

Analyzing Risk Impact Factors Using Extended Fuzzy Cognitive Maps

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Abstract—One of the challenges in *Risk Analysis and Management* (RAM) is identifying the relationships between risk factors and risks. The complexity of the method to analyze these relationships, the time to complete the analysis, the robustness and trustworthiness of the method are important features to be considered. In this paper, we propose using *Extended Fuzzy Cognitive Maps* (E-FCMs) to analyze the relationships between risk factors and risks, and adopting a pessimistic approach to assess the overall risk of a system or a project. E-FCMs are suggested by Hagiwara to represent causal relationships more naturally. The main differences between E-FCMs and conventional *Fuzzy Cognitive Maps* (FCMs) are the following: E-FCMs have nonlinear membership functions, conditional weights, and time delay weights. Therefore E-FCMs are suitable for risk analysis as all features of E-FCMs are more informative and can fit the needs of Risk Analysis. In this paper we suggest a framework to analyze risks using E-FCMs and extend E-FCMs themselves by introducing a special graphical representation for risk analysis. We also suggest a framework for group decision making using E-FCMs. Particularly, we explore the *Software Project Management* (SPM) and discuss risk analysis of SPM applying E-FCMs.

Index Terms— Fuzzy cognitive maps, project management, risk analysis.

I. INTRODUCTION

THE most important factors contributing to the risk of failure for any type of organization or system are related to poor performance, time pressure, low quality and high cost. Every year there is evidence of a large number of cases where a project or an organization fails, and the consequences of those failures are sometimes fatal. Notwithstanding all the technological advances and a huge amount of risk analysis approaches, risks associated with a project or a system cannot yet be analyzed precisely and the risk analysis itself still contains high risks.

The major problem associated with the estimation of risks is that the input data are imprecise by nature and it is difficult to represent them with crisp numbers. Usually the risk analyst prefers to estimate in linguistic terms such as *High* or *Low* rather

than in exact probabilistic terminology. To this end, the application of *Fuzzy Set Theory* (FST) to risk analysis seems appropriate, as such analysis can handle subjectivity as well as inexact and vague information.

The need for establishing FST is the fact that there is a huge amount of problems that are difficult to express in exact mathematical terms as they contain ambiguity and uncertainty. The basic idea of the fuzzy approach is to allow an element to belong to a set with degrees of membership ranging in the continuous real interval $[0, 1]$, rather than in the set $\{0, 1\}$. The use of fuzzy descriptions corresponds to the vagueness that can always be found in risk analysis and management. The review of FST is out of the scope of this paper. The interested reader can refer to [1], [2].

Research in risk analysis and management exploiting fuzzy logic has produced several different models in the recent years. In [3] the authors summarize the application areas (information technology, environment, engineering, bank, tourism, e-commerce, etc) where risk analysis is done applying fuzzy logic concepts. In [4] the authors use fuzzy logic to provide a strategy for reducing the possibility of adopting insufficient risk reduction measures. In [5] there was the first attempt to have a computerized risk analysis model based on fuzzy logic. Afterwards, there were several attempts to have a risk analyzer based on fuzzy logic like in [6], [7]. In [8], risk analysis in decision making is discussed using ranking of generalized fuzzy numbers. However, as far as we found, there are very few approaches that are representative enough to be used for complex problems. Most of the available methods are mainly case-specific targeting a particular field and concentrating on certain types of risks.

We believe that FCMs are powerful tools for RAM as one of the most important features of FCMs is their capability to be used in decision support as prediction tool. Given an initial state of a system, an FCM is able to simulate and predict the behavior of a system over time. However there are some drawbacks of FCMs that make them less attractive for RAM. Particularly, there is a lack of time concept, the relationships between nodes are linear, and there is no possibility to have conditional relationships. These drawbacks are solved by introducing E-FCMs. In this paper we suggest using E-FCMs for describing the relationships between risk factors and risks concerning *low performance*, *time delay*, *low quality* and *high cost*. We also suggest a framework for group decision making using E-FCMs. Once we identify the four mentioned risks, we use a pessimistic approach to evaluate the overall risk [8] of a project or a system. Particularly, we discuss our approach for SPM, which is one of the most risky industries nowadays. To the best of our knowledge there is no previous research that uses E-FCMs for RAM though

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there are several examples of exploiting FCMs for RAM like in [9], [10]. We believe that using E-FCMs has higher modeling power and captures real life cases more precisely.

The rest of this paper is organized as follows. In Section II we introduce the theoretical background of Cognitive Maps (CMs), FCMs and E-FCMs, in Section III we give an illustrative example of the application of E-FCMs for SPM, in Section IV we detail our approach to group decision making using E-FCMs, in Section V we explain the pessimistic assessment of the overall risk, in Section VI we provide an application example, and finally we give concluding remarks in Section VII.

II. FUZZY COGNITIVE MAPS

In this section we introduce the basic concepts of FCMs and outline their drawbacks. First we briefly discuss CMs focusing on their use for risk analysis. CMs were first introduced by Axelroide [11] who focused on the policy domain. Since that time many researchers have used CMs applying them in various fields where the problems are ill-structured or not well-defined.

The application areas of CMs or FCMs are very broad and diverse, and, in general, CMs can be used in any kind of decision making problems (e.g., failure system modeling [12], network security [13], electrical circuits [14], environmental management [15], [16], information management [10], [17]–[19], risk analysis and management [9], [20], social sciences [11], industry applications [21], [22]).

A CM has two types of elements: *Concepts* and *Causal Beliefs*. The former are variables while the latter are relationships between variables. Causal relationships can be either positive or negative, as specified by a “+,” respectively a “–,” sign on the arrow connecting two variables. The variables that cause a change are called *cause variables* and the ones that undergo the effect of the change are called *effect variables*. If the relationship is positive, an increase or decrease of the cause variable causes the effect variable to change in the same direction (e.g., an increase in the cause variable causes increase of the effect variable). In the case of negative relationship, the change of the effect variable is in the opposite direction (e.g., an increase in the cause variable causes decrease of the effect variable).

