Edexcel A Level Maths

Core 1, Core 2, Core 3, Core 4 and two units from (Decision 1, Decision 2, Mechanics 1, Mechanics 2, Statistics 1 and Statistics 2). Each unit is out of 100 UMS, giving a total of 600 UMS. 480 and above is grade A. To obtain an A*, total score must be 480 or more, and total of C3 and C4 must be 180 or more.

Unit 1 Core 1

Chapter 1 Sequence and Series 1

Lesson 1 Sequence and Summation

Question 1 Experience: 10 Order: Level: Question-ID: 26

A sequence is defined by $a_n = 3n^2 - 4$, find the value of a_2.

Solution 1

$a_2 = 3(2)^2 - 4 = 8$

Choice 1: $a_2 = -1$ false
Choice 2: $a_2 = 23$ false
Choice 3: $a_2 = 5$ false
Choice 4: $a_2 = 10$ false
Choice 5: $a_2 = 8$ true

Question 2 Experience: 25 Order: Level: Question-ID: 30

A sequence is defined by $x_n = 3n^2 - 5n + 2$, find the value of n such that $x_n = 14$.

Solution 2

$x_n = 3n^2 - 5n + 2 = 14$

$3n^2 - 5n - 12 = 0 \qquad S = -5 \quad P = -36$

$\left(n + \frac{4}{3}\right)(n - 3) = 0 \qquad (4, -9) \quad \left(\frac{4}{3}, -3\right)$

$n = 3$

Choice 1: $n = 4$ false
Choice 2: $n = 2$ false
Choice 3: $n = 5$ false
Choice 4: $n = 6$ false
Choice 5: $n = 3$ true

Question 3 Experience: 15 Order: Level: Question-ID: 27

A sequence is defined by $x_n = 6n - 3$, find the value of x_3 and x_5.

Solution 3

$x_3 = 6(3) - 3 = 15$

$x_5 = 6(5) - 3 = 27$

Choice 1: $x_3 = 3\ x_5 = 15$ false
Choice 2: $x_3 = 9\ x_5 = 21$ false
Choice 3: $x_3 = 15\ x_5 = 21$ false
Choice 4: $x_3 = 3\ x_5 = 21$ false
Choice 5: $x_3 = 15\ x_5 = 27$ true

Question 4 Experience: 15 Order: Level: Question-ID: 31

A sequence is defined by $u_n = an - b$, find the sum of the first four terms in terms of a and b.

Solution 4

$$u_1 + u_2 + u_3 + u_4 = (a - b) + (2a - b) + (3a - b) + (4a - b) = 10a - 4b$$

Choice 1: $10a - 6b$ false
Choice 2: $6a - 4b$ false
Choice 3: $6a - 6b$ false
Choice 4: $10a - 8b$ false
Choice 5: $10a - 4b$ true

Question 5 Experience: 15 Order: Level: Question-ID: 32

A sequence is defined by $x_n = an^2 - 4$, find the sum of the first three terms in terms of a.

Solution 5

$$x_1 + x_2 + x_3 = (a - 4) + (4a - 4) + (9a - 4) = 14a - 12$$

Choice 1: $14a - 8$ false
Choice 2: $5a - 12$ false
Choice 3: $5a - 8$ false
Choice 4: $5a - 14$ false
Choice 5: $14a - 12$ true

Question 6 Experience: 25 Order: Level: Question-ID: 33

A sequence is defined by $y_n = an^2 + bn + c$, find the sum of the first three terms in terms of a, b and c.

Solution 6

$$y_1 + y_2 + y_3 = (a + b + c) + (4a + 2b + c) + (9a + 3b + c) = 14a + 6b + 3c$$

wrong choice

Choice 1: $14a - 6b + 3c$ false
Choice 2: $6a + 6b + 3c$ false
Choice 3: $6a + 4b + 2c$ false
Choice 4: $6a + 4b + 3c$ false
Choice 5: $14a + 6b + 3c$ true

Question 7 Experience: 10 Order: Level: Question-ID: 34

A sequence is defined by $x_n = 4n - b$, find the third term in terms of b .

Solution 7

$x_3 = 4(3) - b$

$x_3 = 12 - b$

Choice 1: $x_3 = 12 - 3b$ false
Choice 2: $x_3 = 6 - b$ false
Choice 3: $x_3 = 6 - 3b$ false
Choice 4: $x_3 = 8 - 3b$ false
Choice 5: $x_3 = 12 - b$ true

Question 8 Experience: 10 Order: Level: Question-ID: 35

A sequence is defined by $U_n = \frac{a}{n} + b$, find the fourth term in terms of a and b .

Solution 8

$$U_4 = \frac{a}{4} + b$$

Choice 1: $U_4 = \frac{a+b}{4}$ false
Choice 2: $U_4 = \frac{a}{4} + 4b$ false
Choice 3: $U_4 = \frac{a}{4} + 2b$ false
Choice 4: $U_4 = \frac{a}{4} + b$ true
Choice 5: $U_4 = \frac{a+4b}{8}$ false

Question 9 Experience: 15 Order: Level: Question-ID: 36

A sequence is defined by $y_n = \frac{a-3b}{n^2}$, find the fifth term in terms of a and b .

Solution 9

$$y_5 = \frac{a-3b}{(5)^2}$$

$$y_5 = \frac{a-3b}{25}$$

Choice 1: $y_5 = \frac{5a-3b}{25}$ false
Choice 2: $y_5 = \frac{5a-3b}{16}$ false
Choice 3: $y_5 = \frac{a-3b}{16}$ false
Choice 4: $y_5 = \frac{5a-b}{25}$ false
Choice 5: $y_5 = \frac{a-3b}{25}$ true

Question 10 Experience: 50 Order: Level: Question-ID: 37

A sequence is defined by $U_n = an + 2b$, given the Sum of the first four terms is 26 and the fifth term is 9, find the values of a and b.

Solution 10

$$S_4 = (a+2b)+(2a+2b)+(3a+2b)+(4a+2b)$$

$$S_4 = 10a+8b \qquad S_4 = 26$$

$$10a+8b = 26$$

$$5a+4b = 13 \quad (1)$$

$$U_5 = 5a+2b \qquad U_5 = 9$$

$$5a+2b = 9 \quad (2)$$

$$(1)-(2) \quad 5a+4b-(5a+2b) = 13-9$$

$$2b = 4$$

$$b = 2$$

$$\text{sub into} \quad (2) \quad 5a+2(2) = 9$$

$$5a = 5$$

$$a = 1$$

Choice 1:	$a=1$	$b=3$	false
Choice 2:	$a=2$	$b=3$	false
Choice 3:	$a=2$	$b=2$	false
Choice 4:	$a=3$	$b=2$	false
Choice 5:	$a=1$	$b=2$	true

Question 11 Experience: 25 Order: Level: Question-ID: 40

A sequence is defined by $U_{n+1} = U_n - 4, \quad U_1 = 20$, find the values of U_2, U_3 and U_4.

