CS598PS – Machine Learning for Signal Processing

Learning Time Series

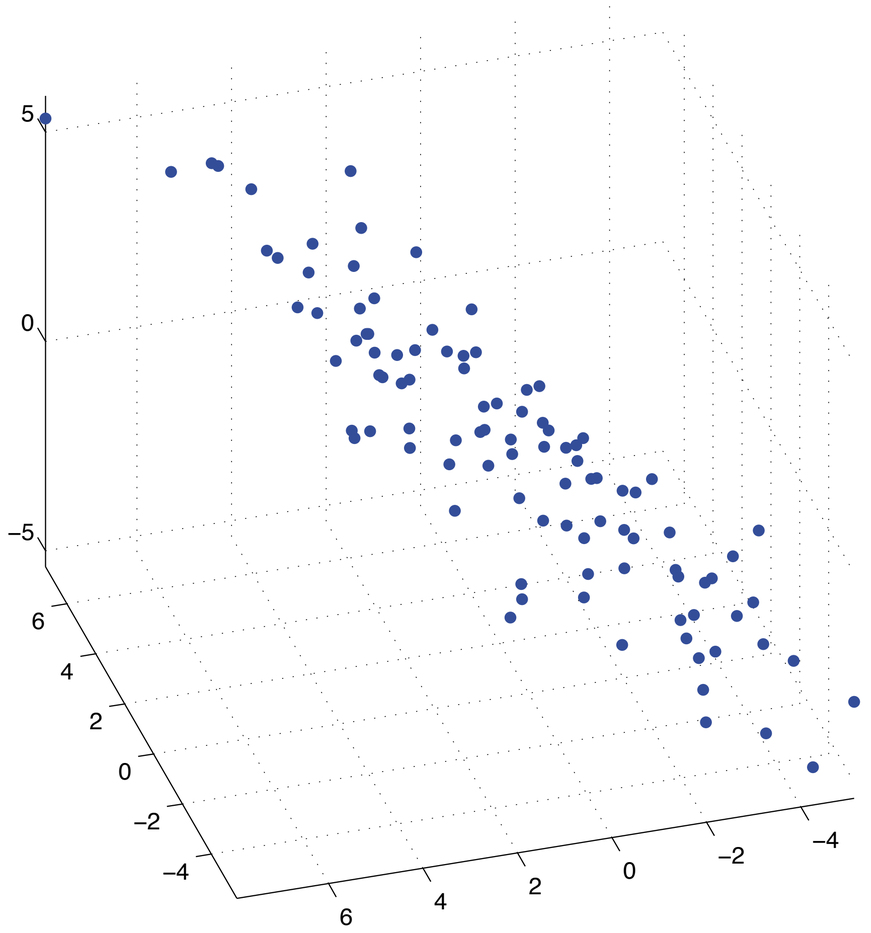
11 October 2017

Today's lecture

- Doing machine learning on time series
- Dynamic Time Warping
- Hidden Markov Models

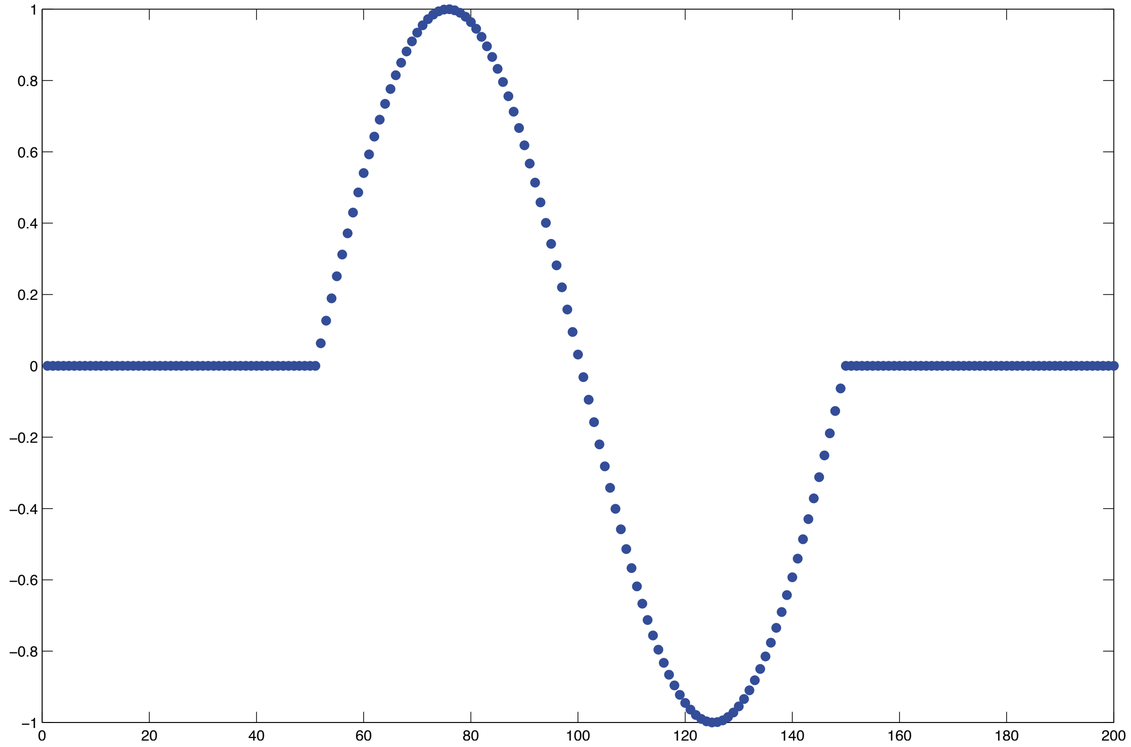
Our view of data so far

- Data are points in a high-dimensional space



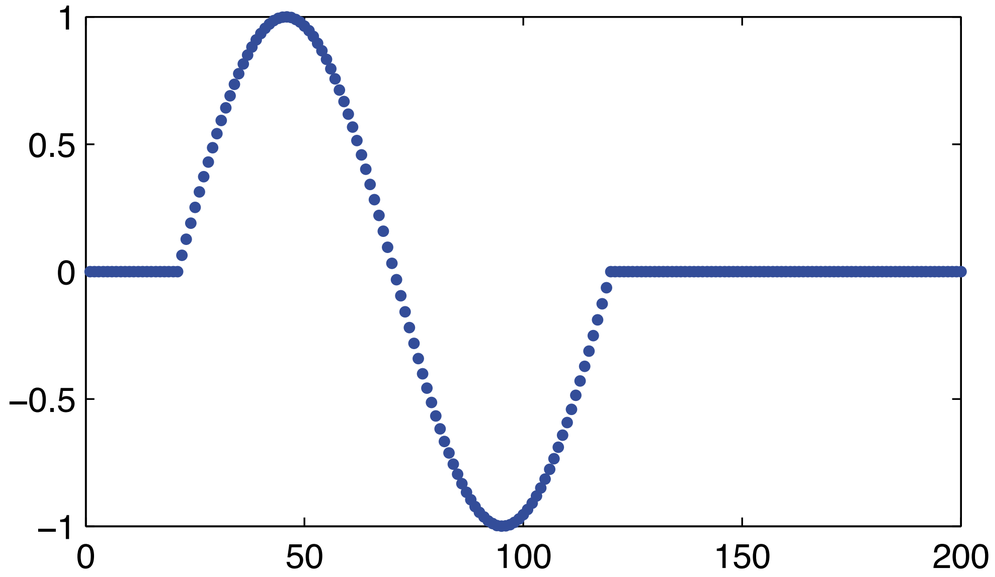
What time series are

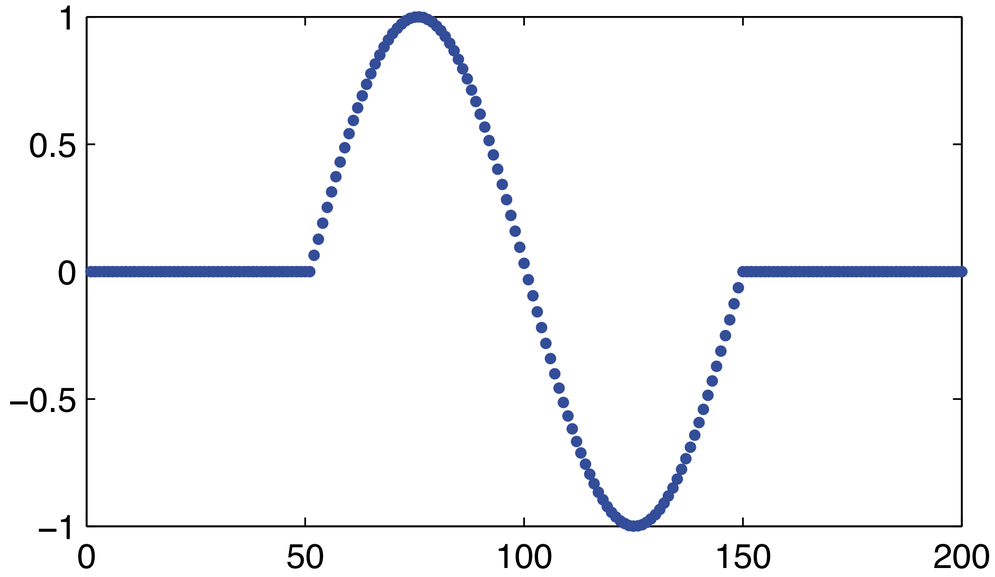
- A sequence of samples, can be thought of as a point in a very very high-D space
  - Often a bad idea
  - Remember the curse!!



Shift variance

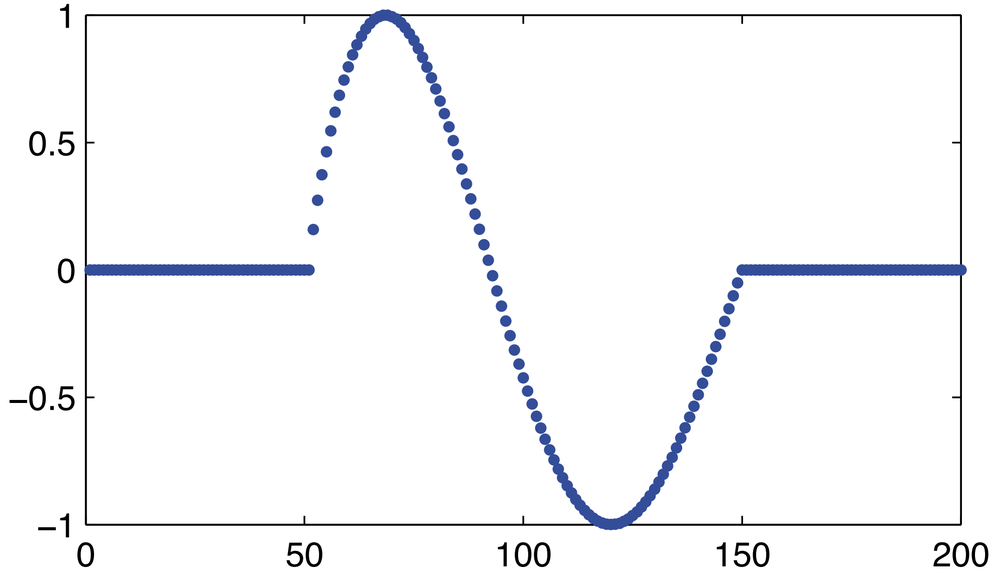
- Time series have shift variance
  - Are these two points close?

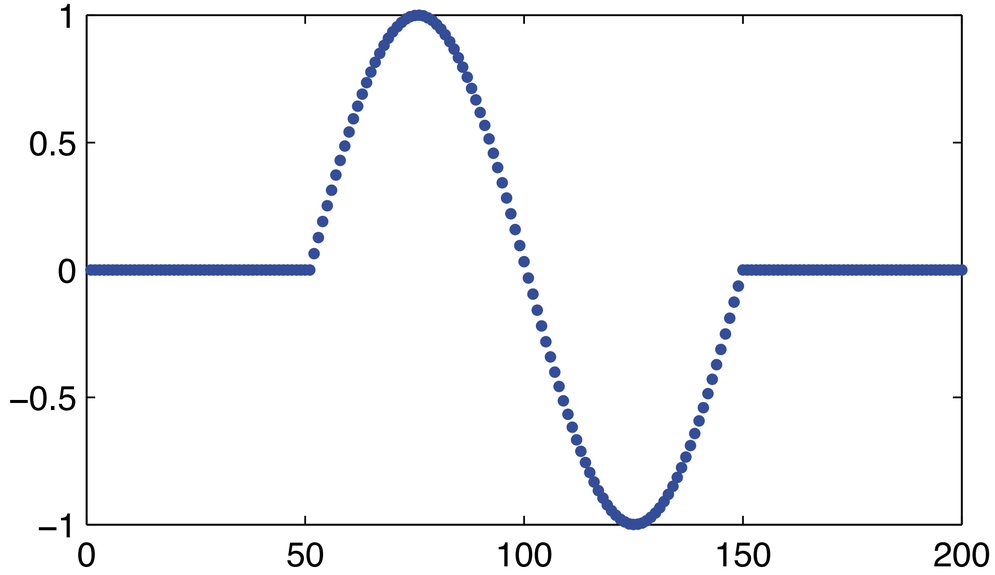




Time warp variance

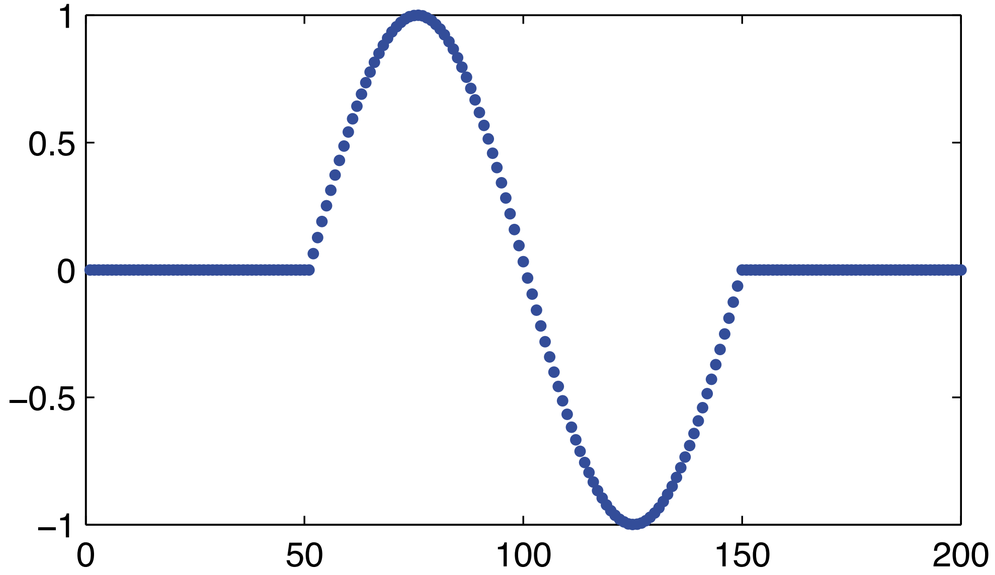
- Slight changes in timing are not relevant
  - Are these two points close?

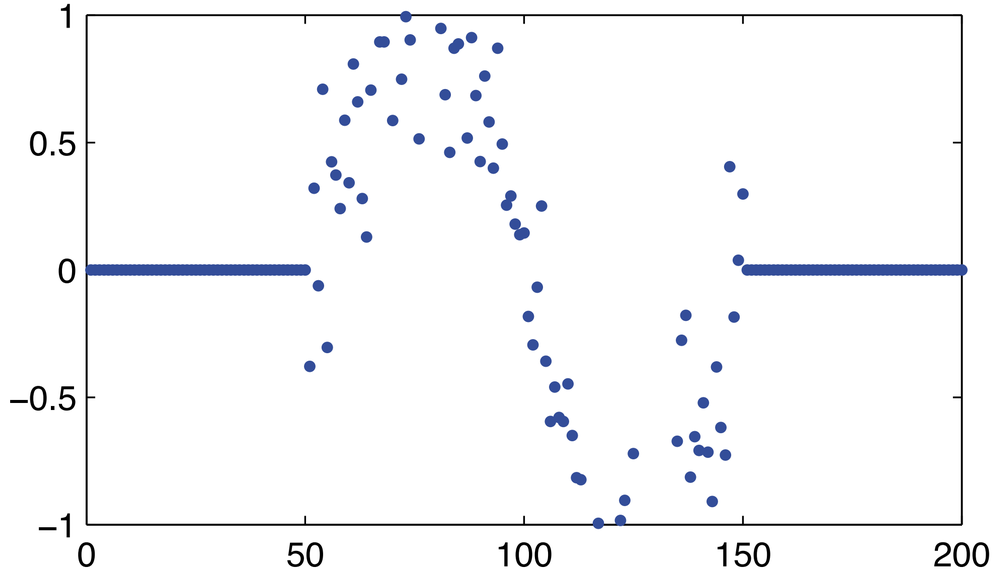




Noise/filtering variance

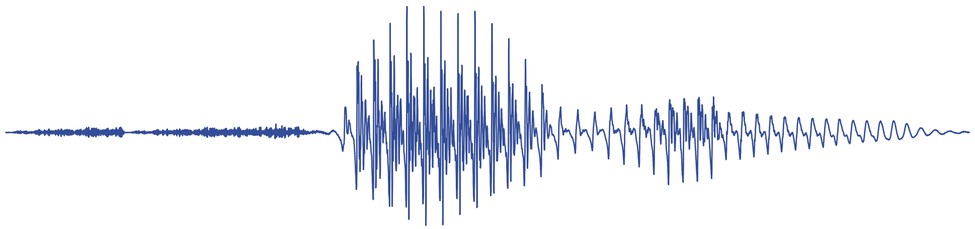
- Small changes can look serious
  - How about these two points?

