Question 1

A sequence is defined by $a_n = 3n^2 - 4$, find the value of a_2.

Solution 1

$$a_2 = 3(2)^2 - 4 = 8$$

Question 2

A sequence is defined by $x_n = 3n^2 - 5n + 2$, find the value of n such that $x_n = 14$.

Solution 2

$$\begin{aligned} x_n = 3n^2 - 5n + 2 &= 14 \\ 3n^2 - 5n - 12 &= 0 \qquad S = -5 \quad P = -36 \\ \left(n + \frac{4}{3}\right)(n-3) &= 0 \qquad (4, -9) \quad \left(\frac{4}{3}, -3\right) \\ n &= 3 \end{aligned}$$

Question 3

A sequence is defined by $2U_{n+4} = 3U_{n+3} - U_n, \quad U_1 = 2, U_5 = 5, U_2 = 2U_4$, find the values of U_2, U_4 and U_6.

Solution 3

 © Joe Zhou, 2016

$$
\begin{aligned}
2U_5 &= 3U_4 - U_1 \\
2(5) &= 3U_4 - 2 \\
3U_4 &= 12 \\
U_4 &= 4
\end{aligned}
$$

$$
\begin{aligned}
U_2 &= 2U_4 \\
U_2 &= 2(4) \\
U_2 &= 8
\end{aligned}
$$

$$
\begin{aligned}
2U_6 &= 3U_5 - U_2 \\
2U_6 &= 3(5) - 8 \\
U_6 &= \frac{7}{2}
\end{aligned}
$$

Question 4

A sequence is defined by $x_n = 6n - 3$, find the value of x_3 and x_5.

Solution 4

$$
\begin{aligned}
x_3 &= 6(3) - 3 = 15 \\
x_5 &= 6(5) - 3 = 27
\end{aligned}
$$

Question 5

A sequence is defined by $X_n = 2n - 1$, find the value of n such that $a_n = 15$.

Solution 5

$$15 = 2n - 1$$
$$n = 8$$

Question 6

A sequence is defined by $u_n = an - b$, find the sum of the first four terms in terms of a and b.

Solution 6

$$u_1 + u_2 + u_3 + u_4 = (a - b) + (2a - b) + (3a - b) + (4a - b) = 10a - 4b$$

Question 7

A sequence is defined by $x_n = an^2 - 4$, find the sum of the first three terms in terms of a.

Solution 7

$$x_1 + x_2 + x_3 = (a - 4) + (4a - 4) + (9a - 4) = 14a - 12$$

Question 8

A sequence is defined by $y_n = an^2 + bn + c$, find the sum of the first three terms in terms of a, b and c.

Solution 8

$$y_1 + y_2 + y_3 = (a + b + c) + (4a + 2b + c) + (9a + 3b + c) = 14a + 6b + 3c$$

wrong choice

Question 9

A sequence is defined by $x_n = 4n - b$, find the third term in terms of b .

Solution 9

$$x_3 = 4(3) - b$$
$$x_3 = 12 - b$$

Question 10

A sequence is defined by $U_n = \frac{a}{n} + b$, find the fourth term in terms of a and b .

Solution 10

$$U_4 = \frac{a}{4} + b$$

Question 11

A sequence is defined by $y_n = \frac{a - 3b}{n^2}$, find the fifth term in terms of a and b .

Solution 11

$$y_5 = \frac{a - 3b}{(5)^2}$$
$$y_5 = \frac{a - 3b}{25}$$

Question 12

A sequence is defined by $U_n = an + 2b$, given the Sum of the first four terms is 26 and the fifth term is 9, find the values of a and b.

Solution 12

$$\begin{aligned} S_4 &= (a+2b) + (2a+2b) + (3a+2b) + (4a+2b) \\ S_4 &= 10a + 8b \qquad S_4 = 26 \\ 10a + 8b &= 26 \\ 5a + 4b &= 13 \quad (1) \end{aligned}$$

$$\begin{aligned} U_5 &= 5a + 2b \qquad U_5 = 9 \\ 5a + 2b &= 9 \quad (2) \end{aligned}$$

$$\begin{aligned} (1) - (2) \quad 5a + 4b - (5a + 2b) &= 13 - 9 \\ 2b &= 4 \\ b &= 2 \end{aligned}$$

$$\begin{aligned} \text{sub into} \quad (2) \quad 5a + 2(2) &= 9 \\ 5a &= 5 \\ a &= 1 \end{aligned}$$

Question 13

A sequence is defined by $U_{n+1} = U_n - 4, \quad U_1 = 20$, find the values of U_2, U_3 and U_4.

Solution 13

$$U_2 = U_1 - 4 = 20 - 4 = 16$$
$$U_3 = U_2 - 4 = 16 - 4 = 12$$
$$U_4 = U_3 - 4 = 12 - 4 = 8$$

Question 14

A sequence is defined by $X_{n+1} = X_n + 5, \quad X_4 = 17$, find the values of X_1, X_2 and X_3.

Solution 14

$$X_4 = X_3 + 5$$
$$17 = X_3 + 5$$
$$X_3 = 12$$

$$X_3 = X_2 + 5$$
$$12 = X_2 + 5$$
$$X_2 = 7$$

$$X_2 = X_1 + 5$$
$$7 = X_1 + 5$$
$$X_1 = 2$$

Question 15

A sequence is defined by $a_{n+1} = (a_n)^2 - 4, \quad a_1 = 2$, find the values of a_2, a_3 and a_4.

Solution 15

$$
\begin{aligned}
a_2 &= (a_1)^2 - 4 = 4 - 4 = 0 \\
a_3 &= (a_2)^2 - 4 = 0 - 4 = -4 \\
a_4 &= (a_3)^2 - 4 = (-4)^2 - 4 = 16 - 4 = 12
\end{aligned}
$$

Question 16

A sequence is defined by $y_{n+2} = 3y_{n+1} - y_n, \quad y_1 = 3, y_2 = 2$, find the values of y_3, y_4 and y_5.

Solution 16

$$
\begin{aligned}
y_3 &= 3(y_2) - y_1 = 3(2) - 3 = 3 \\
y_4 &= 3(y_3) - y_2 = 3(3) - 2 = 7 \\
y_5 &= 3(y_4) - y_3 = 3(7) - 3 = 18
\end{aligned}
$$

Question 17

Calculate the following sum:

$$\sum_{r=2}^{5}(r-1)$$

Solution 17

$$
\begin{aligned}
\sum_{r=2}^{5}(r-1) &= (2-1) + (3-1) + (4-1) + (5-1) \\
&= 1 + 2 + 3 + 4 \\
&= 10
\end{aligned}
$$

 © Joe Zhou, 2016

Question 18

Calculate the following sum:

$$\sum_{r=4}^{8}(r^2 - 2r + 1)$$

Solution 18

$$\begin{aligned}
&\sum_{r=4}^{8}(r^2 - 2r + 1)\\
=\ &\sum_{r=4}^{8}(r-1)^2\\
=\ &(4-1)^2 + (5-1)^2 + (6-1)^2 + (7-1)^2 + (8-1)^2\\
=\ &9 + 16 + 25 + 36 + 49\\
=\ &135
\end{aligned}$$

Question 19

A sequence is defined by $3x_{n+4} = \dfrac{2x_{n+3}}{x_n}, \quad x_1 = 3, x_4 = 9, \dfrac{x_6}{x_3} = 6$, find the value of x_5 and x_7.

Solution 19

 © Joe Zhou, 2016

$$3x_5 = \frac{2x_4}{x_1}$$
$$3x_5 = \frac{2(9)}{3}$$
$$x_5 = 2$$

$$3x_7 = \frac{2x_6}{x_3}$$
$$3x_7 = 2(6)$$
$$x_7 = 4$$

Question 20

A sequence is defined by the recurrence relation $Y_{n+1} = \dfrac{a^2}{Y_n} + b, Y_1 = 3, a, b \in \mathbb{N}$, given that $Y_2 = 7$ and $Y_3 = \dfrac{37}{7}$ find the value of a and b.

Solution 20

$$\begin{aligned}
Y_2 &= \frac{a^2}{Y_1} + b \\
7 &= \frac{a^2}{3} + b \qquad (1) \\[1em]
Y_3 &= \frac{a^2}{Y_2} + b \\
\frac{37}{7} &= \frac{a^2}{7} + b \qquad (2) \\
(1) - (2) \quad 7 - \frac{37}{7} &= \frac{a^2}{3} + b - \left(\frac{a^2}{7} + b\right) \\
\frac{12}{7} &= \frac{4a^2}{21} \\
a^2 &= 9 \\
a &= 3 \\[1em]
\text{Sub into(1)} \quad 7 &= \frac{3^2}{3} + b \\
b &= 4
\end{aligned}$$

Question 21

Calculate the following sum:

$$\sum_{r=5}^{9} U_r \qquad U_r = 3r^2 + 4$$

Solution 21

 © Joe Zhou, 2016

$$\sum_{r=5}^{9} U_r$$
$$= \sum_{r=5}^{9} 3r^2 + 4$$
$$= (3(5)^2 + 4) + (3(6)^2 + 4) + (3(7)^2 + 4) + (3(8)^2 + 4) + (3(9)^2 + 4)$$
$$= 785$$

Question 22

Calculate the following sum:

$$\sum_{r=1}^{3} a_r \qquad a_r = 4r - 1$$

Solution 22

$$\sum_{r=1}^{3} a_r$$
$$= \sum_{r=1}^{3} 4r - 1$$
$$= (4(1) - 1) + (4(2) - 1) + (4(3) - 1)$$
$$= 21$$

Question 23

A sequence is defined by $U_{n+1} = 3(U_n - 1), U_1 = 2$, find the following sum: $\sum_{2}^{4}(U_r + 2)^2$

 © Joe Zhou, 2016

Solution 23

$$U_2 = 3(2-1)$$
$$U_2 = 3$$

$$U_3 = 3(3-1)$$
$$U_3 = 6$$

$$U_4 = 3(6-1)$$
$$U_4 = 15$$

$$\begin{aligned}\sum_{2}^{4}(U_r+2)^2 &= (U_2+2)^2 + (U_3+2)^2 + (U_4+2)^2 \\ &= (3+2)^2 + (6+2)^2 + (15+2)^2 \\ &= 5^2 + 8^2 + 17^2 \\ &= 378\end{aligned}$$

Question 24

Evaluate $\sum_{r=1}^{15}(5r+2)$

Solution 24

$$\sum_{r=1}^{15}(5r+2) = 7 + 12 + 17 + 22 + ... + 77$$

$$a = 7 \quad l = 77 \quad n = 15$$

$$\begin{aligned}\sum_{r=1}^{15}(5r+2) &= \frac{15}{2}(7+77) \\ &= 630\end{aligned}$$

Question 25

A sequence is defined by the recurrence relation $X_{n+1} = \sqrt{k}X_n - 2, X_1 = 2, k > 0$, given that $X_3 = 2$ find the value of k.

