hours

Answer: Option B

Explanation:

Reciprocal of (1/12 + 1/15 - 1/20 = 6/60 =1/10) = 10 Ans

OR
by Chocolate Rule
LCM (12,15,20)=60=Number of Chocolates
Efficiency of A =60/12=5Chocolates per hour
Efficiency of B =60 /15 = 4Chocolates per hour
Efficiency of C (Outlet pipe)= 60/20 = -3 Chocolates per hour

(A+B+C)= 5+ 4 – 3 =6 Chocolates per hour

So working together they will take 60/6 = 10 Hour

Question

Two pipes can fill a tank in 12 hours and 15 hours respectively while a third pipe empties the full tank in 20 hours. Pipe A and B open together at 6:00 AM and C is also open at 10:00 AM, then the tank will be filled at what time?

A.12 PM

B. 2 PM

C. 3 PM

D. 4 PM

- LCM (12,15,20)= 60 =Number of Chocolates
- Efficiency of A = 60/12=5 Chocolates per hour
- Efficiency of B = 60 /15 = 4Chocolates per hour
- Efficiency of C (outlet pipe)= 60/20 = -3 Chocolates per hour
- A and B worked for four hours will consume = (5+4)*4=36 C
- So out of 60 now only 60-36=24 Chocolates left which must be consumed by all together i.e A and B and C
- Working together they all consume 5+4-3=6 C per hour
- So they will take 24/6 = 4 hours after 10 am so tank will be filled by 2 pm ans

- Two pipes can fill a tank in 12 hours and 15 hours respectively while a third pipe empties the full tank in 20 hours. They all open together at 6:00AM and C was closed 2 hours before the completion of work. How much time it will take to fill the tank
- A 9 Hours B 10 Hours C 11 Hours D 5 Hours

- LCM (12,15,20)= 60 =Number of Chocolates
- Efficiency of A = 60/12=5 Chocolates per hour
- Efficiency of B = 60 /15 = 4Chocolates per hour
- Efficiency of C (outlet pipe)= 60/20 = -3 Chocolates per hour
- In last 2 hour only A and B was working so they must have consumed (5+4)*2=18 Chocolates
- Which means earlier while working together they all must have consumed 60-18=42 Chocolates
- Working together they all consume 5+4-3=6 C per hour
- So they must have taken 42/6 = 7 hours earlier
- which means total number of hours =7 + 2 = 9 Hours Ans

Rule 4:

A pipe can fill a cistern in x hours. Because of a leak in the bottom, it is filled in y hours. If it is full, the time taken by the leak to empty the cistern is

$$\frac{XY}{Y - X} \text{ hours.}$$

Example:

A pipe can fill a tank in 3 hours. Because of leak in the bottom, it is filled in 4 hours. If the tank is full, how much time will the leak take to empty it?

Solution:

Work done by leak in one hour = 1/3 – 1/4 = 1/12

So leak will empty the tank in 12 hours.

By formula

Let's say x = 3 and y = 4

$$= \frac{3*4}{4-3} = 12 \text{ hours.}$$

Rule 5:

A cistern has a leak which can empty it in X hours. A pipe which admits Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hours. The capacity of the cistern is

$$\left\{\frac{XYZ}{Z-X}\right\}$$ litres.

Example:

A leak in the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 litres per minute. When the tank is full, the inlet is opened and due to leak, the tank is empty in 8 hours. Find the capacity of the tank.

Solution:

Here, X=6, Y =4*60 =240 and Z = 8.

The capacity of the tank is

$$= \frac{6 * 240 * 8}{8 - 6} = 5760 \text{ litres.}$$

Another Shortcut

- Inlet pipe + Outlet pipe working together takes = 8 hours
- Only Outlet pipe takes 6 Hours
- So only Inlet Pipe = X * Y / Y-X
- = 8*6 / 8-6=24 Hours
- So capacity of tank = 4 * 60 * 24 =5760 Litres

Rule 6:

One fill pipe A is K times faster than the other fill pipe B. If B can fill a cistern in X hours, then the time in which the cistern will be full, if both the fill pipes are opened together, is

$$\left\{\frac{X}{K+1}\right\} \text{ hours.}$$

Example:

One fill pipe A is 10 times faster than second fill pipe B. If B can fill a cistern in 55 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

Solution:

Here, K =10 and X = 55

$$= \frac{55}{10 + 1} = 5 \text{ mins.}$$

- If B takes 55 minutes
- A is 10 times faster than B so A will take = 55/10 minutes
- Working together A and B will take

reciprocal of (1/55 + 1/55/10=11/55=1/5) = 5 minutes or even you can apply (X*Y / X+Y)

Rule 7:

One fill pipe A is K times faster than the other fill pipe B. If A can fill a cistern in X hours, then the time in which the cistern will be full, if both the fill pipes are opened together, is

$$\left\{\frac{K}{K+1}\right\} X \quad \text{hours.}$$

Example:

One fill pipe A is 4 times faster than second fill pipe B. If A can fill a cistern in 15 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

Solution:

Here, K = 4 and X = 15

$= (4/ 4 + 1) 15$

$= 12$ minutes.

- A takes 15 Minutes
- Since A is 4 times faster than B so B takes=60 minutes
- Working together A and B will take reciprocal of (1/15 + 1/60= 5/60) 1/12 =12 minutes
- Or even you can apply X*Y/X+Y

Question

A tank is filled in 5 hours by three pipes A,B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?

A. 20 hours

B. 25 hours

C. 35 hours

D. None of these

Answer: Option **C**

Explanation:

Suppose pipe A alone takes x hours to fill the tank.

Then pipe B and C will take $\frac{x}{2}$ and $\frac{x}{4}$ hours respectively to fill the tank.

So, $\frac{1}{x} + \frac{2}{x} + \frac{4}{x} = \frac{1}{5}$

$\frac{7}{x} = \frac{1}{5}$ and x = 35 hours

Question

Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time A will take to fill the cistern separately?

A. 1 hour

B. 2 hours

C. 6 hours

D. 8 hours

Answer: Option C

Explanation:

Let the cistern filled by A alone in x hours.

Then, B will fill it in (x + 6) hours

So, $\frac{1}{x} + \frac{1}{x+6} = \frac{1}{4}$ and $\frac{x+6+x}{x(x+6)} = \frac{1}{4}$

$$x^2 - 2x - 24 = 0$$

$$(x - 6)(x + 4) = 0$$