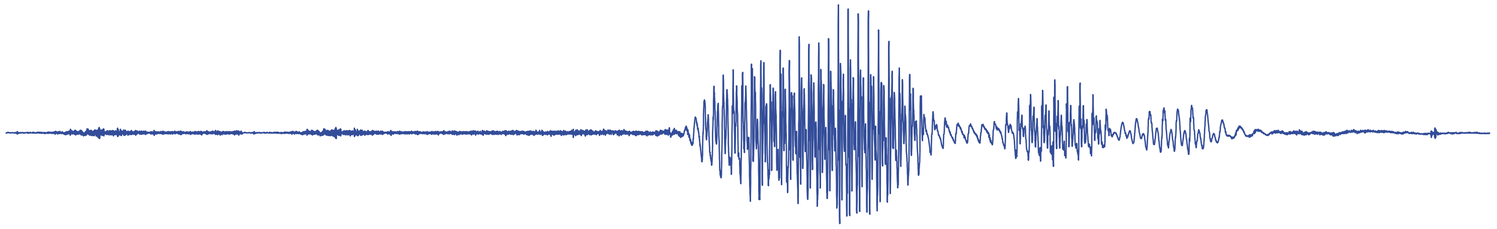


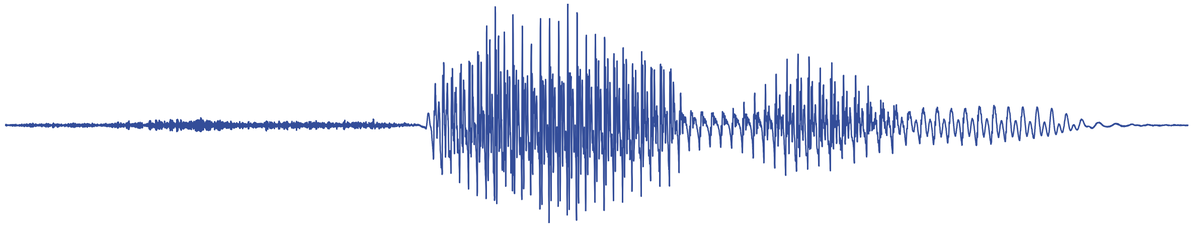


A real-world case

- Spoken digits







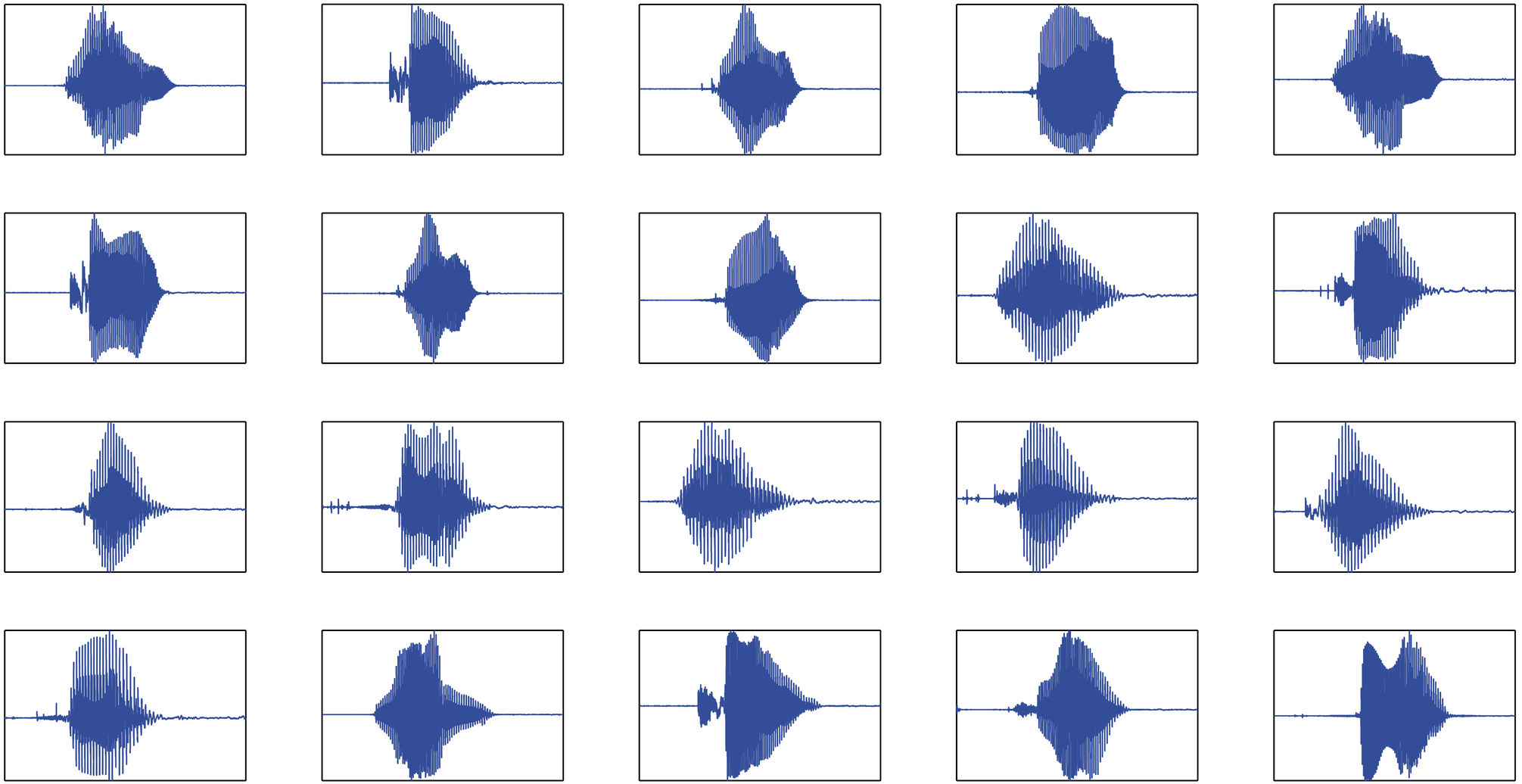
What now?

- Our models so far were too simple
- How do we incorporate time?
- How do we get around all these problems?

A small case study

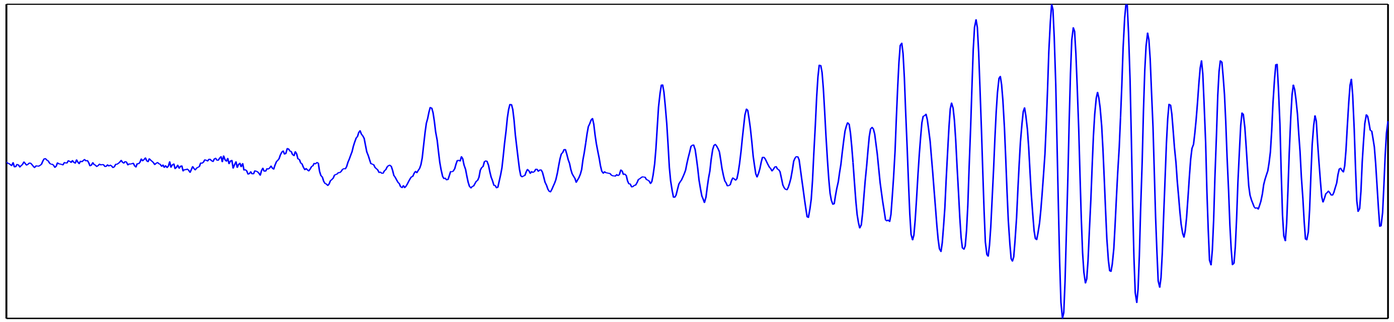
- How to recognize words
  - e.g. yes/no, or spoken digits
- Build reliable features
  - Invariant to minor differences in inputs
- Build a classifier that can deal with a time index
  - Invariant to temporal differences in inputs

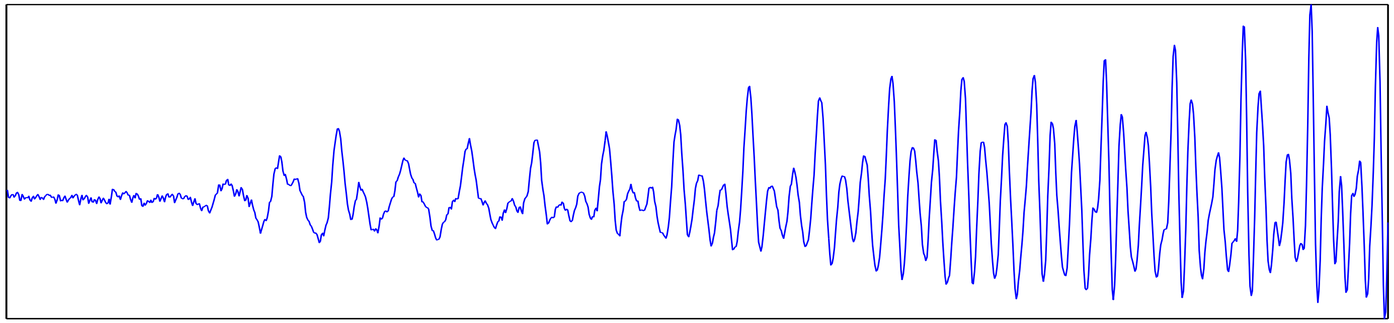
Example data



Going from fine to coarse

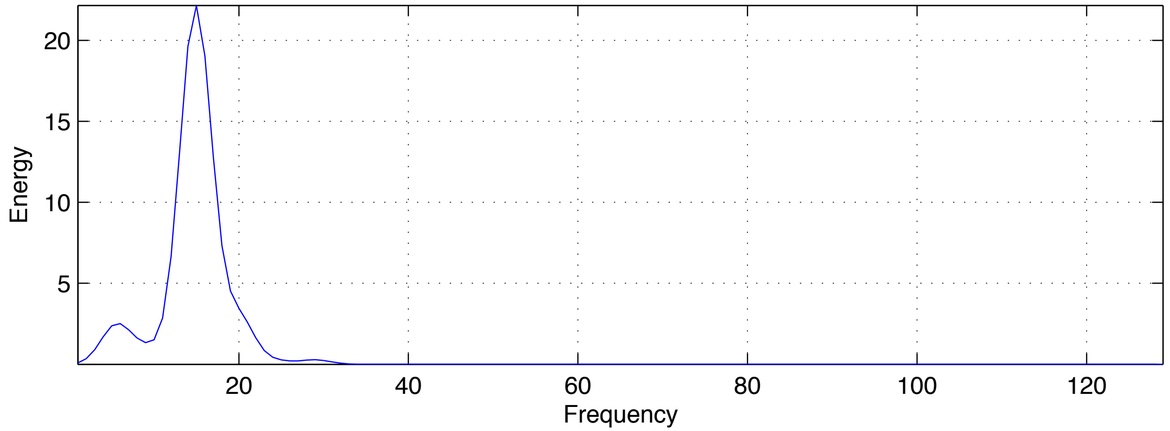
- Small differences are not important
  - Find features that obscure them

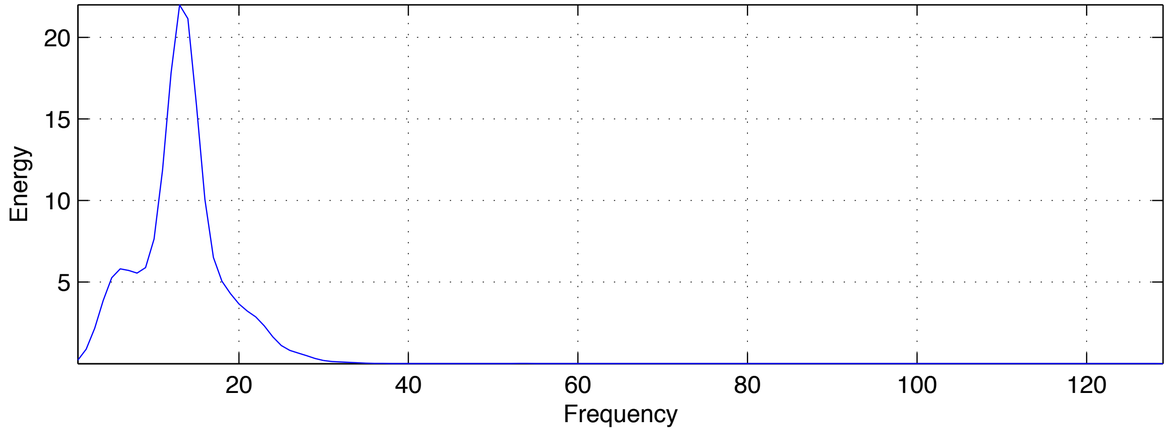




Frequency domain

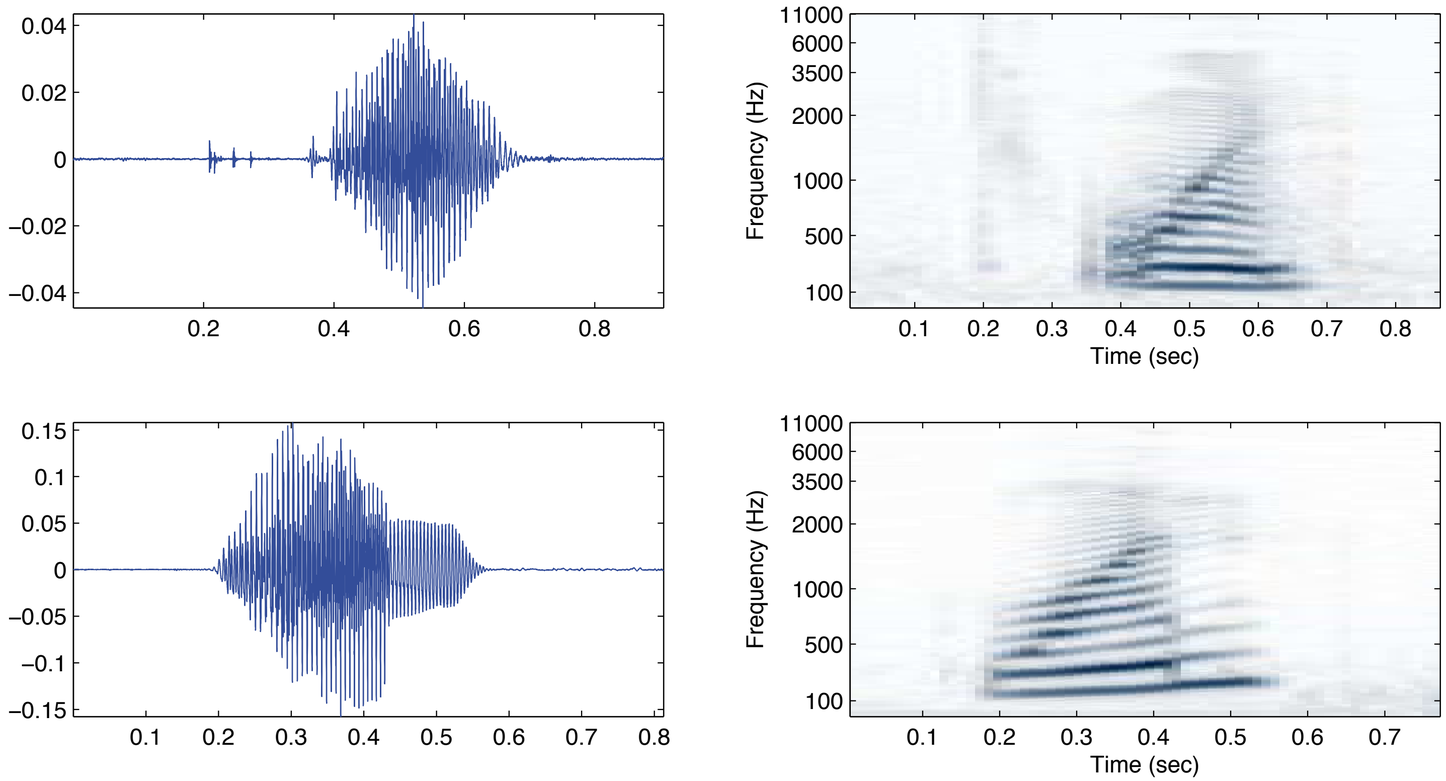
- Look at the magnitude Fourier transform





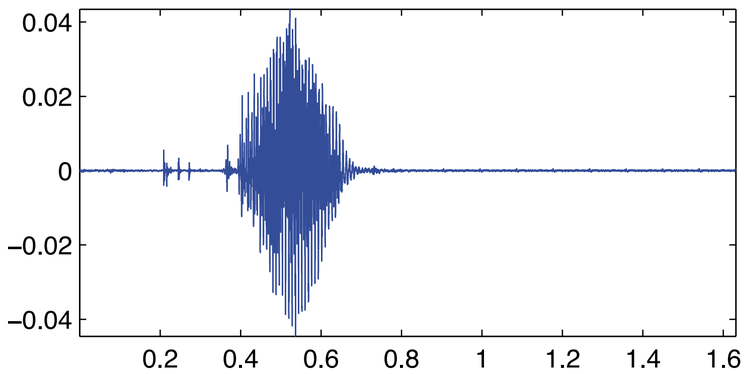
Time/Frequency features

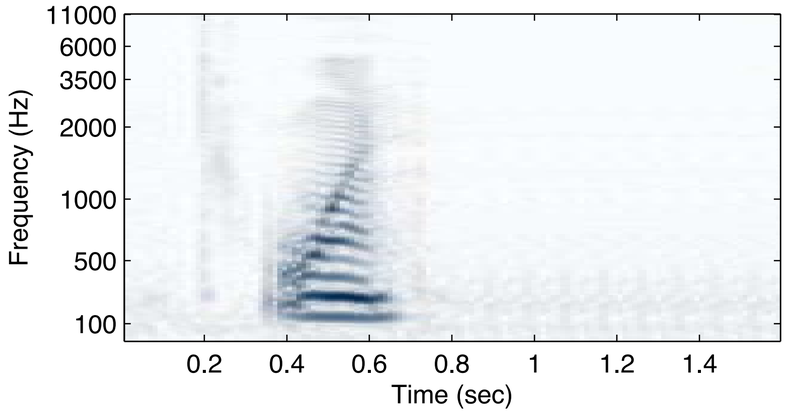
- A more robust representation
  - Bypassing small waveform differences

