CS598PS – Machine Learning for Signal Processing

Deep Learning and Stochastic Neural Models

7 November 2017

Today's lecture

- Shallow vs. Deep learning
- Stochastic neural models
  - Deep learning structures
- Varying network architectures

“Shallow” models

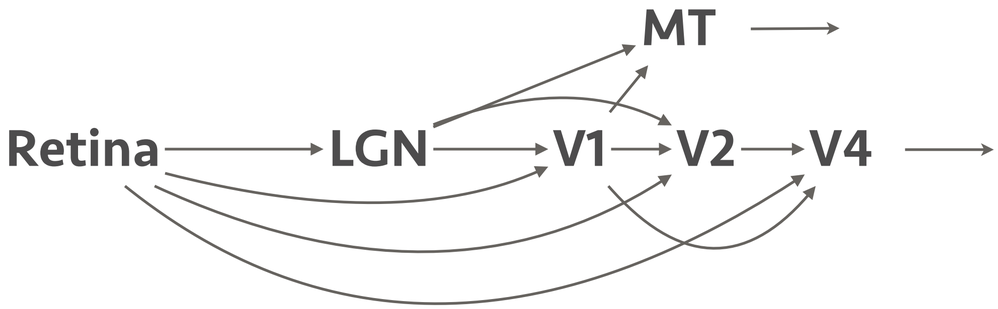
- Most models we used so far were “shallow”

$$\mathbf{y} = \mathbf{W} \cdot \mathbf{x}$$

- Single level of processing (PCA, linear classifier, …)
  - Allows us to use simple representations of the input
    - e.g. linear features, likelihoods, etc.

Looking for depth

- The way we think is inherently hierarchical (i.e. “deep”)
  - e.g. remember the perceptual pathways?
    - Features of features of features, …



  - We don’t really make algorithms like that
    - Maybe we should!

Trying to get some depth

- How about we try multilayer PCA?

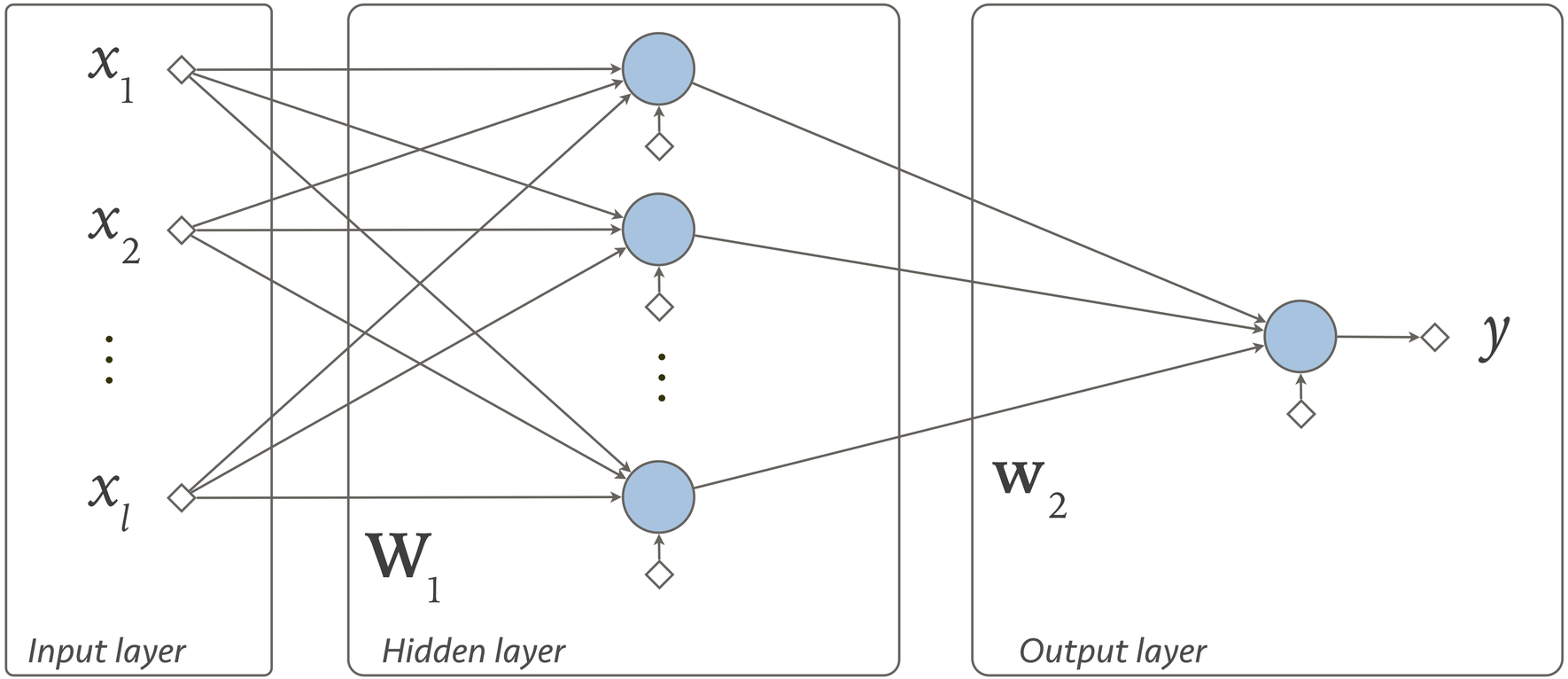
$$\mathbf{y} = \mathbf{W}_1 \cdot \mathbf{W}_2 \cdot \mathbf{x}$$

  - $\mathbf{W}_1$ can contain eigenvectors of eigenvectors!
    - Is that a good idea?

- Not really, $\mathbf{W}_1 = \mathbf{I}$ since $\mathbf{W}_2 \cdot \mathbf{x}$ is whitened already
  - Also with other linear approaches, extra layers make no sense
    - They all collapse to a single linear transform

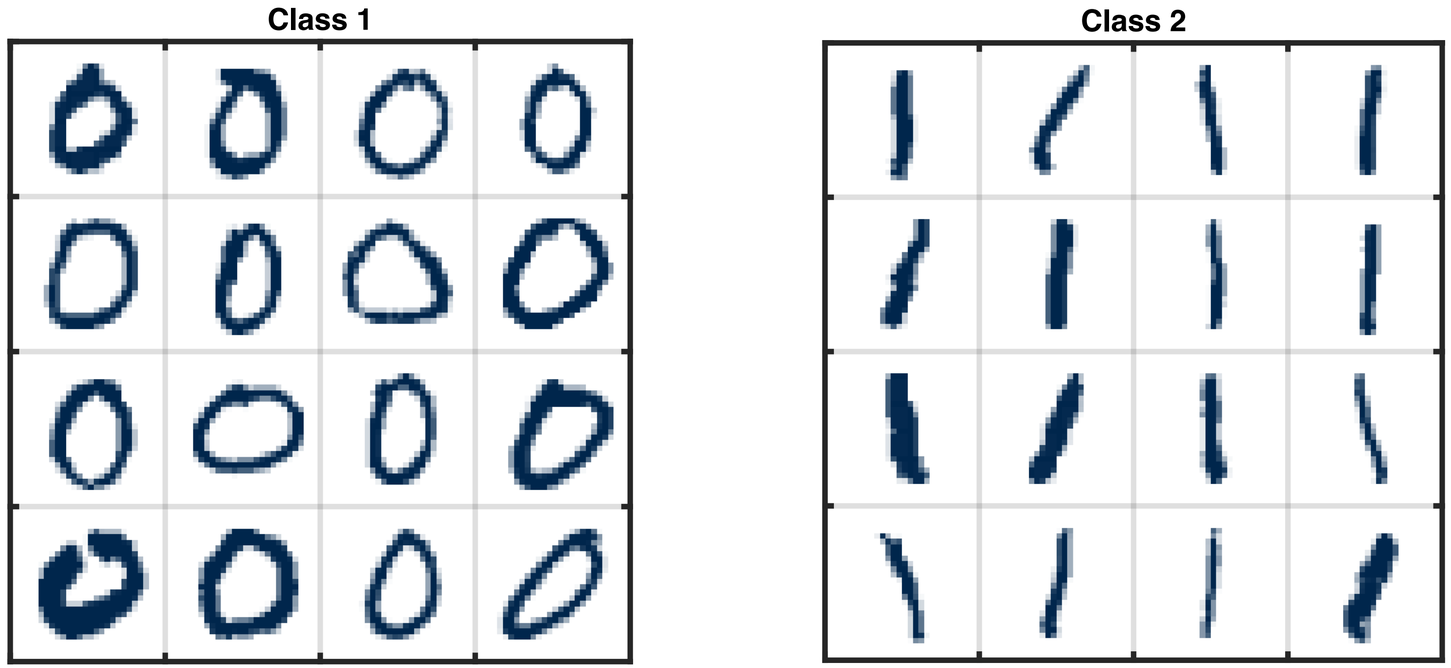
Revisiting the neural net

- An example of a "deep" architecture



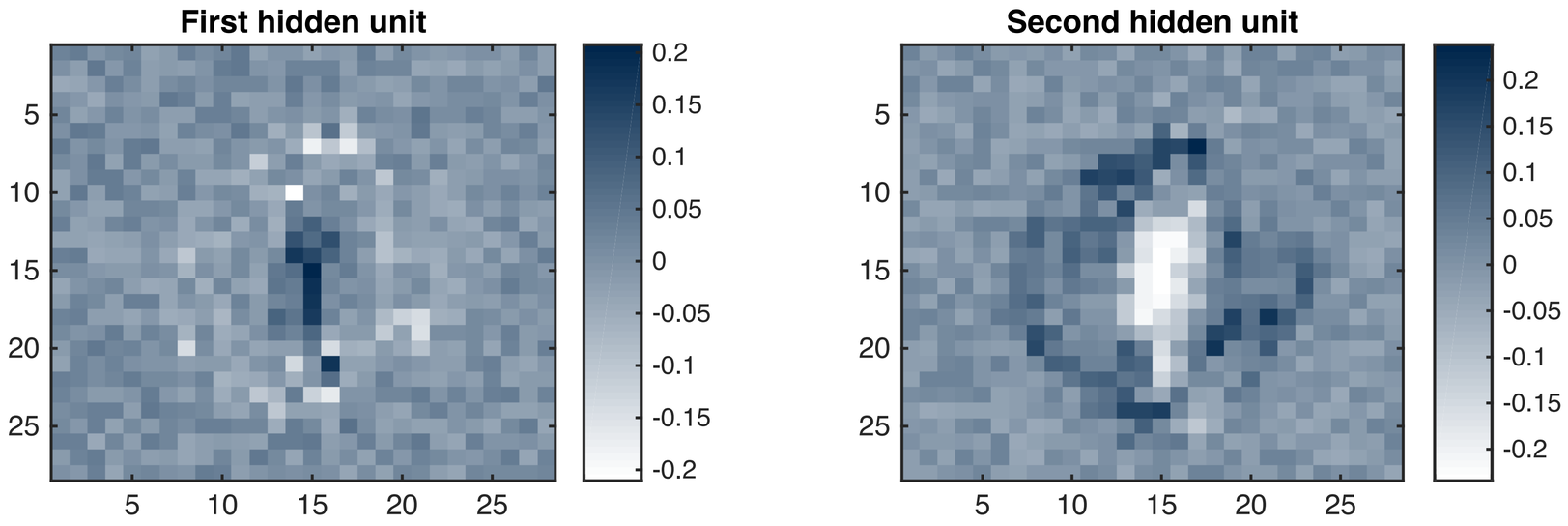
Example classifier

- A “0” vs. “1” digit classifier
  - 2-layer neural net with 2 hidden units and one output