Fig. 1(a) shows a simple cognitive map in a public health domain [23]. If we consider the relationship between C_5 and C_6 , C_5 is a *cause variable* whereas C_6 is an *effect variable*, so that the increase/decrease in C_5 will cause the decrease/increase in C_6 . Indeed, the increase of *Sanitation facilities* decreases *Number of diseases per 1000 residents*, and vice versa. On the other hand, if we consider the relationship between C_4 and C_7 , in this case the increase/decrease of *cause variable* C_4 will cause the increase/decrease of *effect variable* C_7 . In fact, increasing *Garbage per area* causes increase of *Bacteria per area*.

A path between two variables in a cognitive map is a sequence of connected nodes [11].

In his work Axelroide introduces also *Weighted CMs* and *Functional CMs*. In *weighted cognitive maps* the sign in the map is replaced with a positive or a negative number, which shows the direction of the effect as well as its magnitude. *Functional cognitive maps* are CMs in which a function is associated with each causal relationship showing more precisely

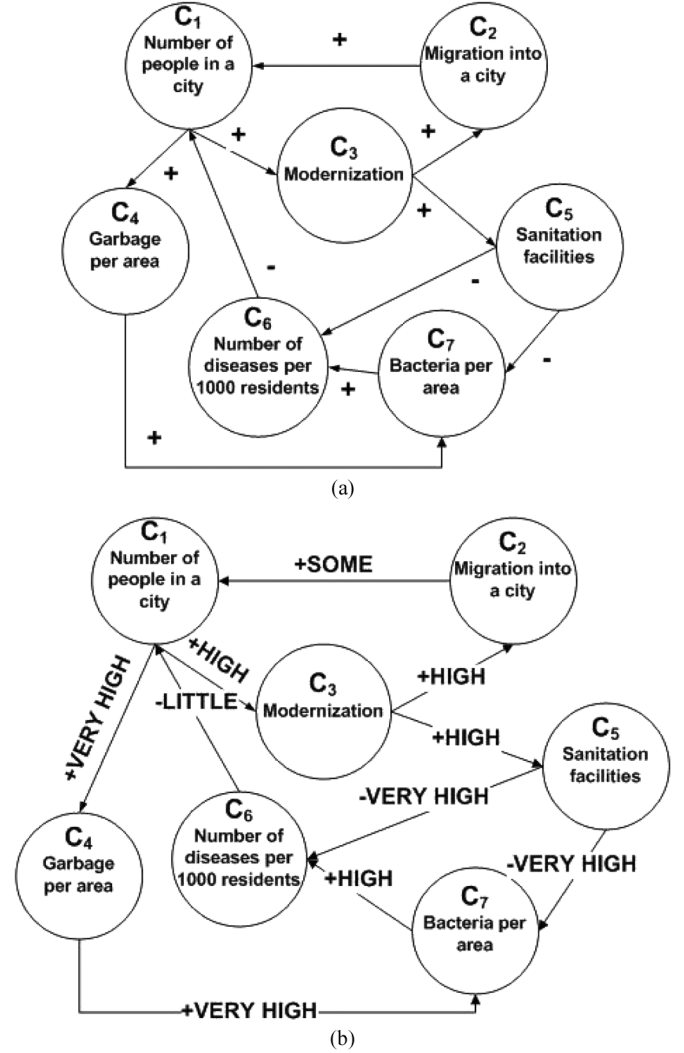


Fig. 1. An illustration of (a) a CM and (b) an FCM.

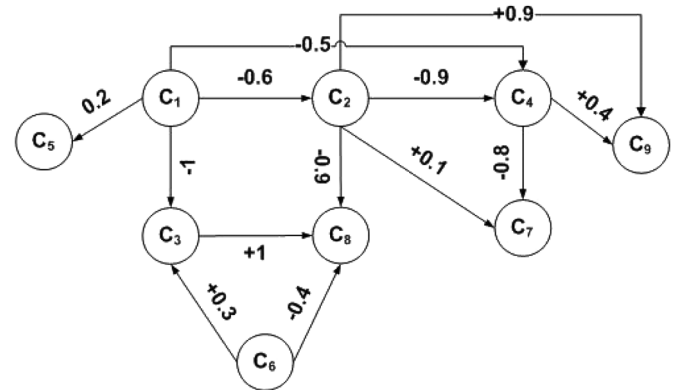


Fig. 2. An example of a map with complex structure.

the direction and the magnitude of the effect. These two types of CMs give more flexibility as they can handle and provide more detailed information.

However, CMs, whatever their type, are not easy to define and the magnitude of the effect is difficult to express in numbers. Usually CMs are constructed by gathering information from experts and generally experts are more likely to express themselves in qualitative rather than quantitative terms. To this

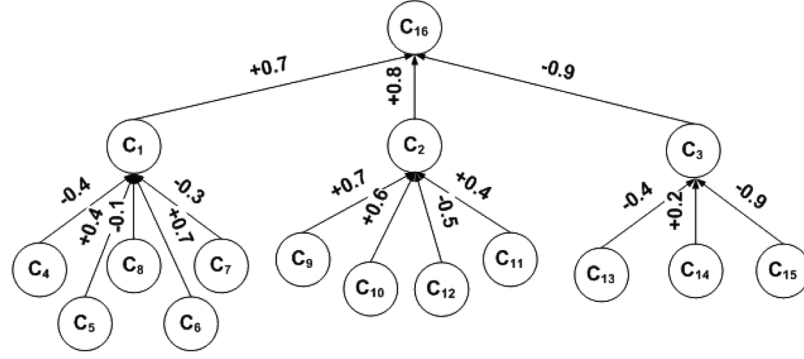


Fig. 3. An example of a map with hierarchical structure.

end, it may be more appropriate to use Fuzzy CMs (FCMs), suggested by Kosko [24] to represent the concepts linguistically with an associated fuzzy set. Actually, FCMs are weighted cognitive maps with fuzzy weights.

The degree of relationship between concepts in an FCM is either a number in $[0, 1]$ or $[-1, 1]$, or a linguistic term, such as *often*, *extremely*, *some*, etc. Fig. 1(b) shows a simple example of FCM where the causal relationships are expressed by using fuzzy linguistic terms. For example, if we consider again the relationship between C_5 and C_6 , the increase/decrease of cause variable C_5 will cause *very high* decrease/increase in C_6 .