Solution 25

$$
\begin{aligned}
X_2 &= \sqrt{k}X_1 - 2 \\
X_2 &= 2\sqrt{k} - 2
\end{aligned}
$$

$$
\begin{aligned}
X_3 &= \sqrt{k}X_2 - 2 \\
X_3 &= \sqrt{k}(2\sqrt{k} - 2) - 2 \\
X_3 &= 2k - 2\sqrt{k} - 2 \quad \text{set} \quad x = \sqrt{k} \\
X_3 &= 2x^2 - 2x - 2 \quad X_3 = 2 \\
2 &= 2x^2 - 2x - 2 \\
1 &= x^2 - x - 1 \\
0 &= x^2 - x - 2 \qquad S = -1 \quad P = -2 \\
0 &= (x-2)(x+1) \qquad (-2, 1) \\
\sqrt{k} &= 2 \\
k &= 4
\end{aligned}
$$

Question 26

A sequence is defined by the recurrence relation $U_{n+1} = aU_n + \dfrac{1}{b}, U_1 = 3$, given that $U_2 = 7$ and $U_3 = 15$ find the value of a and b.

Solution 26

$$U_2 = aU_1 + \frac{1}{b} \quad U_2 = 7$$
$$7 = 3a + \frac{1}{b} \quad (1)$$

$$U_3 = aU_2 + \frac{1}{b} \quad U_2 = 7, U_3 = 15$$
$$15 = 7a + \frac{1}{b} \quad (2)$$

$$(2) - (1) \quad 15 - 7 = 7a + \frac{1}{b} - \left(3a + \frac{1}{b}\right)$$
$$8 = 4a$$
$$a = 2$$
$$\text{Sub into (1)} \quad 7 = 3(2) + \frac{1}{b}$$
$$\frac{1}{b} = 1$$
$$b = 1$$

Question 27

Calculate the following sum:

$$\sum_{r=1}^{4}(2r + 4)$$

Solution 27

$$\begin{aligned}
&\sum_{r=1}^{4}(2r+4)\\
&= \sum_{r=1}^{4} 2(r+2)\\
&= 2\sum_{r=1}^{4}(r+2)\\
&= 2[(1+2)+(2+2)+(3+2)+(4+2)]\\
&= 2(3+4+5+6)\\
&= 36
\end{aligned}$$

Question 28

Calculate the following sum:

$$\sum_{r=3}^{6}(r^2-1)$$

Solution 28

$$\begin{aligned}
&\sum_{r=3}^{6}(r^2-1)\\
&= (3^2-1)+(4^2-1)+(5^2-1)+(6^2-1)\\
&= 8+15+24+35\\
&= 82
\end{aligned}$$

Question 29

Calculate the following sum:

$$\sum_{r=1}^{45} 2$$

Solution 29

$$\sum_{r=1}^{45} 2$$

$= 2 + 2 + 2 + 2 + 2 + ... + 2$

$= 2 \, \text{x} \, 45$

$= 90$

Question 30

Calculate the following sum:

$$\sum_{r=1}^{100} 5$$

Solution 30

$$\sum_{r=1}^{100} 5$$

$= 5 + 5 + 5 + 5 + 5 + 5 + ... + 5$

$= 5 \, \text{x} \, 100$

$= 500$

 © Joe Zhou, 2016

Question 31

A sequence is defined by the recurrence relation $u_{n+1} = \sqrt{a}\left(u_n - \frac{1}{b}\right)$, $5u_1 = 4$, given that $u_2 = 7$ and $u_3 = 13$ find the value of a and b .

Solution 31

$$u_2 = \sqrt{a}\left(u_1 - \frac{1}{b}\right)$$

$$7 = \sqrt{a}\left(4 - \frac{1}{b}\right) \qquad (1)$$

$$7 = 4\sqrt{a} - \frac{\sqrt{a}}{b} \qquad (2)$$

$$u_3 = \sqrt{a}\left(u_2 - \frac{1}{b}\right)$$

$$13 = \sqrt{a}\left(7 - \frac{1}{b}\right)$$

$$13 = 7\sqrt{a} - \frac{\sqrt{a}}{b} \qquad (3)$$

$$(3) - (2) \quad 13 - 7 = 7\sqrt{a} - \frac{\sqrt{a}}{b} - \left(4\sqrt{a} - \frac{\sqrt{a}}{b}\right)$$

$$6 = 3\sqrt{a}$$

$$2 = \sqrt{a}$$

$$a = 4$$

$$\text{Sub into (1)} \quad 7 = \sqrt{4}\left(4 - \frac{1}{b}\right)$$

$$\frac{7}{2} = 4 - \frac{1}{b}$$

$$-\frac{1}{2} = -\frac{1}{b}$$

$$b = 2$$

Question 32

A sequence is defined by the recurrence relation $Y_{n+1} = \dfrac{a^2}{Y_n} + b, Y_1 = 3, a, b > 0$, given that $Y_2 = 7$ and $Y_3 = \dfrac{37}{7}$ find the value of a and b.

Solution 32

$$\begin{aligned}
Y_2 &= \frac{a^2}{Y_1} + b \\
7 &= \frac{a^2}{3} + b \qquad (1)
\end{aligned}$$

$$\begin{aligned}
Y_3 &= \frac{a^2}{Y_2} + b \\
\frac{37}{7} &= \frac{a^2}{7} + b \qquad (2) \\
(1) - (2) \quad 7 - \frac{37}{7} &= \frac{a^2}{3} + b - \left(\frac{a^2}{7} + b\right) \\
\frac{12}{7} &= \frac{4a^2}{21} \\
a^2 &= 9 \\
a &= 3
\end{aligned}$$

$$\begin{aligned}
\text{Sub into(1)} \quad 7 &= \frac{3^2}{3} + b \\
b &= 4
\end{aligned}$$

Question 33

How many terms are there in the arithmetic sequence 19,21,23,...,87

Solution 33

$$\begin{aligned} a &= 19 \quad d = 2 \\ U_n &= a + (n-1)d \\ 87 &= 19 + (n-1)2 \\ n - 1 &= 34 \\ n &= 35 \end{aligned}$$

Question 34

How many terms are there in the arithmetic sequence 21,26,31,...,256

Solution 34

$$\begin{aligned} a &= 21 \quad d = 5 \\ U_n &= a + (n-1)d \\ 256 &= 21 + (n-1)5 \\ n - 1 &= 47 \\ n &= 48 \end{aligned}$$

Question 35

A sequence is defined by the recurrence relation $a_{n+1} = ka_n - 4, k > 0, a_1 = 5$, given that $\sum_{r=1}^{3} a_r = 19$, find the value of k.

Solution 35

$$a_2 = ka_1 - 4$$
$$a_2 = 5k - 4$$

$$a_3 = ka_2 - 4$$
$$a_3 = k(5k - 4) - 4$$
$$a_3 = 5k^2 - 4k - 4$$

$$\sum_{r=1}^{3} a_r = a_1 + a_2 + a_3$$
$$\sum_{r=1}^{3} a_r = (5) + (5k - 4) + (5k^2 - 4k - 4)$$
$$\sum_{r=1}^{3} a_r = 5k^2 + k - 3 \qquad \sum_{r=1}^{3} a_r = 19$$
$$19 = 5k^2 + k - 3$$
$$0 = 5k^2 + k - 22 \qquad S = 1 \quad P = -110$$
$$0 = \left(k + \frac{11}{5}\right)(k - 2) \qquad (11, -10) \quad \Rightarrow \quad \left(\frac{11}{5}, -2\right)$$
$$k = 2$$

Question 36

A sequence is defined by the recurrence relation $U_{n+1} = 5U_n - \dfrac{1}{k}, k > 0, U_1 = 2$, given that $\displaystyle\sum_{r=1}^{4} U_r = 293$, find the value of k.

Solution 36

$$U_2 = 5U_1 - \frac{1}{k}$$
$$U_2 = 5(2) - \frac{1}{k}$$
$$U_2 = 10 - \frac{1}{k}$$

$$U_3 = 5U_2 - \frac{1}{k}$$
$$U_3 = 5\left(10 - \frac{1}{k}\right) - \frac{1}{k}$$
$$U_3 = 50 - \frac{6}{k}$$

$$U_4 = 5U_3 - \frac{1}{k}$$
$$U_4 = 5\left(50 - \frac{6}{k}\right) - \frac{1}{k}$$
$$U_4 = 250 - \frac{31}{k}$$

$$\sum_{r=1}^{4} U_r = U_1 + U_2 + U_3 + U_4$$
$$\sum_{r=1}^{4} U_r = (2) + \left(10 - \frac{1}{k}\right) + \left(50 - \frac{6}{k}\right) + \left(250 - \frac{31}{k}\right)$$
$$\sum_{r=1}^{4} U_r = 312 - \frac{38}{k} \qquad \sum_{r=1}^{4} U_r = 293$$
$$312 - \frac{38}{k} = 293$$
$$19 = \frac{38}{k}$$
$$k = 2$$

Question 37

A sequence is defined by the recurrence relation $X_{n+1} = \dfrac{k}{X_n} + 3, X_1 = 1$, given that $2\sum_{r=1}^{3} X_r = 21$, find the value of k.