Learned weights

- First layer is a “feature” transform
- Second layer is a simple classifier



The magic ingredient is the non-linearity

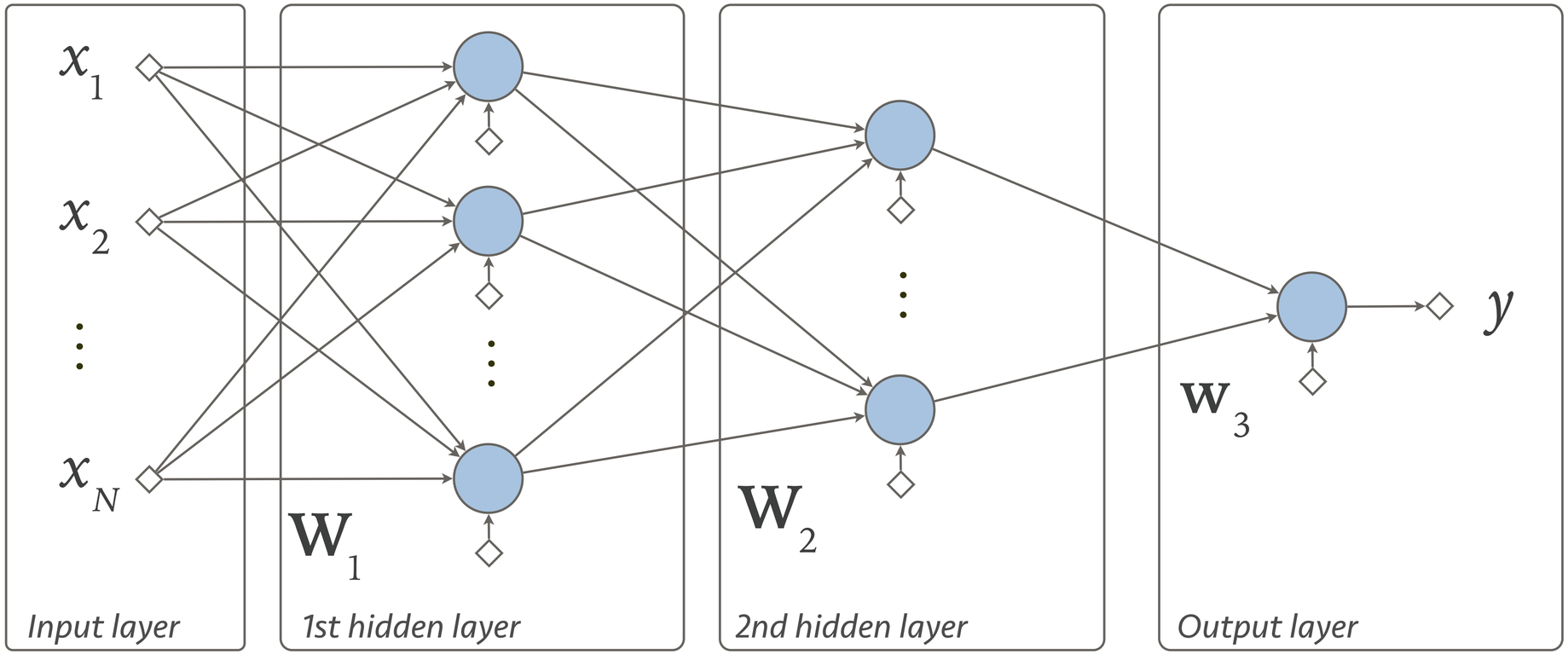
- At every layer we apply a sigmoid:

$$\mathbf{y} = \mathrm{g}\left(\mathbf{W}_1 \cdot \mathrm{g}\left(\mathbf{W}_2 \cdot \mathbf{x}\right)\right)$$

- This allows us to meaningfully stack transforms
  - Hence to obtain a deep architecture

Multiple levels of features

- First layer contains low-level features, second layer contains mid-level features, …, final layer is a classifier



Arbitrarily deep architectures

- We can stack as many transforms as we like
  - The goal is to find rich representations
  - Thus we make a "deep" model

- The goal is to get features of features of features of ...
  - "Like the brain does"

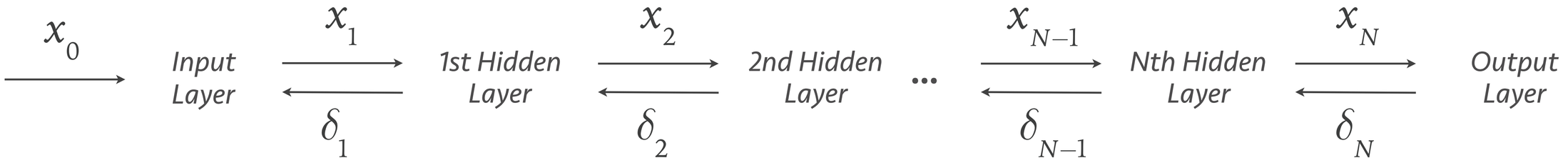
Whoa, wait a minute ...

- You told us that 3 layers are enough for anything!
  - Why should we use more layers than that?
  - Aren't shallow models good enough?

- Yes, a shallow model is fine, but
  - There is no guarantee that you'll easily find the parameters!
  - Nor that you won't need a bazillion units

But there is a problem with depth

- Deep architectures have lots of parameters
  - In some cases in the billions!
- Typical neural net optimization becomes a problem



$$\mathbf{x}_i = \mathrm{g}\left(\mathbf{W}_i \cdot \mathbf{x}_{i-1}\right)$$

$$\delta_i = \left(\mathbf{W}_{i+1}^{\top} \cdot \delta_{i+1}\right) \cdot \mathrm{g}'\left(\mathbf{W}_i \cdot \mathbf{x}_{i-1}\right)$$

Problems with backpropagation

- Through many layers gradient becomes too small
  - We can't propagate errors too far back
- Lots of local minima (lots of parameters!)
  - Even with shallow networks this can be a problem
- Biologically it's a stretch
  - Does the visual cortex influence the retina?
  - Do we really have target values?

Desiderata

- We need a stable learning procedure
- We like biological plausibility
  - Computations should be mostly local
  - We like distributed systems
- We thus need to move away from backprop methods

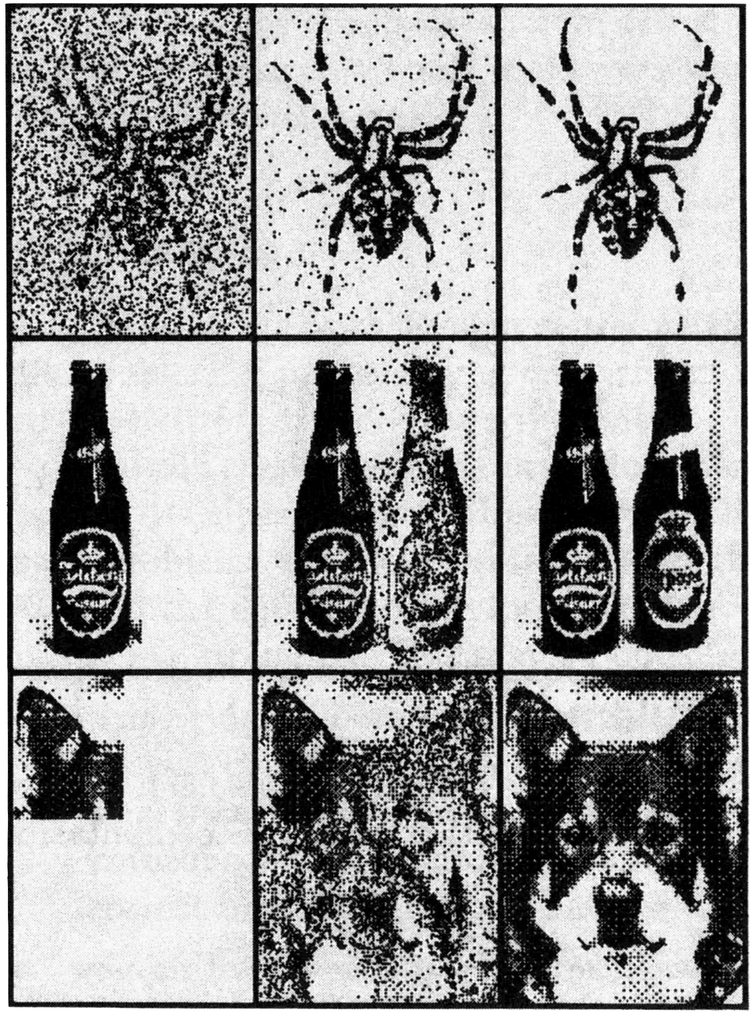
A digression

- A bit on stochastic neural networks
  - A different structure from what we've seen so far
  - Hopfield and Boltzmann models

- Strong influences from physics and neuroscience
  - Recently resurgent models

The Hopfield/Boltzmann networks

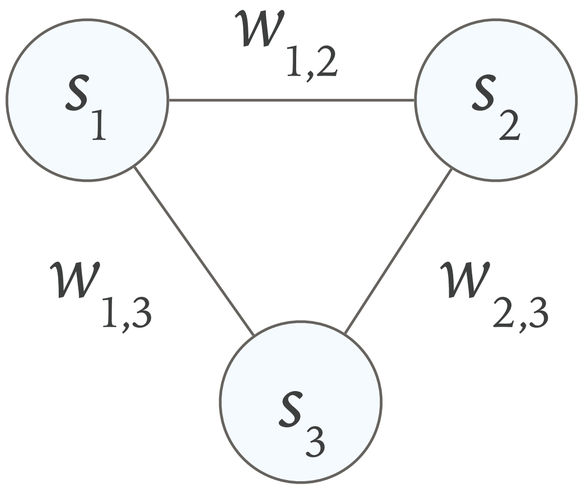
- Auto-associative memory
  - Learns patterns by finding stable equilibria
- Fully connected & recurrent
  - All nodes have binary states {0,1}
- Model can learn to recall patterns



Energy minimization

- Parameters: State $\mathbf{s}_i$, Weight $\mathbf{w}_{ij}$, Threshold θ_i
- After assigning patterns we minimize model "energy"

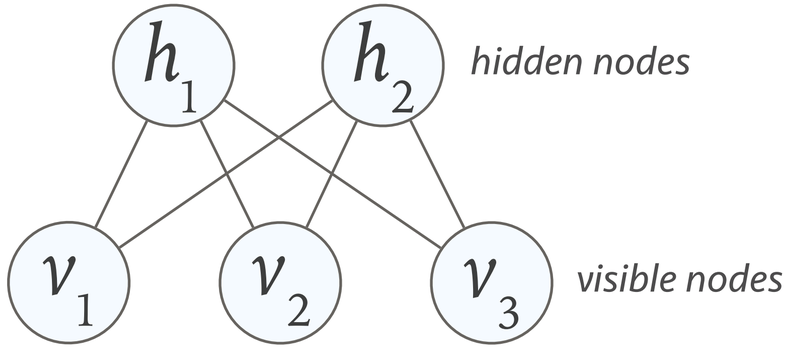
$$E = -\sum_{i<j} w_{i,j} s_i s_j + \sum_i \theta_i s_i$$



- Painful optimization problem!
  - Gradient solution to Hopfield
  - Stochastic solution to Boltzmann

The Restricted Boltzmann Machine (RBM)

