

Alloy

Vatche Ishakian

Boston University- CS511

March/24/2008

Contributors: Andrei Lapets, Michalis Potamias, Mark
Reynolds

Why write a Model

- To describe formally the components of a system and relationships between them
- Check properties about the model
- Exclude ill-formed examples of Models.
- Two kind of problems might arise
 - Bugs (in the logic of the model)
 - Errors (in the subject that you are modeling)

Why Alloy

- Conceptual simplicity and minimalism
 - Very little to learn
 - WYSIWYG: no special semantics
- high-level notation
 - Constraints -- can build up incrementally
 - Relations flexible and powerful
 - Much more succinct than most model checking notations

Alloy Case Studies

- In Industry
 - Animating requirements (Venkatesh, Tata)
 - Military simulation (Hashii, Northrop Grumman)
 - Role-based access control (Zao, BBN)
 - Generating network configurations (Narain, Telcordia)
- In Research
 - security features (Pincus, MSR)
 - exploring design of switching systems (Zave, AT&T)

Alloy Characteristics

- Finite scope check:
 - The analysis is sound, but incomplete
- Infinite model:
 - Finite checking does not get reflected in your model
- Declarative: first-order relational logic
- Automatic analysis:
 - visualization a big help
- Structured data:

Bad Things

- Sequences are awkward
- Recursive functions hard to express

Alloy model structures

- Structures are expressed as a set of tuples (vectors of atoms)
 - individual elements are treated as singleton sets
- Atoms are Alloy's primitive entities
 - Indivisible: can't be broken down into smaller parts
 - Immutable: it does not change.
- Relations
 - Associate atoms with one another.
 - Consists of a set of tuples each tuple being a sequence of atoms.

Relations

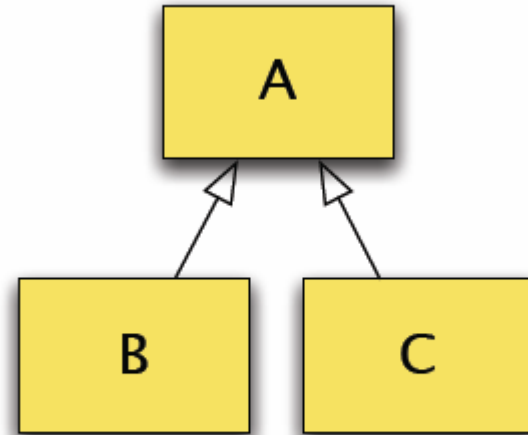
- The "arity" of the relation is the number of atoms in each tuple
- A relation with no tuples is empty

Alloy declarations

- First line module declaration
 - module chapter4/filesystem
- sig Object {}
 - Defines a set named Object represents all objects
 - abstract sig Object{} abstract sig has no elements except those belonging to its extensions.
- sig File extends Object {}
 - A set can be introduced as a subset of another set

Signatures

- sig A {}
- sig B extends A {}
- sig C extends A {}
- Means
 - B in A
 - C in A
 - no B & C



Signatures

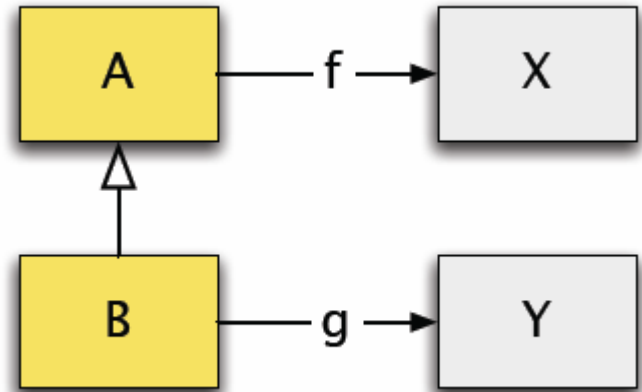
- **abstract sig A {}**
- **sig B extends A {}**
- **sig C extends A {}**
- *B in A*
- *C in A*
- *no B & C*
- *A = (B + C)*

Declaring relations

- Relations are declared as fields of signatures
- ```
sig Object{}
sig Dir extends Object {
 entries: set Object,
 parent: lone Dir
}
```

# Fields

- **sig A {f: set X}**
- **sig B extends A {g: set Y}**
- means
  - B in A
  - $f: A \rightarrow X$
  - $g: B \rightarrow Y$



# Set Multiplicities

- set : any number.
- one: exactly one.
- lone: zero or one.
- some: one or more.
  - sig Man extends Person { Wife: lone Woman}
  - sig Woman extends Person {Husband: lone Man}
  - sig Object{}  
sig Dir extends Object {  
    entries: set Object,  
    parent: lone Dir  
}

# Facts and Assertions

- fact:
  - Constraints that are assumed always to hold (used to restrict space of possible counterexamples)
  - A Model can have any number of facts
    - sig A {
    - fact { all z: A | F }
- assert:
  - Constraints that are intended to follow from the facts of the model
  - Can be checked using the check command

# assert, check

- **fact F {...}**
- **assert A {...}**
- **check A**
- means
  - fact: assume constraint F holds
  - assert: believe that A follows from F
  - check: find an instance that satisfies F *and not* A



# Quantifiers

- $\text{all } x: e \mid F$
- $\text{some } x: e \mid F$
- $\text{no } x: e \mid F$
- $\text{lone } x: e \mid F$
- $\text{one } x: e \mid F$ 
  - $\text{fact EntriesSameName } \{ \text{all } e1, e2 : \text{entries} \mid e1.\text{name} = e2.\text{name} \Rightarrow e1 = e2 \}$
  - $\text{fact nametoaddress } \{ \text{all } n:\text{Name} \mid \text{lone } d:\text{Address} \mid d \text{ in } n.\text{address} \}$
  - $\text{fact } \{ \text{no } p: \text{Person} \mid p \text{ in } p.^{\wedge}(\text{mother} + \text{father}) \}$
  - $\text{fact FourPassengersPerCar } \{ \text{all } c:\text{Car} \mid \# \{ c.\text{contents} \} \leq 4 \}$
  - $\text{fact OneVehiclePerPassenger } \{ \text{all } p:\text{Passenger} \mid \text{one } v:\text{Vehicle} \mid p \text{ in } v.\text{contents} \}$