Solution 11

$$U_2 = U_1 - 4 = 20 - 4 = 16$$

$$U_3 = U_2 - 4 = 16 - 4 = 12$$

$$U_4 = U_3 - 4 = 12 - 4 = 8$$

Choice 1:	$U_2=16\ U_3=12\ U_4=4$	false
Choice 2:	$U_2=12\ U_3=8\ U_4=4$	false
Choice 3:	$U_2=12\ U_3=4\ U_4=0$	false
Choice 4:	$U_2=16\ U_3=4\ U_4=4$	false
Choice 5:	$U_2=16\ U_3=12\ U_4=8$	true

Question 12 Experience: 25 Order: Level: Question-ID: 41

A sequence is defined by $X_{n+1} = X_n + 5, \quad X_4 = 17$, find the values of X_1, X_2 and X_3.

Solution 12

$X_4 = X_3 + 5$

$17 = X_3 + 5$

$X_3 = 12$

$X_3 = X_2 + 5$

$12 = X_2 + 5$

$X_2 = 7$

$X_2 = X_1 + 5$

$7 = X_1 + 5$

$X_1 = 2$

Choice 1: $X_1 = 2\ X_2 = 6\ X_3 = 12$ false
Choice 2: $X_1 = 5\ X_2 = 8\ X_3 = 11$ false
Choice 3: $X_1 = 5\ X_2 = 7\ X_3 = 9$ false
Choice 4: $X_1 = 5\ X_2 = 6\ X_3 = 10$ false
Choice 5: $X_1 = 2\ X_2 = 7\ X_3 = 12$ true

Question 13 Experience: 30 Order: Level: Question-ID: 42

A sequence is defined by $a_{n+1} = (a_n)^2 - 4, \quad a_1 = 2$, find the values of a_2, a_3 and a_4.

Solution 13

$a_2 = (a_1)^2 - 4 = 4 - 4 = 0$

$a_3 = (a_2)^2 - 4 = 0 - 4 = -4$

$a_4 = (a_3)^2 - 4 = (-4)^2 - 4 = 16 - 4 = 12$

Choice 1: $a_2 = 0\ a_3 = 4\ a_4 = 12$ false
Choice 2: $a_2 = 4\ a_3 = -4\ a_4 = 8$ false
Choice 3: $a_2 = 4\ a_3 = 8\ a_4 = -8$ false
Choice 4: $a_2 = 0\ a_3 = 8\ a_4 = 12$ false
Choice 5: $a_2 = 0\ a_3 = -4\ a_4 = 12$ true

Question 14 Experience: 25 Order: Level: Question-ID: 43

A sequence is defined by $y_{n+2} = 3y_{n+1} - y_n, \quad y_1 = 3, y_2 = 2$, find the values of y_3, y_4 and y_5.

Solution 14

$y_3 = 3(y_2) - y_1 = 3(2) - 3 = 3$

$y_4 = 3(y_3) - y_2 = 3(3) - 2 = 7$

$y_5 = 3(y_4) - y_3 = 3(7) - 3 = 18$

Choice 1: $y_3 = 3\ y_4 = 5\ y_5 = 18$ false
Choice 2: $y_3 = 7\ y_4 = 4\ y_5 = 5$ false
Choice 3: $y_3 = 7\ y_4 = 8\ y_5 = 5$ false
Choice 4: $y_3 = 3\ y_4 = 7\ y_5 = 5$ false

Choice 5: $y_3 = 3\ y_4 = 7\ y_5 = 18$ true

Question 15 Experience: 15 Order: Level: Question-ID: 46

Calculate the following sum:

$$\sum_{r=2}^{5}(r-1)$$

Solution 15

$$\begin{aligned}\sum_{r=2}^{5}(r-1) &= (2-1)+(3-1)+(4-1)+(5-1)\\ &= 1+2+3+4\\ &= 10\end{aligned}$$

Choice 1: 9 false
Choice 2: 8 false
Choice 3: 11 false
Choice 4: 12 false
Choice 5: 10 true

Question 16 Experience: 15 Order: Level: Question-ID: 47

Calculate the following sum:

$$\sum_{r=4}^{8}(r^2-2r+1)$$

Solution 16

$$\begin{aligned}&\sum_{r=4}^{8}(r^2-2r+1)\\ =&\sum_{r=4}^{8}(r-1)^2\\ =&(4-1)^2+(5-1)^2+(6-1)^2+(7-1)^2+(8-1)^2\\ =&9+16+25+36+49\\ =&135\end{aligned}$$

Choice 1: 137 false
Choice 2: 128 false
Choice 3: 130 false
Choice 4: 136 false
Choice 5: 135 true

Question 17 Experience: 10 Order: Level: Question-ID: 28

A sequence is defined by $X_n = 2n - 1$, find the value of n such that $a_n = 15$.

Solution 17

$$\begin{aligned}15 &= 2n-1\\ n &= 8\end{aligned}$$

Choice 1:	$n = 6$	false
Choice 2:	$n = 7$	false
Choice 3:	$n = 3$	false
Choice 4:	$n = 9$	false
Choice 5:	$n = 8$	true

Question 18 Experience: 30 Order: Level: Question-ID: 53

Calculate the following sum:

$$\sum_{r=5}^{9} U_r \qquad U_r = 3r^2 + 4$$

Solution 18

$$\begin{aligned}
&\sum_{r=5}^{9} U_r \\
&= \sum_{r=5}^{9} 3r^2 + 4 \\
&= (3(5)^2 + 4) + (3(6)^2 + 4) + (3(7)^2 + 4) + (3(8)^2 + 4) + (3(9)^2 + 4) \\
&= 785
\end{aligned}$$

Choice 1:	795	false
Choice 2:	790	false
Choice 3:	780	false
Choice 4:	800	false
Choice 5:	785	true

Question 19 Experience: 30 Order: Level: Question-ID: 52

Calculate the following sum:

$$\sum_{r=1}^{3} a_r \qquad a_r = 4r - 1$$

Solution 19

$$\begin{aligned}
&\sum_{r=1}^{3} a_r \\
&= \sum_{r=1}^{3} 4r - 1 \\
&= (4(1) - 1) + (4(2) - 1) + (4(3) - 1) \\
&= 21
\end{aligned}$$

Choice 1:	22	false
Choice 2:	19	false
Choice 3:	20	false
Choice 4:	18	false
Choice 5:	21	true

Question 20 Experience: 45 Order: Level: Question-ID: 54

A sequence is defined by $U_{n+1} = 3(U_n - 1), U_1 = 2$, find the following sum: $\sum_{2}^{4}(U_r + 2)^2$