Solution 37

 © Joe Zhou, 2016

$$X_2 = \frac{k}{X_1} + 3$$
$$X_2 = \frac{k}{1} + 3$$
$$X_2 = k + 3$$

$$X_3 = \frac{k}{X_2} + 3$$
$$X_3 = \frac{k}{k+3} + 3$$

$$\sum_{r=1}^{3} X_r = X_1 + X_2 + X_3$$
$$\sum_{r=1}^{3} X_r = (1) + (k+3) + \left(\frac{k}{k+3} + 3\right)$$
$$\sum_{r=1}^{3} X_r = k + 7 + \frac{k}{k+3} \quad 2\sum_{r=1}^{3} X_r = 21$$
$$21 = 2\left(k + 7 + \frac{k}{k+3}\right)$$
$$21 = 2k + 14 + \frac{2k}{k+3}$$
$$7 = 2k + \frac{2k}{k+3}$$
$$7(k+3) = 2k(k+3) + 2k$$
$$7k + 21 = 2k^2 + 6k + 2k$$
$$0 = 2k^2 - k - 21 \qquad S = -1 \quad P = -42$$
$$0 = \left(k + \frac{7}{2}\right)(k-3) \quad (7, -6) \quad \Rightarrow \quad \left(\frac{7}{2}, -3\right)$$
$$k = 3$$

Question 38

A sequence is defined by the recurrence relation $a_{n+1} = a_n^2 - a_n$, given that a_n is a positive sequence and that $a_3 = 132$ find the value of a_1.

Solution 38

$$
\begin{aligned}
a_3 &= a_2^2 - a_2 \\
132 &= a_2^2 - a_2 \\
0 &= a_2^2 - a_2 - 132 \qquad S = 1 \quad P = -132 \\
0 &= (a_2 + 11)(a_2 - 12) \quad (11, -12) \\
a_2 &= 12
\end{aligned}
$$

$$
\begin{aligned}
a_2 &= a_1^2 - a_1 \\
12 &= a_1^2 - a_1 \\
0 &= a_1^2 - a_1 - 12 \\
0 &= (a_1 - 4)(a_1 + 3) \\
a_1 &= 4
\end{aligned}
$$

Question 39

A sequence is defined by the recurrence relation $U_{n+1} = 5U_n - \dfrac{6}{U_n}$, given that $U_3 = 13, U_2 > 0$, find the value of U_2.

Solution 39

$$
\begin{aligned}
U_3 &= 5U_2 - \frac{6}{U_2} \\
13 &= 5U_2 - \frac{6}{U_2} \\
0 &= 5U_2 - 13 - \frac{6}{U_2} \\
0 &= 5(U_2)^2 - 13U_2 - 6 \qquad S = -13 \quad P = -30 \\
0 &= \left(U_2 + \frac{2}{5}\right)(U_2 - 3) \quad (2, -15) \quad \left(\frac{2}{5}, -3\right) \\
U_2 &= 3
\end{aligned}
$$

Question 40

A sequence is defined by the recurrence relation $Y_{n+1} = 3Y_n - 5$, given that $Y_3 = 7$, find the value of Y_1.

Solution 40

$$
\begin{aligned}
Y_3 &= 3Y_2 - 5 \\
7 &= 3Y_2 - 5 \\
Y_2 &= 4
\end{aligned}
$$

$$
\begin{aligned}
Y_2 &= 3Y_1 - 5 \\
4 &= 3Y_1 - 5 \\
Y_1 &= 3
\end{aligned}
$$

Question 41

A sequence is defined by the recurrence relation $a_{n+1} = a_n - \dfrac{2a_n + 6}{a_n + 3}$, given that

$a_2 = 5$, find the value of a_1.

Solution 41

$$\begin{aligned}
a_2 &= a_1 - \frac{2a_1+6}{a_1+3} \\
5 &= a_1 - \frac{2a_1+6}{a_1+3} \\
5(a_1+3) &= a_1(a_1+3) - (2a_1+6) \\
5a_1 + 15 &= (a_1)^2 + 3a_1 - 2a_1 - 6 \\
0 &= (a_1)^2 - 4a_1 - 21 \qquad S = -4 \quad P = -21 \\
0 &= (a_1+3)(a_1-7) \qquad (3, -7) \\
a_1 &= 7
\end{aligned}$$

Question 42

A sequence is defined by the recurrence relation $X_{n+1} = 3(X_n)^2 - 11$, given that $X_1 = 2$, find $\displaystyle\sum_{r=1}^{4} X_r$.

Solution 42

$$X_2 = 3(X_1)^2 - 11$$
$$X_2 = 3(2)^2 - 11$$
$$X_2 = 1$$

$$X_3 = 3(X_2)^2 - 11$$
$$X_3 = 3(1)^2 - 11$$
$$X_3 = -8$$

$$X_4 = 3(X_3)^2 - 11$$
$$X_4 = 3(-8)^2 - 11$$
$$X_4 = 181$$

$$\sum_{r=1}^{4} X_r = X_1 + X_2 + X_3 + X_4$$
$$\sum_{r=1}^{4} X_r = (2) + (1) + (-8) + (181)$$
$$\sum_{r=1}^{4} X_r = 176$$

Question 43

A sequence is defined by the recurrence relation $U_{n+2} = 3U_{n+1} - U_n + 5$, given that $U_1 = 4, U_2 = 2$, find $\sum_{r=1}^{4} U_r$.

Solution 43

$$U_3 = 3U_2 - U_1 + 5$$
$$U_3 = 3(2) - (4) + 5$$
$$U_3 = 7$$

$$U_4 = 3U_3 - U_2 + 5$$
$$U_4 = 3(7) - (2) + 5$$
$$U_4 = 24$$

$$\sum_{r=1}^{4} U_r = U_1 + U_2 + U_3 + U_4$$
$$\sum_{r=1}^{4} U_r = 4 + 2 + 7 + 24$$
$$\sum_{r=1}^{4} U_r = 37$$

Question 44

A sequence is defined by the recurrence relation $Y_{n+1} = 21 - 2Y_n$, given that $Y_1 = 5$, find $\sum_{r=2}^{4} Y_r$.

Solution 44

$$Y_2 = 21 - 2Y_1$$
$$Y_2 = 21 - 2(5)$$
$$Y_2 = 11$$

$$Y_3 = 21 - 2Y_2$$
$$Y_3 = 21 - 2(11)$$
$$Y_3 = -1$$

$$Y_4 = 21 - 2Y_3$$
$$Y_4 = 21 - 2(-1)$$
$$Y_4 = 23$$

$$\sum_{r=2}^{4} Y_r = Y_2 + Y_3 + Y_4$$
$$\sum_{r=2}^{4} Y_r = 11 + (-1) + 23$$
$$\sum_{r=2}^{4} Y_r = 33$$

Question 45

How many terms are there in the arithmetic sequence 88,86,84,...,22

Solution 45

Reverse the order of the sequence 22,24,26,28...88

$$a = 88 \quad d = 2$$
$$U_n = a + (n-1)d$$
$$88 = 22 + (n-1)2$$
$$n - 1 = 33$$
$$n = 34$$

Question 46

A sequence is defined by the recurrence relation $X_{n+1} = 5 - X_n$, given that $X_1 = 7$, find $\sum_{r=1}^{20} X_r$.

Solution 46

$$X_2 = 5 - X_1 = 5 - 7 = -2$$

$$X_3 = 5 - X_2 = 5 - (-2) = 7$$

$$X_4 = 5 - X_3 = 5 - 7 = -2$$

$$X_5 = 5 - X_4 = 5 - (-2) = 7$$

$$\sum_{r=1}^{20} X_r = X_1 + X_2 + X_3 + X_4 + ... + X_{20}$$

$$\sum_{r=1}^{20} X_r = -2 + 7 + -2 + 7 + -2 + ... + 7$$

$$\sum_{r=1}^{20} X_r = 10(-2) + 10(7)$$

$$\sum_{r=1}^{20} X_r = 50$$

Question 47

Evaluate $S = 1 + 2 + 3 + 4 + ... + 50$

Solution 47

$$
\begin{aligned}
S &= 1 + 2 + 3 + 4 + ... + 50 \\
S &= 50 + 49 + 48 + 47 + ... + 1 \\
2S &= 51 \text{ x } 50 \\
S &= \frac{51 \text{ x } 50}{2} \\
S &= 1275
\end{aligned}
$$

Question 48

Evaluate $T = 2 + 4 + 6 + 8 + ... + 100$

Solution 48

$$
\begin{aligned}
T &= 2 + 4 + 6 + 8 + ... + 100 \\
T &= 100 + 98 + 96 + 94 + ... + 2 \\
2T &= 102 \text{ x } 50 \\
T &= \frac{102 \text{ x } 50}{2} \\
T &= 2550
\end{aligned}
$$

Question 49

Evaluate $R = 1 + 3 + 5 + 7 + ... + 99$

Solution 49

$$R = 1 + 3 + 5 + 7 + ... + 99$$
$$R = 99 + 97 + 95 + 93 + ... + 1$$
$$2R = 100 \text{ x } 100$$
$$R = \frac{100 \text{ x } 100}{2}$$
$$R = 5000$$

