Theory Homework 5

3137 - Data Structures and Algorithms in JAVA
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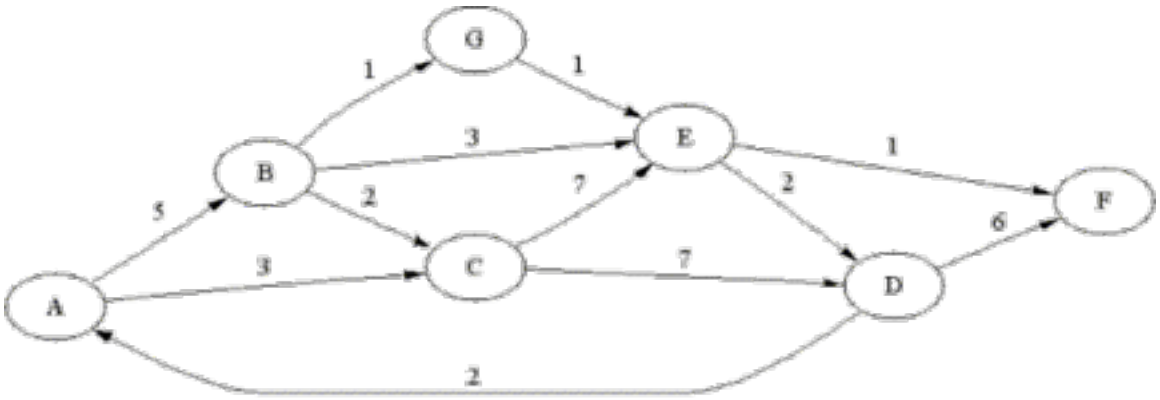
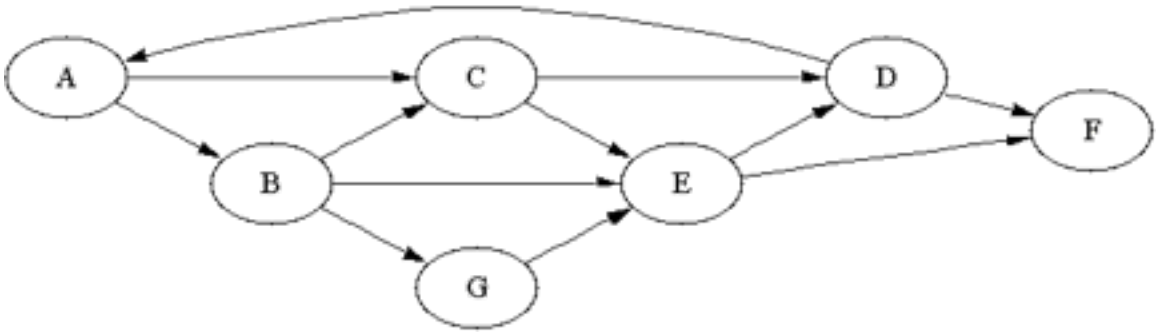
Fall 2013

Out: Oct 30, 2013
Due: Nov 21, midnight.

1) (10) Show the execution of Dijkstra's algorithm from node A in the following graph, show some work and the resulting table
   A,C,3
   A,B,5
   B,G,1
   B,C,2
   B,E,3
   G,E,1
   C,E,7
   C,D,7
   E,D,2
   E,F,1
   D,F,6
   D,A,2:



2) (10) Show the stages of the shortest unweighted path algorithm, starting from node B in the following graph:



3) (5) Suppose you choose to represent the graph as an adjacency list, with *n* nodes and *m* edges. What is the runtime for insertVertex and what is the runtime for deleteVertex ?

4) (16) show the result of the following sequence of instructions: union(1,2), union(3,4), union(3,5), union(1,7), union(3,6), union(8,9), union (1,8), union(3,10), union(3,11), union(3,12), union(3,13), union(14,15), union(16,0), union(14,16), union(1,3), union(1,14)…when

   a. Arbitrary union
   b. Union by height
   c. Union by size

5) (8) In your opinion, what is the difference between merge-sort and quick-sort and why would you prefer one over the other ??

6) (10) Draw a simple, connected, undirected weighted graph with 8 nodes and 16 edges, each edge with a random unique weight. Show how kruskal's algorithm run on this graph. (Note for this example, there is one minimum spanning tree for the graph).

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	240	120	215	180	190	150
3	-	-	-	260	110	350	430	190
4	-	-	-	-	160	330	295	200
5	-	-	-	-	-	350	400	180
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	300
8	-	-	-	-	-	-	-	-

(15) You are the president of a very small country with 8 cities (told you it was small) and want to build 7 roads to connect the cities so that any city can be reached from any other via one or more roads. The cost of construction is proportional to the length of the road, the distances between pairs of cities are given in the following table:

Find which roads to build between which cities so that you spend the minimum for the entire project. Show work!

7) (15) Given the following adjacency list graph representation, with nodes 1 .. 8

Assume that when you move through the graph, the adjacent vertices of a given node are returned in the same order as in the table,

Vertex	Adjacencies
1	[2, 3, 4]
2	[1, 3, 4]
3	[1, 2, 4]
4	[1, 2, 3, 6]
5	[6, 7, 8]
6	[4, 5, 7]
7	[5, 6, 8]
8	[5, 7]

a. Draw the graph

b. Give the sequence of nodes visited using a DFS starting from 1

c. Give the BFS traversal starting at vertex 1