Solution 20

$$U_2 = 3(2-1)$$
$$U_2 = 3$$

$$U_3 = 3(3-1)$$
$$U_3 = 6$$

$$U_4 = 3(6-1)$$
$$U_4 = 15$$

$$\sum_{2}^{4}(U_r + 2)^2 = (U_2 + 2)^2 + (U_3 + 2)^2 + (U_4 + 2)^2$$
$$= (3+2)^2 + (6+2)^2 + (15+2)^2$$
$$= 5^2 + 8^2 + 17^2$$
$$= 378$$

Choice 1:	380	false
Choice 2:	377	false
Choice 3:	379	false
Choice 4:	381	false
Choice 5:	378	true

Question 21 Experience: 15 Order: Level: Question-ID: 48

Calculate the following sum:

$$\sum_{r=1}^{4}(2r + 4)$$

Solution 21

$$\sum_{r=1}^{4}(2r + 4)$$
$$= \sum_{r=1}^{4} 2(r + 2)$$
$$= 2\sum_{r=1}^{4}(r + 2)$$
$$= 2[(1+2) + (2+2) + (3+2) + (4+2)]$$
$$= 2(3 + 4 + 5 + 6)$$
$$= 36$$

Choice 1:	37	false
Choice 2:	35	false
Choice 3:	34	false
Choice 4:	33	false

Choice 5: 36 true

Question 22 Experience: 25 Order: Level: Question-ID: 50

Calculate the following sum:

$$\sum_{r=3}^{6}(r^2-1)$$

Solution 22

$$\sum_{r=3}^{6}(r^2-1)$$

$$= (3^2-1)+(4^2-1)+(5^2-1)+(6^2-1)$$

$$= 8+15+24+35$$

$$= 82$$

Choice 1: 81 false
Choice 2: 80 false
Choice 3: 83 false
Choice 4: 84 false
Choice 5: 82 true

Question 23 Experience: 15 Order: Level: Question-ID: 51

Calculate the following sum:

$$\sum_{r=1}^{45}2$$

Solution 23

$$\sum_{r=1}^{45}2$$

$$= 2+2+2+2+2+...+2$$

$$= 2\,\mathrm{x}\,45$$

$$= 90$$

Choice 1: 94 false
Choice 2: 92 false
Choice 3: 88 false
Choice 4: 86 false
Choice 5: 90 true

Question 24 Experience: 15 Order: Level: Question-ID: 49

Calculate the following sum:

$$\sum_{r=1}^{100}5$$

Solution 24

$$\sum_{r=1}^{100} 5$$

$= 5 + 5 + 5 + 5 + 5 + 5 + ... + 5$

$= 5 \,\text{x}\, 100$

$= 500$

Choice 1:	495	false
Choice 2:	490	false
Choice 3:	480	false
Choice 4:	500	true
Choice 5:	485	false

Question 25 Experience: 25 Order: Level: Question-ID: 25

A sequence is defined by $U_n = 2n + 3$, find the value of U_2, U_4 and U_5.

Solution 25

$U_2 = 2(2) + 3 = 7$

$U_4 = 2(4) + 3 = 11$

$U_5 = 2(5) + 3 = 13$

Choice 1:	$U_2 = 5\ U_4 = 9\ U_5 = 11$	false
Choice 2:	$U_2 = 7\ U_4 = 9\ U_5 = 13$	false
Choice 3:	$U_2 = 7\ U_4 = 10\ U_5 = 15$	false
Choice 4:	$U_2 = 7\ U_4 = 11\ U_5 = 13$	true
Choice 5:	$U_2 = 5\ U_4 = 11\ U_5 = 13$	false

Question 26 Experience: 25 Order: Level: Question-ID: 29

A sequence is defined by $u_n = 2n^2 - 5n - 3$, find the value of n such that $u_n = 9$.

Solution 26

$u_n = 2n^2 - 5n - 3 = 9$

$2n^2 - 5n - 12 = 0 \qquad S = -5 \quad P = -24$

$\left(n + \frac{3}{2}\right)(n - 4) = 0 \qquad (3, -8) \quad \left(\frac{3}{2}, -4\right)$

$n = 4$

Choice 1:	$n = 5$	false
Choice 2:	$n = 2$	false
Choice 3:	$n = 3$	false
Choice 4:	$n = 6$	false
Choice 5:	$n = 4$	true

Question 27 Experience: 50 Order: Level: Question-ID: 39

A sequence is defined by $a_n = an^2 + b$, given the Sum of the first five terms is -5 and the sixth term is 4, find the

values of a and b.

Solution 27

$$S_5 = (a+b) + (4a+b) + (9a+b) + (16a+b)$$

$$S_5 = 30a + 5b \qquad S_5 = -5$$

$$30a + 5b = -5$$

$$6a + b = -1 \quad (1)$$

$$a_6 = 25a + b \qquad a_6 = 4$$

$$36a + b = 4 \quad (2)$$

$$(2) - (1) \quad 36a + b - (6a + b) = 4 - (-1)$$

$$30a = 5$$

$$a = \frac{1}{6}$$

$$\text{sub into} \quad (1) \quad 6\left(\frac{1}{6}\right) + b = -1$$

$$b = -2$$

Choice 1: $a = \frac{1}{6} \quad b = 2$ false
Choice 2: $a = 1 \quad b = -2$ false
Choice 3: $a = 1 \quad b = 2$ false
Choice 4: $a = 2 \quad b = \frac{1}{6}$ false
Choice 5: $a = \frac{1}{6} \quad b = -2$ true

Lesson 2 Arithmetic Sequence 1

Question 1 Experience: 30 Order: Level: Question-ID: 114

Evaluate $\sum_{r=1}^{15}(5r+2)$

Solution 1

$$\sum_{r=1}^{15}(5r+2) = 7 + 12 + 17 + 22 + ... + 77$$

$$a = 7 \quad l = 77 \quad n = 15$$

$$\sum_{r=1}^{15}(5r+2) = \frac{15}{2}(7+77)$$

$$= 630$$

Choice 1: $\sum_{r=1}^{15}(5r+2) = 625$ false
Choice 2: $\sum_{r=1}^{15}(5r+2) = 620$ false
Choice 3: $\sum_{r=1}^{15}(5r+2) = 615$ false

Choice 4: $\sum_{r=1}^{15}(5r+2) = 635$ false

Choice 5: $\sum_{r=1}^{15}(5r+2) = 630$ true

Question 2 Experience: 30 Order: Level: Question-ID: 100

How many terms are there in the arithmetic sequence 19,21,23,...,87

Solution 2

$$a = 19 \quad d = 2$$
$$U_n = a + (n-1)d$$
$$87 = 19 + (n-1)2$$
$$n - 1 = 34$$
$$n = 35$$