# Examples of Asserts

- `assert NoSelfFather { no m: Man | m = m.father }`
- `check NoSelfFather`
- `assert FileInDir { all f: File | some d: Dir | f in d.contents }`
- `check FileInDir`
- `assert RootTop { no o: Object | Root in o.contents }`
- `check RootTop`

# Constants and Operators

- The language of relations has its own constants and operators
- Constants
  - none = empty set
  - univ = universal set
  - iden = identity

# Constants and Operators

- Name = {(N0), (N1)}  
Addr = {(D0), (D1)}
- none={}  
univ = {(N0), (N1), (D0), (D1)}  
iden = {(N0,N0), (N1,N1), (D0,D0), (D1,D1)}
- Operators fall into Two Categories
  - Set
  - Relational

# Set Operators

- **+ : union**
  - `sig Vehicle {} {Vehicle = Car + Truck}`
- **& : intersection**
  - `fact DisjSubtrees { all t1, t2:Tree | (t1.^children) & (t2.^children) = none }`
- **- : difference**
  - `fact dirinothers { all d: Directory - Root | some contents.d }`
- **in : subset**
  - `Sibling = (brother + sister)`
  - `sister in sibling`
- **= : equality**

# Relational operators

- **.** : dot (Join)
  - $\{(N0), (A0)\} . \{(A0), (D0)\} = \{(N0), (D0)\}$
- **->** : arrow (product)
  - $s \rightarrow t$  is their cartesian product
  - $r: s \rightarrow t$  says  $r$  maps atoms in  $s$  to atoms in  $t$
- **^** : transitive closure
  - `fact DisjSubtrees { all t1, t2:Tree | (t1.^children) & (t2.^children) = none }`
  - `fact {no p: Person | p in p.^(mother + father) }`
- **\*** : reflexive-transitive closure
  - `fact {Object in Root.*contents}`
- **~** : transpose
  - Takes its mirror image  $s.\sim r = r.s$  (image of  $s$  navigating backwards through rel  $r$ )

# Relational Operators

- $\sim$  : transpose (continued)
  - `fact dirinothers{ all d: Directory - Root | some d.~contents }`
  - `fact dirinothers { all d: Directory - Root | some contents.d }`
- `[]` : box (join)
  - Semantically identical to join, takes arguments in different order.  
Expressions: `e1[e2] = e2.e1`
- `<:` : domain restriction
  - Contains those tuples of `r` that start with an element in `s`.
- `::>` : range restriction
  - Contains the tuples of `r` that end with an element in `s`
- `++` : Override
  - Override `p ++ q` (just like union), except that tuples of `q` can replace tuples of `p` rather than augmenting them.

# Relational Operators

- $\text{Alias} = \{(N0), (N1)\}$
- $\text{Addr} = \{(A0)\}$
- $\text{address} = \{(N0, N1), (N1, N2), (N2, A0)\}$
- Domain restriction
- $\text{Alias} <: \text{address} = \{(N0, N1), (N1, N2)\}$
- Range restriction:
- $\text{address} :> \text{Addr} = \{(N2, A0)\}$
- $\text{workAddress} = \{(N0, N1), (N1, A0)\}$
- $\text{address} ++ \text{workAddress} = \{(N0, N1), (N1, A0), (N2, A0)\}$



# Logical operators

- ! : negation
- && : conjunction (and)
- || : disjunction (OR)
- ==> : implication
- else : alternative
- <=> : bi-implication (IFF)

# Logic Cardinalities

- *= equals*
- *< less than*
- *> greater than*
- *=< less than or equal to*
- *>= greater than or equal to*
  - all b: Bag | #b.marbles =< 3  
all bags have 3 or less marbles
  - fact FourPassengersPerCar { all c:Car | #{c.contents} <= 4 }
- *#r number of tuples in r*
- *0,1,... integer literal*
- *+ plus*
- *- minus*

# Functions and Predicates

- A function is a named expression
- Zero or more declarations for arguments
- ```
fun grandpas [p: Person]: set Person {  
    p.(mother + father).father  
}
```
- ```
fun colorSequence: Color -> Color {
 Color <: iden + Red->Green + Green->Yellow + Yellow->Red
}
```

# Predicates and functions

- A predicate is a named constraint
- Zero or more declarations for arguments
- Can be used to represent an operation
- Only holds when invoked (unlike fact)
  - ```
sig Name, Addr {}  
sig Book { addr: Name -> Addr }  
pred add (b, b': Book, n: Name, a: Addr) {  
  b'.addr = b.addr + n->a  
}
```
 - ```
pred Above(m, n: Man) {m.floor = n.ceiling}
```

Man m is “above” Man n if m's floor is n's ceiling

# fact, pred, run

- **fact F {...}**
- **pred P () {...}**
- **run P**
- means
  - fact: assume constraint  $F$  holds
  - pred: define constraint  $P$
  - run: find an instance that satisfies  $P$  *and*  $F$

# Analyzing the model

- Write a command to analyze a model
- A run command searches for an instance of a predicate
  - run ownGrandpa
- A check command searches for a counter example of an assertion
  - check NoSelfFather
- You may give a scope that bounds the size of instances.
  - run solve for 5 State

- Demo The File System.