In the following we give the main definitions and concepts of CMs and FCMs [25].

Definition: An FCM is a signed directed graph that allows feedback and employs concepts (nodes) and weighted edges (arcs) between concepts.

Definition: Consider the nodes C_1, C_2, \dots, C_n of the CM/FCM. The matrix $E = (e_{ij})$, where e_{ij} is the weight of the directed edge $C_i \rightarrow C_j$, is called *adjacency* or *connection* matrix of the CM/FCM.

The following matrix shows the adjacency matrix of the *public health* example with reference to Fig. 1(a):

$$E = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{matrix} & \begin{bmatrix} 0 & 0 & +1 & +1 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 \end{bmatrix} \end{matrix}$$

Notice that all matrices associated with CMs/FCMs are always square matrices with diagonal entries equal to zero.

Definition: Let C_1, C_2, \dots, C_n be the nodes of a CM/FCM. $A = (a_1, a_2, \dots, a_n)$, where $a_i \in \{0, 1\}$, is called the *instantaneous state vector* and denotes the on-off position of the nodes at any instant; $a_i = 0$ is an off state while $a_i = 1$ is an on state for $i = 1, \dots, n$.

Definition: The *conceptual centrality* of a node C_i is denoted by $CEN(C_i)$ and is defined by $CEN(C_i) = IN(C_i) + OUT(C_i)$, where $IN(C_i) = \sum_{k=1}^n e_{ki}$ and $OUT(C_i) = \sum_{k=1}^n e_{ik}$.

The *conceptual centrality* represents the importance of the node for the causal flow on the cognitive map.

Definition: A finite number of CMs/FCMs with the same nodes can be combined together to produce the joint effect of all the CMs/FCMs. Let E_1, E_2, \dots, E_p be the adjacency matrices of the p CMs/FCMs with nodes C_1, C_2, \dots, C_n ; then the combined map is got by adding all the adjacency matrices. The adjacency matrix E of the joint maps will be $E = E_1 + E_2 + \dots + E_p$.

Definition: A node C_i is called *transmitter* if $IN(C_i) = 0$ and $OUT(C_i) > 0$, and is called *receiver* if $IN(C_i) > 0$ and $OUT(C_i) = 0$.

The total number of receiver nodes in a map is considered an index of its *complexity*. A large number of receiver variables indicates a complex map (Fig. 2), while a large number of transmitters shows a formal hierarchical system (Fig. 3) where causal nodes do not collaborate with each other [15].

The CMs/FCMs pertinent to the same problem but drawn by different experts may be different. Each expert can be assigned a weight in the $[0, 1]$ range that shows the importance, experience or trustworthiness of the expert.

In general, if the CM/FCM adjacency matrices are different (in the sense that they include different concepts), each matrix may be augmented by including any missing concept(s) through the addition of extra rows and columns of all zeros. The final matrix, representing the group opinion, becomes

$$E = \sum_{i=1}^m w_i \cdot E_i \quad (1)$$

where m is the number of experts, $w_i \geq 0$ shows the i th expert's weight, and E_i is the adjacency matrix of the map provided by the i th expert.

Suppose two experts evaluate the same problem and the maps provided by them differ by sizes, by nodes and by their structures as in Fig. 4. Expert e_1 provided six nodes whereas expert e_2 evaluated the relationships only among five nodes (which are a subset of the first map's nodes). Table I shows the adjacency matrices of each expert according to their maps.

Note that as expert e_2 did not consider node c_6 , in the augmented matrix the sixth row and column will be filled with zeros.

Assume that the two experts have different weights: $w_{e_1} = 0.4$ and $w_{e_2} = 0.6$. Table II shows the aggregated adjacency matrix by using (1), and Fig. 5 shows the final map generated from Table II.

Among several ways of developing CMs and FCMs, the most common methods are

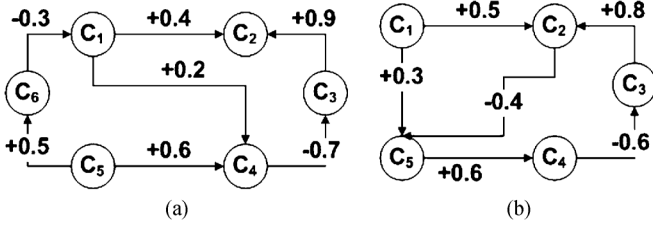


Fig. 4. Evaluations of the same problem provided by two different experts (C_i , $i = 1, \dots, 5$, in the two figures refers to the same concept). (a) FCM by expert e_1 . (b) FCM by expert e_2

TABLE I
ADJACENCY MATRICES OF TWO EXPERTS FROM Fig. 4:
(a) EXPERT e_1 . (b) EXPERT e_2

(a)

	c_1	c_2	c_3	c_4	c_5	c_6
c_1	0	0.4	0	0.2	0	0
c_2	0	0	0	0	0	0
c_3	0	0.9	0	0	0	0
c_4	0	0	-0.7	0	0	0
c_5	0	0	0	0.6	0	0.5
c_6	-0.3	0	0	0	0	0

(b)

	c_1	c_2	c_3	c_4	c_5
c_1	0	0.5	0	0	0.3
c_2	0	0	0	0	-0.4
c_3	0	0.8	0	0	0
c_4	0	0	-0.6	0	0
c_5	0	0	0	0.6	0

TABLE II
AGGREGATED RESULTS OF TWO EXPERTS' OPINION AS A GROUP

	c_1	c_2	c_3	c_4	c_5	c_6
c_1	0	0.46	0	0.08	0.18	0
c_2	0	0	0	0	-0.24	0
c_3	0	0.84	0	0	0	0
c_4	0	0	-0.64	0	0	0
c_5	0	0	0	0.6	0	0.2
c_6	-0.12	0	0	0	0	0

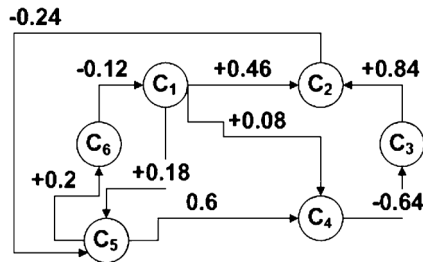


Fig. 5. Group map generated from Table II.