Question 50

Evaluate $S = 1 + 2 + 3 + 4 + ... + 200$

Solution 50

$$S = 1 + 2 + 3 + 4 + ... + 200$$
$$S = 200 + 199 + 198 + 197 + ... + 1$$
$$2S = 201 \text{ x } 200$$
$$S = \frac{201 \text{ x } 200}{2}$$
$$S = 20100$$

Question 51

Evaluate $T = 102 + 104 + 106 + 108 + ... + 200$

Solution 51

$$T = 102 + 104 + 106 + 108 + ... + 200$$
$$T = 200 + 198 + 196 + 194 + ... + 102$$
$$2T = 302 \text{ x } 50$$
$$T = \frac{302 \text{ x } 50}{2}$$
$$T = 7550$$

Question 52

Find the sum of all numbers divisible by 5 between 1 and 300

Solution 52

$$300 \div 5 = 60$$
$$\Rightarrow \quad \text{last term } = 300$$
$$S = 5 + 10 + 15 + ... + 300$$

$$a = 5 \quad d = 5 \quad U_n = 300$$
$$U_n = a + (n-1)d$$
$$300 = 5 + (n-1)5$$
$$n = 60$$

$$S = \frac{n}{2}(a + l)$$
$$S = \frac{60}{2}(5 + 300)$$
$$S = 9150$$

Question 53

Find the sum of all numbers divisible by 7 between 1 and 200

Solution 53

$$200 \div 7 = 28 \text{ remainder } 4$$

$$\Rightarrow \quad \text{last term } = 7 \text{ x } 28 = 196$$

$$S = 7 + 14 + 21 + ... + 196$$

$$a = 7 \quad d = 7 \quad U_n = 196$$

$$U_n = a + (n-1)d$$

$$196 = 7 + (n-1)7$$

$$n = 28$$

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{28}{2}(7 + 196)$$

$$S = 2842$$

Question 54

Evaluate $S = 27 + 31 + 35 + 39 + ... + 107$

Solution 54

$$S = 27 + 31 + 35 + 39 + ... + 107$$

$$a = 27 \quad d = 4 \quad U_n = 107$$

$$U_n = a + (n-1)d$$

$$107 = 27 + (n-1)4$$

$$n = 21$$

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{21}{2}(27 + 107)$$

$$S = 1407$$

Question 55

Evaluate $T = 31 + 33 + 35 + 37 + ... + 81$

Solution 55

$$T = 31 + 33 + 35 + 37 + ... + 81$$
$$a = 31 \quad d = 2 \quad U_n = 81$$
$$U_n = a + (n-1)d$$
$$81 = 31 + (n-1)2$$
$$n = 26$$
$$S = \frac{n}{2}(a + l)$$
$$S = \frac{26}{2}(31 + 81)$$
$$S = 1456$$

Question 56

Evaluate $R = 97 + 92 + 87 + 82 + ... + 22$

Solution 56

Reverse the sequence: $R = 22 + 27 + 32 + 27 + ... + 97$

$$R = 22 + 27 + 32 + 27 + ... + 97$$
$$a = 22 \quad d = 5 \quad U_n = 97$$
$$U_n = a + (n-1)d$$
$$97 = 22 + (n-1)5$$
$$n = 16$$
$$S = \frac{n}{2}(a + l)$$
$$S = \frac{16}{2}(22 + 97)$$
$$S = 952$$

Question 57

Evaluate $\sum_{r=9}^{35}(3r - 1)$

Solution 57

$$\sum_{r=9}^{35}(3r-1) = 26 + 29 + 32 + 35 + ... + 104$$

$$a = 26 \quad l = 104 \quad n = 27$$

$$\sum_{r=9}^{35}(3r-1) = \frac{27}{2}(26 + 104)$$

$$= 1755$$

Question 58

Evaluate $\sum_{r=1}^{20}(3r-1)$

Solution 58

$$\sum_{r=1}^{20}(3r-1) = 2 + 5 + 8 + 11 + ... + 59$$

$$a = 2 \quad l = 59 \quad n = 20$$

$$\sum_{r=1}^{20}(3r-1) = \frac{20}{2}(2 + 59)$$

$$= 610$$

Question 59

Evaluate $\sum_{r=21}^{45}(2r-25)$

Solution 59

 © Joe Zhou, 2016

$$\sum_{r=21}^{45}(2r-25) = 17+19+21+23+...+65$$

$$a = 17 \quad l = 65 \quad n = 25$$

$$\sum_{r=21}^{45}(2r-25) = \frac{25}{2}(17+65)$$

$$= 1025$$

Question 60

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 11$ and $U_7 = 23$, find S_{11}

Solution 60

$$U_n = a + (n-1)d$$

$$U_4 = a + (4-1)d = a + 3d = 11 \quad (1)$$

$$U_7 = a + (7-1)d = a + 6d = 23 \quad (2)$$

$$(2) - (1) \quad a + 6d - (a + 3d) = 23 - 11$$

$$3d = 12$$

$$d = 4$$

$$\text{Sub into} \quad (1) \quad a + 3(4) = 11$$

$$a = -1$$

$$S_n = \frac{n}{2}(2(a) + (n-1)d)$$

$$S_{11} = \frac{11}{2}(2(-1) + (11-1)4) = 209$$

Question 61

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_3 = -5$ and $U_5 = -11$, find S_7

Solution 61

$$\begin{aligned}
U_n &= a + (n-1)d \\
U_3 &= a + (3-1)d = a + 2d = -5 \quad (1) \\
U_5 &= a + (5-1)d = a + 4d = -11 \quad (2) \\
(2) - (1) \quad a + 4d - (a + 2d) &= -11 - (-5) \\
2d &= -6 \\
d &= -3 \\
\text{Sub into} \quad (1) \quad a + 2(-3) &= -5 \\
a &= 1
\end{aligned}$$

$$\begin{aligned}
U_7 &= 1 + (7-1)(-3) = -17 \\
S_7 &= \frac{7}{2}(1 + (-17)) = -56
\end{aligned}$$

Question 62

The first three terms of an arithmetic sequence are $60, 58, 56...$, there exists a k^{th} term which $= 0$, find the value of k, hence of otherwise find the maximum value of S_n

Solution 62

$$
\begin{aligned}
U_n &= a + (n-1)d \\
U_k &= 60 + (k-1)(-2) = 0 \\
k - 1 &= 30 \\
k &= 31
\end{aligned}
$$

maimum value of $S_n = S_k$ as any term after U_k is negative

$$
\begin{aligned}
S_n &= \frac{n}{2}(a+d) \\
S_k &= \frac{31}{2}(60+0) \\
S_k &= 930
\end{aligned}
$$

Question 63

A sequence is defined by the recurrence relation $Y_{n+1} = 5 + 5Y_n - 2(Y_n)^3$, given that $Y_1 = 2$, find Y_{1000}.

Solution 63

$$Y_2 = 5 + 5Y_1 - 2(Y_1)^3 = 5 + 5(2) - 2(2)^3 = -1$$

$$Y_3 = 5 + 5Y_2 - 2(Y_2)^3 = 5 + 5(-1) - 2(-1)^3 = 2$$

$$Y_4 = 5 + 5Y_3 - 2(Y_3)^3 = 5 + 5(2) - 2(2)^3 = -1$$

Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
-1	2	-1	2	-1	2

We can see that $Y_2 = Y_4 = Y_6 = Y_8 = ... = 2$

Every numbered term divisible by 2 is 2

Find a numbered term that is close to Y_{1000} that is divisible by 2

$Y_2 = 2 \quad Y_4 = 2 \quad Y_{100} = 2 \quad Y_{1000} = 2$

Question 64

Given $\sum_{r=1}^{n} x_r = 5n^2 - 3$, find $\sum_{r=1}^{7} x_r$

Solution 64

$$\sum_{r=1}^{7} x_r = 5(7)^2 - 3 = 242$$

Question 65

A sequence is defined by the recurrence relation $U_{n+1} = \dfrac{13 - 5U_n}{7 - 3U_n}$, given that $U_1 = 1$, find U_{50}.

Solution 65

$$U_2 = \frac{13 - 5U_1}{7 - 3U_1} = \frac{13 - 5(1)}{7 - 3(1)} = \frac{8}{4} = 2$$

$$U_3 = \frac{13 - 5U_2}{7 - 3U_2} = \frac{13 - 5(2)}{7 - 3(2)} = \frac{3}{1} = 3$$

$$U_4 = \frac{13 - 5U_3}{7 - 3U_3} = \frac{13 - 5(3)}{7 - 3(3)} = \frac{-2}{-2} = 1$$

$$U_5 = \frac{13 - 5U_4}{7 - 3U_4} = \frac{13 - 5(1)}{7 - 3(1)} = \frac{8}{4} = 2$$

U_1	U_2	U_3	U_4	U_5	U_6
1	2	3	1	2	3

We can see that $U_3 = U_6 = U_9 = U_{12} = ... = 3$
Every numbered term divisible by 3 is 3
Find a numbered term that is close to U_{50} that is divisible by 3
$U_3 = 3 \quad U_9 = 3 \quad U_{30} = 3 \quad U_{51} = 3$
$U_{51} = 3 \quad \Rightarrow \quad U_{50} = 2$ since 2 is the term before 3 in the sequence
i.e. $1, 2, 3, 1, 2, 3, 1,$