Choice 1: $n = 38$ false

Choice 2: $n = 37$ false

Choice 3: $n = 36$ false

Choice 4: $n = 34$ false

Choice 5: $n = 35$ true

Question 3 Experience: 30 Order: Level: Question-ID: 101

How many terms are there in the arithmetic sequence 21,26,31,...,256

Solution 3

$$a = 21 \quad d = 5$$
$$U_n = a + (n-1)d$$
$$256 = 21 + (n-1)5$$
$$n - 1 = 47$$
$$n = 48$$

Choice 1: $n = 51$ false

Choice 2: $n = 47$ false

Choice 3: $n = 50$ false

Choice 4: $n = 49$ false

Choice 5: $n = 48$ true

Question 4 Experience: 35 Order: Level: Question-ID: 102

How many terms are there in the arithmetic sequence 88,86,84,...,22

Solution 4

Reverse the order of the sequence 22,24,26,28...88

$$a = 88 \quad d = 2$$

Choice 1: $U_n = a + (n-1)d$ false

Choice 2: $n = 35$ false

Choice 3: $88 = 22 + (n-1)2$ false

$n - 1 = 33$

$n = 34$

Choice 4: $n = 33$ false
Choice 5: $n = 34$ true

Question 5 Experience: 30 Order: Level: Question-ID: 103

Evaluate $S = 1 + 2 + 3 + 4 + ... + 50$

Solution 5

$$S = 1 + 2 + 3 + 4 + ... + 50$$

$$S = 50 + 49 + 48 + 47 + ... + 1$$

$$2S = 51 \text{ x } 50$$

$$S = \frac{51 \text{ x } 50}{2}$$

$$S = 1275$$

Choice 1: $S = 1270$ false
Choice 2: $S = 1280$ false
Choice 3: $S = 1285$ false
Choice 4: $S = 1290$ false
Choice 5: $S = 1275$ true

Question 6 Experience: 30 Order: Level: Question-ID: 104

Evaluate $T = 2 + 4 + 6 + 8 + ... + 100$

Solution 6

$$T = 2 + 4 + 6 + 8 + ... + 100$$

$$T = 100 + 98 + 96 + 94 + ... + 2$$

$$2T = 102 \text{ x } 50$$

$$T = \frac{102 \text{ x } 50}{2}$$

$$T = 2550$$

Choice 1: $T = 2565$ false
Choice 2: $T = 2560$ false
Choice 3: $T = 2555$ false
Choice 4: $T = 2545$ false
Choice 5: $T = 2550$ true

Question 7 Experience: 30 Order: Level: Question-ID: 105

Evaluate $R = 1 + 3 + 5 + 7 + ... + 99$

Solution 7

$$R = 1 + 3 + 5 + 7 + ... + 99$$

$$R = 99 + 97 + 95 + 93 + ... + 1$$

$$2R = 100 \text{ x } 100$$

$$R = \frac{100 \text{ x } 100}{2}$$

$$R = 5000$$

Choice 1: $R = 5015$ false
Choice 2: $R = 5010$ false
Choice 3: $R = 5005$ false
Choice 4: $R = 4995$ false
Choice 5: $R = 5000$ true

Question 8 Experience: 30 Order: Level: Question-ID: 106

Evaluate $S = 1 + 2 + 3 + 4 + ... + 200$

Solution 8

$$S = 1 + 2 + 3 + 4 + ... + 200$$

$$S = 200 + 199 + 198 + 197 + ... + 1$$

$$2S = 201 \text{ x } 200$$

$$S = \frac{201 \text{ x } 200}{2}$$

$$S = 20100$$

Choice 1: $S = 20115$ false
Choice 2: $S = 20110$ false
Choice 3: $S = 20105$ false
Choice 4: $S = 20095$ false
Choice 5: $S = 20100$ true

Question 9 Experience: 30 Order: Level: Question-ID: 107

Evaluate $T = 102 + 104 + 106 + 108 + ... + 200$

Solution 9

$$T = 102 + 104 + 106 + 108 + ... + 200$$

$$T = 200 + 198 + 196 + 194 + ... + 102$$

$$2T = 302 \text{ x } 50$$

$$T = \frac{302 \text{ x } 50}{2}$$

$$T = 7550$$

Choice 1: $T = 7565$ false
Choice 2: $T = 7560$ false
Choice 3: $T = 7555$ false
Choice 4: $T = 7545$ false
Choice 5: $T = 7550$ true

Question 10 Experience: 40 Order: Level: Question-ID: 109

Find the sum of all numbers divisible by 5 between 1 and 300

Solution 10

$300 \div 5 = 60$

$\Rightarrow$ last term $= 300$

$S = 5 + 10 + 15 + ... + 300$

$a = 5 \quad d = 5 \quad U_n = 300$

$U_n = a + (n-1)d$

$300 = 5 + (n-1)5$

$n = 60$

$S = \frac{n}{2}(a + l)$

$S = \frac{60}{2}(5 + 300)$

$S = 9150$

Choice 1: $S = 9165$ false
Choice 2: $S = 9160$ false
Choice 3: $S = 9155$ false
Choice 4: $S = 9145$ false
Choice 5: $S = 9150$ true

Question 11 Experience: 40 Order: Level: Question-ID: 110

Find the sum of all numbers divisible by 7 between 1 and 200

Solution 11

$200 \div 7 = 28$ remainder 4

$\Rightarrow$ last term $= 7$ x $28 = 196$

$S = 7 + 14 + 21 + ... + 196$

$a = 7 \quad d = 7 \quad U_n = 196$

$U_n = a + (n-1)d$

$196 = 7 + (n-1)7$

$n = 28$

$S = \frac{n}{2}(a + l)$

$S = \frac{28}{2}(7 + 196)$

$S = 2842$

Choice 1: $S = 2863$ false
Choice 2: $S = 2856$ false
Choice 3: $S = 2849$ false
Choice 4: $S = 2835$ false
Choice 5: $S = 2842$ true

Question 12 Experience: 45 Order: Level: Question-ID: 111

Evaluate $S = 27 + 31 + 35 + 39 + ... + 107$

Solution 12

$S = 27 + 31 + 35 + 39 + ... + 107$

$a = 27 \quad d = 4 \quad U_n = 107$

$U_n = a + (n-1)d$

$107 = 27 + (n-1)4$

$n = 21$

$S = \frac{n}{2}(a + l)$

$S = \frac{21}{2}(27 + 107)$

Choice 1: $S = 1414$ false
Choice 2: $S = 1386$ false

Choice 3: $S = 1393$ false
Choice 4: $S = 1400$ false
Choice 5: $S = 1407$ true

Question 13 Experience: 45 Order: Level: Question-ID: 112

Evaluate $T = 31 + 33 + 35 + 37 + ... + 81$

Solution 13

$$T = 31 + 33 + 35 + 37 + ... + 81$$

$$a = 31 \quad d = 2 \quad U_n = 81$$

$$U_n = a + (n-1)d$$

$$81 = 31 + (n-1)2$$

$$n = 26$$

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{26}{2}(31 + 81)$$

$$S = 1456$$

Choice 1: $S = 1462$ false
Choice 2: $S = 1460$ false
Choice 3: $S = 1458$ false
Choice 4: $S = 1454$ false
Choice 5: $S = 1456$ true