- extracting knowledge from questionnaires;
- extracting knowledge from written texts;
- conducting interviews;
- drawing maps from data.

To obtain a cognitive map from questionnaires requires to first identify the most important variables for the given problem, and then give experts ordered pairs of variables in a questionnaire format. Afterward, experts decide the strength of causal links relying on their knowledge and experience.

The second method is a type of content analysis in which the causal relationships are identified by analyzing texts. The main

problem related to this method is that usually the relationships are not explicitly stated, and the language structure in which texts are written can vary from one language to another.

The detailed description as how to construct a cognitive map through interviews is provided in [15]. The methodology is composed of the following steps: decide the most important variables, provide experts with a sample map, unrelated with the problem at hand, and finally, ask them to draw their own maps of the issue under investigation.

The automatic construction of cognitive maps (particularly FCMs) based on user provided data is discussed in [26]. This method first finds the degree of similarity between any two variables, then decides whether the relation between the two variables is direct or inverse, and with the use of the fuzzy expert system tool determines the causality among variables.

a. Extended Fuzzy Cognitive Maps

In [27] Hagiwara mentions three important drawbacks of FCMs:

- 1) connections of FCMs are just linear;
- 2) there is a lack of time modeling (i.e., it is not possible to model the fact that different causal effects can have different time delay);
- 3) FCMs cannot deal with co-occurrence of multiple causes such as those expressed by “and” conditions.

To overcome the above mentioned drawbacks, Hagiwara [27] suggested the extension of FCMs including three additional features:

- 1) weights with nonlinear membership functions;
- 2) time delay weights;
- 3) conditional weights.

In E-FCMs, the total input to node C_j at time t can be expressed as follows:

$$net_j = \sum_{i=1}^n w_{ij} (C_i(t - delay_{ij})) C_i(t - delay_{ij})$$

where $C_i(t)$ is a causal concept at time t , $w_{ij}(\cdot)$ is a weight function from concept $C_i(t)$ to concept $C_j(t)$, and $delay_{ij}$ is a time delay from concept $C_i(t)$ to concept $C_j(t)$.

Fig. 6 [27] shows an example of E-FCM with reference to Fig. 1 (the only difference between the two maps is the existence of the node *Government*) where α is a nonlinear weight, β is a conditional weight and γ is a time delay weight. Indeed, if we increase/decrease *Number of people in a city* (C_1) the *Modernization* (C_3) will be increased/decreased as well, however, this relationship cannot be linear as the further change in cause node (C_1) cannot have further influence in effect node (C_3). If the *Government* (C_8) controls *Migration into a city* (C_2), and then if *Modernization* (C_3) advances then *Migration into a city* increases (C_2). Finally, the time delay relationship exists only between C_5 and C_6 . In fact, the change in *Sanitation facilities* (C_5) will effect the *Number of diseases per 1000 residents* only after some time delay.

In the next section we present E-FCMs applied to risk analysis in the domain of SPM.

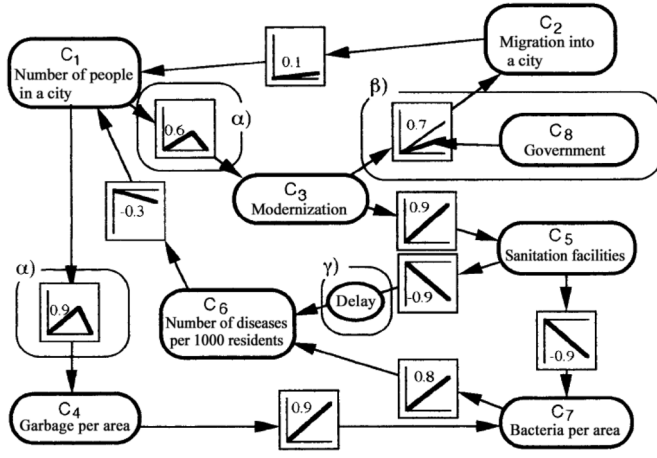


Fig. 6. EFCM of public health domain.

III. E-FCMs FOR SPM

SPM is considered one of the most risky industries today as risks exist in SPM in all stages of the product development, starting from specification documents and ending with product delivery to the client. In SPM one of the main issues concerns the reliability of the project in terms of project cost, time to complete the project, quality and performance. These features of any project are influenced by many factors and the relationships between them in the majority of the cases are very complex.

In [28] the authors used taxonomy-based questionnaires to identify the risk factors. They organized the software taxonomy in three classes:

- 1) *Product Engineering*, which includes technical aspects of the work to be accomplished;
- 2) *Development Environment*, which includes the methods, procedures, and tools to produce the product;
- 3) *Program Constraints*, which include contractual, organizational, and operational factors within which the software is developed but which are generally outside the direct control of the local management.

In this paper we refer to their taxonomy of risks of software development as we consider that it provides quite comprehensive information about all risk factors. For the application example we break down the taxonomy into a more detailed hierarchy as suggested in [29]. We used this taxonomy to analyze the impact of the risk factors in each class of the taxonomy on the overall cost, time delay, performance and quality of the product. As we mentioned before, we used E-FCMs to describe the relationships between risk factors and risks, which in our case are cost, time, performance and quality.

In Fig. 7 we show a small example of using E-FCMs for SPM. We considered only the risk factors concerning the resources (i.e., human resources, financial resources, etc.). We introduce special symbols that we will use for RAM. We use different symbols for risks and risk factors as well as different arrows for describing the time delay, conditional and nonlinear weights. We have to mention that for the simplicity of the diagram we did not draw all relationships but only some of them. Furthermore, we simply associate a ‘+’/‘-’ sign with each arrow.