- A more manageable form of Boltzmann machines
  - Two-layer, no intra-layer connections



- Visible and hidden nodes
  - Visible nodes represent known data
  - Hidden nodes are used for internal representation

- Easier to train and very useful for many tasks

Getting a handle on RBMs

- Define a probability of the network energy:

$$E(\mathbf{v},\mathbf{h}) = -\mathbf{a}^\top \cdot \mathbf{v} - \mathbf{b}^\top \cdot \mathbf{h} - \mathbf{h}^\top \cdot \mathbf{W} \cdot \mathbf{v}$$

$$P(v,h) = \frac{1}{Z} e^{-E(v,h)}$$

  - Z is a partition function and $\mathbf{a},\mathbf{b}$ are node biases

- We also define the node probabilities

$$P(h_i \mid v) = \mathrm{g}\left(b_i + \sum_j w_{i,j} v_j\right), \quad P(v_i \mid h) = \mathrm{g}\left(a_i + \sum_j w_{i,j} h_j\right)$$

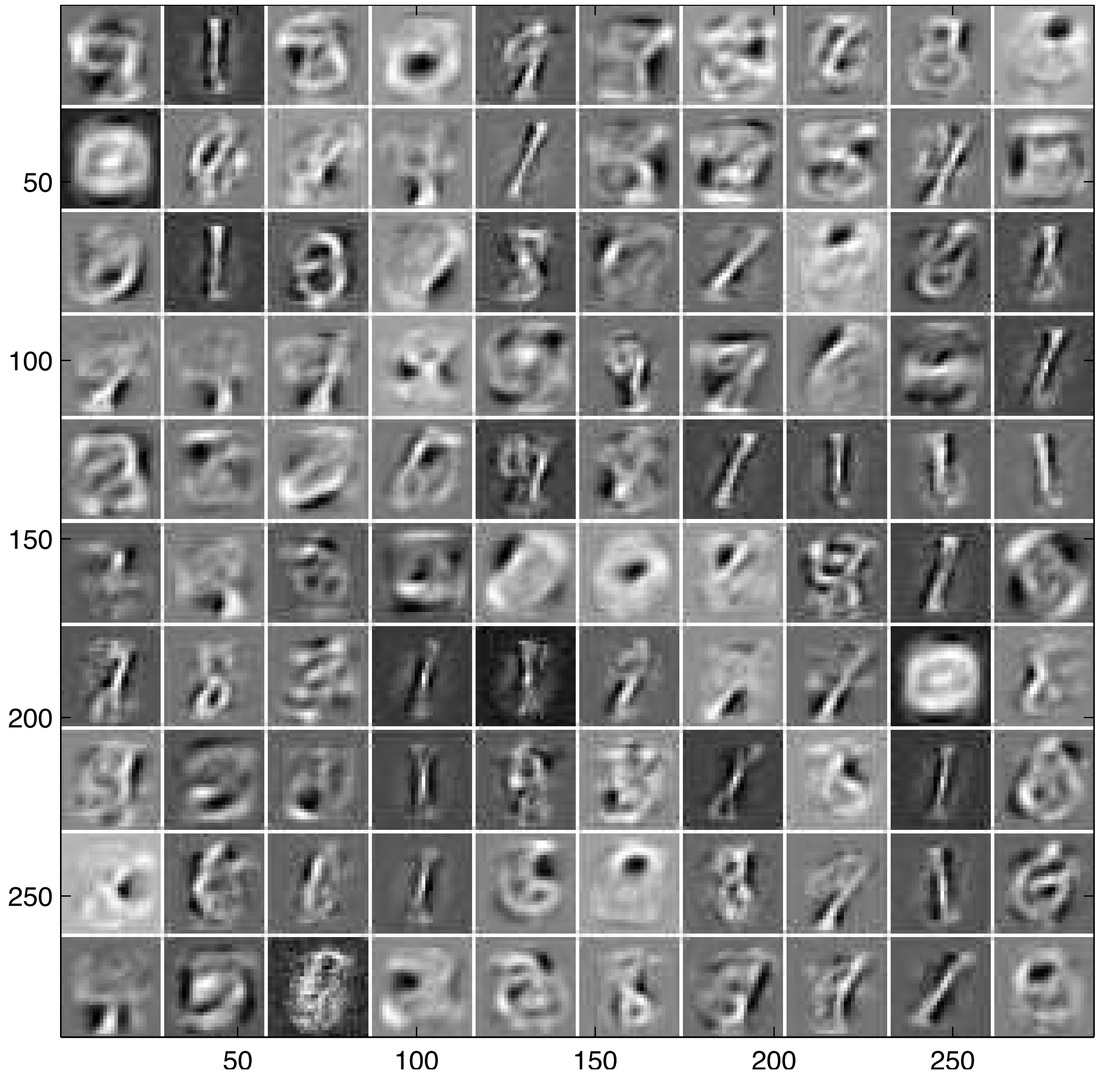
Contrastive Divergence learning

- Maximize product of all $P(\mathbf{v})$
  - 1) For a training sample v_1 compute hidden node probabilities
  - 2) Generate a hidden layer activation vector h_1 from above
  - 3) Go back and generate a new input vector v_2 based on h_1
  - 4) Go forth and generate a new activation vector h_2 from v_2
  - 5) Update corresponding $\mathbf{w}$ using: $\Delta \mathbf{w} \propto v_1 h_1 - v_2 h_2$

"Positive gradient" *"Negative gradient"*

So what does this do?

- Example on digit data
- 28×28 visible nodes
  - Set each pattern to a digit
- 100 hidden nodes
- $\mathbf{W}$ is 784×100
  - i.e. 100 "basis" functions

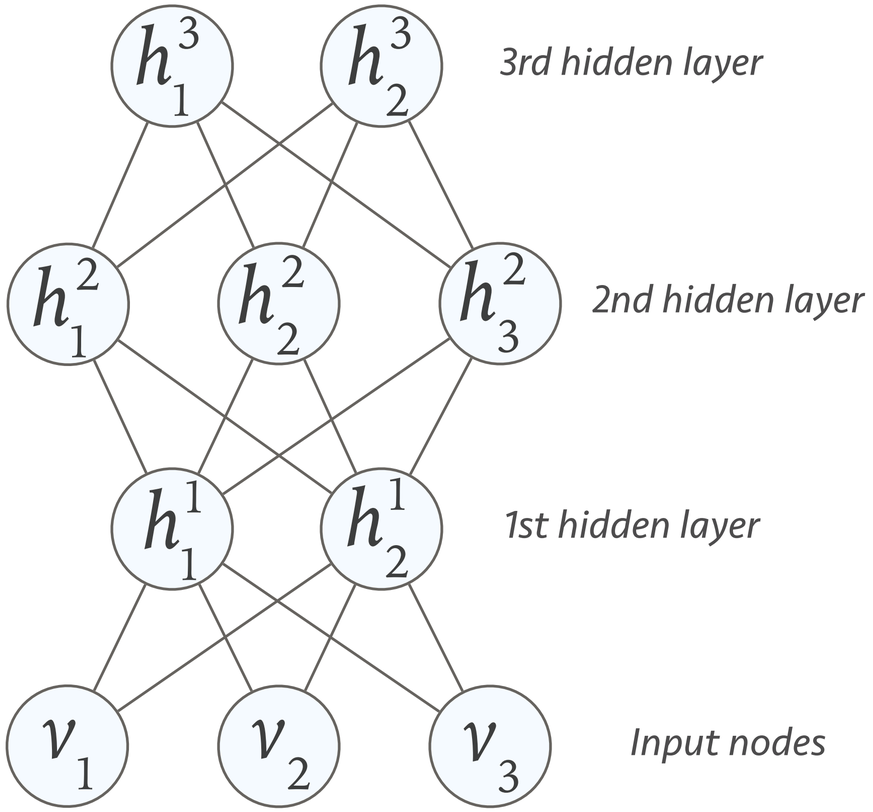


Back to deep learning

- The RBM is a shallow learner
- But we can use it to connect multiple layers
  - Treat each layer in a multi-layer input as an RBM
- The big idea: Train locally, group globally
  - This helps computational complexity and is biologically plausible

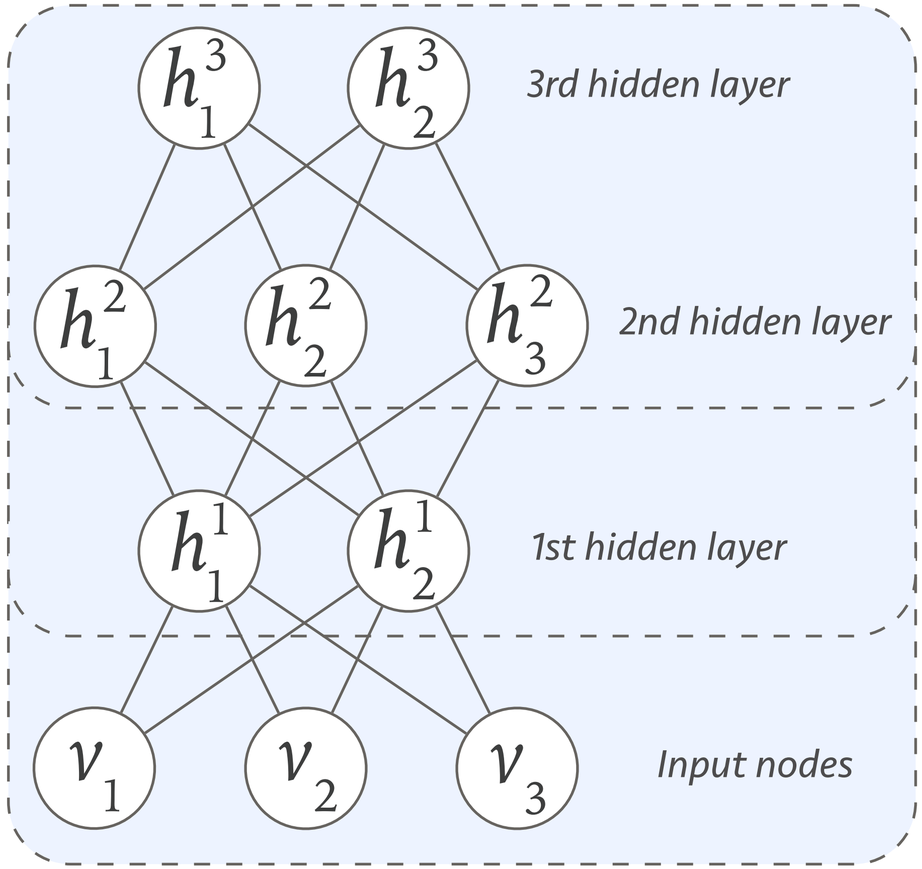
Deep Belief Networks (DBN)

- A stack of multiple RBMs
  - A deep generative model
- Initial weights are set using RBM training
- Further refinement using backpropagation



Greedy learning

- Step 1: Train an RBM for first layer
  - Visible nodes are the inputs
- Step 2: Fix hidden layer
  - Pretend it's visible and train next layer
- Step 3: Keep going



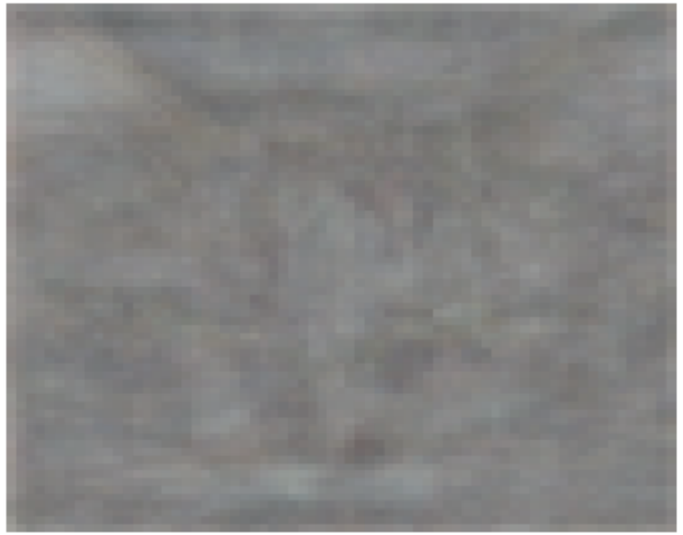
So what is it good for?