Question 66

Given $\displaystyle\sum_{r=1}^{n} a_r = 2n^3 + 5$, find a_2

Solution 66

$$\sum_{r=1}^{n} a_r = 2n^3 + 5$$

$$a_2 = \sum_{r=1}^{2} a_r - \sum_{r=1}^{1} a_r$$

$$a_2 = 2(2)^3 + 5 - (2(1)^3 + 5)$$

$$a_2 = 21 - 7$$

$$a_2 = 14$$

Question 67

Given $\displaystyle\sum_{r=1}^{n} U_r = 6n^2 + 11$, find U_1

Solution 67

$$\sum_{r=1}^{1} U_r = U_1 = 6(1)^2 + 11 = 17$$

Question 68

Given $\sum_{r=1}^{n} u_r = n^3 + 4$, find $\sum_{r=1}^{5} u_r$

Solution 68

$$\sum_{r=1}^{5} u_r = (5)^3 + 4 = 129$$

Question 69

Given $\sum_{r=1}^{n} Y_r = 3n^3 - 2$, find Y_3

Solution 69

$$Y_3 = \sum_{r=1}^{3} - \sum_{r=1}^{2}$$
$$Y_3 = 3(3)^3 - 2 - (3(2)^3 - 2)$$
$$Y_3 = 57$$

Question 70

Given $\sum_{r=1}^{n} U_r = 3n + 7$, find U_5

Solution 70

$$U_5 = \sum_{r=1}^{5} U_r - \sum_{r=1}^{4} U_r$$

$$U_5 = 3(5) + 7 - (3(4) + 7)$$

$$U_5 = 3$$

Question 71

The first three terms of an arithmetic sequence are 3,5,7, find U_{10}

Solution 71

$$a = 3 \quad n = 10 \quad d = 5 - 3 = 2$$

$$U_n = a + (n-1)d$$

$$U_{10} = 3 + (10-1)2 = 21$$

Question 72

The first four terms of an arithmetic sequence are 5,9,13,17, find A_7

Solution 72

$$a = 5 \quad n = 7 \quad d = 9 - 5 = 4$$

$$A_n = a + (n-1)d$$

$$A_7 = 5 + (7-1)4 = 29$$

Question 73

The first three terms of an arithmetic sequence are 22,19,16, find X_6

Solution 73

$$
\begin{aligned}
a = 22 \quad n = 6 \quad d = 22 - 19 = 3 \\
X_n = a + (n-1)d \\
X_7 = 22 + (6-1)3 = 37
\end{aligned}
$$

Question 74

a_n is an arithmetic sequence, given that $a_3 = 13$ and $a_6 = 19$, find a_{11}

Solution 74

$$
\begin{aligned}
a_n &= a + (n-1)d \\
a_3 &= a + (3-1)d = a + 2d = 13 \quad (1) \\
a_6 &= a + (6-1)d = a + 5d = 19 \quad (2) \\
(2) - (1) \quad a + 5d - (a + 2d) &= 19 - 13 \\
3d &= 6 \\
d &= 2
\end{aligned}
$$

$$
\begin{aligned}
\text{Sub into(1)} \quad a + 2(2) &= 13 \\
a &= 9
\end{aligned}
$$

$$
a_{11} = 9 + (11-1)2 = 29
$$

Question 75

U_n is an arithmetic sequence, given that $U_4 = 25$ and $U_9 = 40$, find U_{13}

Solution 75

$$
\begin{aligned}
U_n &= a + (n-1)d \\
U_4 &= a + (4-1)d = a + 3d = 25 \quad (1) \\
U_9 &= a + (9-1)d = a + 8d = 40 \quad (2) \\
(2)-(1) \quad a + 8d - (a + 3d) &= 40 - 25 \\
5d &= 15 \\
d &= 3
\end{aligned}
$$

$$
\begin{aligned}
\text{Sub into(1)} \quad a + 3(3) &= 25 \\
a &= 16
\end{aligned}
$$

$$U_{13} = 16 + (13-1)3 = 52$$

Question 76

X_n is an arithmetic sequence, given that $X_{13} = 51$ and $X_{19} = 33$, find X_{10}

Solution 76

$$X_n = a + (n-1)d$$
$$X_{13} = a + (13-1)d = a + 12d = 51 \quad (1)$$
$$X_{19} = a + (19-1)d = a + 18d = 33 \quad (2)$$
$$(2)-(1) \quad a + 18d - (a + 12d) = 33 - 51$$
$$6d = -18$$
$$d = -3$$

$$\text{Sub into(1)} \quad a + 12(-3) = 51$$
$$a = 87$$

$$X_{10} = 87 + (10-1)(-3) = 60$$

Question 77

u_n is an arithmetic sequence, given that $u_3 = 5$ and $u_7 = 13$, for what value of n is $a_n = 71$

Solution 77

$$u_n = a + (n-1)d$$
$$u_3 = a + (3-1)d = a + 2d = 5 \quad (1)$$
$$u_7 = a + (7-1)d = a + 6d = 13 \quad (2)$$
$$(2)-(1) \quad a + 6d - (a + 2d) = 13 - 5$$
$$4d = 8$$
$$d = 2$$

$$\text{Sub into(1)} \quad a + 2(2) = 5$$
$$a = 1$$

$$u_n = 1 + (n-1)2 = 71$$
$$n - 1 = 35$$
$$n = 36$$

Question 78

The first three terms of an arithmetic sequence are 11,14,17, find a n for which $U_n = 83$

Solution 78

$$
\begin{aligned}
u_n &= 83 \quad a = 11 \quad d = 3 \\
u_n &= a + (n-1)d \\
83 &= 11 + (n-1)3 \\
n - 1 &= 24 \\
n &= 25
\end{aligned}
$$

Question 79

Y_n is an arithmetic sequence, given that $Y_{15} = 51$ and $X_{19} = 71$, find Y_{26}

Solution 79

$$
\begin{aligned}
Y_n &= a + (n-1)d \\
Y_{15} &= a + (15-1)d = a + 14d = 51 \quad (1) \\
Y_{19} &= a + (19-1)d = a + 18d = 71 \quad (2) \\
(2) - (1) \quad a + 18d - (a + 14d) &= 71 - 51 \\
4d &= 20 \\
d &= 5
\end{aligned}
$$

$$
\begin{aligned}
\text{Sub into(1)} \quad a + 14(5) &= 51 \\
a &= -19
\end{aligned}
$$

$$
Y_{26} = -19 + (26-1)5 = 106
$$

Question 80

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $S_5 = 85$ and $S_8 = 184$, find U_6

Solution 80

$$\begin{aligned}
S_n &= \frac{n}{2}(2a + (n-1)d) \\
S_5 &= \frac{5}{2}(2a + (5-1)d) = 85 \\
2a + 4d &= 34 \quad (1) \\
S_8 &= \frac{8}{2}(2a + (8-1)d) = 184 \\
2a + 7d &= 46 \quad (2) \\
(2) - (1) \quad 2a + 7d - (2a + 4d) &= 46 - 34 \\
3d &= 12 \\
d &= 4 \\
\text{Sub into} \quad (1) \quad 2a + 4(4) &= 34 \\
a &= 9
\end{aligned}$$

$$U_6 = a + (6-1)d = 9 + 5(4) = 29$$

Question 81

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_5 = 19$ and $S_{10} = 170$, find U_4

Solution 81

$$U_n = a + (n-1)d$$
$$U_5 = a + 4d = 19 \qquad (1)$$
$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_{10} = \frac{10}{2}(2a + (10-1)d) = 170$$
$$2a + 9d = 34 \qquad (2)$$
$$(2) - 2(1) \quad 2a + 9d - 2(a + 4d) = 34 - 38$$
$$d = -4$$
$$d = -4$$
$$\text{Sub into} \quad (1) \quad a + 4(-4) = 19$$
$$a = 35$$
$$U_4 = a + (4-1)d = 35 + (3)(-4) = 23$$

Question 82

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 8$ and $S_{12} = 0$, find S_9

Solution 82

 © Joe Zhou, 2016

$$
\begin{aligned}
U_n &= a + (n-1)d \\
U_4 &= a + 3d = 8 \qquad\qquad (1) \\
S_n &= \frac{n}{2}(2a + (n-1)d) \\
S_{12} &= \frac{12}{2}(2a + (12-1)d) = 0 \\
2a + 11d &= 0 \qquad\qquad (2) \\
(2) - 2(1) \quad 2a + 11d - 2(a + 3d) &= 0 - 16 \\
8d &= -16 \\
d &= -2 \\
\text{Sub into} \quad (1) \quad a + 3(-2) &= 8 \\
a &= 14 \\
S_9 &= \frac{9}{2}(2(14) + (9-1)(-2)) = 54
\end{aligned}
$$

Question 83

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_3 = 4$ and $U_7 = 0$, find S_{10}

Solution 83

$$
\begin{aligned}
U_n &= a + (n-1)d \\
U_3 &= a + 2d = 4 \quad (1) \\
U_7 &= a + 6d = 0 \quad (2) \\
(2) - (1) \quad a + 6d - (a + 2d) &= 0 - 4 \\
4d &= -4 \\
d &= -1 \\
\text{Sub into} \quad (1) \quad a + 2(-1) &= 4 \\
a &= 6
\end{aligned}
$$

$$
\begin{aligned}
S_n &= \frac{n}{2}(2a + (n-1)d) \\
S_{10} &= \frac{10}{2}(2(6) + (10-1)(-1)) = 15
\end{aligned}
$$

Question 84

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 10$ and $S_6 = 57$, find S_{11}

Solution 84

$$
\begin{aligned}
U_n &= a + (n-1)d \\
U_4 &= a + 3d = 10 \qquad (1) \\
S_n &= \frac{n}{2}(2a + (n-1)d) \\
S_6 &= \frac{6}{2}(2a + (5-1)d) = 57 \\
a + 2d &= \frac{19}{2} \qquad (2) \\
(1) - (2) \quad a + 3d - (a + 2d) &= 10 - \frac{19}{2} \\
d &= \frac{1}{2} \\
\text{Sub into} \quad (1) \quad a + 3\left(\frac{1}{2}\right) &= 10 \\
a &= \frac{17}{2} \\
S_{11} &= \frac{11}{2}\left(2\left(\frac{17}{2}\right) + (11-1)\left(\frac{1}{2}\right)\right) = 121
\end{aligned}
$$