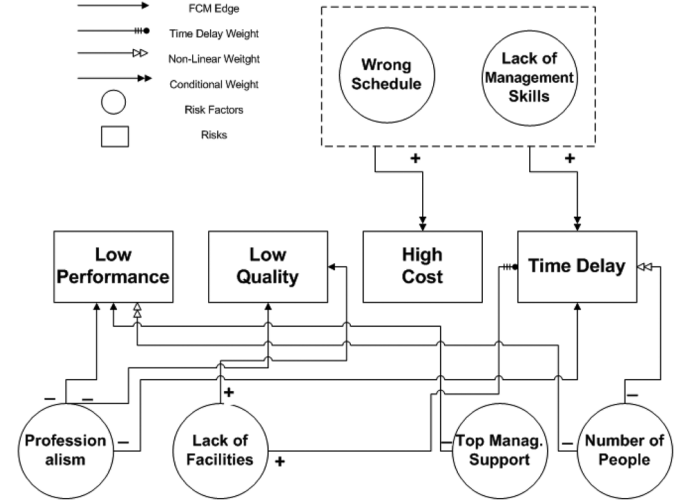


Fig. 7. E-FCM for SPM.

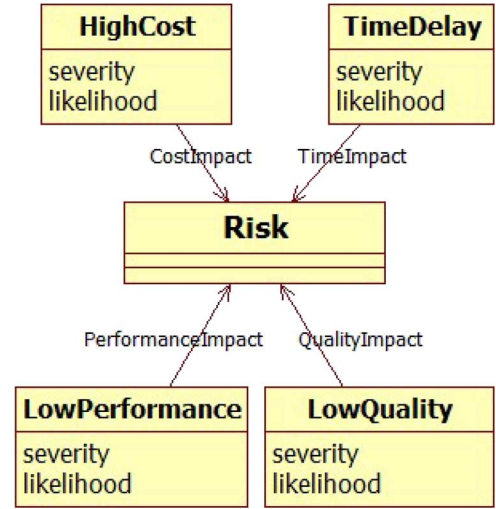


Fig. 8. System high level design.

TABLE III
FUZZY SETS FOR LIKELIHOOD OF RISK FACTORS, AND
CORRESPONDING LINGUISTIC TERMS

Linguistic Variables	Triangular Fuzzy Sets
Very low (VL)	(0, 0, 0.25)
Low (L)	(0, 0.25, 0.5)
Medium (M)	(0.25, 0.5, 0.75)
High (H)	(0.5, 0.75, 1.0)
Very High (VH)	(0.75, 1.0, 1.0)

As we can see we have one *time delay weight*, two *conditional weights* and two *nonlinear weights*. Below we discuss them in more details.

- Time delay weights: risk factor *Lack of Facilities* will not necessarily have immediate effect on the risk *Time Delay* but it will affect after some time delay. Indeed, if, for instance, the computers that are used to develop software are old, the immediate effect on time delay will not be so obvious, but in the long term it will definitely cause an increase of risk *Time Delay*. Notice that the same observation holds also for performance, cost and quality.
- Non-linear weights: if we increase the risk factor *Number of People*, initially it may help to meet deadlines, but if we

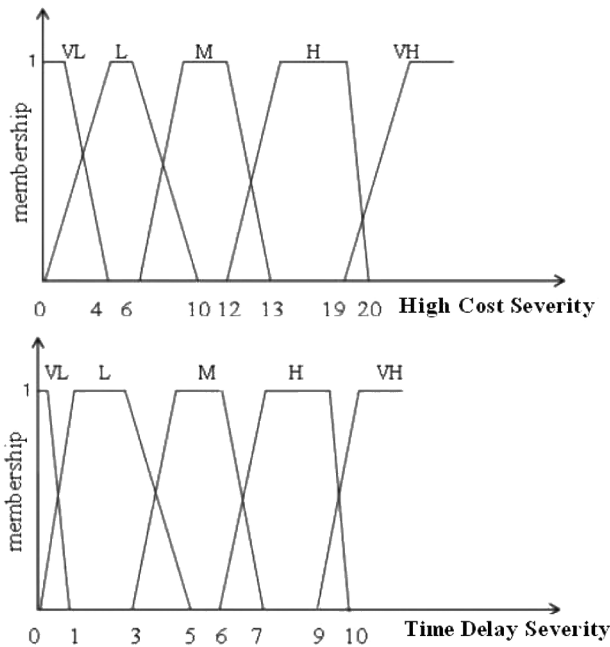


Fig. 9. A possible definition of linguistic variables for ‘The Severity of High Cost’ and ‘The Severity of Time Delay’ (VL = Very Low, L = Low, M = Medium, H = High, VH = Very High).

TABLE IV
A POSSIBLE REPRESENTATION OF RISK IMPACT CALCULATION
BASED ON RISK FACTOR SEVERITY AND LIKELIHOOD

	Sev.				
Likel.	Very Low	Low	Medium	High	Very High
Very Low	Ignorable	Ignorable	Ignorable	Low	Low
Low	Ignorable	Ignorable	Low	Medium	Medium
Medium	Ignorable	Low	Medium	High	High
High	Low	Medium	High	High	Critical
Very High	Low	Medium	High	Critical	Critical

increase more than required, it might not help anymore, and might even lead to the opposite result. Therefore, the relationship here should be nonlinear. Notice that the same observation holds for the performance.

- Conditional weights: if initially there is a *Wrong Schedule* and if there is also *Lack of Management Skills* then they will affect the *Time Delay* as well as the *High Cost*. Notice that we categorize them as conditional weights because they affect only if they both happen. For example, if the schedule is wrong, but on the other hand, the management is very experienced to handle those extreme situations, the effect would surely be different.

After constructing E-FCMs, we define a fuzzy rule-based model for the two attributes of risk: *likelihood* of occurrence and *severity* of impact. In other words, we use E-FCM to define fuzzy rules related to *likelihood* and *severity*.

Constructing fuzzy rules requires intensive and comprehensive knowledge of the SPM area. This knowledge can be achieved by asking the experts of the field and/or the experienced managers. The aim is to find the best set of rules that describe the overall behavior of the SPM and especially meet the specific needs of the given software company or the given

software project. Having E-FCMs helps to define fuzzy rules more precisely as they represent a more comprehensive view of a system or a project.