- We can learn complex representations of data
  - Learn a deep model with multiple feature levels
- We can learn to classify
  - Use visible nodes to represent classes
- Example simulations on digit data:
  - Hinton's Neural Network Simulation (Generative)
    - Demo: https://www.youtube.com/watch?v=KuPai0ogiHk#t=47s
  - 10 labels / 2000 l3 units / 500 l2 units / l1 500 units / 784 pixels

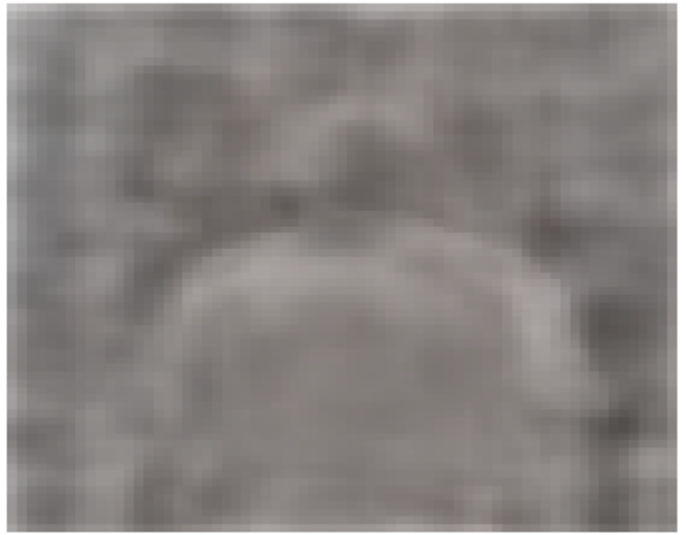
Some well-known press exposure

- A billion weights network trained on 10M YouTube frames
  - 1,000 machines for 3 days!
- Conclusion: YouTube has lots of cats! :)
  - and that we can get some great features that way

The cat neuron



The human body neuron

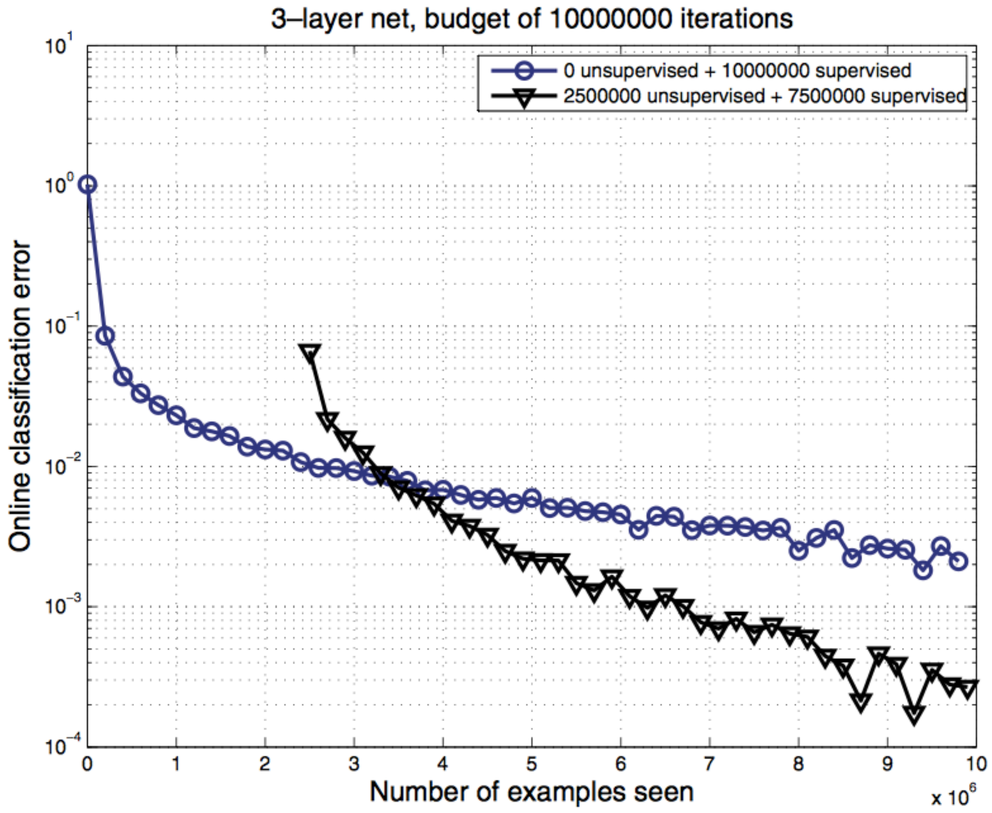


Helping out backpropagation

- We can also use this approach to learn large networks
  - e.g. a multiple layer classification network
- Use greedy learning to find initial values for weights
  - Treat each layer set as an RBM
- Once trained use as initial values for backprop

The importance of a good start

- Find "sensible" weight values
  - Don't start from irrelevant points
- Starts from a space that is well-tuned to the data at hand
- Reduces the amount of required computations



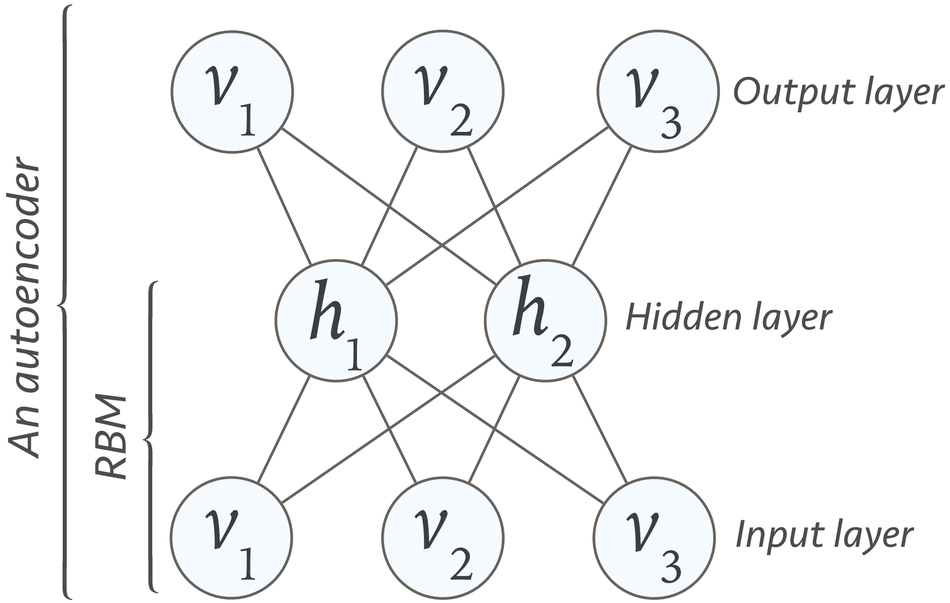
Ugh, that RBM business is difficult …

- There's no reason to stick to RBMs
  - The math is a little tricky and the optimization costly
- Instead we could use another type of "shallow" learner
  - But it has to have a structure conducive to what we want
    - E.g. overcomplete ICA so that we can have more output nodes
- Or we can use an "autoencoder"

Autoencoders

- A very simple approach to designing a shallow non-linear feature extractor

- Try to learn an identity mapping
  - but we won't make it that easy!
  - We won't give it enough resources



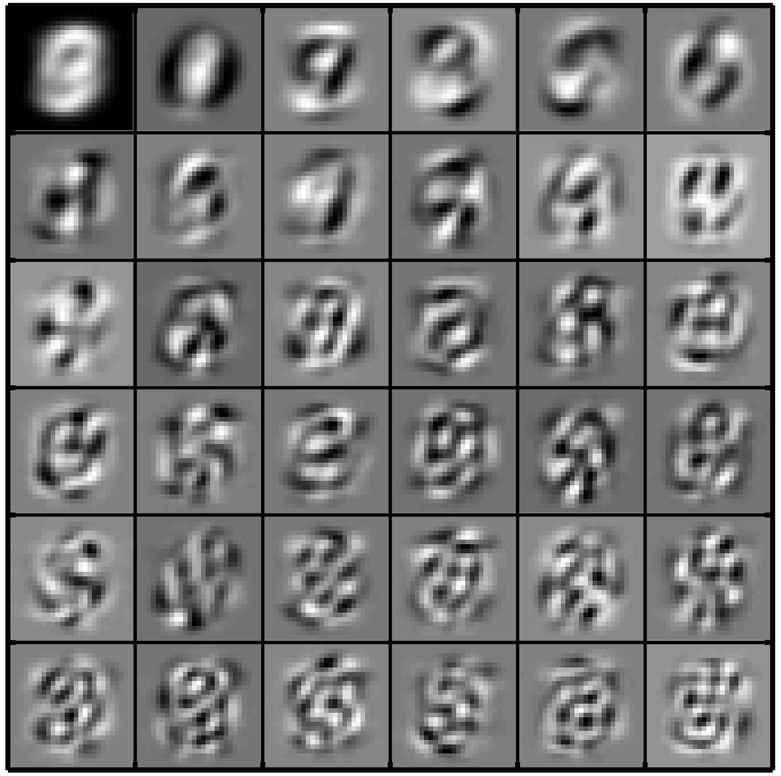
Ways to constraint an autoencoder

- Restrict the number of hidden nodes
  - Creates an information bottleneck
    - Resulting in an informative low-rank representation
- Go to higher dimensions but use sparsity
  - Creates informative “bases” and projects to high-D space
- Add some structure to the form of the layers
  - e.g. orthogonality, independence, etc.