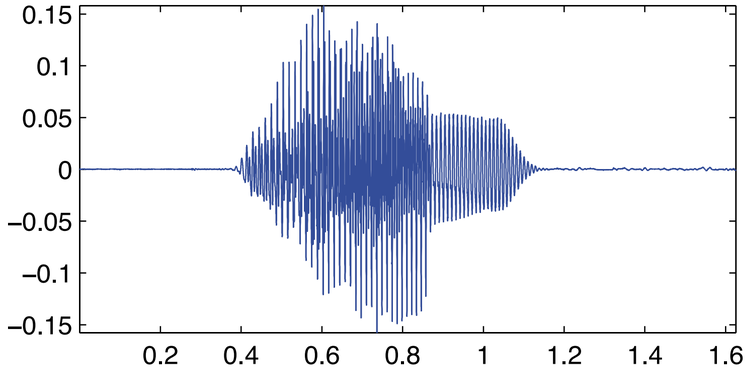


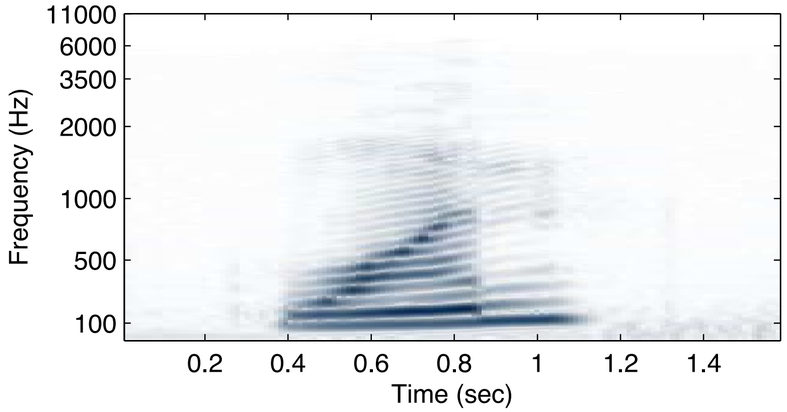
A new problem

- What about time warping?



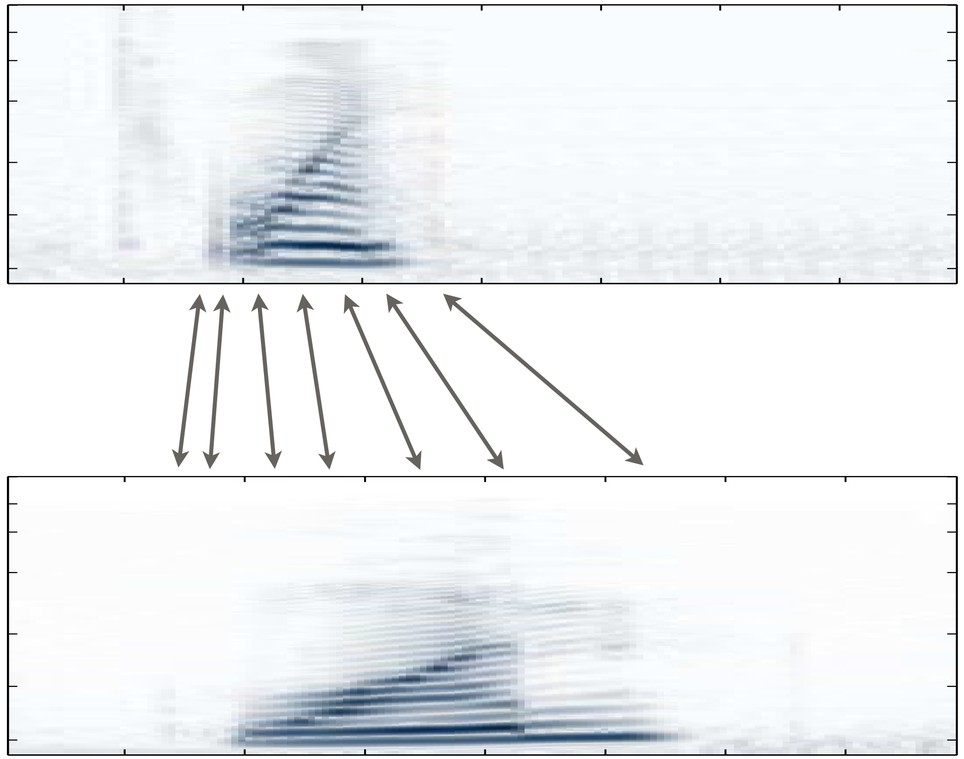






Time warping

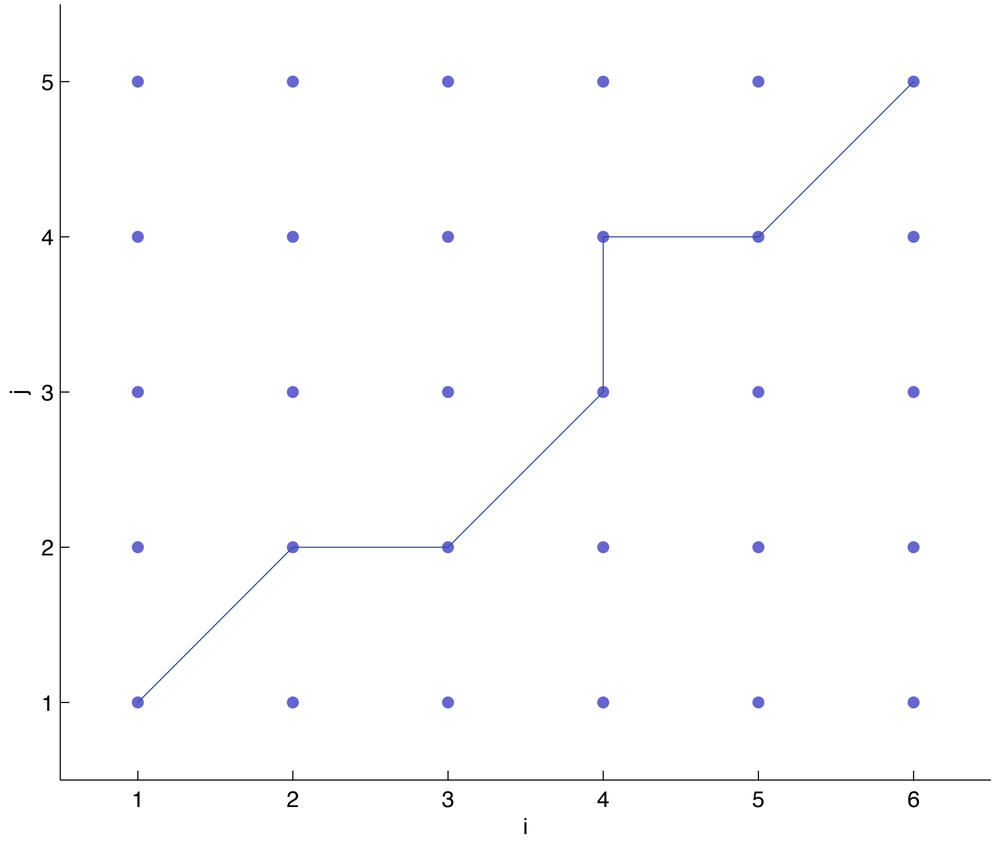
- There is a “warped” time map
  - How do we find it?



Matching warped series

- Represent the warping with a path on a grid

$$\mathbf{r}(i), i = 1,2,\ldots,6 \quad \mathbf{t}(j), j = 1,2,\ldots,5$$

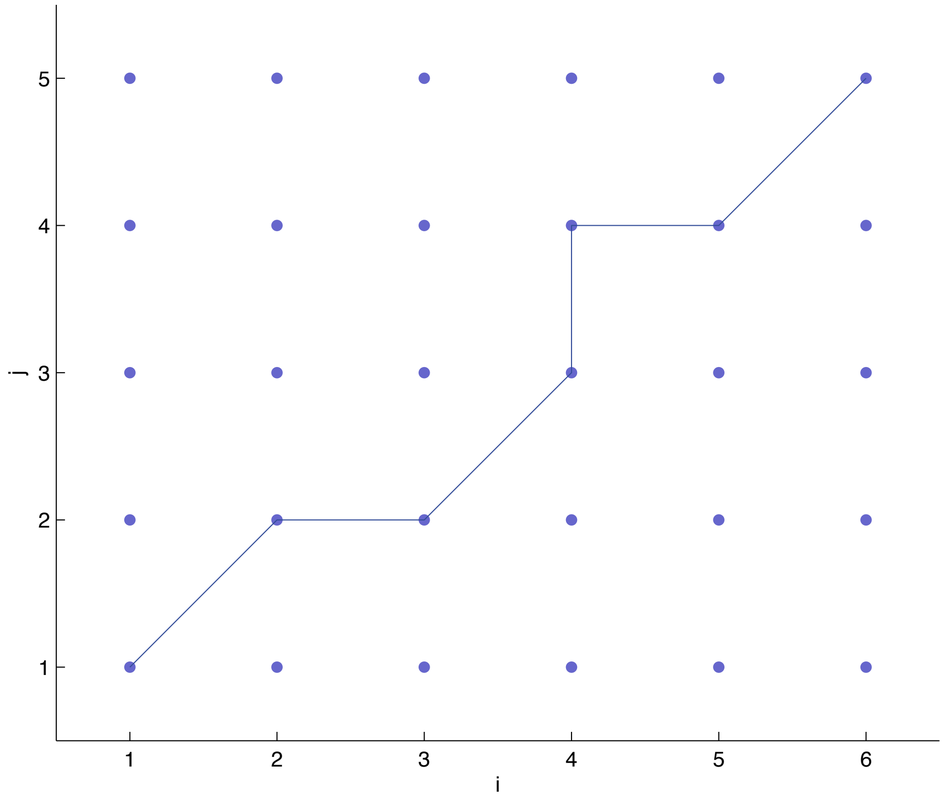


Finding the overall "distance"

- Each node will have a cost
  - e.g., $d(i,j) = \left\| \mathbf{r}(i) - \mathbf{t}(j) \right\|$
- Overall path cost is:

$$D = \sum_k d(i_k, j_k)$$

- Optimal path (i_k, j_k) defines the "distance" between two given sequences
- But how do we find the optimal path?
  - Big search space …



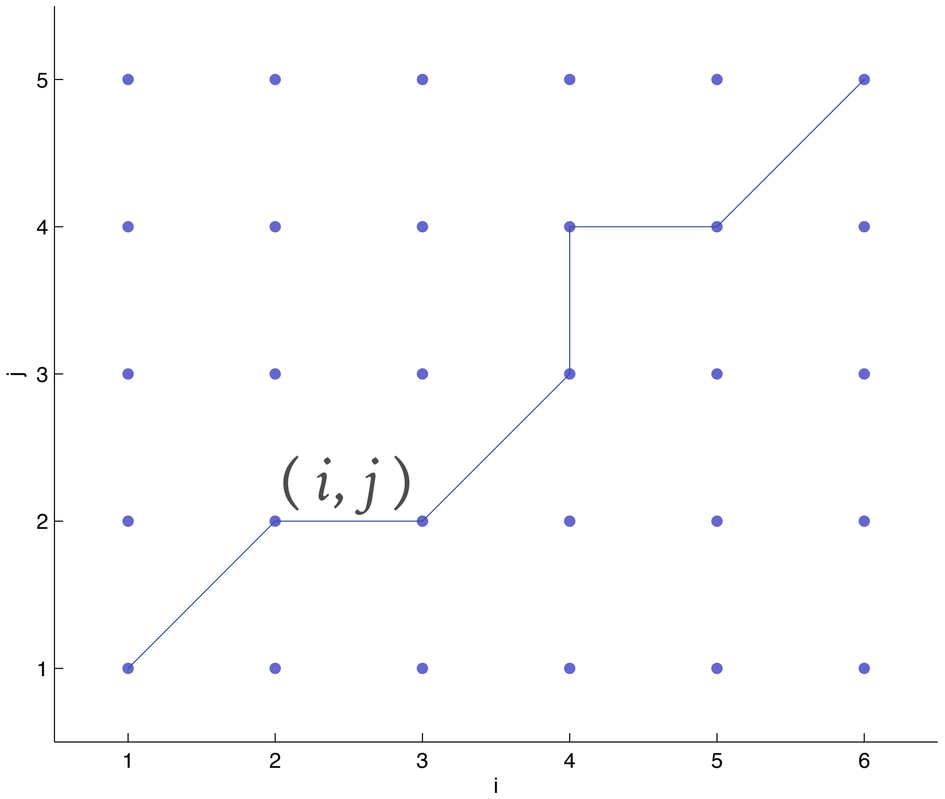
Bellman's optimality principle

- For an optimal path passing through

$$(i,j):(i_0,j_0)\overset{opt}{\longrightarrow}(i_f,j_f)$$

- Then:

$$(i_0,j_0)\overset{opt}{\longrightarrow}(i_f,j_f)=$$
$$\left\{(i_0,j_0)\overset{opt}{\longrightarrow}(i,j),(i,j)\overset{opt}{\longrightarrow}(i_f,j_f)\right\}$$



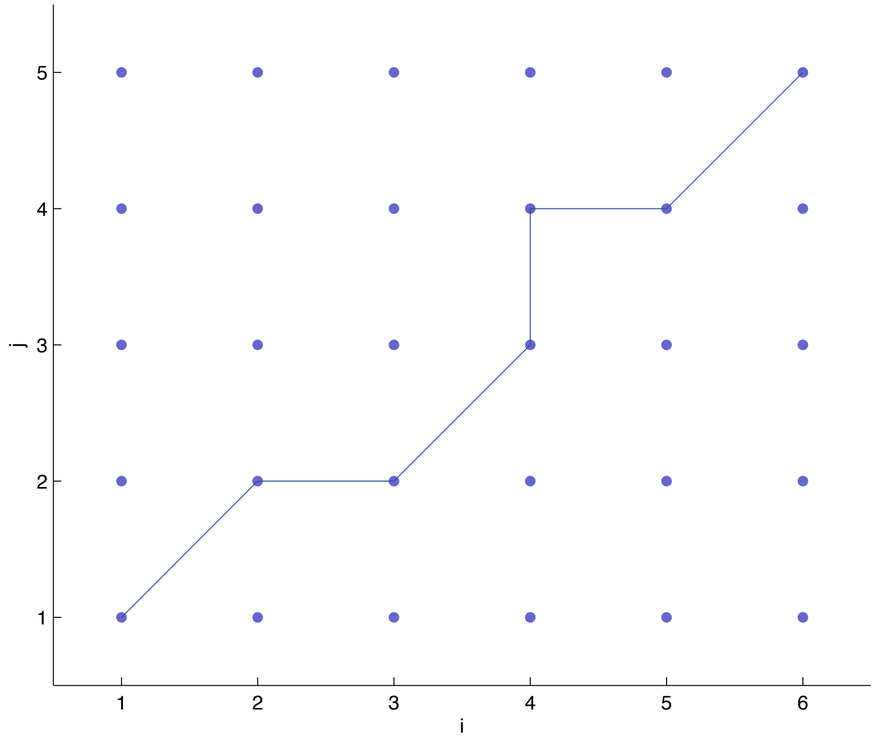
Finding an optimal path

- Optimal path to (i_k, j_k):

$$D_{\min}(i_k, j_k) = \min_{i_k-1, j_k-1} D_{\min}(i_k - 1, j_k - 1) + d(i_k, j_k \mid i_k - 1, j_k - 1)$$

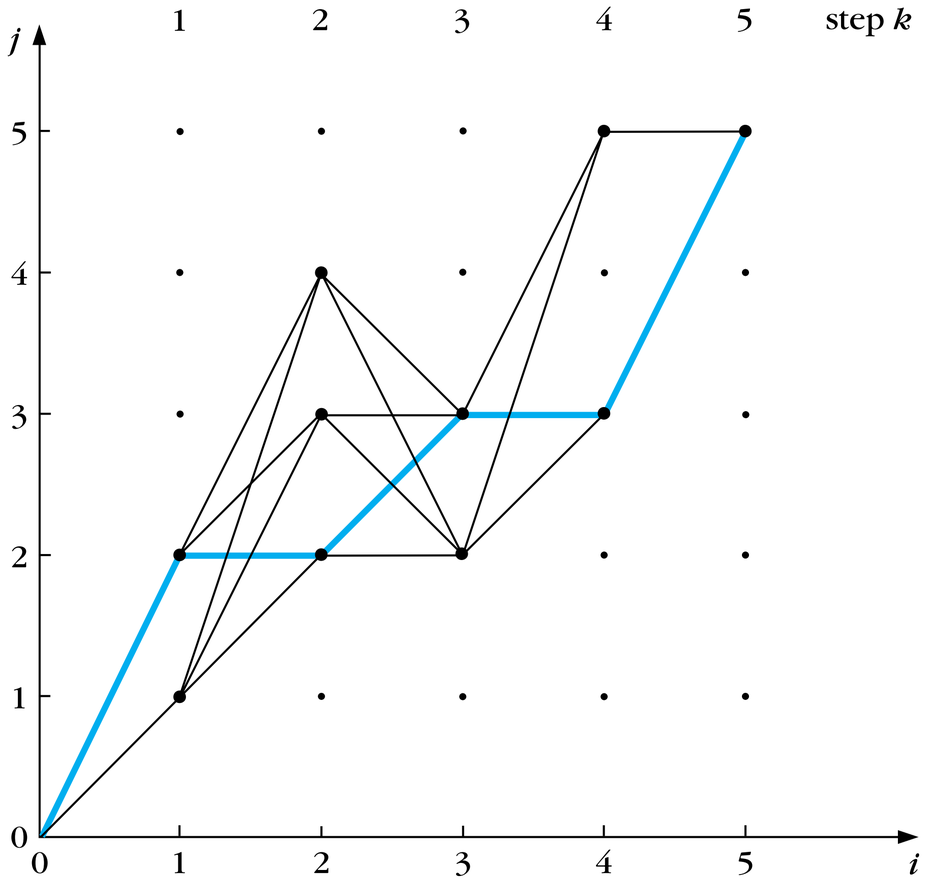
  - Smaller search!