IV. GROUP DECISION MAKING USING E-FCMs

One of the main advantages of using FCMs is that they provide a framework to collect information from the experts of a given domain and to have a more realistic decision based on experts’ knowledge. In [30] the authors suggest a framework to combine different FCMs produced by different experts to design a final FCM which is used for decision making.

In this paper we suggest a framework to have group decision making based on E-FCMs. We discuss the use of *Extended Group Fuzzy Cognitive Maps* (E-GFCMs) for RAM but it is obvious that the suggested framework can be used in any kind of decision process that involves opinions of several experts.

The experts are assigned different weights which show their experience and knowledge in the given domain. Suppose we have n experts e_i with the corresponding weights w_i . For large projects, each E-FCM can represent the opinion of an entire group or entire department. For simplicity we assume that one E-FCM represents the opinion of one expert who, in his turn, can represent a group or a department (financial, management, marketing, etc).

Each expert draws an E-FCM first defining the concepts that are important for the given problem, then designing the causal relationships between the concepts based on his experience and knowledge. Each expert is free to use any linguistic term to describe concepts and to design as many concepts as he considers necessary to completely represent the given domain. The causal strengths can be assigned by using linguistic terms or numbers depending on the convention among group members.

The first draft of E-GFCM is constructed by putting together all E-FCMs. To this aim, we start identifying and merging the repeated concepts; these concepts may be written in different terms but in reality have the same meaning. We then insert all the concepts representing risks into the E-GFCM.

For each risk in the E-GFCM, we identify all the risk factors present in at least one E-FCM. Then, for each causal relationship involving a risk and one of its risk factors, we check whether the type of relation is the same or not in all the E-FCMs. In the former case, a relation of the same type is retained in the global E-GFCM, otherwise we have to decide the specific type of the relation.

Indeed, different experts may have different ideas about the type of each edge. For instance, an expert might use time delay weight to express a given cause-effect relationship, while another might adopt a traditional FCM edge to represent the same relationship. To solve conflicting opinions, we base ourselves on heuristic considerations and adopt the following strategy to decide the type of causal relationship in the final E-GFCM:

- 1) a relationship is considered linear or nonlinear essentially based on the weights and/or the number of experts expressing the same opinion (we will better explain this point in the application example in Section VI);
- 2) a relation is considered time-delayed or not based on the same considerations as in previous point 1);

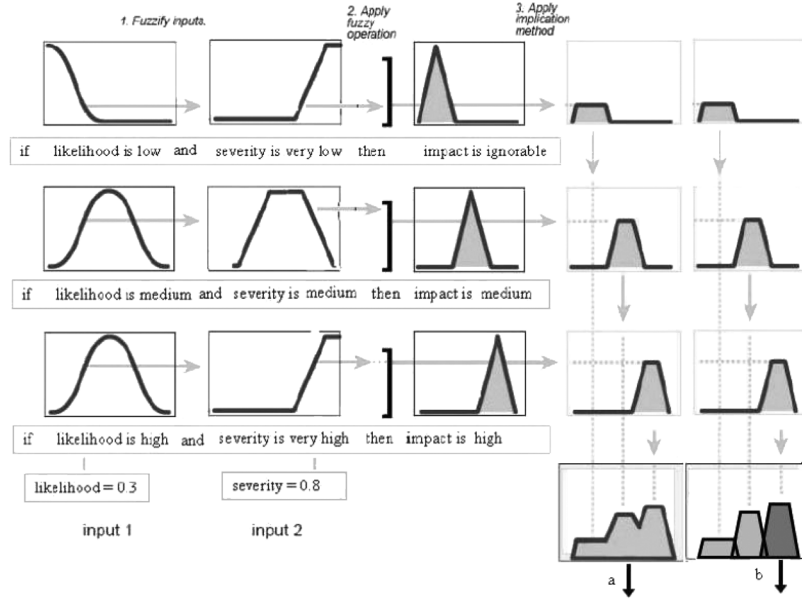


Fig. 10. Risk calculation with traditional method (a) and our method (b).

- 3) all conditional relationships are inserted in the first draft of E-GFCM (each of them will be maintained or not in the final E-GFCM as explained in the following point);
- 4) *redundant* relationships are eliminated. More precisely, let *Rel1* and *Rel2* be two different causal relationships between the same pair (risk factor, risk). Assume that the risk factors contained in *Rel1* are a subset of the risk factors contained in *Rel2*. In this case, we say that *Rel1* is *more general* than *Rel2* or, equivalently, *Rel2* is *less general* than *Rel1*. We consider a relation that is less general than (at least) another relation as candidate to be redundant, we eliminate or keep it from the final E-GFCM depending on the weights and/or the number of experts expressing the same opinion.

Finally, for each relationship *Rel* present in the final E-GFCM, we compute the strength (i.e., the weight) associated with the corresponding edge as the weighted average of the strengths of the relationships that contributed to the creation of *Rel*. More precisely, these concepts may be written in different terms but in reality have the same meaning. If for a given causal link experts assign different strengths s_i , then the final strength is decided by the following equation:

$$s = \frac{\sum_{i=1}^n w_i \cdot s_i}{\sum_{i=1}^n w_i} \quad (2)$$

where n is the number of experts and w_i is the expert's weight.

We wish to point out that, with reference to previous point 3), also the weights associated with any removed relations may be considered in the computation of the weights to be associated with the relationships retained in the final E-GFCM.

As a final remark, we would like to observe that in the context of this paper we were not interested in adopting the optimal fusion strategy of two or more E-FCMs, but rather in showing the high modeling power of the proposed idea of applying E-FCMs to risk analysis and management.

V. PESSIMISTIC ASSESSMENT OF THE OVERALL RISK OF A PROJECT

After identifying the likelihood and the severity of *time*, *cost*, *performance*, and *quality*, we assess the attractiveness of the project (or, better, the *risk of non attractiveness*) by using pessimistic evaluation as suggested in [31]. The high-level architecture is given in Fig. 8. For a given risk that is influenced by a set of risk factors, one of the most used fuzzy techniques for identifying the total effect of risk impacts is the *fuzzy weighted average* operator. Another widely adopted method is the traditional way to solve fuzzy inference systems using aggregation of the outputs of fuzzy inference rules. We believe that our approach is more realistic compared to other existing methods. Particularly, the methods that use fuzzy weighted average often give a result smaller than the real risk, especially in the case of a large amount of input variables. Furthermore, the traditional approaches may give the same result for different cases ([8], [31]).