Question 85

Three consecutive terms in an arithmetic sequence are $3k+2, 2k+5, 4k+5$, find the value of k

Solution 85

$$
\begin{aligned}
2k + 5 - (3k + 2) = d = 4k + 5 - (2k + 5) \\
-k + 3 &= 2k \\
3k &= 3 \\
k &= 1
\end{aligned}
$$

Question 86

Three consecutive terms in an arithmetic sequence are $k^2 + 3, -k, k - 1$, find the possible values of k

Solution 86

$$\begin{aligned} -k-(k^2+3) = d &= k-1-(-k) \\ -k-k^2-3 &= 2k-1 \\ 0 &= k^2+3k+2 \\ 0 &= (k+2)(k+1) \end{aligned}$$

Question 87

Three consecutive terms in an arithmetic sequence are $k+16, 3k+12, 7k-2$, find the value of k

Solution 87

$$\begin{aligned} 3k+12-(k+16) = d &= 7k-2-(3k+12) \\ 2k-4 &= 4k-14 \\ 2k &= 10 \\ k &= 5 \end{aligned}$$

Question 88

The first three terms in an arithmetic sequence are $2k, k+9, 3k$, find the smallest n such that $S_n > 117$

Solution 88

$$
\begin{aligned}
k + 9 - 2k = d &= 3k - (k + 9) \\
-k + 9 &= 2k - 9 \\
3k &= 18 \\
k &= 6
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad U_1 &= 12 \quad U_2 = 15 \quad U_3 = 18 \\
S_n &= \frac{n}{2}(2a + (n-1)d) \\
\frac{n}{2}(2(12) + (n-1)3) &> 117 \\
n(24 + 3n - 3) &> 234 \\
3n^2 + 21n - 234 &> 0 \\
n^2 + 7n - 78 &> 0 \quad P = -78 \quad S = 7 \\
(n+13)(n-6) &> 0 \quad (13, -6) \\
n &= 6
\end{aligned}
$$

Question 89

The first three terms of an arithmetic sequence are $99, 96, 93...$, there exists a k^{th} term which $= 0$, find the value of k, hence of otherwise find the maximum value of S_n

Solution 89

 © Joe Zhou, 2016

$$
\begin{aligned}
U_n &= a + (n-1)d \\
U_k &= 99 + (k-1)(-3) = 0 \\
k - 1 &= 33 \\
k &= 34
\end{aligned}
$$

maimum value of $S_n = S_k$ as any term after U_k is negative

$$
\begin{aligned}
S_n &= \frac{n}{2}(a + l) \\
S_k &= \frac{34}{2}(99 + 0) \\
S_k &= 1683
\end{aligned}
$$

Question 90

The first three terms in an arithmetic sequence are $5, 7, 9$, find the smallest n such that $S_n > 252$

Solution 90

$$
\begin{aligned}
S_n &= \frac{n}{2}(2a + (n-1)d) \\
S_n &= \frac{n}{2}(2(5) + (n-1)2) > 252 \\
n(5 + n - 1) &> 252 \\
n^2 + 4n - 252 &> 0 \quad P = -252 \quad S = 4 \\
(n + 18)(n - 14) &> 0 \quad (18, -14) \\
n &= 14
\end{aligned}
$$

Question 91

The first three terms in an arithmetic sequence are $9, 12, 15$, find the smallest n

such that $S_n > 750$

Solution 91

$$\begin{aligned}
S_n &= \frac{n}{2}(2a + (n-1)d) \\
S_n &= \frac{n}{2}(2(9) + (n-1)3) > 750 \\
n(18 + 3n - 3) &> 1500 \\
3n(5 + n) &> 1500 \\
n(5 + n) &> 500 \\
n^2 + 5n - 500 &> 0 \quad P = -500 \quad S = 5 \\
(n + 25)(n - 20) &> 0 \quad (25, -20) \\
n &= 20
\end{aligned}$$

Question 92

The first three terms in an arithmetic sequence are $12, 16, 20, 24$, find the smallest n such that $S_n > 672$

Solution 92

$$\begin{aligned}
S_n &= \frac{n}{2}(2a + (n-1)d) \\
S_n &= \frac{n}{2}(2(12) + (n-1)4) > 672 \\
n(12 + 2n - 2) &> 672 \\
2n^2 + 10n - 672 &> 0 \\
n^2 + 5n - 336 &> 0 \quad P = -336 \quad S = 5 \\
(n + 21)(n - 16) &> 0 \quad (21, -16) \\
n &= 16
\end{aligned}$$

 © Joe Zhou, 2016

Question 93

Kendrick decides to open up a savings account. He puts in £100 for the first month, £120 for the second month and an extra £20 for subsequent months till he's putting in £300 a month. Find the total amount he's saved in 2 years.

Solution 93

Sequence goes: 100,120,140,160,180,200...300,300,300,300...

$$U_n = a + (n-1)d$$
$$U_n = 300 \quad a = 100 \quad d = 20$$
$$300 = 100 + (n-1)20$$
$$n = 11$$

$$S_n = \frac{n}{2}(a + l)$$
$$n = 11 \quad a = 100 \quad l = 300$$
$$S_{11} = \frac{11}{2}(100 + 300)$$
$$S_{11} = 2200$$

Every term after is 300

$$\sum_{r=12}^{24} 300 = 13 \text{ x } 300$$
$$= 3900$$

$$\Rightarrow \quad \text{Total days} = 2200 + 3900 = 6100$$

Question 94

Avery is playing with 340 sticks, she puts them in rows. The first row has 7 sticks, next row has 13 sticks, subsequent rows have 6 more sticks then the previous row.

She has enough for k rows but not enough for $k+1$ rows. Find k.

Solution 94

Sequence goes: 7,13,19,25,31,37....

Not having enough for k+1 rows means that $S_k \leq 340$

$$\begin{aligned}
S_n &= \frac{n}{2}(2a + (k-1)d) \\
S_k &= \frac{k}{2}(2(7) + (k-1)6) \\
S_k &= k(7 + 3(k-1)) \\
S_k &= k(3k+4) \\
S_k &= 3k^2 + 4k \qquad (1)
\end{aligned}$$

$$\begin{aligned}
S_k &\leq 340 \\
(1) \qquad 3k^2 + 4k &\leq 340 \\
3k^2 + 4k - 340 &\leq 0 \qquad P = -1020 \quad S = 4 \\
\left(k + \frac{34}{3}\right)(k-10) &\leq 0 \qquad (34, -30) \qquad \left(\frac{34}{3}, -10\right) \\
k &= 10
\end{aligned}$$

Question 95

Express $\log_{x+5} 10 = 4$ in power form

Solution 95

$$\begin{aligned}
\log_{x+5} 10 &= 4 \\
(x+5)^4 &= 10
\end{aligned}$$

Question 96

Express $\log_{a+b} 6 = c$ in power form

Solution 96

$$\log_{a+b} 6 = c$$
$$(a+b)^c = 6$$

Question 97

Express $\log_{xy} 3 = 2$ in power form

Solution 97

$$\log_{xy} 3 = 2$$
$$(xy)^2 = 3$$

Question 98

Express $a^b = c$ in log form

Solution 98

$$a^b = c$$
$$\log_a c = b$$

Question 99

Express $5^2 = 25$ in log form

Solution 99

$$5^2 = 25$$
$$\log_5 25 = 2$$

 © Joe Zhou, 2016

Question 100

Express $(xy)^5 = 20$ in log form

Solution 100

$$\begin{aligned} (xy)^5 &= 20 \\ \log_{xy} 20 &= 5 \end{aligned}$$

Question 101

Express $\log_2(x^2y) - \log_2 x$ as a single logarithm

Solution 101

$$\begin{aligned} &\log_2(x^2y) - \log_2 x \\ =&\log_2((x^2y) \div x) \\ =&\log_2 xy \end{aligned}$$

Question 102

Express $a^{bc} = 6$ in log form

Solution 102

$$\begin{aligned} a^{bc} &= 6 \\ \log_a 6 &= bc \end{aligned}$$

Question 103

Griffin is training daily for a cycling marathon in 100 days. He cycles 10km on the first day, 11km on the second day and 1 more km then the previous day till he's cycling 40km a day. Calculate the total number of km he's cycled as training for the marathon.

Solution 103

Sequence goes: 10,11,12,13,14,15...40,40,40,40...

$$\begin{aligned}
U_n &= a + (n-1)d \\
U_n &= 40 \quad a = 10 \quad d = 1 \\
40 &= 10 + (n-1)1 \\
n &= 31
\end{aligned}$$

$$\begin{aligned}
S_n &= \frac{n}{2}(a + l) \\
n &= 31 \quad a = 10 \quad l = 40 \\
S_{31} &= \frac{31}{2}(10 + 40) \\
S_{31} &= 775
\end{aligned}$$

Every term after is 40

$$\begin{aligned}
\sum_{r=32}^{100} 40 &= 69 \text{ x } 40 \\
&= 2760
\end{aligned}$$

$$\Rightarrow \quad \text{Total days} = 775 + 2760 = 3535$$

Question 104

Express $(a + b)^4 = 15$ in log form

Solution 104

$$
\begin{aligned}
(a+b)^4 &= 15 \\
\log_{(a+b)} 15 &= 4
\end{aligned}
$$

Question 105

Express $(x+4)^4 = 5$ in log form

Solution 105

$$
\begin{aligned}
(x+4)^4 &= 5 \\
\log_{(x+4)} 5 &= 4
\end{aligned}
$$

Question 106

Express $\log_4(x+y) + \log_4 6$ as a single logarithm

Solution 106

$$
\begin{aligned}
&\log_4(x+y) + \log_4 6 \\
=&\log_4((x+y) \text{ x } 6) \\
=&\log_4 6(x+y)
\end{aligned}
$$

Question 107

Judith is playing with 294 sticks, she puts them in rows. The first row has 8 sticks, next row has 10 sticks, subsequent rows have 2 more sticks then the previous row.