- Local/global constraints
  - Limited transitions
  - Nodes we never visit



And adding some constraints

- Global constraints
  - Can only visit these
    - Bold dots
- Local constraints
  - Can only transition using them
    - Black lines
- Optimal path
  - Blue line

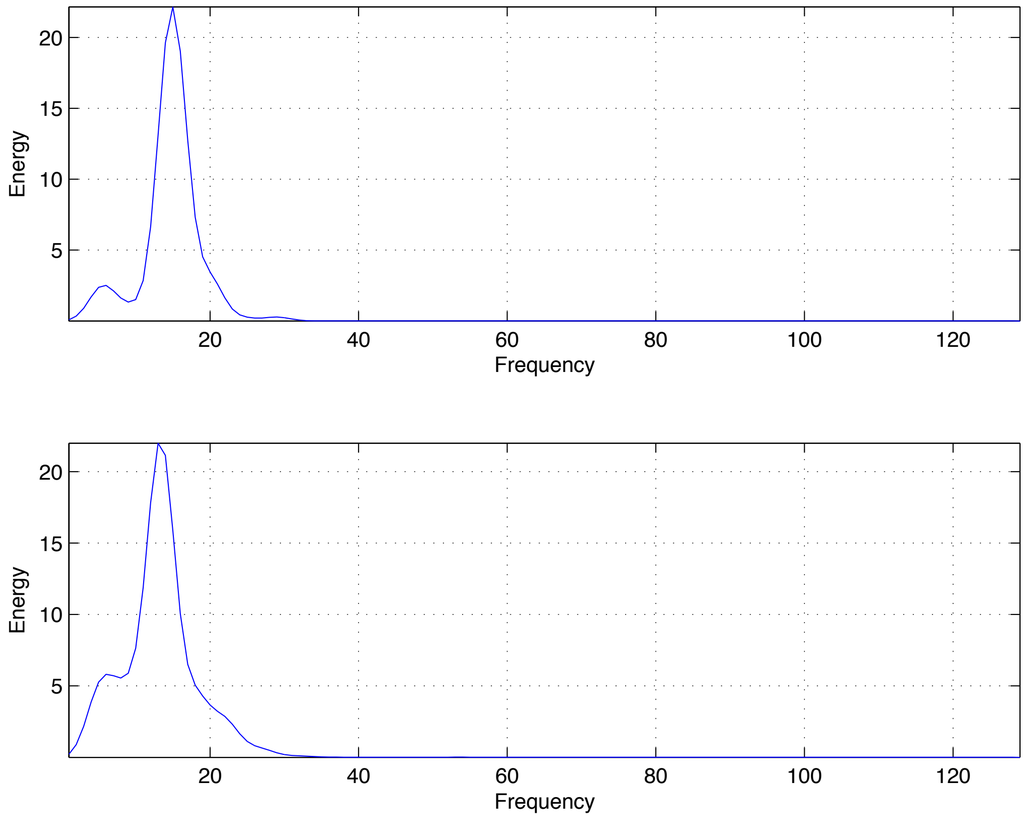


Making this work for speech recognition

- Define a distance function
- Define local constraints
- Define global constraints

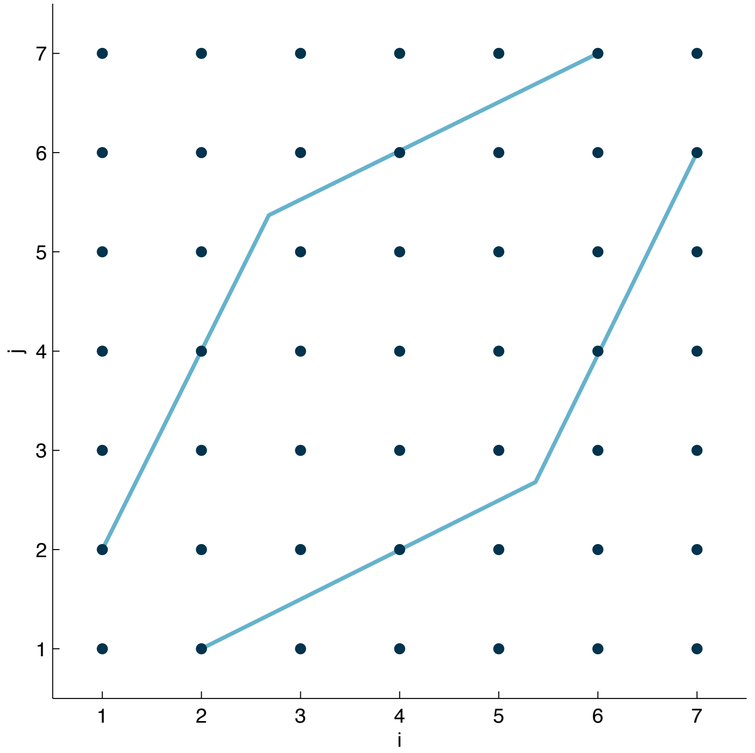
Distance function

- Given our robust feature we can use a simple measure like Euclidean distance: $d(i,j) = \left\| \mathbf{f}_1(i) - \mathbf{f}_2(j) \right\|$



Global constraints

- Define time ratios that make sense
  - e.g. no sequence can be half as slow as the other one



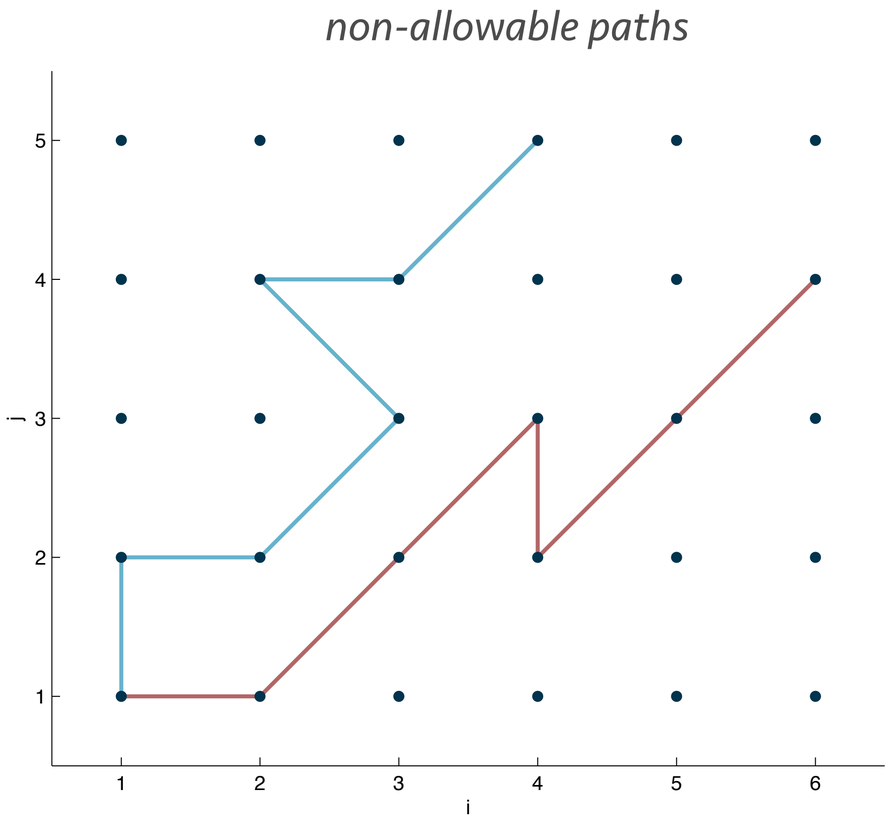
Local constraints

- Monotonicity:

$$i_{k-1} \leq i_k \quad j_{k-1} \leq j_k$$

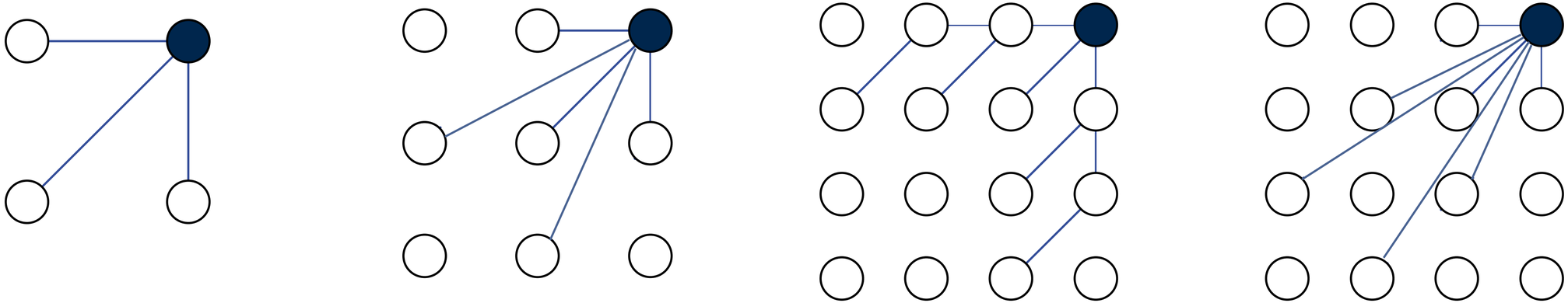
  - Repeat, but don't go backwards in time

- This enforces time order
  - Don't match"ban" with "banana"



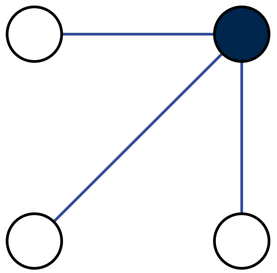
More local constraints

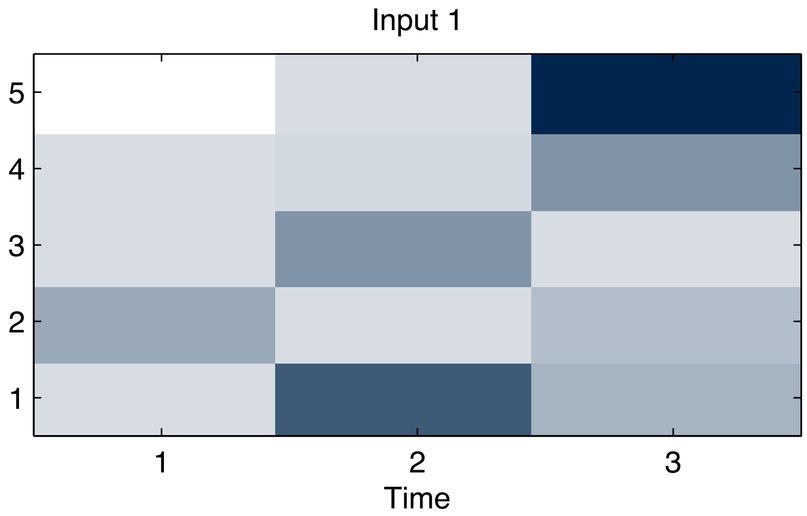
- Define acceptable local subpaths
  - Consider only these patterns
  - Application dependent