Question 14 Experience: 45 Order: Level: Question-ID: 113

Evaluate $R = 97 + 92 + 87 + 82 + ... + 22$

Solution 14

Reverse the sequence: $R = 22 + 27 + 32 + 27 + ... + 97$

$$R = 22 + 27 + 32 + 27 + ... + 97$$

$$a = 22 \quad d = 5 \quad U_n = 97$$

$$U_n = a + (n-1)d$$

$$97 = 22 + (n-1)5$$

$$n = 16$$

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{16}{2}(22 + 97)$$

$$S = 952$$

Choice 1: $S = 950$ false
Choice 2: $S = 954$ false
Choice 3: $S = 948$ false
Choice 4: $S = 950$ false
Choice 5: $S = 952$ true

Question 15 Experience: 30 Order: Level: Question-ID: 115

Evaluate $\sum_{r=9}^{35}(3r - 1)$

Solution 15

$$\sum_{r=9}^{35}(3r-1) = 26 + 29 + 32 + 35 + ... + 104$$

$$a = 26 \quad l = 104 \quad n = 27$$

$$\sum_{r=9}^{35}(3r-1) = \frac{27}{2}(26 + 104)$$

$$= 1755$$

Choice 1: $\sum_{r=9}^{35}(3r-1) = 1760$ false

Choice 2: $\sum_{r=9}^{35}(3r-1) = 1740$ false

Choice 3: $\sum_{r=9}^{35}(3r-1) = 1745$ false

Choice 4: $\sum_{r=9}^{35}(3r-1) = 1750$ false

Choice 5: $\sum_{r=9}^{35}(3r-1) = 1755$ true

Question 16 Experience: 30 Order: Level: Question-ID: 116

Evaluate $\sum_{r=1}^{20}(3r-1)$

Solution 16

$$\sum_{r=1}^{20}(3r-1) = 2 + 5 + 8 + 11 + ... + 59$$

$$a = 2 \quad l = 59 \quad n = 20$$

$$\sum_{r=1}^{20}(3r-1) = \frac{20}{2}(2 + 59)$$

$$= 610$$

Choice 1: $\sum_{r=1}^{20}(3r-1) = 625$ false

Choice 2: $\sum_{r=1}^{20}(3r-1) = 605$ false

Choice 3: $\sum_{r=1}^{20}(3r-1) = 615$ false

Choice 4: $\sum_{r=1}^{20}(3r-1) = 620$ false

Choice 5: $\sum_{r=1}^{20}(3r-1) = 610$ true

Question 17 Experience: 30 Order: Level: Question-ID: 117

Evaluate $\sum_{r=21}^{45}(2r-25)$

Solution 17

$$\sum_{r=21}^{45}(2r-25) = 17+19+21+23+...+65$$

$$a = 17 \quad l = 65 \quad n = 25$$

$$\sum_{r=21}^{45}(2r-25) = \frac{25}{2}(17+65)$$

$$= 1025$$

Choice 1: $\sum_{r=21}^{45}(2r-25) = 1020$ false

Choice 2: $\sum_{r=21}^{45}(2r-25) = 1015$ false

Choice 3: $\sum_{r=21}^{45}(2r-25) = 1010$ false

Choice 4: $\sum_{r=21}^{45}(2r-25) = 1030$ false

Choice 5: $\sum_{r=21}^{45}(2r-25) = 1025$ true

Question 18 Experience: 20 Order: Level: Question-ID: 84

The first three terms of an arithmetic sequence are 3,5,7, find U_{10}

Solution 18

$$a = 3 \quad n = 10 \quad d = 5-3 = 2$$

$$U_n = a + (n-1)d$$

$$U_{10} = 3 + (10-1)2 = 21$$

Choice 1: $U_{10} = 20$ false

Choice 2: $U_{10} = 17$ false

Choice 3: $U_{10} = 18$ false

Choice 4: $U_{10} = 19$ false

Choice 5: $U_{10} = 21$ true

Question 19 Experience: 20 Order: Level: Question-ID: 85

The first four terms of an arithmetic sequence are 5,9,13,17, find A_7

Solution 19

$$a = 5 \quad n = 7 \quad d = 9-5 = 4$$

$$A_n = a + (n-1)d$$

$$A_7 = 5 + (7-1)4 = 29$$

Choice 1: $A_7 = 28$ false

Choice 2: $A_7 = 27$ false

Choice 3: $A_7 = 30$ false

Choice 4: $A_7 = 26$ false

Choice 5: $A_7 = 29$ true

Question 20 Experience: 20 Order: Level: Question-ID: 89

The first three terms of an arithmetic sequence are 22,19,16, find X_6

Solution 20

$$a = 22 \quad n = 6 \quad d = 22 - 19 = 3$$

$$X_n = a + (n-1)d$$

$$X_7 = 22 + (6-1)3 = 37$$

Choice 1: $A_7 = 28$ false
Choice 2: $A_7 = 27$ false
Choice 3: $A_7 = 30$ false
Choice 4: $A_7 = 26$ false
Choice 5: $A_7 = 29$ true

Question 21 Experience: 40 Order: Level: Question-ID: 90

a_n is an arithmetic sequence, given that $a_3 = 13$ and $a_6 = 19$, find a_{11}

Solution 21

$$a_n = a + (n-1)d$$

$$a_3 = a + (3-1)d = a + 2d = 13 \quad (1)$$

$$a_6 = a + (6-1)d = a + 5d = 19 \quad (2)$$

$$(2) - (1) \quad a + 5d - (a + 2d) = 19 - 13$$

$$3d = 6$$

$$d = 2$$

$$\text{Sub into(1)} \quad a + 2(2) = 13$$

$$a = 9$$

$$a_{11} = 9 + (11-1)2 = 29$$

Choice 1: $a_{11} = 25$ false
Choice 2: $a_{11} = 26$ false
Choice 3: $a_{11} = 27$ false
Choice 4: $a_{11} = 28$ false
Choice 5: $a_{11} = 29$ true

Question 22 Experience: 40 Order: Level: Question-ID: 91

U_n is an arithmetic sequence, given that $U_4 = 25$ and $U_9 = 40$, find U_{13}

Solution 22

$$U_n = a + (n-1)d$$

$$U_4 = a + (4-1)d = a + 3d = 25 \quad (1)$$

$$U_9 = a + (9-1)d = a + 8d = 40 \quad (2)$$

$(2) - (1)$ $a + 8d - (a + 3d) = 40 - 25$

$$5d = 15$$

$$d = 3$$

Sub into(1) $a + 3(3) = 25$

$$a = 16$$

$U_{13} = 16 + (13-1)3 = 52$

Choice 1: $U_{13} = 51$ false
Choice 2: $U_{13} = 50$ false
Choice 3: $U_{13} = 49$ false
Choice 4: $U_{13} = 53$ false
Choice 5: $U_{13} = 52$ true