 © Joe Zhou, 2016

She has enough for k rows but not enough for $k+1$ rows. Find k.

Solution 107

Sequence goes: 8,10,12,14,18,20....

Not having enough for k+1 rows means that $S_k \leq 294$

$$\begin{aligned}
S_n &= \frac{n}{2}(2a + (k-1)d) \\
S_k &= \frac{k}{2}(2(8) + (k-1)2) \\
S_k &= k(8 + k - 1) \\
S_k &= k(k+7) \\
S_k &= k^2 + 7k \qquad (1)
\end{aligned}$$

$$\begin{aligned}
S_k &\leq 294 \\
(1) \qquad k^2 + 7k &\leq 294 \\
k^2 + 7k - 294 &\leq 0 \qquad P = 294 \quad S = 7 \\
(k+21)(k-14) &\leq 0 \qquad (21, -14) \\
k &= 14
\end{aligned}$$

Question 108

Heidi is training daily for a swimming competition in 60 days. She swims 10 laps on the first day, 12 laps on the second day and 2 more laps then the previous day till she's swimming 30 laps a day. Calculate the total number of laps she's swum as training for the competition.

Solution 108

Sequence goes: 10,12,14,16,18,20...30,30,30,30...

$$U_n = a + (n-1)d$$
$$U_n = 30 \quad a = 10 \quad d = 2$$
$$30 = 10 + (n-1)2$$
$$n = 11$$

$$S_n = \frac{n}{2}(a + l)$$
$$n = 11 \quad a = 10 \quad l = 30$$
$$S_{11} = \frac{11}{2}(10 + 30)$$
$$S_{11} = 220$$

Every term after is 30

$$\sum_{r=12}^{60} 30 = 49 \text{ x } 30$$
$$= 1470$$

$$\Rightarrow \quad \text{Total days} = 220 + 1470 = 1690$$

Question 109

Express $\log_x 9 = 2$ in power form

Solution 109

$$\log_x 9 = 2$$
$$x^2 = 9$$

Question 110

A sequence is defined by the recurrence relation $U_{n+1} = kU_n - 4, U_1 = 3, k > 0$, given that $U_3 = 0$ find the value of k

Solution 110

$$U_2 = kU_1 - 4$$
$$U_2 = 3k - 4$$

$$U_3 = kU_2 - 4$$
$$U_3 = k(3k-4) - 4$$
$$U_3 = 3k^2 - 4k - 4 \quad U_3 = 0$$

$$0 = 3k^2 - 4k - 4 \qquad S = -4 \quad P = -12$$
$$0 = \left(k + \frac{2}{3}\right)(k-2) \quad (2, -6) \quad \Rightarrow \quad \left(\frac{2}{3}, -2\right)$$
$$k = 2$$

Question 111

A sequence is defined by the recurrence relation $a_{n+1} = \dfrac{a_n}{k} + 3, a_1 = 3, k > 0$, given that $a_3 = 9$ find the value of k

Solution 111

$$a_2 = \frac{a_1}{k} + 3$$
$$a_2 = \frac{3}{k} + 3$$
$$a_3 = \frac{a_2}{k} + 3$$
$$a_3 = \frac{\left(\frac{3}{k} + 3\right)}{k} + 3$$
$$a_3 = \frac{3}{k^2} + \frac{3}{k} + 3 \quad a_3 = 9$$
$$9 = \frac{3}{k^2} + \frac{3}{k} + 3$$
$$6 - \frac{3}{k^2} - \frac{3}{k} = 0$$
$$6k^2 - 3k - 3 = 0$$
$$2k^2 - k - 1 = 0 \qquad S = -1 \quad P = -2$$
$$\left(x + \frac{1}{2}\right)(k-1) = 0 \qquad (1, -2) \quad \Rightarrow \quad \left(\frac{1}{2}, -1\right)$$
$$k = 1$$

Question 112

A sequence is defined by the recurrence relation $u_{n+1} = \sqrt{a}\left(u_n - \frac{1}{b}\right)$, $5u_1 = 4$, given that $u_2 = 7$ and $u_3 = 13$ find the value of a and b .

Solution 112

$$u_2 = \sqrt{a}\left(u_1 - \frac{1}{b}\right)$$

$$7 = \sqrt{a}\left(4 - \frac{1}{b}\right) \qquad (1)$$

$$7 = 4\sqrt{a} - \frac{\sqrt{a}}{b} \qquad (2)$$

$$u_3 = \sqrt{a}\left(u_2 - \frac{1}{b}\right)$$

$$13 = \sqrt{a}\left(7 - \frac{1}{b}\right)$$

$$13 = 7\sqrt{a} - \frac{\sqrt{a}}{b} \qquad (3)$$

$$(3) - (2) \quad 13 - 7 = 7\sqrt{a} - \frac{\sqrt{a}}{b} - \left(4\sqrt{a} - \frac{\sqrt{a}}{b}\right)$$

$$6 = 3\sqrt{a}$$

$$2 = \sqrt{a}$$

$$a = 4$$

$$\text{Sub into (1)} \quad 7 = \sqrt{4}\left(4 - \frac{1}{b}\right)$$

$$\frac{7}{2} = 4 - \frac{1}{b}$$

$$-\frac{1}{2} = -\frac{1}{b}$$

$$b = 2$$

Question 113

A sequence is defined by the recurrence relation $a_{n+1} = 3 - a_n$, given that $a_1 = 1$, find $\displaystyle\sum_{r=1}^{100} a_r$.

Solution 113

$$a_2 = 3 - a_1 = 3 - 1 = 2$$

$$a_3 = 3 - a_2 = 3 - 2 = 1$$

$$a_4 = 3 - a_3 = 3 - 1 = 2$$

$$a_5 = 3 - a_4 = 3 - 2 = 1$$

$$\sum_{r=1}^{100} a_r = a_1 + a_2 + a_3 + a_4 + ... + a_{100}$$

$$\sum_{r=1}^{100} a_r = 1 + 2 + 1 + 2 + 1 + 2 + ... + 2$$

$$\sum_{r=1}^{100} a_r = 50(2) + 50(1)$$

$$\sum_{r=1}^{100} a_r = 150$$

Question 114

A sequence is defined by the recurrence relation $A_{n+1} = \dfrac{4A_n - 16}{3A_n - 8}$, given that $A_1 = 0$, find A_{100}.

Solution 114

$$A_2 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(0) - 16}{3(0) - 8} = \frac{-16}{-8} = 2$$

$$A_3 = \frac{4A_2 - 16}{3A_2 - 8} = \frac{4(2) - 16}{3(2) - 8} = \frac{-8}{-2} = 4$$

$$A_4 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(4) - 16}{3(4) - 8} = 0$$

$$A_5 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(0) - 16}{3(0) - 8} = \frac{-16}{-8} = 2$$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6$

$0 \quad 2 \quad 4 \quad 0 \quad 2 \quad 4$

We can see that $a_3 = a_6 = a_9 = a_{12} = ... = 4$

Every numbered term divisible by 3 is 4

Find a numbered term that is close to a_{100} that is divisible by 3

$a_3 = 4 \quad a_9 = 4 \quad a_{30} = 4 \quad a_{99} = 4$

$a_{99} = 4 \quad \Rightarrow \quad a_{100} = 0$ since 0 is the next term after 4 in the sequence

i.e. $0, 2, 4, 0, 2, 4, 0$

Question 115

Express $3\log_3(a+b) + \log_3 4$ as a single logarithm

Solution 115

$$\begin{aligned}
&3\log_3(a+b) + \log_3 4 \\
=&\log_3(a+b)^3 + \log_3 4 \\
=&\log_3((a+b)^3 \text{ x } 4) \\
=&\log_3 4(a+b)^3
\end{aligned}$$

Question 116

Express $\log_4(a^2 - b^2) - 2\log_4(a+b)$ as a single logarithm

Solution 116

$$
\begin{aligned}
& \log_4(a^2 - b^2) - 2\log_4 a + b \\
= & \log_4(a^2 - b^2) - \log_4(a + b)^2 \\
= & \log_3((a^2 - b^2) \ \div \ (a + b)^2) \\
= & \log_3 \left(\frac{(a + b)(a - b)}{(a + b)^2} \right) \\
= & \log_3 \frac{a - b}{a + b}
\end{aligned}
$$

Question 117

Express $\log_x(4a - 6b) + \log_x \frac{1}{2}$ as a single logarithm

Solution 117

$$
\begin{aligned}
& \log_x(4a - 6b) + \log_x \frac{1}{2} \\
= & \log_x \frac{1}{2}(4a - 6b) \\
= & \log_x(2a - 3b)
\end{aligned}
$$

Question 118

Express $\log_4(6a) - \log_4(2a)$ as a single logarithm

Solution 118

$$
\begin{aligned}
& \log_4(6a) - \log_4(2a) \\
= & \log_4(6a \ \div \ 2a) \\
= & \log_4 3
\end{aligned}
$$

Question 119

Express $\log_{10}(15) - \log_{10}(3)$ as a single logarithm

Solution 119

$$\begin{aligned}
&\log_{10}(15) - \log_{10}(3)\\
=&\log_{10}(15 \ \div \ 3)\\
=&\log_{10} 5
\end{aligned}$$