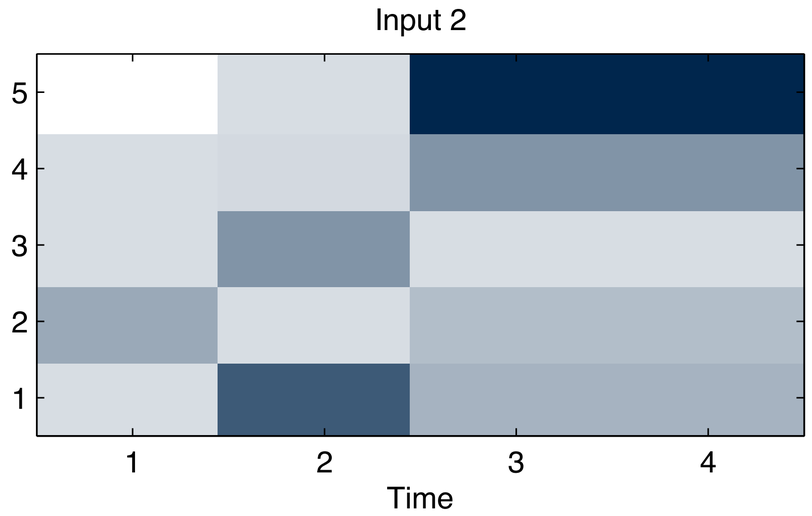


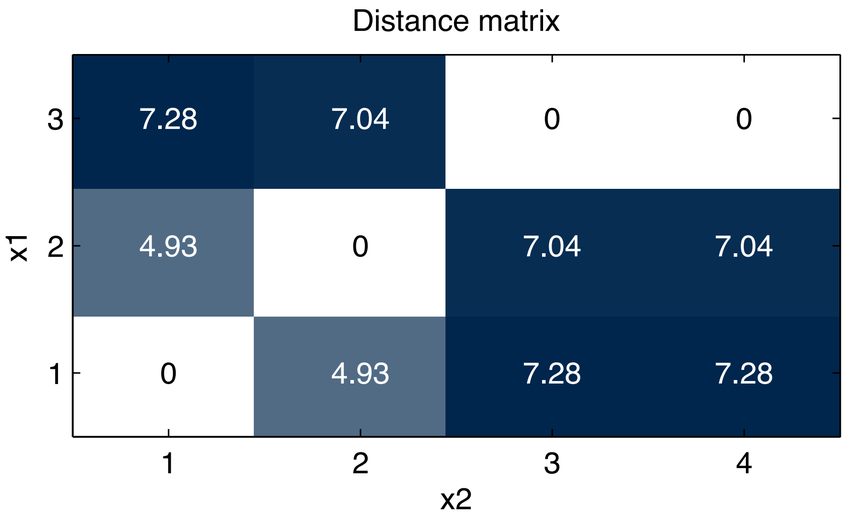
Toy data run

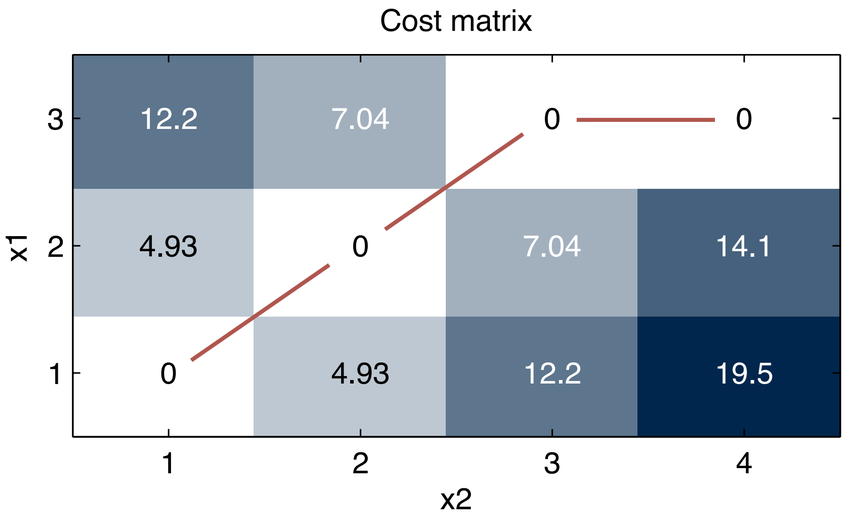
Local Constraint



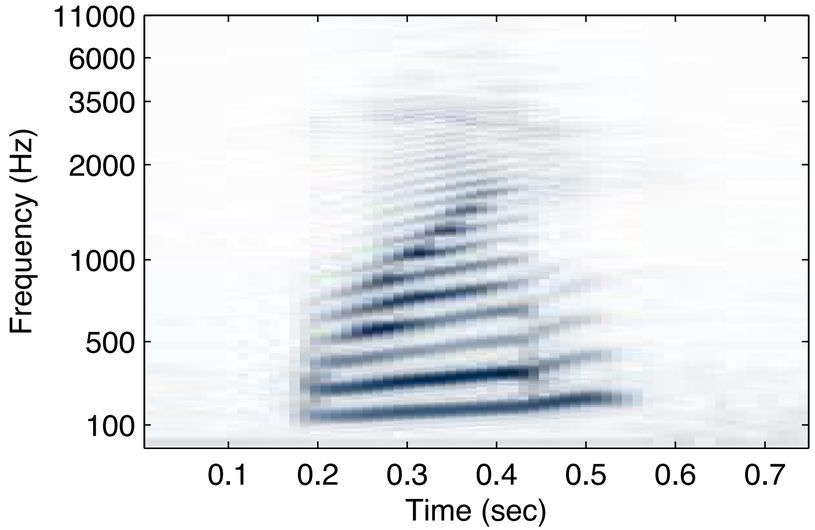


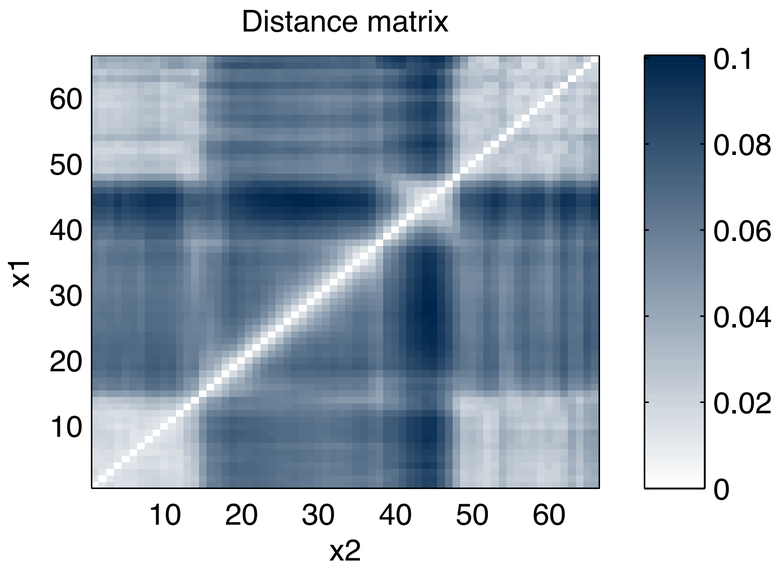


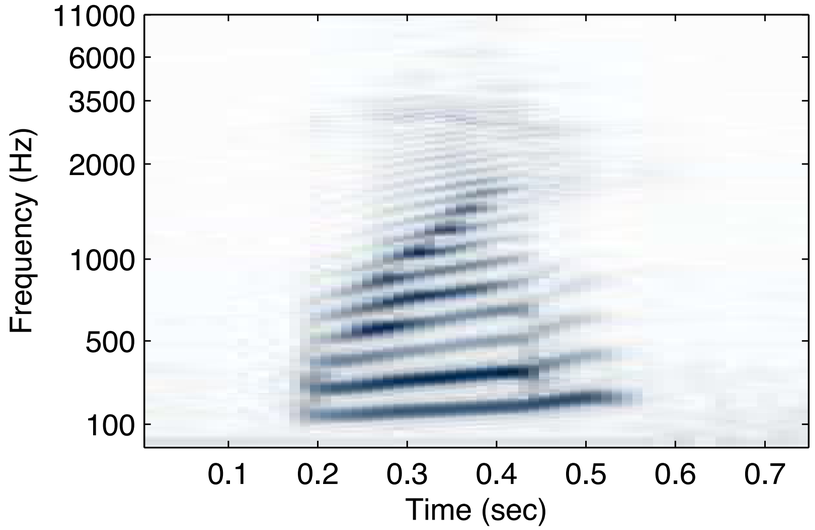


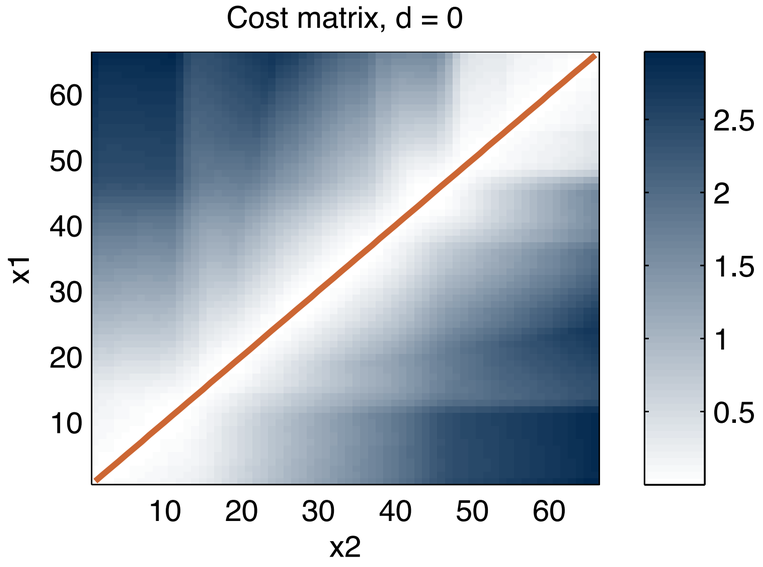


Speech example with identical utterance

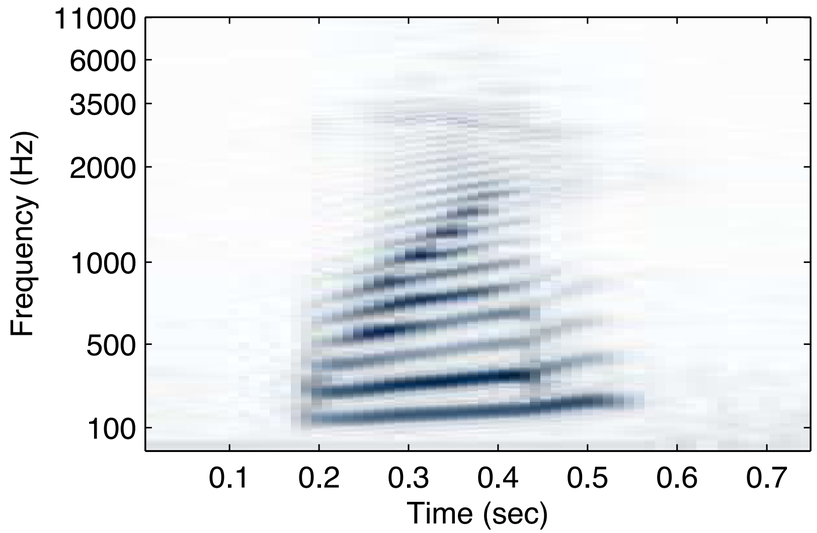


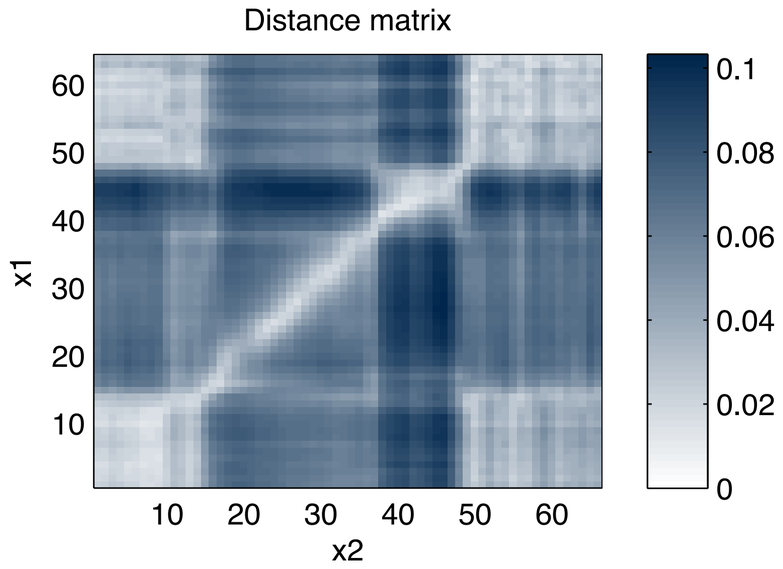


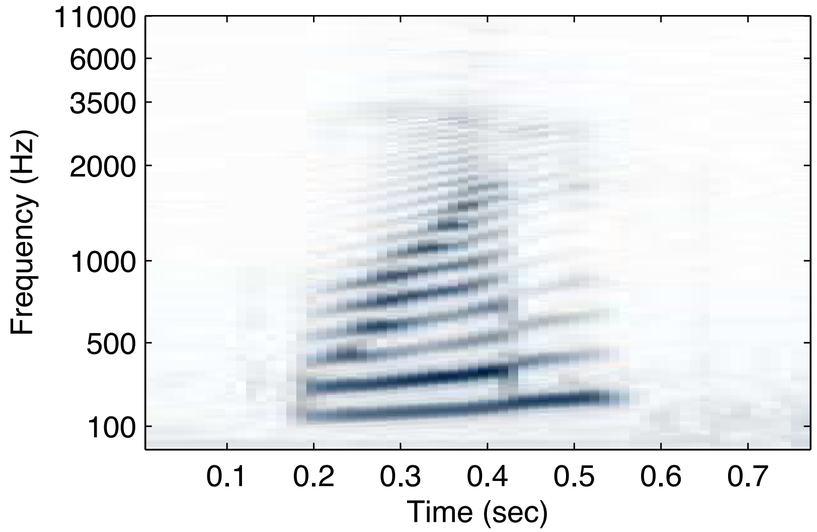


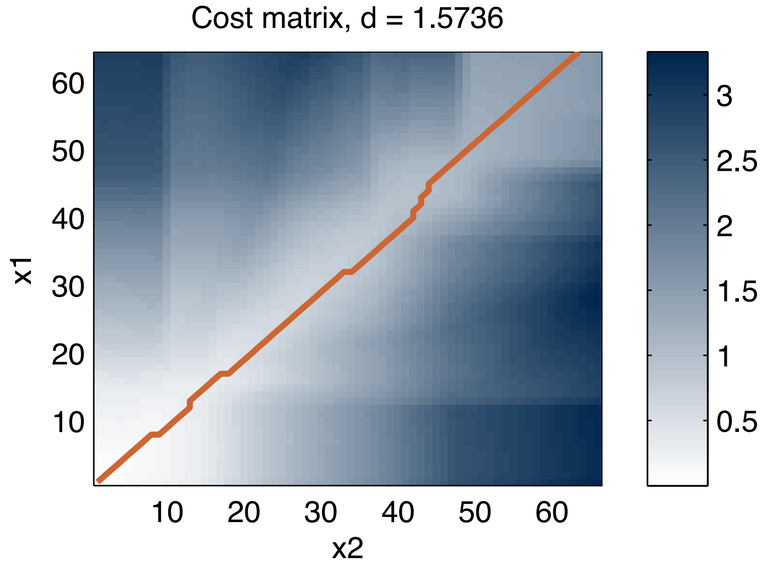


Ditto with similar utterance

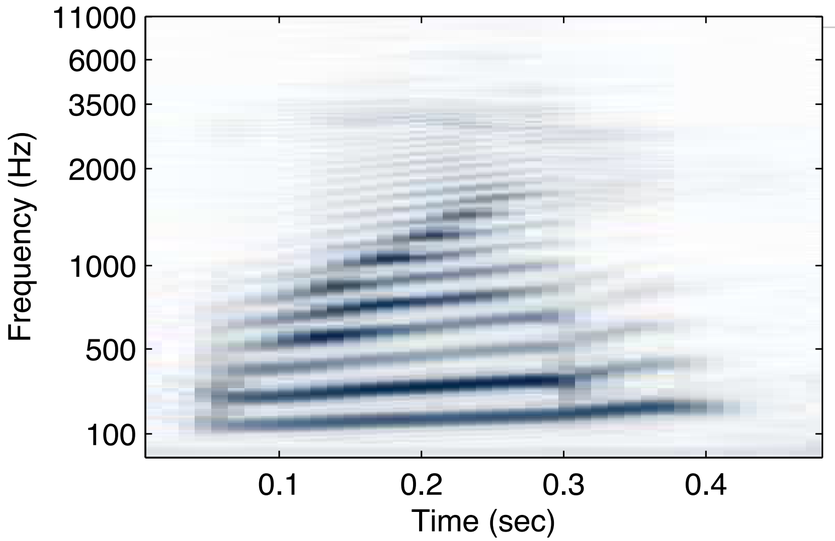


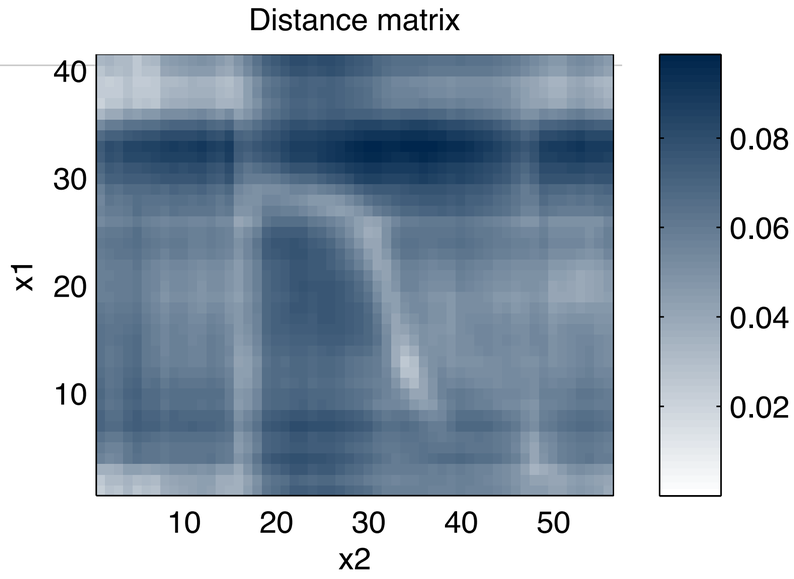


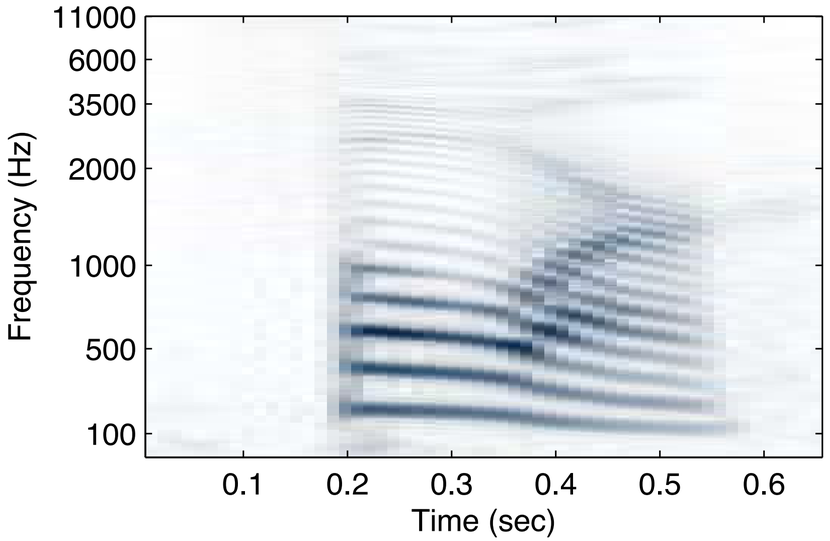


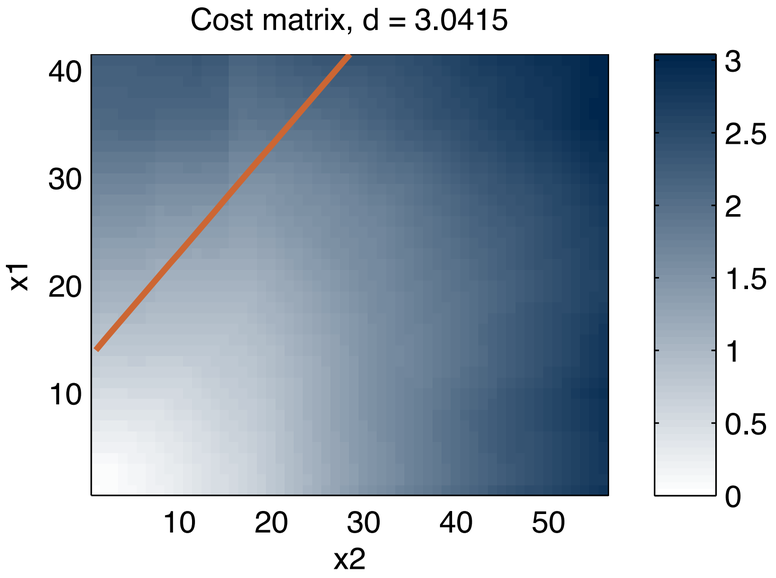


Ditto with different utterance



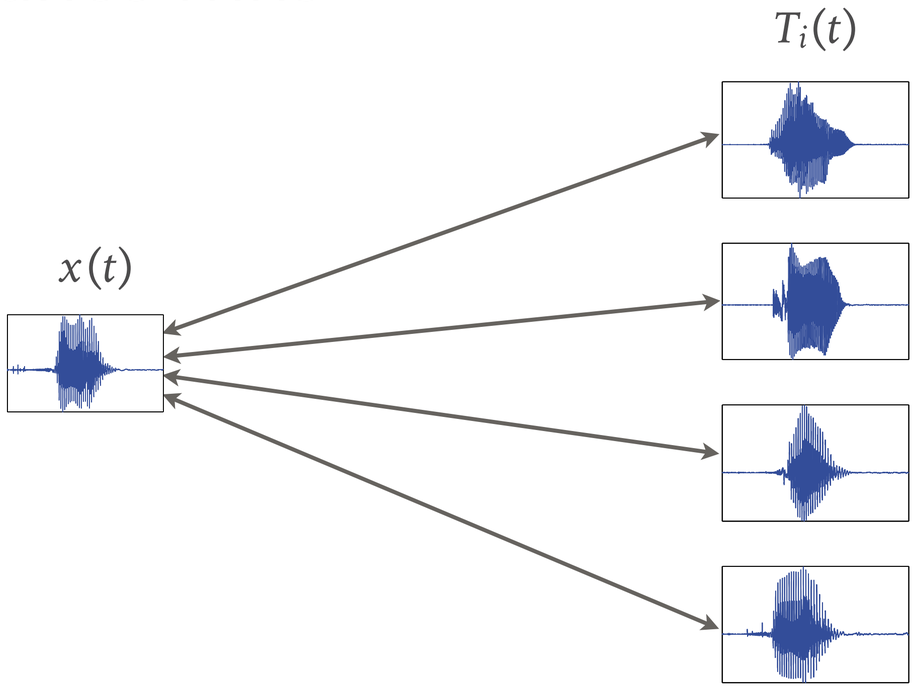






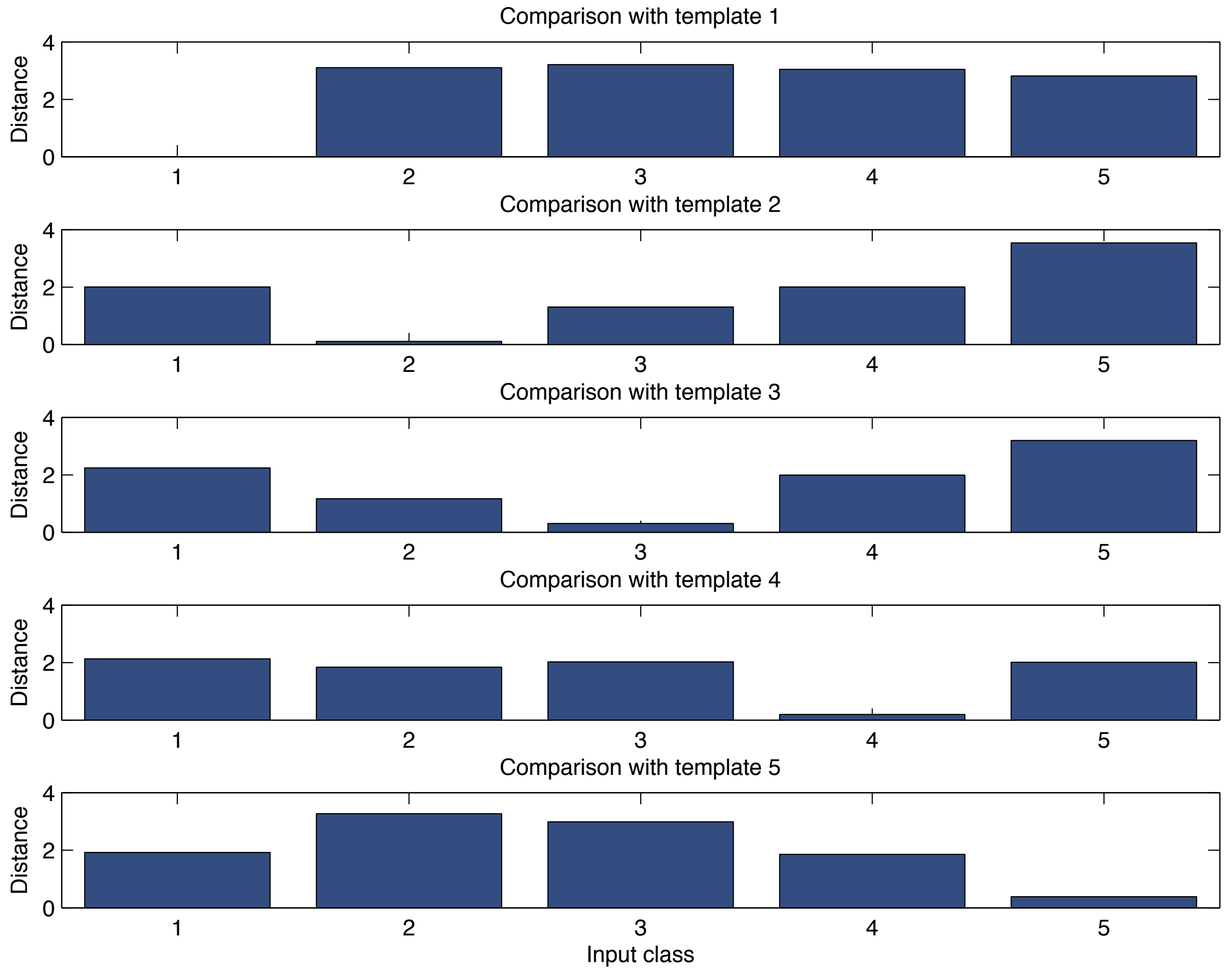
A basic speech recognizer

- Collect template spoken words $T_i(t)$
- Get their DTW distances from input $x(t)$
  - Smallest distance wins



Example case

DTW-derived distances



Defining a distance for time series

- Dynamic Time Warping computes time series distances
  - Dependent on the constraints, etc

- Having a time series distance gives us options
  - Nearest-neighbor classifiers (just did that)
  - Clustering of time series
  - Manifold embeddings of time series