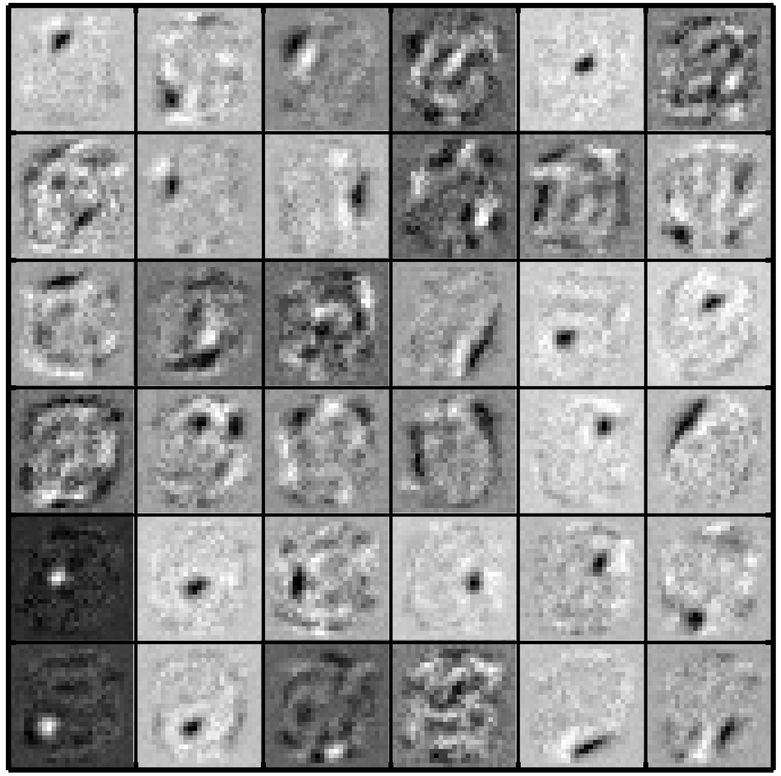
Example case

- Running on digit data

PCA

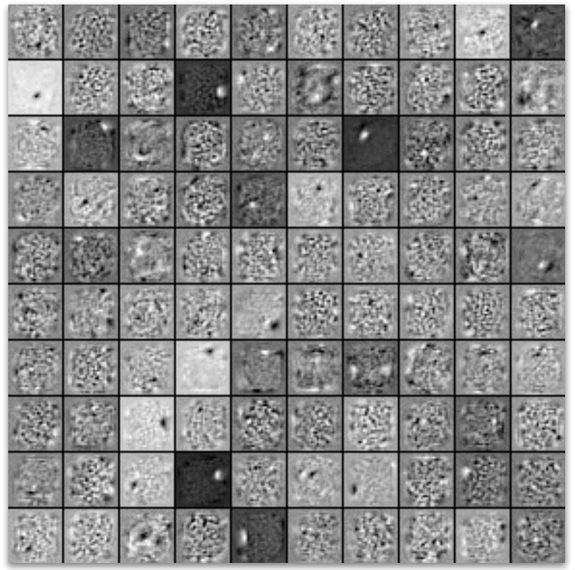


Autoencoder

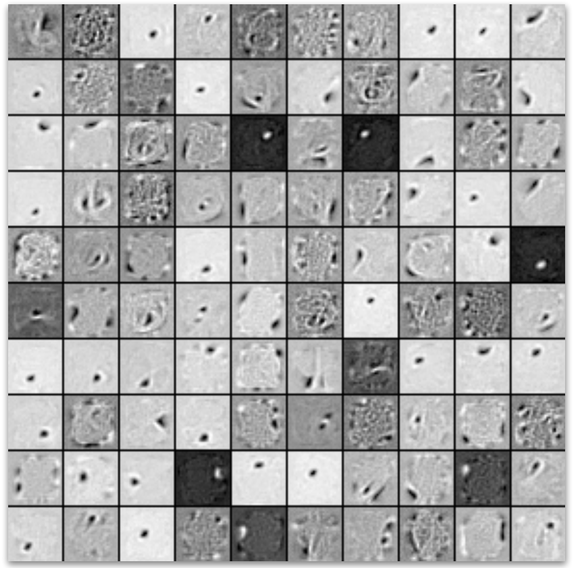


Noisy autoencoders

- Find a “robust” representation
  - But stochastically corrupt the input
  - e.g. adding noise, removing random bits, transform it in non-linear ways, etc.



- Now the input is not always the same as the output
  - We need robust features that map all the noisy inputs to the proper output

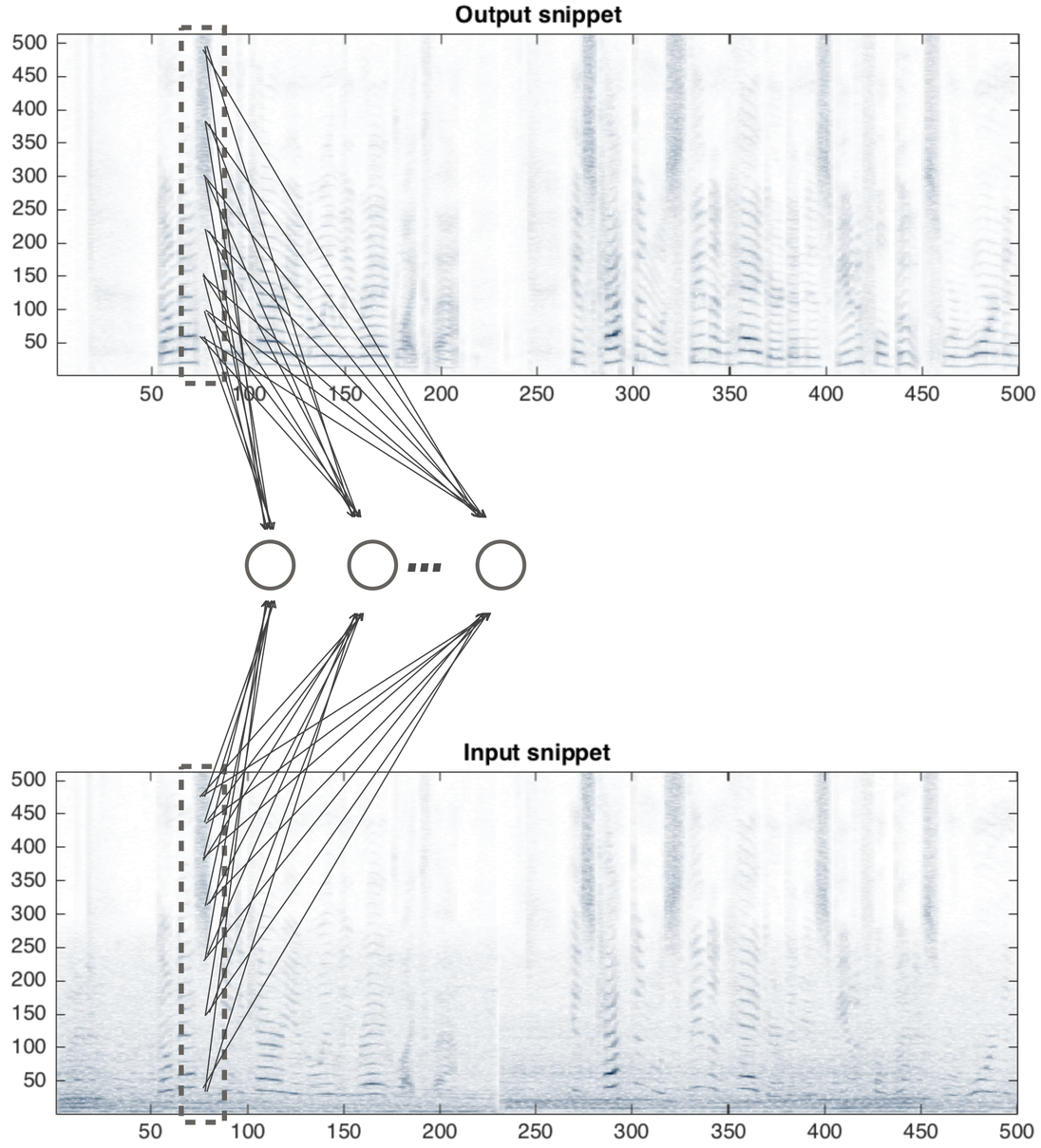


Noisy autoencoders for enhancement

- We can also use noisy autoencoders to clean signals
  - Learn to predict a desirable output for a noisy input
- Can be used for multiple enhancement tasks
  - Removing noise, recovering higher resolution, …
- Generate noisy/clean training data and learn a network

Toy example: Speech denoising

- Trained on 30sec inputs
  - Speech + street noise
  - Known speaker
  - Takes 30sec to train
    - on a laptop (2-3sec with GPU)
- Parameters
  - 1024pt spectra
  - 1 hidden layer, 100 nodes
  - Leaky ReLU activations

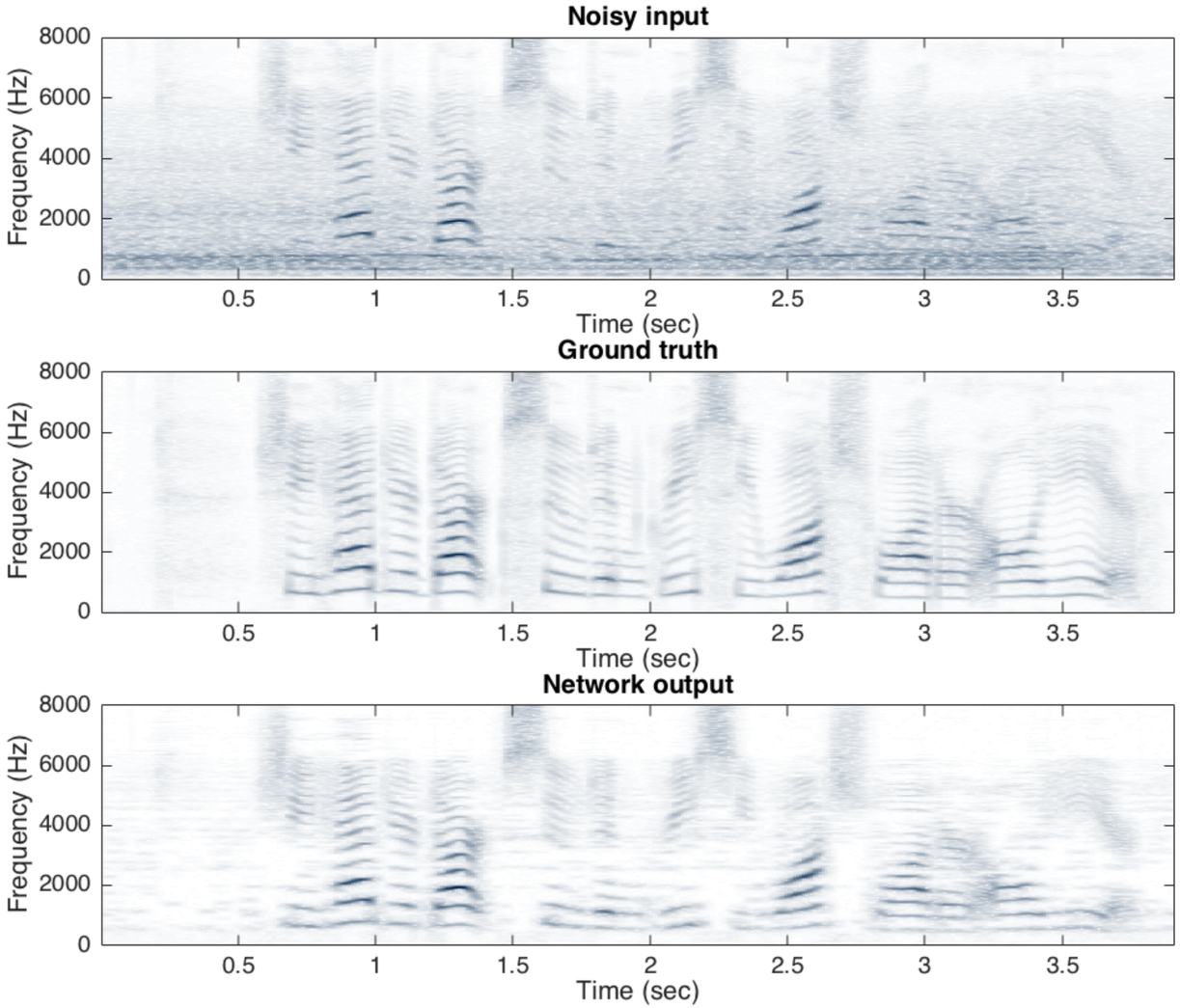


Runtime denoising

- Very lightweight process
  - ~300x real-time
    - 0.01sec in this case
- Works better than NMF
  - But can't generalize to new noise types!