For the likelihood and severity of all risk factors we define fuzzy linguistic terms *{Very Low, Low, Medium, High, Very High}*. Similarly, for the overall risk we assign fuzzy linguistic terms *{Ignorable, Low, Medium, High, Critical}*.

The number and shapes of membership functions can vary depending on the specific problem. For simplicity we use triangular or trapezoidal membership functions and we fix the number of membership functions to five. For likelihood of risk factors we define membership functions based on uniform partition (see Table III). Fig. 9 shows an example of a possible definition of the severity of two risk factors using trapezoidal membership functions.

The membership functions of risk factor severity are not uniformly partitioned but depend on the risk assessment approach of the given organization. Notice that different risk factors can have different scales as shown in Fig. 9.

With reference to Fig. 8, for each risk factor we use fuzzy inference rules such as: “if the severity of *High Cost* is low and

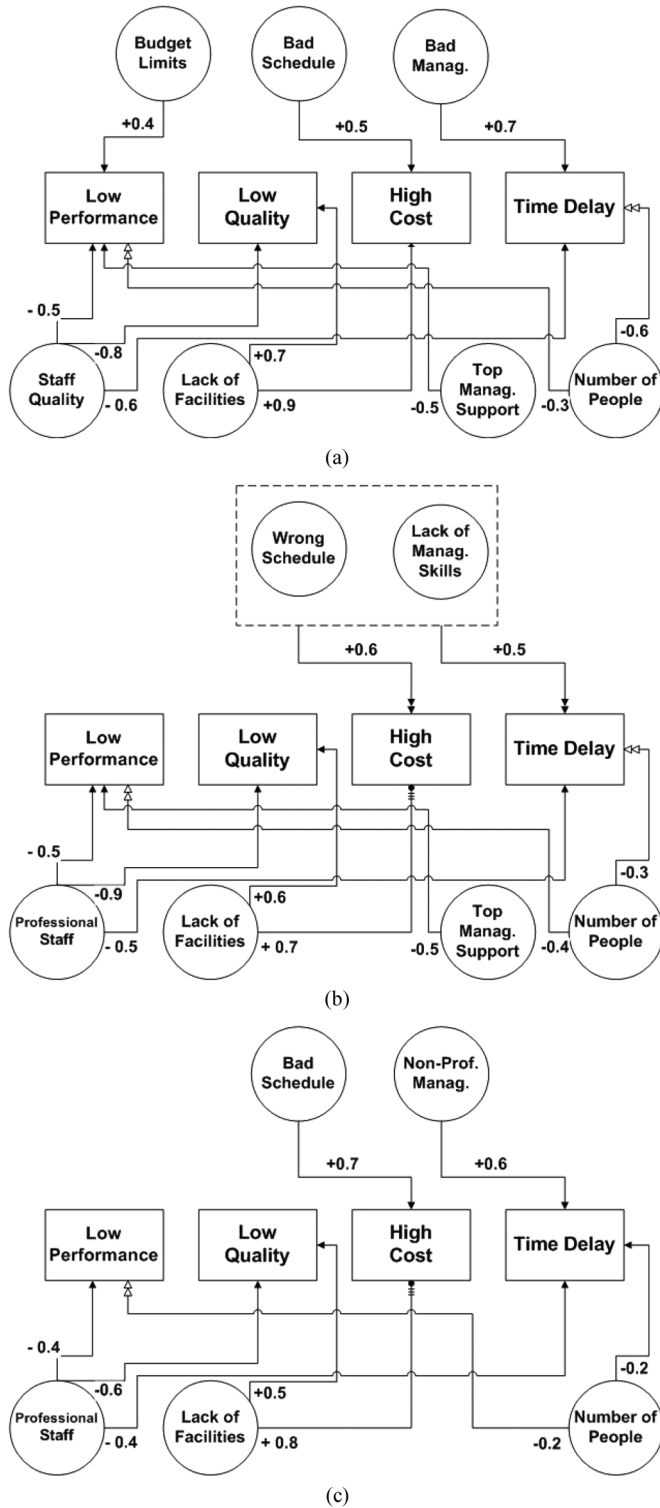


Fig. 11. E-FCMs drawn by three different experts. (a) E-FCM of Expert1; (b) E-FCM of Expert2; (c) E-FCM of Expert3.

the likelihood of *High Cost* is high then the impact of *High Cost* is medium”.

Table IV, where the columns and rows represent possible values for risk severity and likelihood, shows one possible fuzzy-valued calculation of risk impact. Notice that this table can vary for different risk factors.

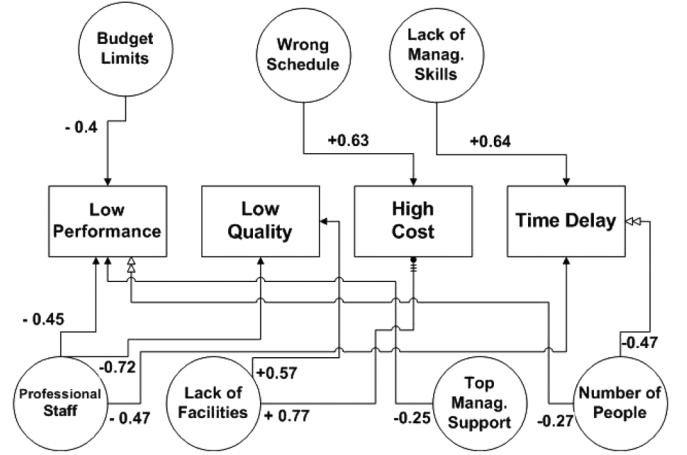


Fig. 12. The final E-GFCM.

Our approach is based on pessimistic assumption as we believe that taking the maximum value of impacts of all risk factors is more realistic and more precise [31].