  - …

A real-world application of DTW

- Simplifying ADR
  - Costs time and money
- Automatically align audio tracks

Good take, lousy audio

Good audio, lousy sync

Good take, good audio!

What's not quite right so far?

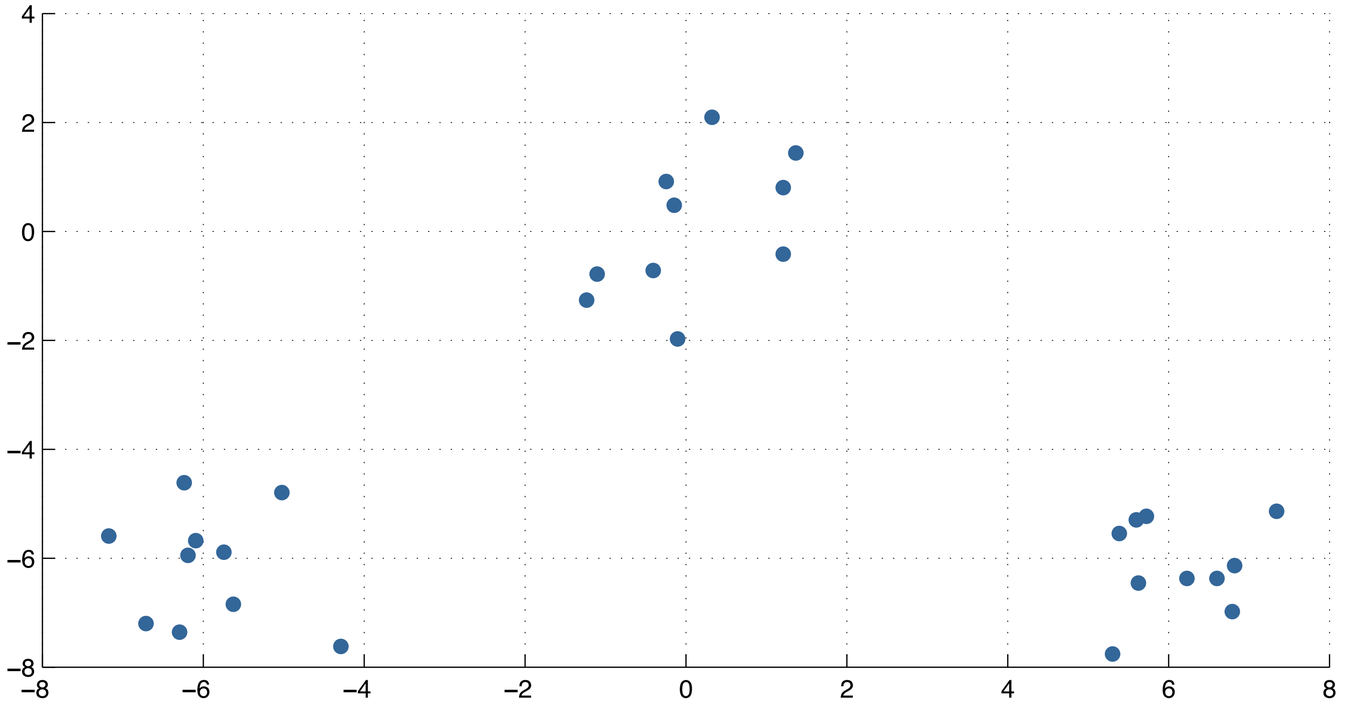
- Something I keep constantly complaining about …

We prefer soft models

- The paths are hard decisions
- We'd rather have soft assignments
- Also obtaining probabilities would help

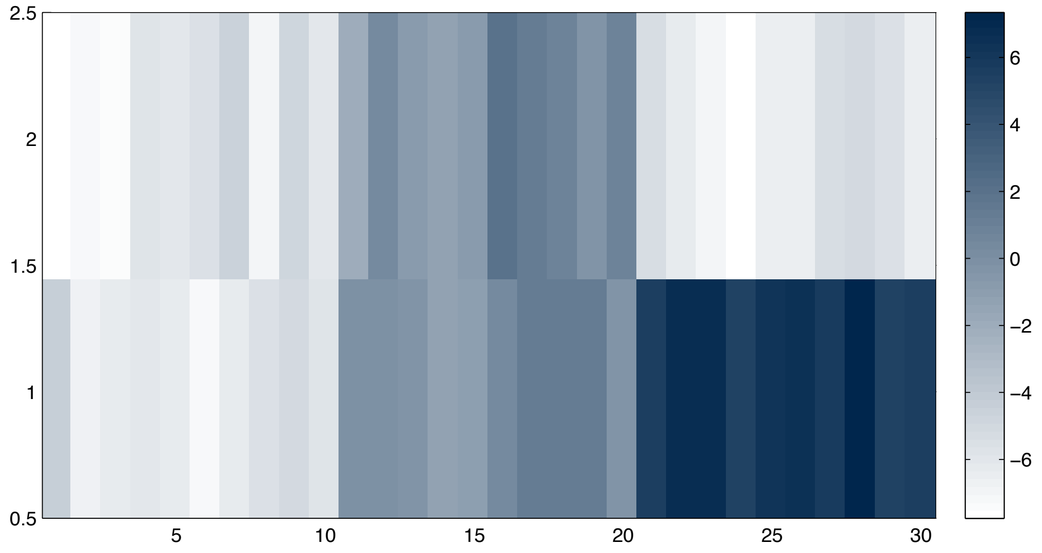
Starting with the GMM

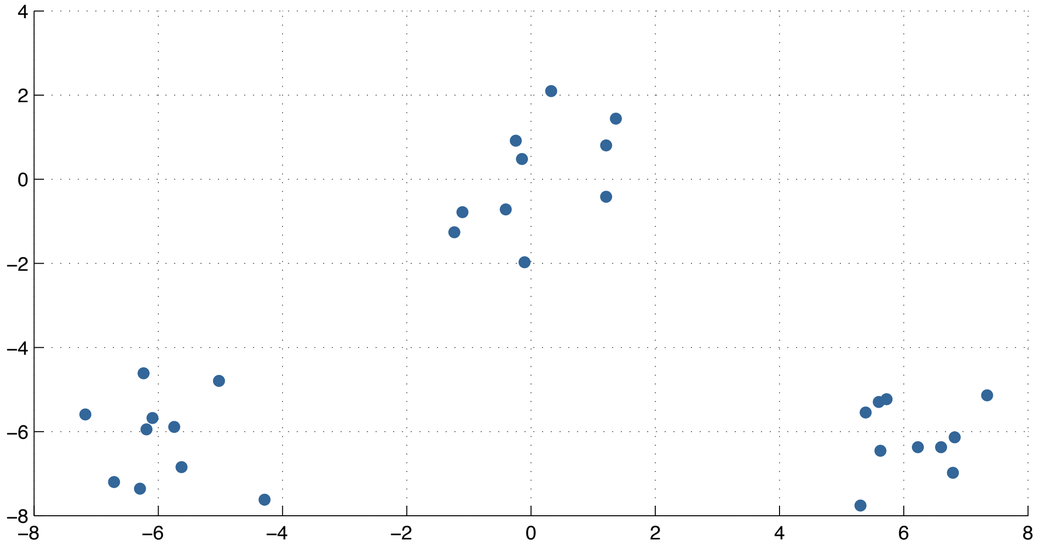
- GMMs are "time agnostic"
  - We don't have a notion of a sequence



What if there is time order?

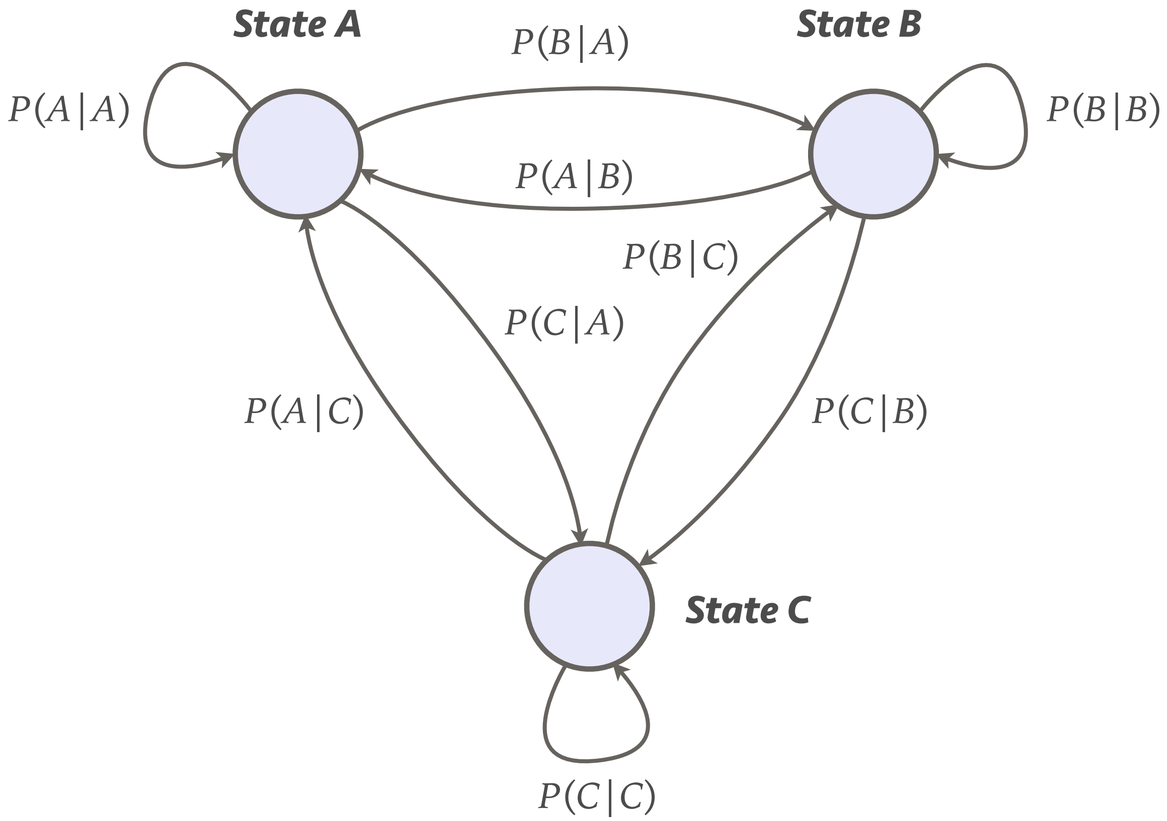
- Both plots are the same data
  - But we see time structure on the left





Representing time

- Hidden Markov Model
  - Gaussian variant for now
- Each state is represented by a Gaussian distribution
- Transition probabilities between all states
  - Only the last state matters



Initial Probabilities: $P(A), P(B), P(C)$

Problems to solve

- Evaluation
  - Given a model, evaluate likelihoods
- Decoding
  - Given a model, find state sequences
- Learning
  - Given the data, find the model parameters

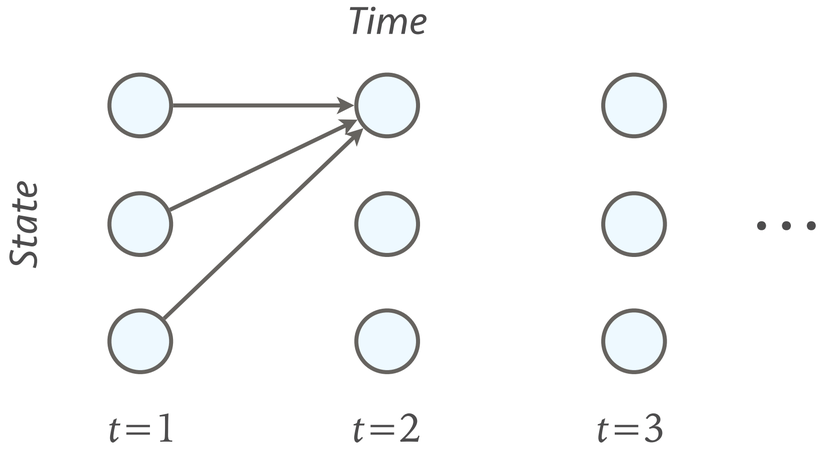
Evaluation

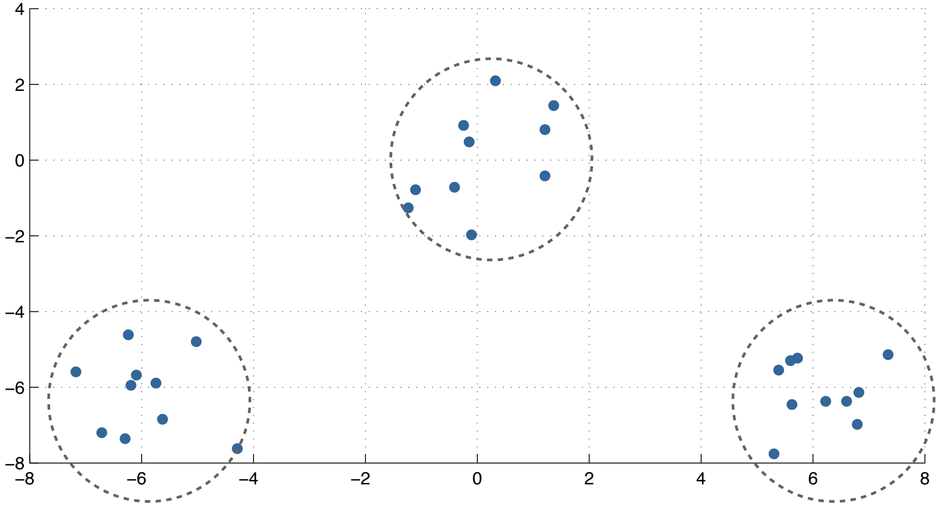
- Propagate likelihoods using the *forward algorithm*:

Probability of initial state *Likelihood of data given state*

$$\alpha_i(1) = P_1(i)P(x_1 \mid i)$$

$$\alpha_i(t+1) = P(x_{t+1} \mid i)\sum_j \alpha_j(t)P(i \mid j)$$





Evaluation

- Forward pass provides probability of a specific state and time, given past inputs

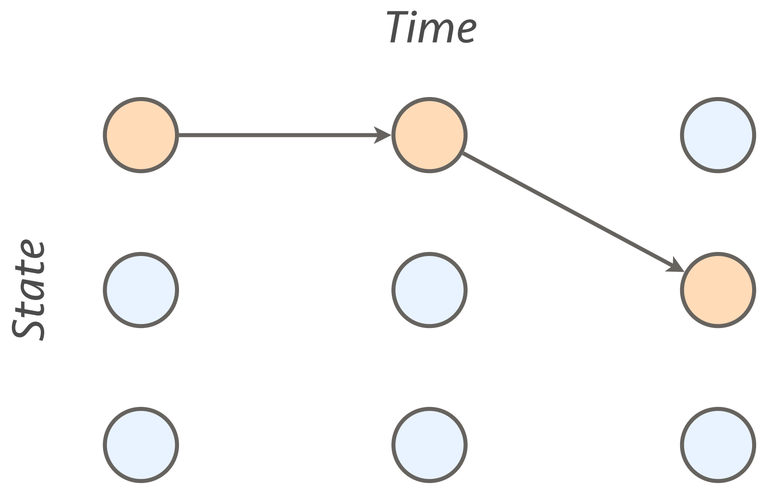
$$\alpha_i(t) = P(x_t \mid i) \sum_j \alpha_j(t-1) P(i \mid j)$$

- Terminal time point values provide the overall likelihood of model given an input

$$P(\mathbf{x}) = \sum_i \alpha_i(T)$$

Decoding

- The forward pass provides us with the state likelihoods
  - "soft" decisions
- With a backwards pass we can find most likely transitions
  - Just as we did with DTW
- The Viterbi algorithm



The Viterbi algorithm

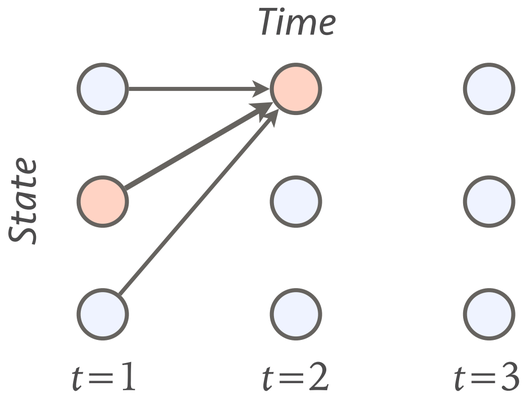
- Similar to forward algorithm, but with a hard decision:

$$v_i(1) = P_1(i)P(x_1 \mid i)$$

$$v_i(t+1) = P(x_{t+1} \mid i)\max\Big(P(j \mid i)v_j(t-1)\Big)$$

  - Probability of most likely path up to time $t+1$ that ends in state i
  - For every step remember most likely transition (as with DTW)

- Backwards pass
  - Get most likely terminal state
  - Work backwards using the most likely transitions



Learning

- An EM-type approach
  - Known as Baum-Welch training
- E-step
  - Find likelihood of each data point being associated with each state
- M-step
  - Estimate each state's parameters based on the associations from the E-step

E-step: Forward-backward pass

- Forward pass provides probability of a specific state and time given past inputs $\alpha_i(t)$
- A backward pass will provide probability of given state and time given future inputs $\beta_i(t)$
  - Same as forward pass, but computed backwards in time
- The product of these will be the probability of a state and time given all inputs $\gamma_i(t) = \alpha_i(t)\,\beta_i(t)$

M-step: Parameter estimation

- State models
  - Use $\gamma_i(t)$ as weights to compute state parameters, e.g.:

$$\mu_i = \frac{\sum_t x_t(t)\gamma_i(t)}{\sum_t \gamma_i(t)} \qquad \Sigma_i = \frac{\sum_t \gamma_i(t)\left(x_t - \mu_i\right)\left(x_t - \mu_i\right)^\top}{\sum_t \gamma_i(t)}$$

- Transition matrix and initial probabilities
  - Count potential transitions between states

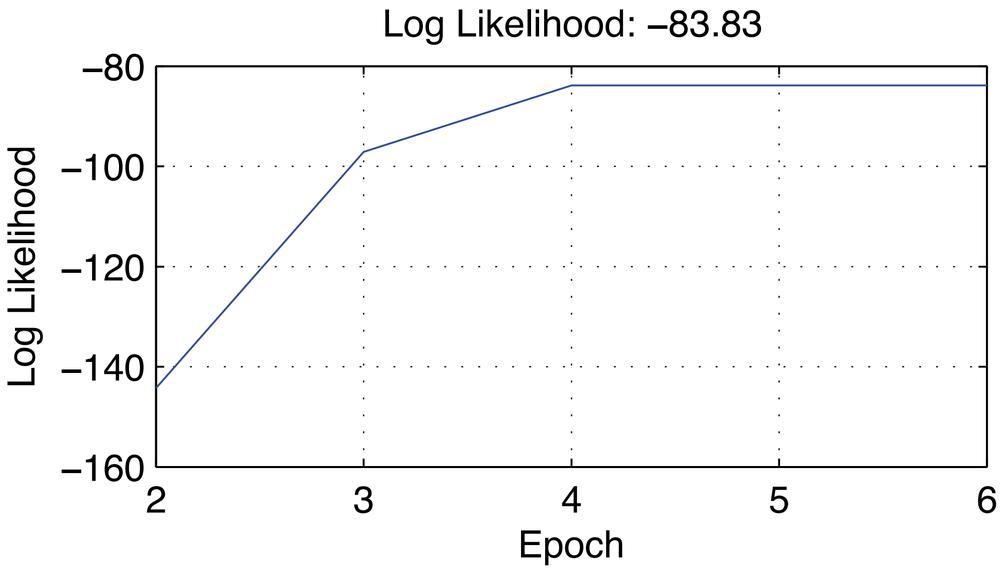
Noteworthy things

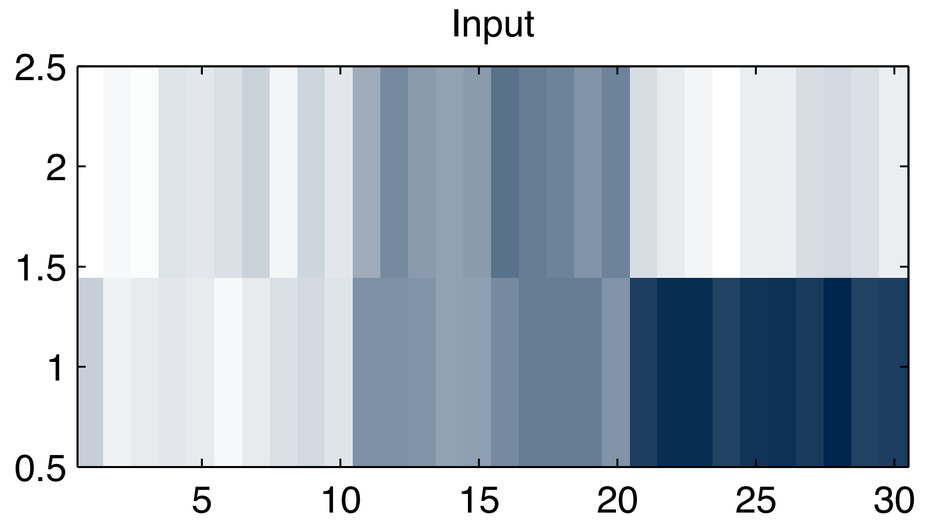
- HMM learning works with multiple training sequences
- Use log probabilities
  - What's a state likelihood after a million time points? Can you represent it?

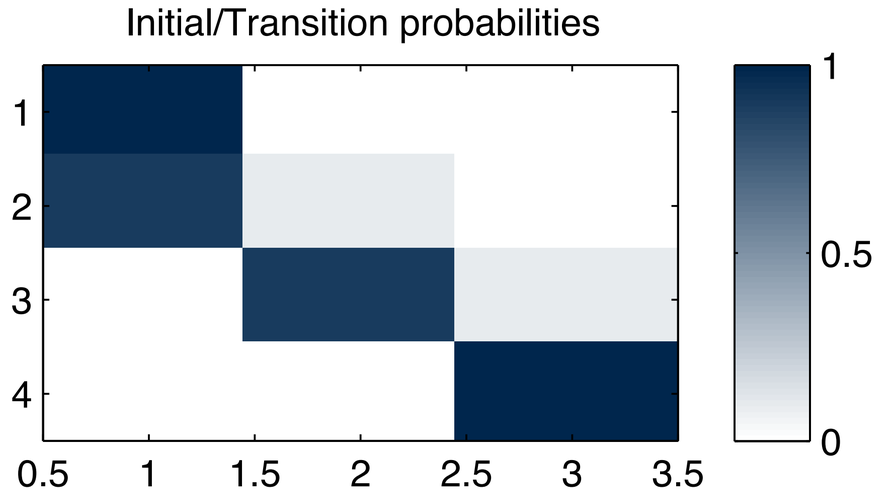
$$\alpha_i(t) = P(x_t \mid i) \sum_j \alpha_j(t-1) P(i \mid j)$$

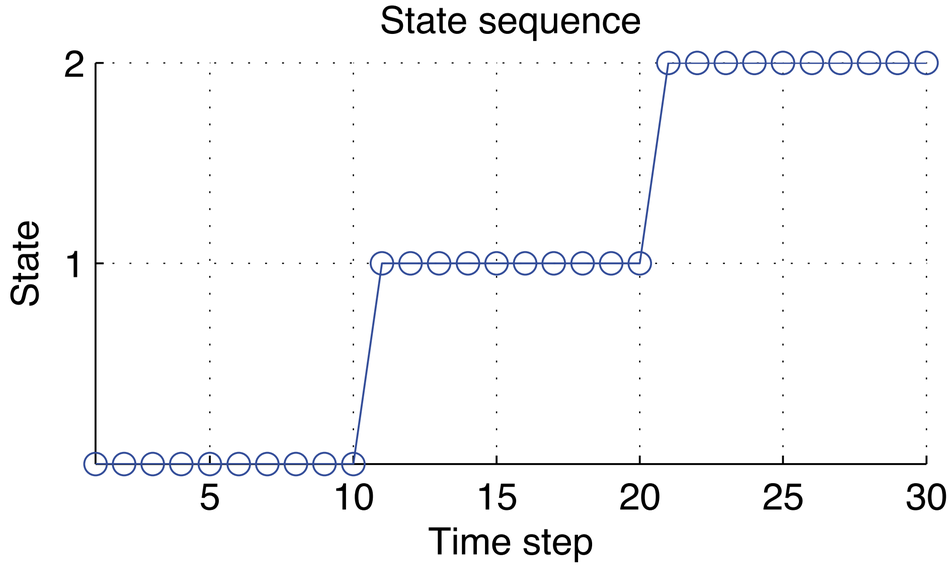
- Use an arbitrary state model
  - e.g. a neural net, or a GMM

Learning an HMM model



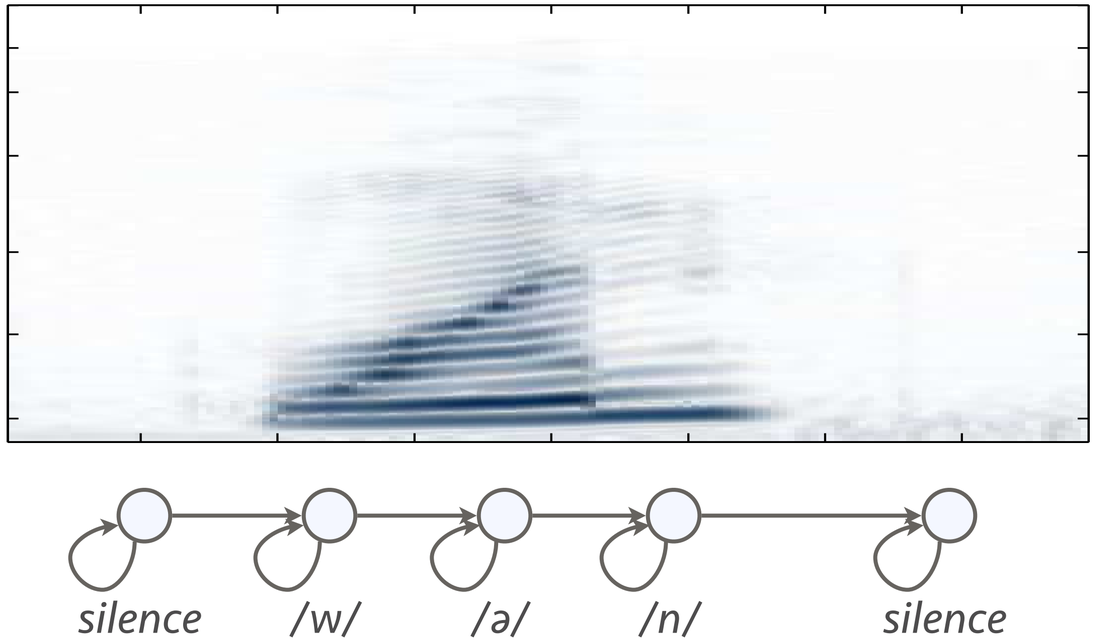




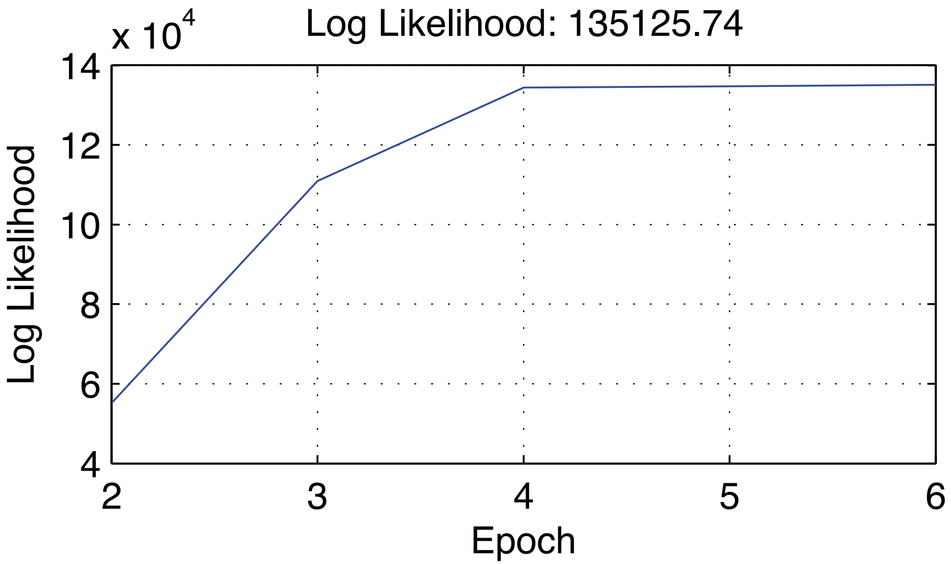


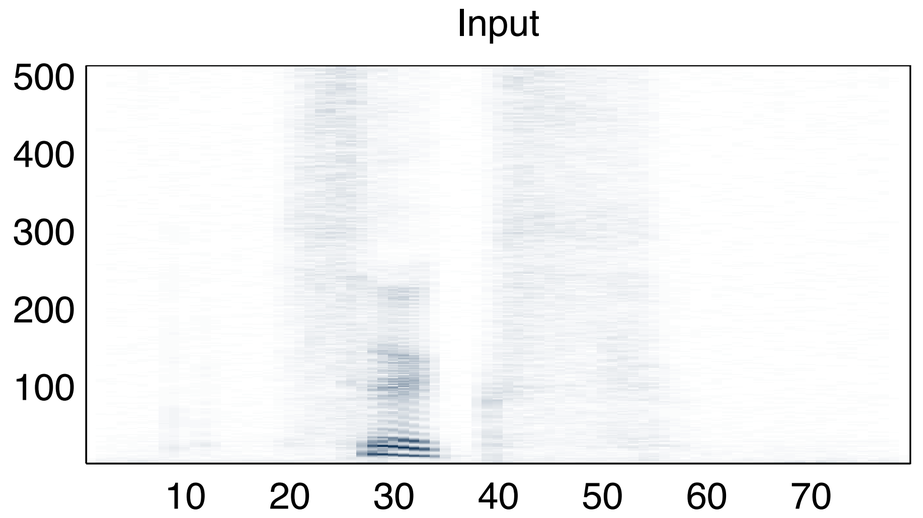
Back to speech

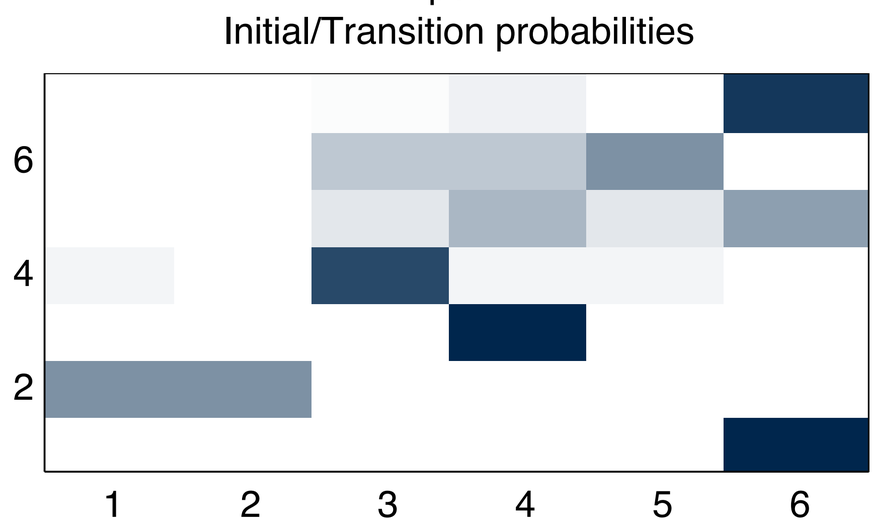
- Learning an HMM for a word
  - Each state should correspond to a coherent part
    - e.g. a syllable, a phoneme, etc