Question 120

Express $3\log_y(5) + \log_y(4)$ as a single logarithm

Solution 120

$$\begin{aligned}
&3\log_y(5) + \log_y(4)\\
=&\log_y 5^3 + \log_y 4\\
=&\log_y(5^3 \text{ x } 4)\\
=&\log_y 500
\end{aligned}$$

Question 121

Express $3\log_a(4) - 4\log_a(2)$ as a single logarithm

Solution 121

$$\begin{aligned}
&\log_a(4^3) - \log_a(2^4)\\
=&\log_a(64) - \log_a(16)\\
=&\log_y(64 \div 16)\\
=&\log_y 4
\end{aligned}$$

Question 122

Express $\log_3 7 = a + b^2$ in power form

Solution 122

$$\begin{aligned} \log_3 7 &= a + b^2 \\ 3^{a+b^2} &= 7 \end{aligned}$$

Question 123

Express $4\log_9 5 - 2\log_3(15)$ as a single logarithm

Solution 123

$$\begin{aligned} &4\log_9 5 - 2\log_3(9) \\ =&4\left(\frac{\log_3 5}{\log_3 9}\right) - 2\log_3(15) \\ =&\left(\frac{4\log_3 5}{2}\right) - \log_3(15^2) \\ =&2\log_3 5 - \log_3(15^2) \\ =&\log_3(5^2 \div 15^2) \\ =&\log_3 \frac{25}{225} \\ =&\log_3 \frac{1}{9} \end{aligned}$$

Question 124

Express $\log_a 4 + \log_a 5$ as a single logarithm

Solution 124

$$
\begin{aligned}
&\log_a 4 + \log_a 5 \\
=&\log_a(4 \text{ x } 5) \\
=&\log_a 20
\end{aligned}
$$

Question 125

A sequence is defined by $U_n = 2n + 3$, find the value of U_2, U_4 and U_5.

Solution 125

$$
\begin{aligned}
U_2 &= 2(2) + 3 = 7 \\
U_4 &= 2(4) + 3 = 11 \\
U_5 &= 2(5) + 3 = 13
\end{aligned}
$$

Question 126

A sequence is defined by $u_n = 2n^2 - 5n - 3$, find the value of n such that $u_n = 9$.

Solution 126

$$
\begin{aligned}
u_n = 2n^2 - 5n - 3 &= 9 \\
2n^2 - 5n - 12 &= 0 \qquad S = -5 \quad P = -24 \\
\left(n + \frac{3}{2}\right)(n - 4) &= 0 \qquad (3, -8) \quad \left(\frac{3}{2}, -4\right) \\
n &= 4
\end{aligned}
$$

 © Joe Zhou, 2016

Question 127

A sequence is defined by $a_n = an^2 + b$, given the Sum of the first five terms is -5 and the sixth term is 4, find the values of a and b.

Solution 127

$$S_5 = (a+b) + (4a+b) + (9a+b) + (16a+b)$$

$$S_5 = 30a + 5b \qquad S_5 = -5$$

$$30a + 5b = -5$$

$$6a + b = -1 \quad (1)$$

$$a_6 = 25a + b \qquad a_6 = 4$$

$$36a + b = 4 \quad (2)$$

$$(2) - (1) \quad 36a + b - (6a + b) = 4 - (-1)$$

$$30a = 5$$

$$a = \frac{1}{6}$$

$$\text{sub into} \quad (1) \quad 6\left(\frac{1}{6}\right) + b = -1$$

$$b = -2$$

Question 128

Express $\log_a b - 4 = 7$ in power form

Solution 128

$$\log_a b - 4 = 7$$

$$a^7 = b - 4$$

Question 129

Express $2\log_{16} 8 - 4\log_4(2)$ as a single logarithm

Solution 129

$$\begin{aligned}
&2\log_{16} 8 - 4\log_4(2)\\
=&2\left(\frac{\log_4 8}{\log_4 16}\right) - 4\log_4(2)\\
=&\left(\frac{2\log_4 8}{2}\right) - \log_4(16)\\
=&\log_4(8 \div 16)\\
=&\log_4 \frac{1}{2}
\end{aligned}$$

Question 130

Express $3\log_4 5 + 4\log_{16}(3)$ as a single logarithm.

Solution 130

$$\begin{aligned}
&3\log_4 5 + 4\log_{16}(3)\\
=&3\log_4 5 + 4\left(\frac{\log_4 3}{\log_4 16}\right)\\
=&\log_4 5^3 + 4\left(\frac{\log_4 3}{2}\right)\\
=&\log_4 5^3 + \log_4 3^2\\
=&\log_4 1125
\end{aligned}$$

Question 131

A sequence is defined by $X_n = \dfrac{a+1}{n} + b$, given the Sum of the first three terms is $\dfrac{2}{3}$ and the fifth term is $-\dfrac{3}{5}$, find the values of a and b.

Solution 131

$$S_3 = \left(\frac{a+1}{(1)} + b\right) + \left(\frac{a+1}{(2)} + b\right) + \left(\frac{a+}{(3)}\right.$$

$$S_3 = a + \frac{a}{2} + \frac{a}{3} + 3b + 1 + \frac{1}{2} + \frac{1}{3}$$

$$S_3 = \frac{11}{6}a + 3b + \frac{11}{6} \quad S_3 = \frac{2}{3}$$

$$\frac{11}{6}a + 3b + \frac{11}{6} = \frac{2}{3}$$

$$11a + 18b + 11 = 4 \quad (1)$$

$$X_5 = \frac{a+1}{5} + b \qquad X_5 = -\frac{3}{5}$$

$$\frac{a+1}{5} + b = -\frac{3}{5}$$

$$a + 1 + 5b = -3 \quad (2)$$

$$11a + 11 + 55b = -33 \quad (3)$$

$$(3) - (1) \quad 11a + 11 + 55b - (11a + 18b + 11) = -33 - 4$$

$$37b = -37$$

$$b = -1$$

$$\text{sub into} \quad (2) \quad a + 1 + 5(-1) = -3$$

$$a - 4 = -3$$

$$a = 1$$

Question 132

Find the sum of all numbers divisible by 3 between 2 and 200

Solution 132

$$200 \div 3 = 66 \text{ remainder } 2$$

$$\Rightarrow \quad \text{last term } = 3 \text{ x } 66 = 198$$

$$S = 3 + 6 + 9 + ... + 198$$

$$a = 3 \quad d = 3 \quad U_n = 198$$

$$U_n = a + (n-1)d$$

$$198 = 3 + (n-1)3$$

$$n = 66$$

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{66}{2}(3 + 198)$$

$$S = 6633$$

Question 133

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $S_{11} = 0$ and $U_2 = 8$, find U_6

Solution 133

$$
\begin{aligned}
U_n &= a + (n-1)d \\
U_2 &= a + (2-1)d = a + d = 8 \qquad\qquad (1) \\
S_n &= \frac{n}{2}(2a + (n-1)d) \\
S_{11} &= \frac{11}{2}(2a + (11-1)d) = 0 \\
S_{11} &= a + 5d = 0 \qquad\qquad (2) \\
(2)-(1) \quad a + 5d - (a+d) &= 0 - 8 \\
4d &= -8 \\
d &= -2 \\
\text{Sub into} \quad (1) \quad a + (-2) &= 8 \\
a &= 10 \\
U_6 &= 1 + (7-1)(-2) = -11
\end{aligned}
$$

Question 134

The first three terms of an arithmetic sequence are $44, 41, 38...$, there exists a k^{th} term which is the smallest positive term in the sequence, find the value of k, hence of otherwise find the maximum value of S_n

Solution 134

 © Joe Zhou, 2016

$$U_n = a + (n-1)d$$
$$U_k = 44 + (k-1)(-3) = 0$$
$$k - 1 = \frac{44}{3}$$
$$k = \frac{44}{3} + 1 = 15.6$$
$$k = 15$$

maimum value of $S_n = S_k$ as any term after U_k is negative

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_k = \frac{34}{2}(2(99) + (15-1)(-3))$$
$$S_k = 2652$$

Question 135

At the start of the year 2000, Tony the farmer has $50m^2$ of land, he buys $7m^2$ of land at the end of each year. At the beginning of this year, Tony owns $141m^2$ of land. What year is it?

Solution 135

Sequence goes from the start of every year: 50,57,64,71,78,85....

$$U_n = a + (n-1)d$$
$$U_n = 141 \quad a = 50 \quad d = 7$$
$$141 = 50 + (n-1)7$$
$$n - 1 = 13$$
$$n = 14$$
$$\text{Year} = 2000 + 14 = 2014$$

Question 136

James is playing with 324 sticks, she puts them in rows. The first row has 5 sticks, next row has 9 sticks, subsequent rows have 4 more sticks then the previous row. She has enough for k rows but not enough for $k+1$ rows. Find k.

Solution 136

Sequence goes: 5,9,13,17,21,25....

Not having enough for k+1 rows means that $S_k \leq 324$

$$\begin{aligned}
S_n &= \frac{n}{2}(2a + (k-1)d) \\
S_k &= \frac{k}{2}(2(5) + (k-1)4) \\
S_k &= k(5 + 2k - 2) \\
S_k &= k(2k+3) \\
S_k &= 2k^2 + 3k \qquad (1)
\end{aligned}$$

$$\begin{aligned}
S_k &\leq 324 \\
(1) \qquad 2k^2 + 3k &\leq 324 \\
2k^2 + 3k - 324 &\leq 0 \qquad P = -648 \quad S = 3 \\
\left(k + \frac{27}{2}\right)(k - 12) &\leq 0 \qquad (27, -24) \qquad \left(\frac{27}{2}, -12\right) \\
k &= 12
\end{aligned}$$

Question 137

dsafsdfa $\geq$

Solution 137

adfsdsf