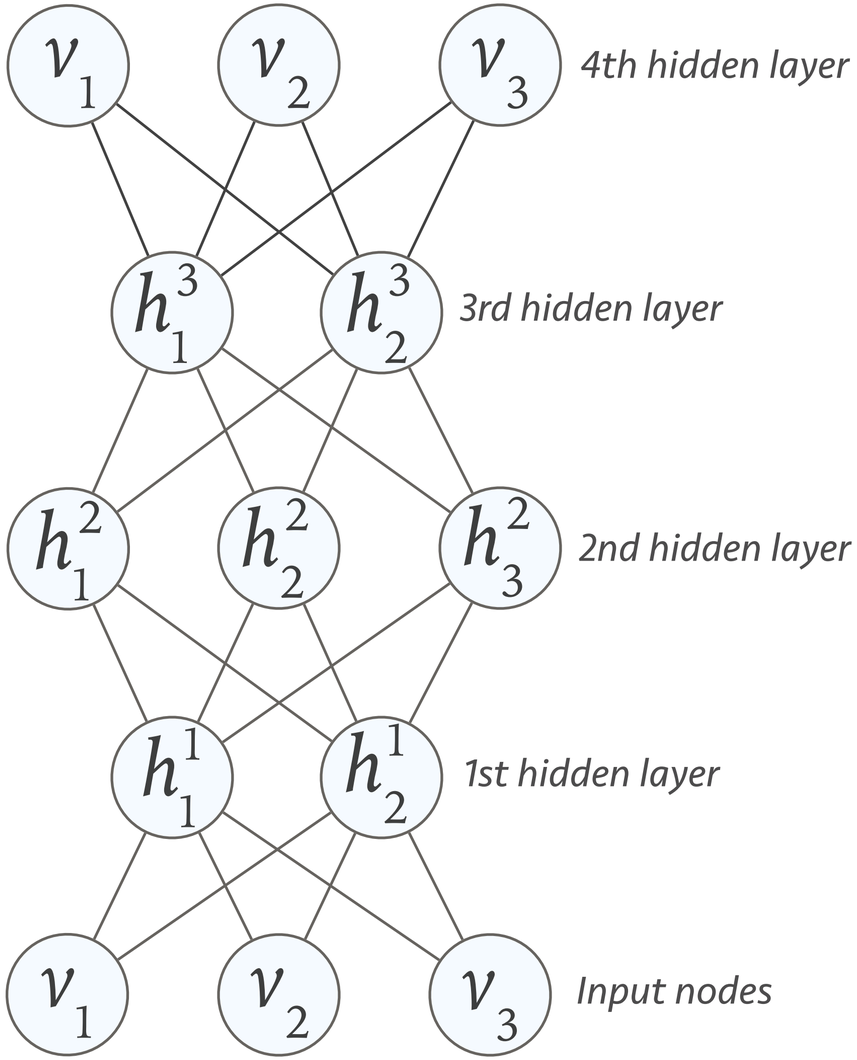
Input mixture

Predicted output



Stacked autoencoders

- Similar to DBNs
  - Multiple-layer architecture
  - Train each layer separately
    - Stack them all in the end
- Can also be used to classify
  - Once trained, add a classification layer and refine with backprop



So what's new from the 90's?

- Cynical view: Not much
  - We have more data and better processors
    - This allows us to train more useful models
  - The RBM/DBN business might have been a distraction …

- But, we now have many more tricks of the trade
  - Better activations, smarter training strategies, many little tricks to assist better convergence, more elaborate models than before, …

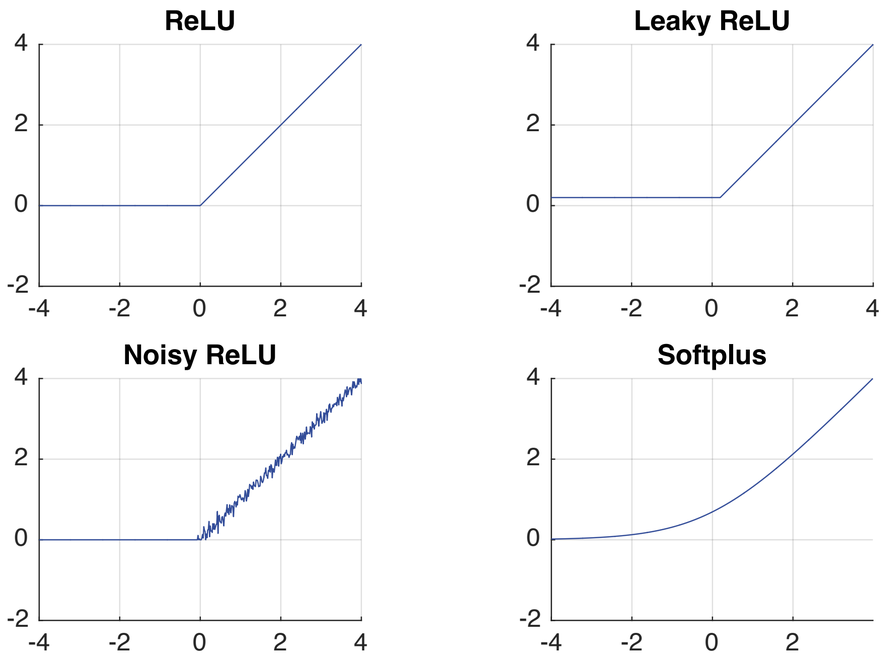
Dropout for better training

- Drop-out training
  - Randomly “turn off” units at every training iteration
    - Usually 50%
  - Puts pressure on units to be useful

- Results in a more robust networks
  - Equivalent to training multiple nets with shared weights, but slightly different connectivity patterns

Modern activation functions

- Rectified Linear Units (ReLU's)
  - Instead of a sigmoid use: $y_i = \max(0, x_i)$
  - Much faster since there is minimal computation
    - Leaky versions: $y_i = \max(\varepsilon, x_i)$
    - Noisy versions: $y_i = \max(0, x_i + n_i)$, …
- Softplus: $y_i = \log(1 + e^{x_i})$
  - "Softer" version of ReLU

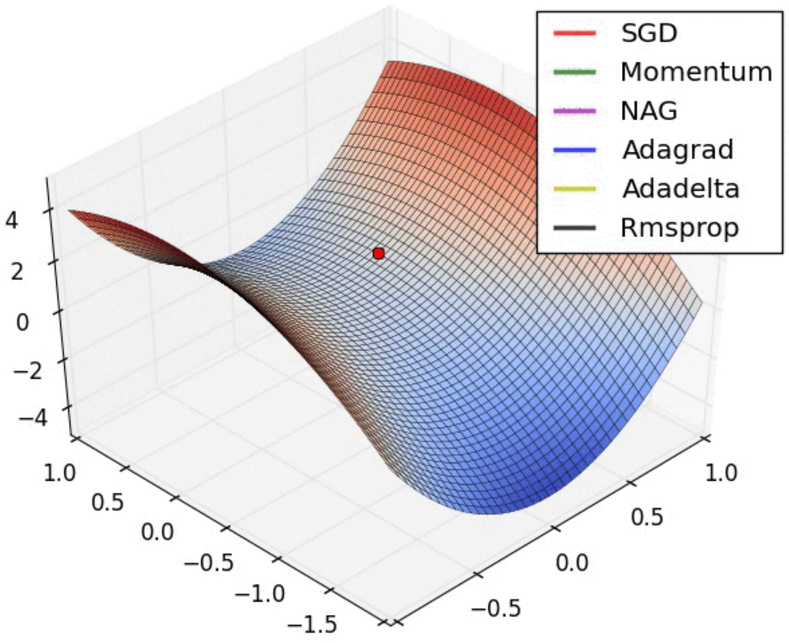


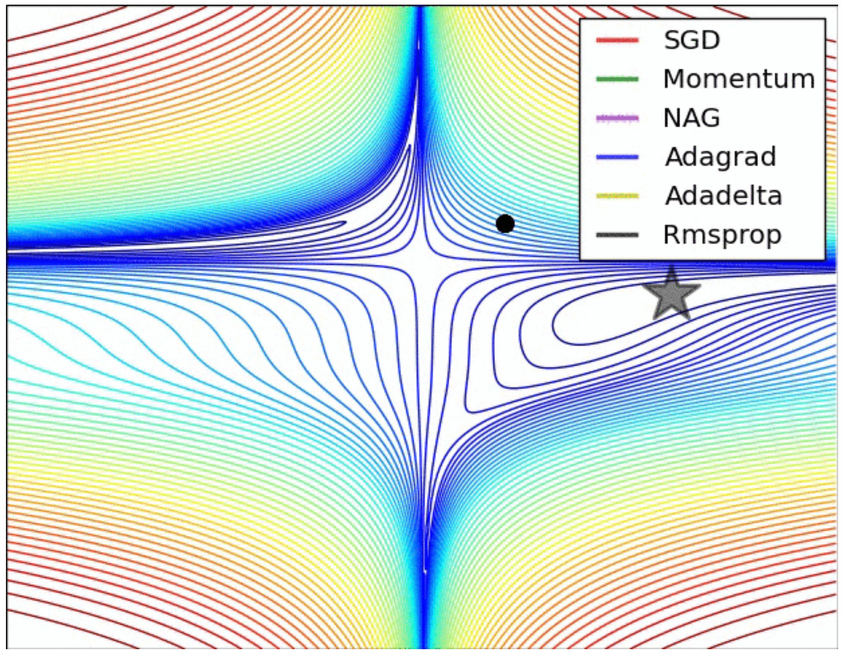
Moving past gradient descent

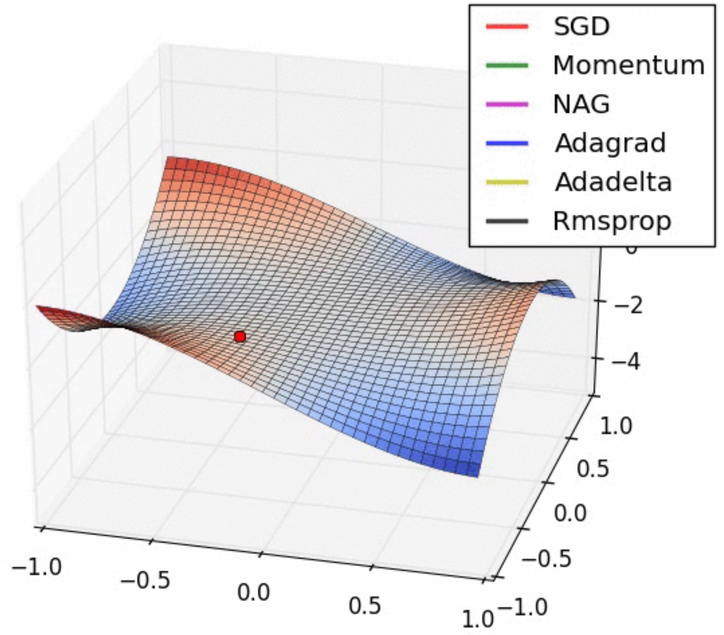
- Stochastic Gradient (with optional momentum)
  - Use random batches, update using: $\Delta w_t = \nabla f(w_t) + \mu \Delta w_{t-1}$
  - Nesterov momentum: $\Delta w_t = \nabla f(w_t + \mu \Delta w_{t-1}) + \mu \Delta w_{t-1}$
- Rprop
  - Use gradient sign, make steps using adaptive learning rate/momentum
  - RMSprop: Normalize individual weight learning rates by the running average of past gradient magnitudes
- Adagrad / Adadelta / Adam
  - Learning rate-free approaches
- …

What difference does it make?