Question 23 Experience: 45 Order: Level: Question-ID: 96

X_n is an arithmetic sequence, given that $X_{13} = 51$ and $X_{19} = 33$, find X_{10}

Solution 23

$$X_n = a + (n-1)d$$

$$X_{13} = a + (13-1)d = a + 12d = 51 \quad (1)$$

$$X_{19} = a + (19-1)d = a + 18d = 33 \quad (2)$$

$(2) - (1)$ $a + 18d - (a + 12d) = 33 - 51$

$$6d = -18$$

$$d = -3$$

Sub into(1) $a + 12(-3) = 51$

$$a = 87$$

$X_{10} = 87 + (10-1)(-3) = 60$

Choice 1: $X_{10} = 63$ false
Choice 2: $X_{10} = 62$ false
Choice 3: $X_{10} = 61$ false
Choice 4: $X_{10} = 59$ false
Choice 5: $X_{10} = 60$ true

Question 24 Experience: 45 Order: Level: Question-ID: 97

u_n is an arithmetic sequence, given that $u_3 = 5$ and $u_7 = 13$, for what value of n is $a_n = 71$

Solution 24

$$u_n = a + (n-1)d$$

$$u_3 = a + (3-1)d = a + 2d = 5 \quad (1)$$

$$u_7 = a + (7-1)d = a + 6d = 13 \quad (2)$$

$$(2) - (1) \quad a + 6d - (a + 2d) = 13 - 5$$

$$4d = 8$$

$$d = 2$$

$$\text{Sub into(1)} \quad a + 2(2) = 5$$

$$a = 1$$

$$u_n = 1 + (n-1)2 = 71$$

$$n - 1 = 35$$

$$n = 36$$

Choice 1:	$n = 35$	false
Choice 2:	$n = 32$	false
Choice 3:	$n = 33$	false
Choice 4:	$n = 34$	false
Choice 5:	$n = 36$	true

Question 25 Experience: 30 Order: Level: Question-ID: 98

The first three terms of an arithmetic sequence are 11,14,17, find a n for which $U_n = 83$

Solution 25

$$u_n = 83 \quad a = 11 \quad d = 3$$

$$u_n = a + (n-1)d$$

$$83 = 11 + (n-1)3$$

$$n - 1 = 24$$

$$n = 25$$

Choice 1:	$n = 24$	false
Choice 2:	$n = 23$	false
Choice 3:	$n = 22$	false
Choice 4:	$n = 26$	false
Choice 5:	$n = 25$	true

Question 26 Experience: 45 Order: Level: Question-ID: 99

Y_n is an arithmetic sequence, given that $Y_{15} = 51$ and $X_{19} = 71$, find Y_{26}

Solution 26

$$Y_n = a + (n-1)d$$

$$Y_{15} = a + (15-1)d = a + 14d = 51 \quad (1)$$

$$Y_{19} = a + (19-1)d = a + 18d = 71 \quad (2)$$

$(2) - (1) \quad a + 18d - (a + 14d) = 71 - 51$

$$4d = 20$$

$$d = 5$$

Sub into(1) $\quad a + 14(5) = 51$

$$a = -19$$

$$Y_{26} = -19 + (26-1)5 = 106$$

Choice 1:	$Y_{26} = 102$	false
Choice 2:	$Y_{26} = 103$	false
Choice 3:	$Y_{26} = 104$	false
Choice 4:	$Y_{26} = 105$	false
Choice 5:	$Y_{26} = 106$	true

Question 27 Experience: 50 Order: Level: Question-ID: 147

Kendrick decides to open up a savings account. He puts in 100 for the first month, 120 for the second month and an extra 20 for subsequent months till he's putting in 300 a month. Find the total amount he's saved in 2 years.

Solution 27

Sequence goes: 100,120,140,160,180,200...300,300,300,300...

$$U_n = a + (n-1)d$$

$$U_n = 300 \quad a = 100 \quad d = 20$$

$$300 = 100 + (n-1)20$$

$$n = 11$$

$$S_n = \frac{n}{2}(a + l)$$

$$n = 11 \quad a = 100 \quad l = 300$$

$$S_{11} = \frac{11}{2}(100 + 300)$$

$$S_{11} = 2200$$

Every term after is 300

$$\sum_{r=12}^{24} 300 = 13 \text{ x } 300$$

$$= 3900$$

$\Rightarrow$ Total days $= 2200 + 3900 = 6100$

Choice 1:	6105	false
Choice 2:	6085	false

Choice 3:	6090	false
Choice 4:	6095	false
Choice 5:	6100	true

Question 28 Experience: 50 Order: Level: Question-ID: 144

Avery is playing with 340 sticks, she puts them in rows. The first row has 7 sticks, next row has 13 sticks, subsequent rows have 6 more sticks then the previous row. She has enough for k rows but not enough for $k+1$ rows. Find k.

Solution 28

Sequence goes: 7,13,19,25,31,37....

Not having enough for k+1 rows means that $S_k \leq 340$

$$S_n = \frac{n}{2}(2a + (k-1)d)$$

$$S_k = \frac{k}{2}(2(7) + (k-1)6)$$

$$S_k = k(7 + 3(k-1))$$

$$S_k = k(3k+4)$$

$$S_k = 3k^2 + 4k \qquad (1)$$

$$S_k \leq 340$$

$$(1) \quad 3k^2 + 4k \leq 340$$

$$3k^2 + 4k - 340 \leq 0 \qquad P = -1020 \quad S = 4$$

$$\left(k + \frac{34}{3}\right)(k - 10) \leq 0 \qquad (34, -30) \qquad \left(\frac{34}{3}, -10\right)$$

$$k = 10$$

Choice 1:	$k = 9$	false
Choice 2:	$k = 11$	false
Choice 3:	$k = 7$	false
Choice 4:	$k = 8$	false
Choice 5:	$k = 10$	true

Question 29 Experience: 50 Order: Level: Question-ID: 146

Griffin is training daily for a cycling marathon in 100 days. He cycles 10km on the first day, 11km on the second day and 1 more km then the previous day till he's cycling 40km a day. Calculate the total number of km he's cycled as training for the marathon.

