Dear 4041 students,

I wanted to send a note to clarify the discussion we had in class regarding where the expression

$$3(\lceil\frac{1}{2}\lceil\frac{n}{5}\rceil\rceil - 2) \geq \frac{3n}{10} - 6$$

comes from.

In class, I gave an unsatisfactory explanation of this.

Recall that we are interested in figuring out how many elements will be greater than or less than our pivot element, x. As discussed in class, we can be confident that since x is a median of other medians, half of those medians will be less than x and half will be greater.

Recall also that as discussed in class, in any of the "sets of 5", two elements will be greater than their median, and two will be less.

How many elements can we guarantee will be bigger than x, the median of the medians?

We know that in any set of 5 where the median of that set is bigger than x, there will then be 3 elements that are bigger than x. And, we know that approximately half of the medians are bigger than x. So, "approximately" $3 * (\frac{1}{2})(\lceil\frac{n}{5})\rceil$ will be greater than the median.

However, this is actually overestimating the number - we have counted the set containing x itself (which will not have 3 elements strictly greater than x), and we have counted the (possible) set that contains fewer than 5 elements.

So, to account for this, the book subtracts these 2 sets from $\lceil\frac{1}{2}\lceil\frac{n}{5}\rceil\rceil$. You can interpret this expression as "half the sets of 5, except for those that contain x itself, and the set that might not contain 5 elements. Then, you are guaranteed that the number of elements that will be greater than the median is greater than or equal to $3(\lceil\frac{1}{2}\lceil\frac{n}{5}\rceil\rceil - 2)$, which itself is greater than or equal to $\frac{3n}{10} - 6$. A symmetric argument applies to finding the number of elements smaller than the pivot.

I was also asked after class where $\frac{7n}{10} + 6$ comes from. We are trying to find the maximum number of elements that will need to be examined (i.e. how many elements could be greater than or less than the pivot). Well, we know that there will be at least $\frac{3n}{10} - 6$ that are *not* in the partition (for example, if we want to find the elements greater than the pivot, by our earlier argument we know there are at least $\frac{3n}{10} - 6$ that are smaller than the pivot). Unfortunately, we can't really say anything more about the other elements - they could be greater or smaller, we just don't know. So, our bound is just the total num-

ber of elements (n), minus those that we know are not greater than the pivot: $n - (\frac{3n}{10} - 6) = \frac{7n}{10} + 6$.
-Kate