- Alec Radford's excellent training animations:







http://imgur.com/a/Hqolp

Model Compression

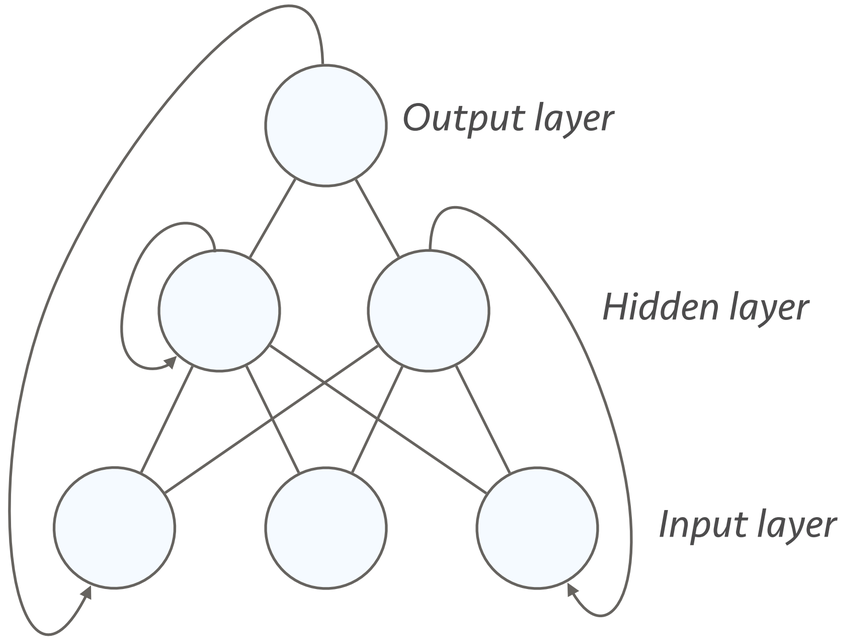
- Is being shallow necessary?
  - Remember that three layers should be all we need
- Using a deep structure tends to facilitate training
  - Despite the introduction of many more parameters
- Train shallow networks to mimic trained deep networks
  - Train a deep network, generate a lot of outputs for random inputs and use them as training data for a shallow network

Getting more into signals

- As we've seen before we care about time!
  - That's what makes a signal
- What we have so far is time-agnostic
  - Therefore a bad idea for signals
- How can we add some temporal structure?

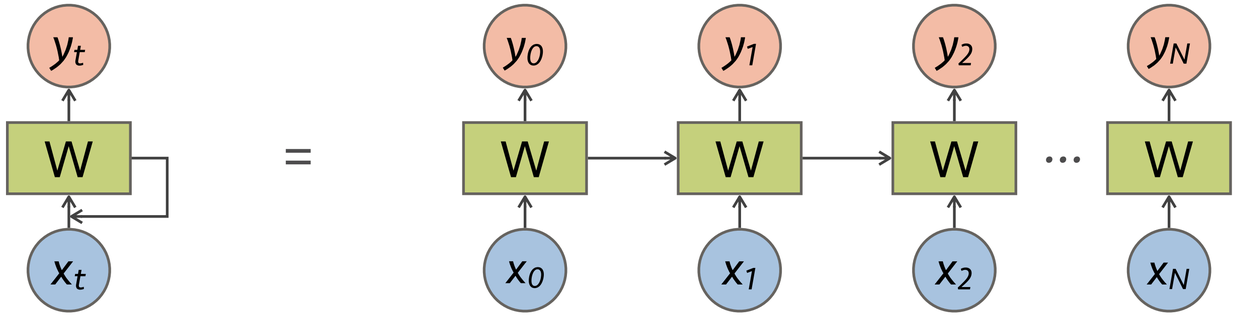
Recursive Neural Networks

- Use node outputs as inputs
  - From anywhere to anywhere
- Some problems
  - Can form an unstable system
  - Can struggle with large data
    - More on RNNs on Friday
- RNNs are Turing complete!
  - In theory they can compute anything!

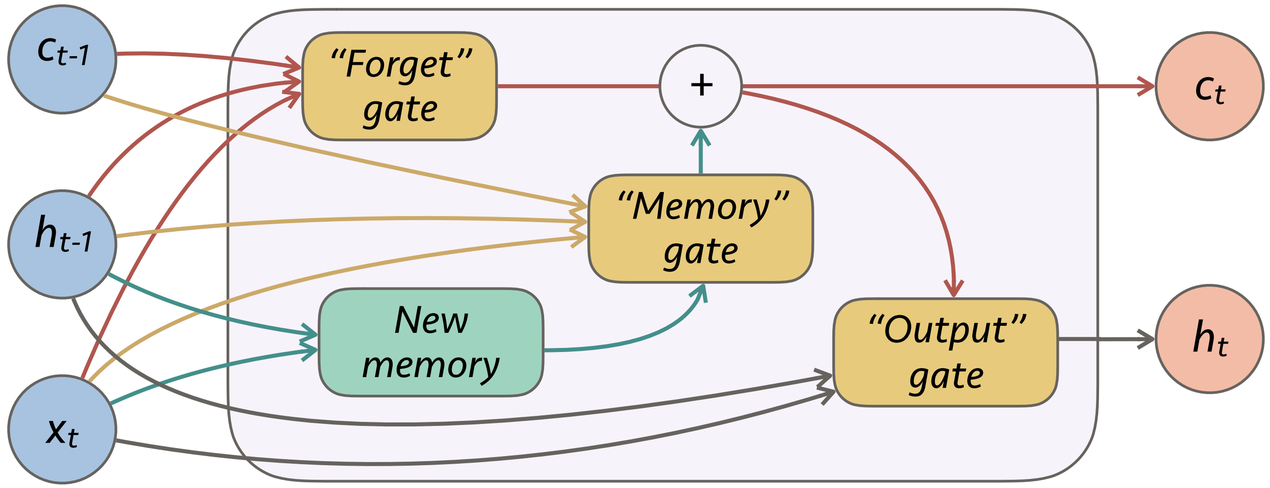


Long Short-Term Memory Networks

- RNNs are notoriously hard to train (inherently deep!)

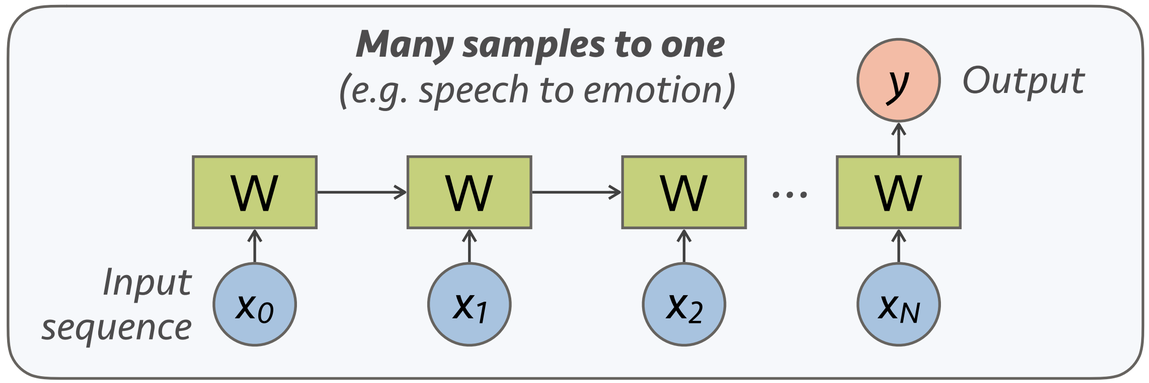


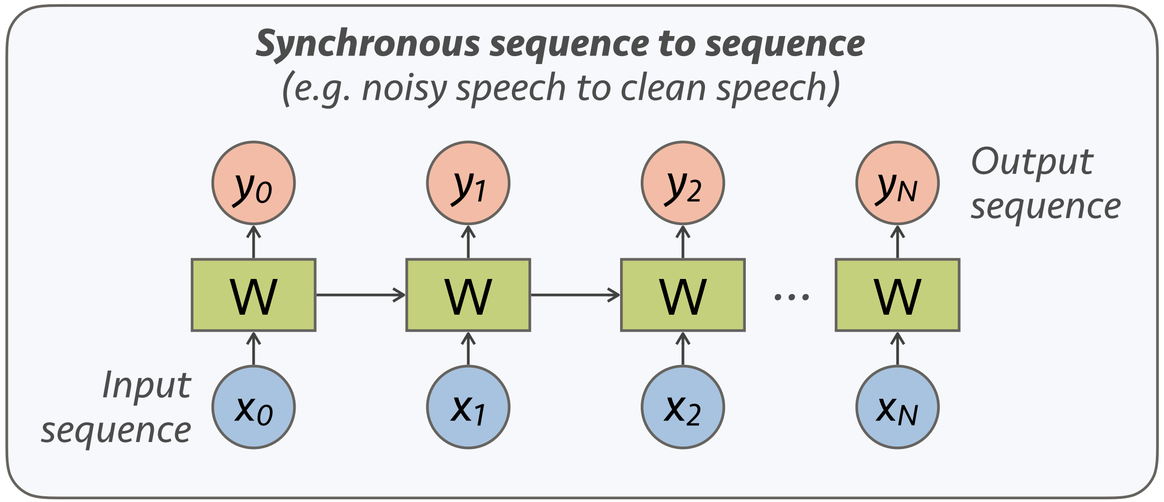
- LSTMs resolve that using gating:

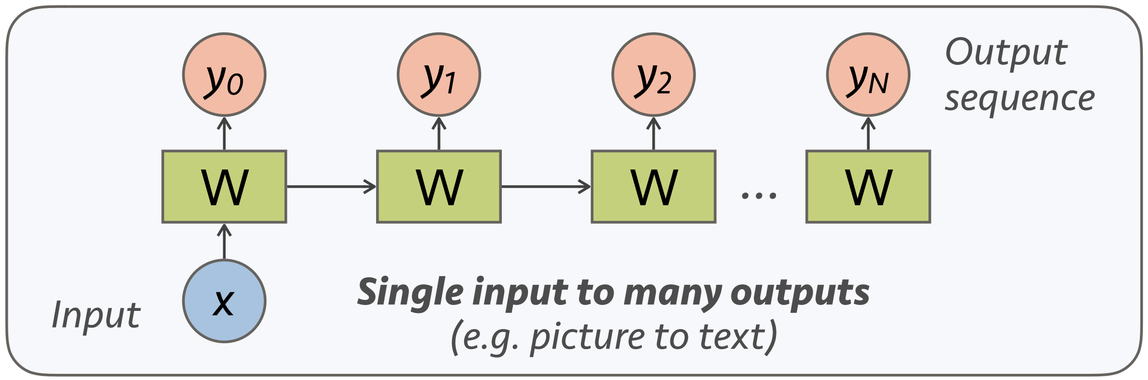


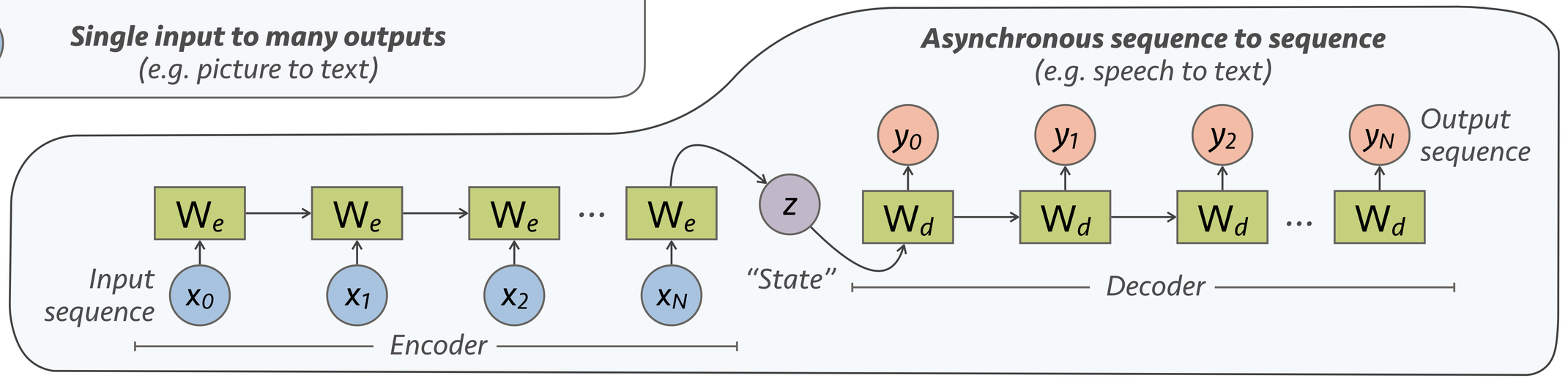
- *New memory from input & past output*
- *Forget gate: how much past memory counts*
- *Memory gate: how much new memory counts*
- *Output gate: combine all to make output*

Lots of time-aware topologies





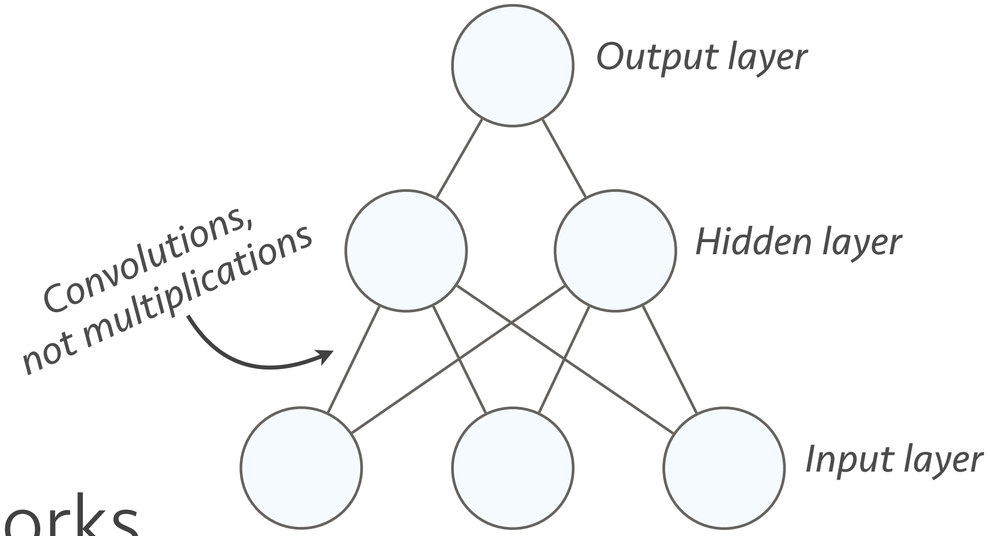




FIR Neural Networks

- Extend the scope of a neural net
  - Instead of weights, use FIR filters

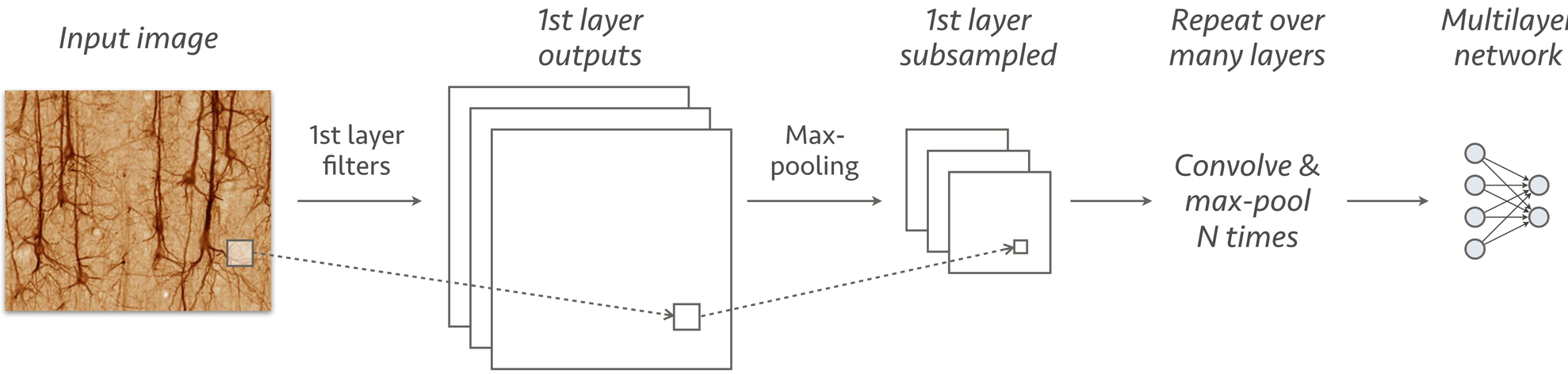
$$x_i^l = \sum_j w_{i,j} * x_j^{l-1}$$



- Convolution is still linear
  - The usual backprop approach still works
    - Convolution is a matrix multiply, i.e. a simple layer
- Good fit for temporal prediction tasks!

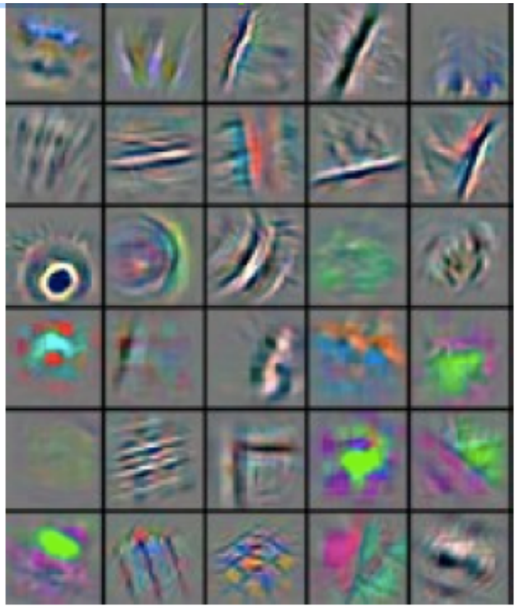
Convolutional Networks

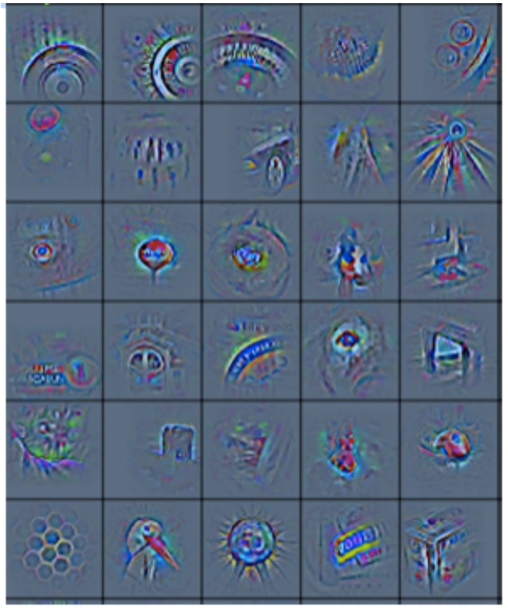
- Generalizing the FIR filter idea
  - Unit weights are 2D filters (or 3D, 4D, …)
- Max-pooling
  - For each neighborhood of filter outputs, keep only the max value



Convolutional networks for vision

- We extract deep features that make sense
  - These features are the filters at each layer
  - Also results in state of the art recognition!





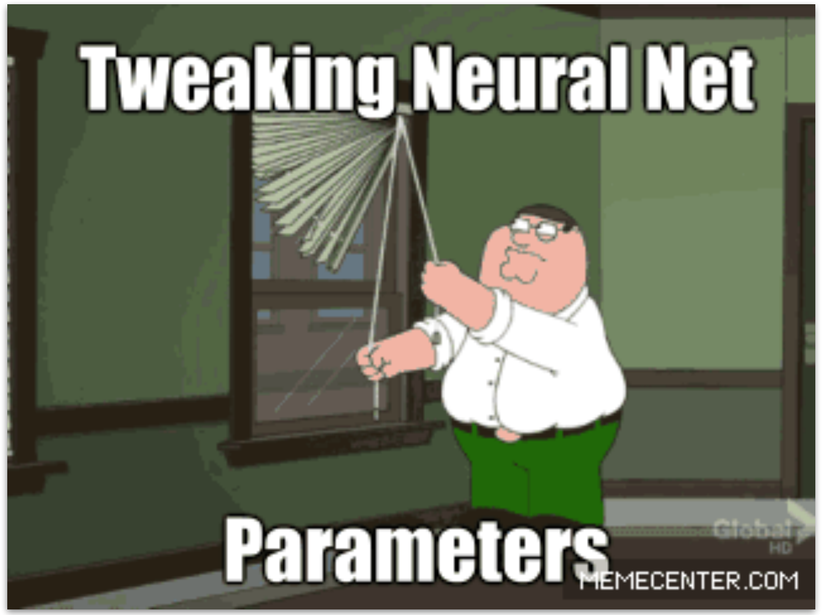
faces, cars, airplanes, motorbikes

And there are many evolving ideas

- Extreme Learning Machines & Echo State Machines
  - Using a lot of fixed random nodes
- Residual Networks
  - Map to output + input, allows for deeper models
- Wavenet
  - Using convolutions over multiple time-scales
- Generative Adversarial Models
  - We'll cover these later
- …

So what's the verdict?

- Good news:
  - Very powerful and flexible approaches
  - Potential biological plausibility
    - And computability implications



- Bad news:
  - Too flexible, picking right structure is very much an art
  - Cumbersome and potentially finicky training procedures

Running deep learning today

- You need data, GPUs and software for it
  - GPUs and software are easy to get!

- Software takes care of learning/deriving models:
  - http://pytorch.org ← *Paris' tool of choice, super flexible*
  - https://www.tensorflow.org ← *Probably the most popular today*
  - http://keras.io ← *Higher-level API, easy for quick prototyping*

Recap

- Deep learning concepts
- Stochastic shallow networks
  - Boltzmann machine
- Common deep architectures
  - DBNs, auto-encoders, recurrent and convolutional networks

More material

- "Deep Learning", the book:
  - http://www.deeplearningbook.org
  - (no PDF but you can read online)
- Learning Convolutional Feature Hierarchies for Visual Recognition
  - http://yann.lecun.com/exdb/publis/pdf/koray-nips-10.pdf
- A Fast Learning Algorithm for Deep Belief Nets
  - http://www.cs.toronto.edu/~hinton/absps/fastnc.pdf

Next lecture

- Probabilistic Graphical Models
  - Cem guest lectures