Fig. 10 is a simple example that shows the difference between the traditional approach adopting centroid defuzzification method (10a) and the pessimistic approach (10b).

In our approach we apply pessimistic evaluation twice: first for identifying the real impact of each impact factor; second, for identifying each risk based on the impacts of risk factors. Indeed, also for each risk assessment, we first define fuzzy inference rules having impacts as inputs, then, after applying the implication operator, we decide the overall risk as maximum of the results of implications.

To find the maximum of fuzzy numbers we have to rank fuzzy numbers, then take the one with the highest ranking. The ranking of fuzzy numbers has long been of interest and received a great deal of attention. There are several issues that make it difficult to choose the ranking technique that is the most appropriate to the given problem. The various shapes of fuzzy numbers, the ranking order, their relative dominance are the important factors to be considered. Notice that for our needs we have to be able to differentiate all nonidentical fuzzy numbers even if they are very close to each other. For that reason, we prefer to use the ranking algorithm suggested in [31].

The detailed description of the used algorithm to find the maximum of generalized fuzzy numbers is out of the scope of this paper. The interested reader can refer to [31] to read the details of the algorithm.

VI. APPLICATION EXAMPLE

As we already mentioned in the previous sections we discuss our approach to SPM, which is considered one of the most risky businesses.

Suppose we have three experts who are asked to draw E-FCMs to show the relationships between risk factors and risks. They are free to use any number of nodes and any kind of arrows between cause and effect. The only restriction is that they are asked to have four risks: *high cost*, *time delay*, *low quality* and *low performance*.

The weights of experts e_i are, respectively, $w_1 = 0.4$, $w_2 = 0.3$ and $w_3 = 0.7$. The E-FCMs drawn by the three experts

are shown in Fig. 11. As we can see, the E-FCMs differ by node names, number of nodes as well as connection arrows. After collecting all E-FCMs from the experts, we put all together having the first draft of E-GFCM. Of course only one node is kept to represent all those nodes that have the same meaning but are expressed with different terms. For example, *Bad Management*, *Lack of Management Skills* and *Non-Professional Management* express the same idea, so they can be merged producing only one node.

For the final draft we have to decide the type of arrows.

For *nonlinear weights* we have that the opinions of the experts are different: two of them consider that increasing of *Number of People* helps to decrease *Time Delay* but not linearly, whereas the third expert considers that it always helps to meet project deadlines. The weights of the two conflicting opinions are $w_{\text{nonlinear}} = 0.4 + 0.3 = 0.7$ and $w_{\text{linear}} = 0.7$, respectively. In this case, having the same weight for the two groups, we take as a final decision the one with more voters; thus for the final E-GFCM we will have that adding more people into the project helps to decrease the time delay but adding more and more people may not have any positive effect.

We have a similar scenario for *time delay weights*. In this case, the weight of the group of experts that think that there is a time delay between risk factor *Lack of Facilities* and the risk *High Cost* is $w_{\text{time-delay}} = 0.3 + 0.7 = 1$ and the weight of the opposite opinion is $w_{\text{notime-delay}} = 0.4$. Obviously, we take as a final decision the one with higher weight.

Notice that similar considerations can be made for *conditional weights*. Indeed, the vote for the conditional dependency between *Wrong Schedule*, *Lack of Management Skills* and *High Cost/Time Delay* nodes is given as follows: $w_{\text{cond}} = 0.3$, $w_{\text{non-cond}} = 0.4 + 0.7 = 1.1$. Therefore, in the final E-GFCM there is not conditional dependency between the mentioned nodes.

Once we have defined the nodes and the arrow types for the E-GFCM, we decide the edge weights based on similar considerations. We will not detail the calculations here. The final result is shown in Fig. 12.

Fig. 12 depicts the final E-GFCM which represents the group decision about the given task merging the E-FCMs produced by three different experts.

For *conditional weights* we use the AND operator to link together the cause nodes, and use fuzzy rules to connect the linked cause nodes to the effect node.

For the relationships with *nonlinear weights* we define fuzzy rules considering the nonlinearity. For example, increasing the risk factor *Number of People* will decrease the risk *Time Delay* but increasing more will have no more effect or will have the opposite result. Consequently, the fuzzy rules to describe these nonlinear relationships can be as follows.

- If the likelihood of risk factor *Number of People* is LOW then the risk *Time Delay* is HIGH.
- If the likelihood of risk factor *Number of People* is MEDIUM then the risk *Time Delay* is MEDIUM.
- If the likelihood of risk factor *Number of People* is HIGH then the risk *Time Delay* is HIGH.

For the relationships with *time delay weights*, we define an additional input variable *time period* in fuzzy inference rules.

For our application example two fuzzy rules indicating the existence of *time delay weight* can be the following.

- If the likelihood of risk factor *Lack of Facilities* is HIGH and *Time Period* is SHORT then the likelihood of the risk *High Cost* is LOW.
- If the likelihood of risk factor *Lack of Facilities* is HIGH and *Time Period* is LONG then the likelihood of the risk *High Cost* is HIGH.

It is obvious that using E-FCMs helps to have more informative fuzzy rules which capture real-life situations more precisely.

Once we have defined all fuzzy rules, we use the pessimistic approach for final risk identification.

VII. CONCLUSIONS

Business environments become more and more dynamic and the nature of business becomes more and more complex, thus the timeliness and efficiency for identifying the negative impacts and failures of the business become a primary task. Mostly risk analysis is highly subjective and the available data contain vague and inexact information. The traditional statistical methods are not powerful to handle uncertainties concerning risks. To this end, fuzzy logic is a more appropriate tool for risk analysis.

One of the most important and challenging tasks in RAM is identifying the relationships between risk factors and risks. The strength of the influence, the time delay between the occurrence of the risk factor and its effect, the possible conditionality of different risk factors are important issues to be considered. To this end E-FCMs are suitable to be used for RAM.

In this paper we presented the use of E-FCMs for RAM and a pessimistic evaluation of risks. We also proposed a group decision approach based on expert opinions and E-FCMs. We particularly discussed our approach for SPM showing its benefits.

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