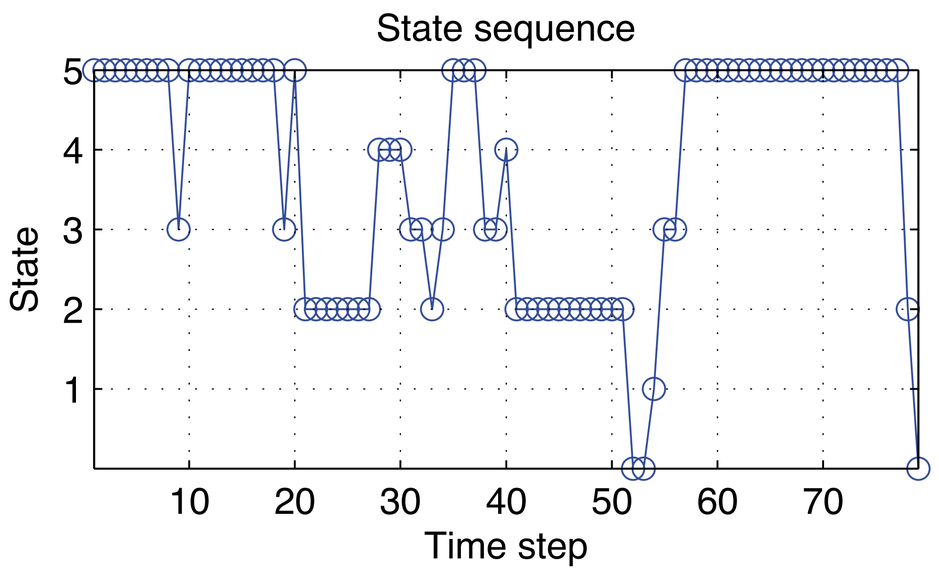


Fully-connected model

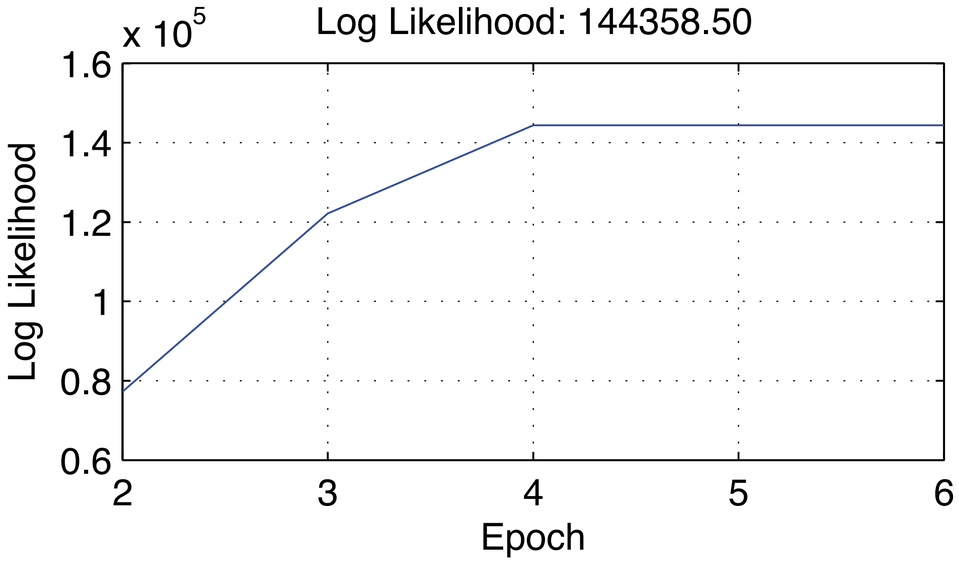


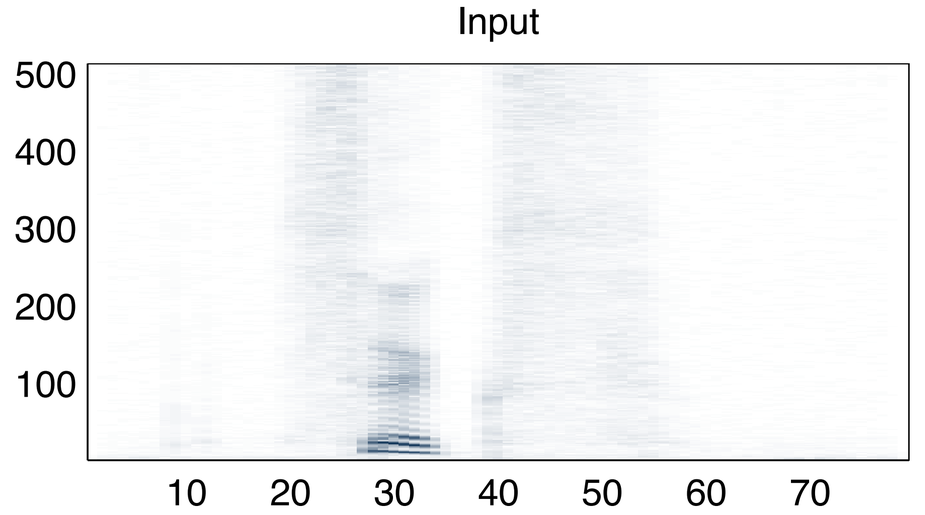


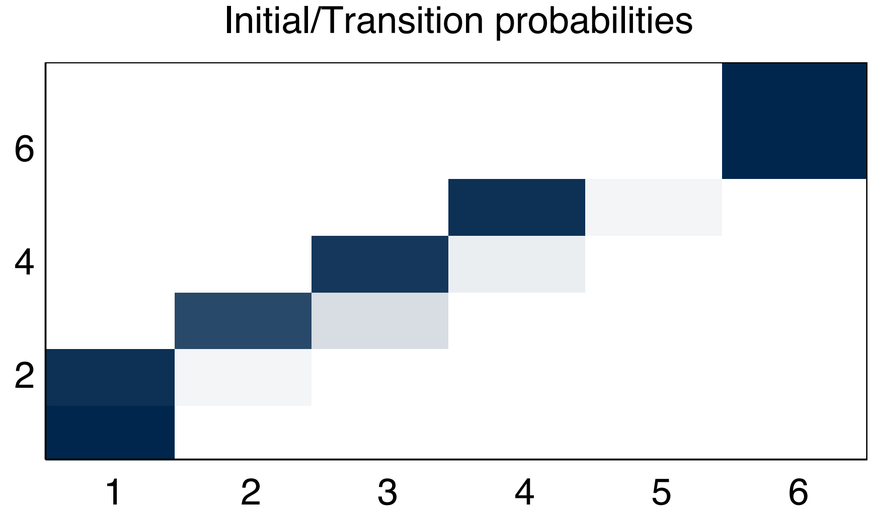


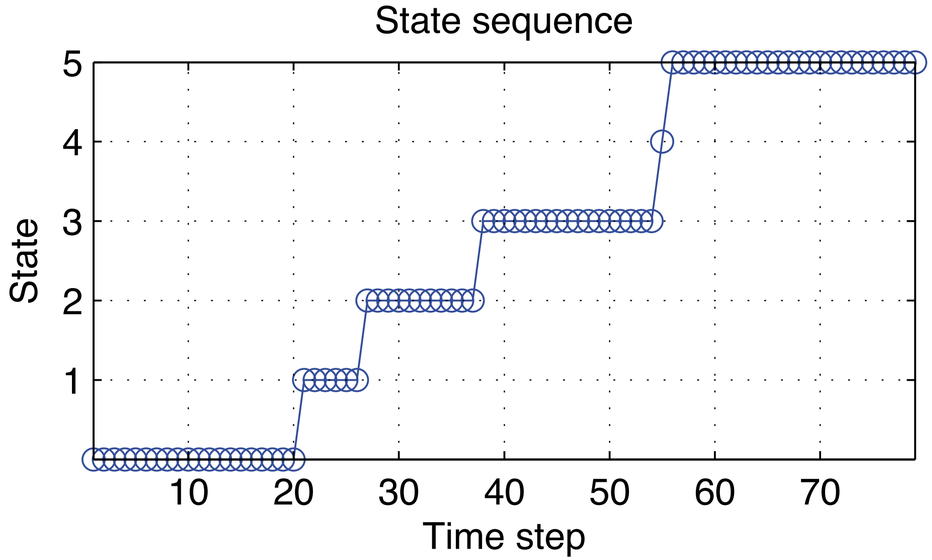


Left-to-right model

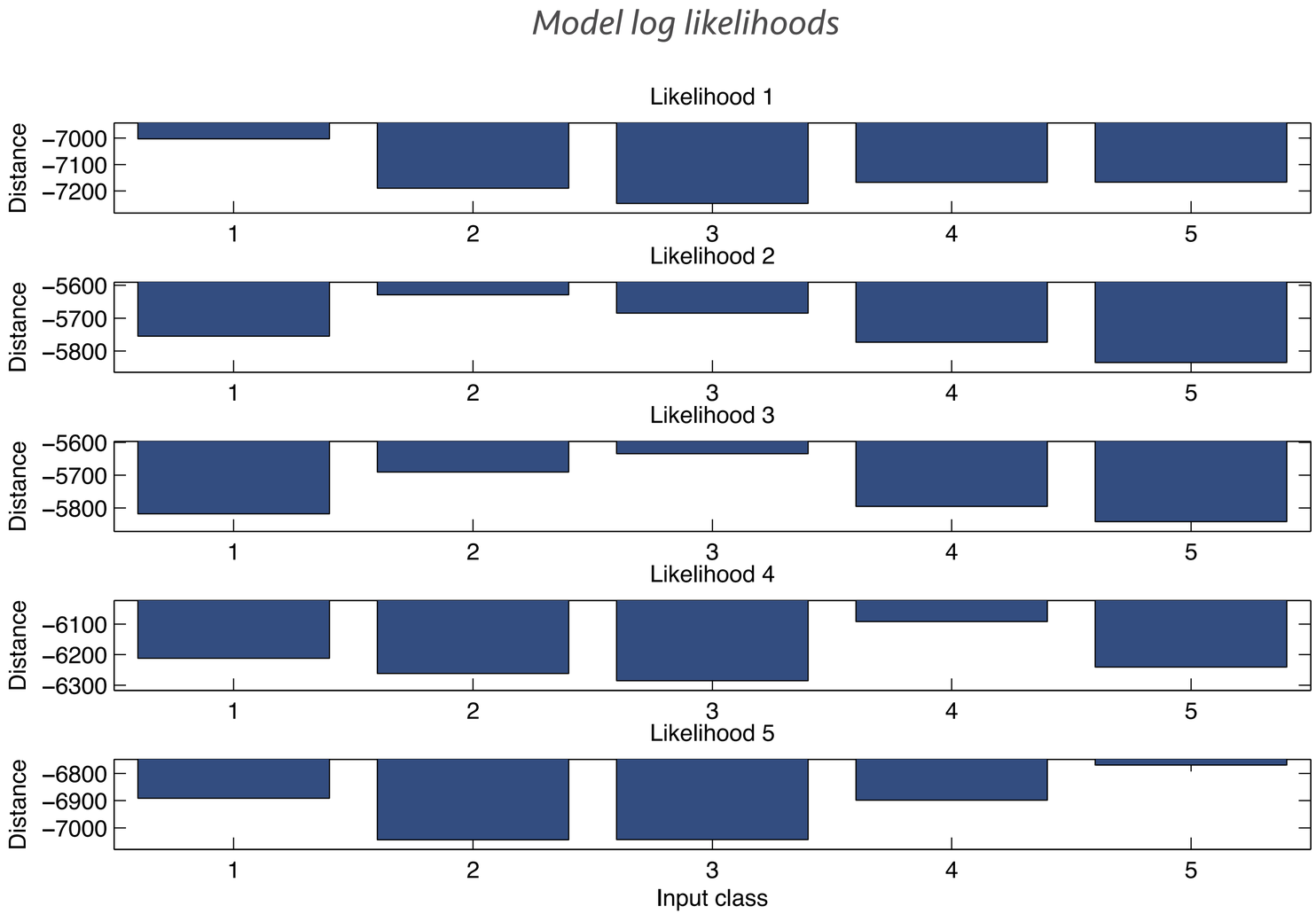






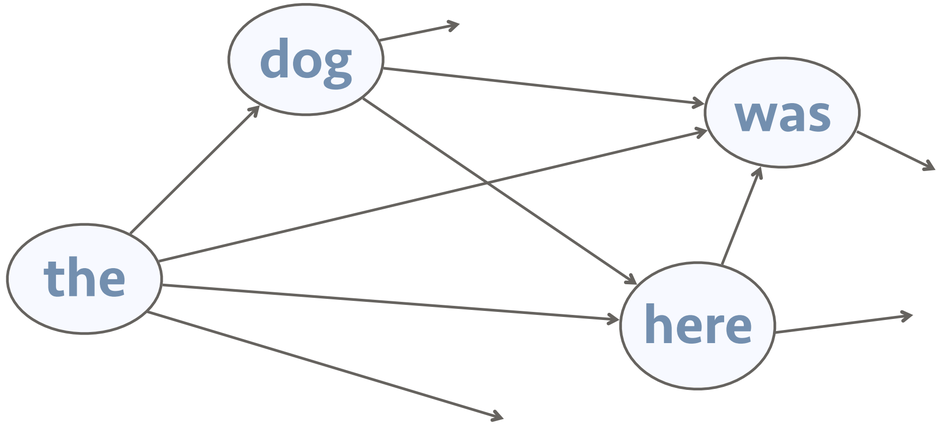


Digit recognition results



Speech recognition in a nutshell

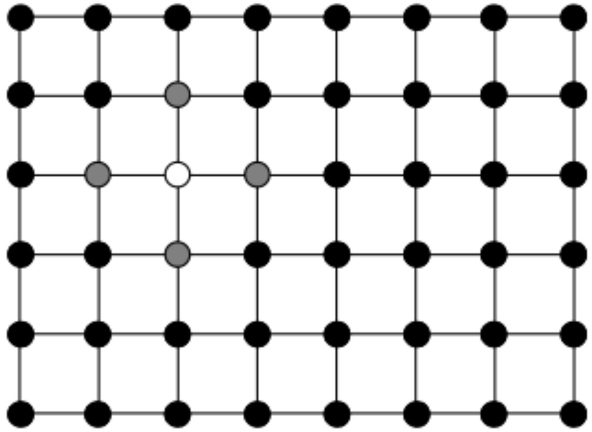
- Small scale speech recognition

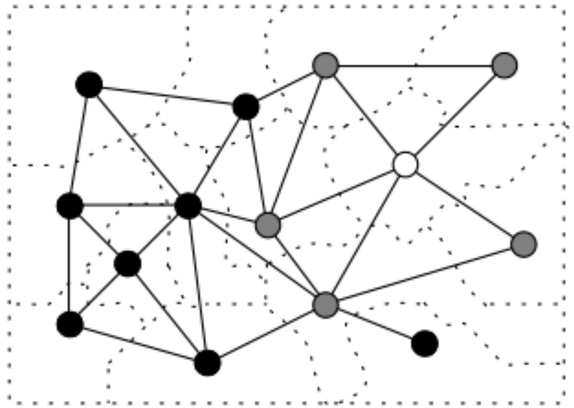


- Large scale systems are another beast …

Elaborations

- Denser interconnections
  - e.g., second order time ties
- Markov random fields





Recap

- Learning with time series
- Dynamic Time Warping
- Hidden Markov Models
- Some basic speech recognition

Next lecture

- Missing data
  - Using temporal dynamics to fill in missing values
- Dynamical systems
  - Learning continuous time-series

Reading

- Textbook chapters 8, 9

- A tutorial on HMMs
  - http://www.cs.ubc.ca/~murphyk/Bayes/rabiner.pdf

Final projects!

- Time to start thinking about projects
  - Send us (me/TAs) an informal proposal by end of next week
    - 1-2 paragraphs
- You have to be part of a team
  - $2 \leq N \leq 3$, no solo projects, $N = 4$ if you have a good excuse
- Deliverables
  - A poster presentation/demo and a 3-4 page writeup

Some project advice

- Work on a domain you have expertise in (if any)
  - Bring your own (signal) data, don't stick to generic projects
- Show me that you learned something in class
  - Demonstrate proper use of the tools we cover, use some signals-fu
- Propose multiple objectives, see how far you can go
  - One objective will be really easy to do (you pass the class)
  - Another will be tough, but doable (you get a potential paper out of it)
  - Final should be an amazing result (here's your dissertation!)