Solution 29

Sequence goes: 10,11,12,13,14,15...40,40,40,40...

$$U_n = a + (n-1)d$$
$$U_n = 40 \quad a = 10 \quad d = 1$$
$$40 = 10 + (n-1)1$$
$$n = 31$$

$$S_n = \frac{n}{2}(a + l)$$
$$n = 31 \quad a = 10 \quad l = 40$$
$$S_{31} = \frac{31}{2}(10 + 40)$$
$$S_{31} = 775$$

Every term after is 40

$$\sum_{r=32}^{100} 40 = 69 \text{ x } 40$$
$$= 2760$$

$\Rightarrow$ Total days $= 775 + 2760 = 3535$

Choice 1:	3540	false
Choice 2:	3520	false
Choice 3:	3525	false
Choice 4:	3530	false
Choice 5:	3535	true

Question 30 Experience: 50 Order: Level: Question-ID: 145

Heidi is training daily for a swimming competition in 60 days. She swims 10 laps on the first day, 12 laps on the second day and 2 more laps then the previous day till she's swimming 30 laps a day. Calculate the total number of laps she's swum as training for the competition.

Solution 30

Sequence goes: 10,12,14,16,18,20...30,30,30,30...

$$U_n = a + (n-1)d$$
$$U_n = 30 \quad a = 10 \quad d = 2$$
$$30 = 10 + (n-1)2$$
$$n = 11$$

$$S_n = \frac{n}{2}(a + l)$$
$$n = 11 \quad a = 10 \quad l = 30$$
$$S_{11} = \frac{11}{2}(10 + 30)$$
$$S_{11} = 220$$

Every term after is 30

$$\sum_{r=12}^{60} 30 = 49 \text{ x } 30$$
$$= 1470$$

$\Rightarrow$ Total days $= 220 + 1470 = 1690$

Choice 1:	1685	false
Choice 2:	1705	false
Choice 3:	1700	false
Choice 4:	1695	false
Choice 5:	1690	true

Question 31 Experience: 40 Order: Level: Question-ID: 108

Find the sum of all numbers divisible by 3 between 2 and 200

Solution 31

$200 \div 3 = 66$ remainder 2

$\Rightarrow$ last term $= 3 \text{ x } 66 = 198$

$$S = 3 + 6 + 9 + ... + 198$$

$$a = 3 \quad d = 3 \quad U_n = 198$$
$$U_n = a + (n-1)d$$
$$198 = 3 + (n-1)3$$
$$n = 66$$

$$S = \frac{n}{2}(a + l)$$
$$S = \frac{66}{2}(3 + 198)$$
$$S = 6633$$

Choice 1: $S = 6642$ false
Choice 2: $S = 6639$ false
Choice 3: $S = 6636$ false
Choice 4: $S = 6630$ false
Choice 5: $S = 6633$ true

Question 32 Experience: 50 Order: Level: Question-ID: 143

James is playing with 324 sticks, she puts them in rows. The first row has 5 sticks, next row has 9 sticks, subsequent rows have 4 more sticks then the previous row. She has enough for k rows but not enough for $k+1$ rows. Find k.

Solution 32

Sequence goes: 5,9,13,17,21,25....

Not having enough for k+1 rows means that $S_k \leq 324$

$$S_n = \frac{n}{2}(2a + (k-1)d)$$
$$S_k = \frac{k}{2}(2(5) + (k-1)4)$$
$$S_k = k(5 + 2k - 2)$$
$$S_k = k(2k + 3)$$
$$S_k = 2k^2 + 3k \qquad (1)$$

$$S_k \leq 324$$
$$(1) \qquad 2k^2 + 3k \leq 324$$
$$2k^2 + 3k - 324 \leq 0 \qquad P = -648 \quad S = 3$$
$$\left(k + \frac{27}{2}\right)(k - 12) \leq 0 \qquad (27, -24) \qquad \left(\frac{27}{2}, -12\right)$$
$$k = 12$$

Choice 1: $k = 11$ false
Choice 2: $k = 15$ false
Choice 3: $k = 14$ false
Choice 4: $k = 13$ false
Choice 5: $k = 12$ true

Lesson 3 Recurrence Relations

Question 1 Experience: 50 Order: Level: Question-ID: 57

A sequence is defined by the recurrence relation $X_{n+1} = \sqrt{k}X_n - 2, X_1 = 2, k > 0$, given that $X_3 = 2$ find the value of k.

Solution 1

$X_2 = \sqrt{k}X_1 - 2$

$X_2 = 2\sqrt{k} - 2$

$X_3 = \sqrt{k}X_2 - 2$

$X_3 = \sqrt{k}(2\sqrt{k} - 2) - 2$

$X_3 = 2k - 2\sqrt{k} - 2 \quad \text{set} \quad x = \sqrt{k}$

$X_3 = 2x^2 - 2x - 2 \quad X_3 = 2$

$2 = 2x^2 - 2x - 2$

$1 = x^2 - x - 1$

$0 = x^2 - x - 2 \qquad S = -1 \quad P = -2$

$0 = (x-2)(x+1) \qquad (-2, 1)$

$\sqrt{k} = 2$

$k = 4$

Choice 1:	$k = 5$	false
Choice 2:	$k = 3$	false
Choice 3:	$k = 6$	false
Choice 4:	$k = 7$	false
Choice 5:	$k = 4$	true

Question 2 Experience: 50 Order: Level: Question-ID: 58

A sequence is defined by the recurrence relation $U_{n+1} = aU_n + \frac{1}{b}, U_1 = 3$, given that $U_2 = 7$ and $U_3 = 15$ find the value of a and b.

Solution 2

$$U_2 = aU_1 + \frac{1}{b} \quad U_2 = 7$$

$$7 = 3a + \frac{1}{b} \quad (1)$$

$$U_3 = aU_2 + \frac{1}{b} \quad U_2 = 7, U_3 = 15$$

$$15 = 7a + \frac{1}{b} \quad (2)$$

$(2) - (1) \quad 15 - 7 = 7a + \frac{1}{b} - \left(3a + \frac{1}{b}\right)$

$8 = 4a$

$a = 2$

Sub into (1) $\quad 7 = 3(2) + \frac{1}{b}$

$\frac{1}{b} = 1$

$b = 1$

Choice 1: $a = 2 \quad b = 3$ false
Choice 2: $a = 3 \quad b = 3$ false
Choice 3: $a = 3 \quad b = 1$ false
Choice 4: $a = 1 \quad b = 1$ false
Choice 5: $a = 2 \quad b = 1$ true

Question 3 Experience: 70 Order: Level: Question-ID: 63

A sequence is defined by the recurrence relation $a_{n+1} = ka_n - 4, k > 0, a_1 = 5$, given that $\sum_{r=1}^{3} a_r = 19$, find the value of k.

Solution 3

$$a_2 = ka_1 - 4$$
$$a_2 = 5k - 4$$

$$a_3 = ka_2 - 4$$
$$a_3 = k(5k - 4) - 4$$
$$a_3 = 5k^2 - 4k - 4$$

$$\sum_{r=1}^{3} a_r = a_1 + a_2 + a_3$$
$$\sum_{r=1}^{3} a_r = (5) + (5k - 4) + (5k^2 - 4k - 4)$$
$$\sum_{r=1}^{3} a_r = 5k^2 + k - 3 \qquad \sum_{r=1}^{3} a_r = 19$$
$$19 = 5k^2 + k - 3$$
$$0 = 5k^2 + k - 22 \qquad S = 1 \quad P = -110$$
$$0 = \left(k + \frac{11}{5}\right)(k - 2) \qquad (11, -10) \quad \Rightarrow \quad \left(\frac{11}{5}, -2\right)$$
$$k = 2$$

Choice 1: $k = 3$ false
Choice 2: $k = 4$ false
Choice 3: $k = 1$ false
Choice 4: $k = 5$ false
Choice 5: $k = 2$ true

Question 4 Experience: 60 Order: Level: Question-ID: 64

A sequence is defined by the recurrence relation $U_{n+1} = 5U_n - \frac{1}{k}, k > 0, U_1 = 2$, given that $\sum_{r=1}^{4} U_r = 293$, find the value of k.

Solution 4

$$U_2 = 5U_1 - \frac{1}{k}$$
$$U_2 = 5(2) - \frac{1}{k}$$
$$U_2 = 10 - \frac{1}{k}$$

$$U_3 = 5U_2 - \frac{1}{k}$$
$$U_3 = 5\left(10 - \frac{1}{k}\right) - \frac{1}{k}$$
$$U_3 = 50 - \frac{6}{k}$$

$$U_4 = 5U_3 - \frac{1}{k}$$
$$U_4 = 5\left(50 - \frac{6}{k}\right) - \frac{1}{k}$$
$$U_4 = 250 - \frac{31}{k}$$

$$\sum_{r=1}^{4} U_r = U_1 + U_2 + U_3 + U_4$$
$$\sum_{r=1}^{4} U_r = (2) + \left(10 - \frac{1}{k}\right) + \left(50 - \frac{6}{k}\right) + \left(250 - \frac{31}{k}\right)$$
$$\sum_{r=1}^{4} U_r = 312 - \frac{38}{k} \quad \sum_{r=1}^{4} U_r = 293$$
$$312 - \frac{38}{k} = 293$$
$$19 = \frac{38}{k}$$
$$k = 2$$

Choice 1: $k = 5$ false
Choice 2: $k = 4$ false
Choice 3: $k = 3$ false
Choice 4: $k = 1$ false
Choice 5: $k = 2$ false

Question 5 Experience: 100 Order: Level: Question-ID: 65

A sequence is defined by the recurrence relation $X_{n+1} = \frac{k}{X_n} + 3, X_1 = 1$, given that $2\sum_{r=1}^{3} X_r = 21$, find the value of k.

Solution 5

$$X_2 = \frac{k}{X_1} + 3$$
$$X_2 = \frac{k}{1} + 3$$
$$X_2 = k + 3$$

$$X_3 = \frac{k}{X_2} + 3$$
$$X_3 = \frac{k}{k+3} + 3$$

$$\sum_{r=1}^{3} X_r = X_1 + X_2 + X_3$$
$$\sum_{r=1}^{3} X_r = (1) + (k+3) + \left(\frac{k}{k+3} + 3\right)$$
$$\sum_{r=1}^{3} X_r = k + 7 + \frac{k}{k+3} \quad 2\sum_{r=1}^{3} X_r = 21$$
$$21 = 2\left(k + 7 + \frac{k}{k+3}\right)$$
$$21 = 2k + 14 + \frac{2k}{k+3}$$
$$7 = 2k + \frac{2k}{k+3}$$
$$7(k+3) = 2k(k+3) + 2k$$
$$7k + 21 = 2k^2 + 6k + 2k$$
$$0 = 2k^2 - k - 21 \qquad S = -1 \quad P = -42$$
$$0 = \left(k + \frac{7}{2}\right)(k-3) \quad (7, -6) \quad \Rightarrow \quad \left(\frac{7}{2}, -3\right)$$
$$k = 3$$

Choice 1:	$k = 5$	false
Choice 2:	$k = 2$	false
Choice 3:	$k = 4$	false
Choice 4:	$k = 1$	false
Choice 5:	$k = 3$	true

Question 6 Experience: 35 Order: Level: Question-ID: 66

A sequence is defined by the recurrence relation $a_{n+1} = a_n^2 - a_n$, given that a_n is a positive sequence and that $a_3 = 132$ find the value of a_1.

Solution 6

$$a_3 = a_2^2 - a_2$$
$$132 = a_2^2 - a_2$$
$$0 = a_2^2 - a_2 - 132 \qquad S = 1 \quad P = -132$$
$$0 = (a_2 + 11)(a_2 - 12) \qquad (11, -12)$$
$$a_2 = 12$$

$$a_2 = a_1^2 - a_1$$
$$12 = a_1^2 - a_1$$
$$0 = a_1^2 - a_1 - 12$$
$$0 = (a_1 - 4)(a_1 + 3)$$
$$a_1 = 4$$

Choice 1: $a_1 = 6$ false
Choice 2: $a_1 = 5$ false
Choice 3: $a_1 = 3$ false
Choice 4: $a_1 = 7$ false
Choice 5: $a_1 = 4$ true

Question 7 Experience: 35 Order: Level: Question-ID: 67

A sequence is defined by the recurrence relation $U_{n+1} = 5U_n - \frac{6}{U_n}$, given that $U_3 = 13, U_2 > 0$, find the value of U_2.

Solution 7

$$U_3 = 5U_2 - \frac{6}{U_2}$$
$$13 = 5U_2 - \frac{6}{U_2}$$
$$0 = 5U_2 - 13 - \frac{6}{U_2}$$
$$0 = 5(U_2)^2 - 13U_2 - 6 \qquad S = -13 \quad P = -30$$
$$0 = \left(U_2 + \frac{2}{5}\right)(U_2 - 3) \qquad (2, -15) \quad \left(\frac{2}{5}, -3\right)$$
$$U_2 = 3$$

Choice 1: $U_2 = 4$ false
Choice 2: $U_2 = 5$ false
Choice 3: $U_2 = 2$ false
Choice 4: $U_2 = 1$ false
Choice 5: $U_2 = 3$ true

Question 8 Experience: 15 Order: Level: Question-ID: 68

A sequence is defined by the recurrence relation $Y_{n+1} = 3Y_n - 5$, given that $Y_3 = 7$, find the value of Y_1.

Solution 8

$$Y_3 = 3Y_2 - 5$$
$$7 = 3Y_2 - 5$$
$$Y_2 = 4$$

$$Y_2 = 3Y_1 - 5$$
$$4 = 3Y_1 - 5$$
$$Y_1 = 3$$

Choice 1:	$Y_1 = 5$	false
Choice 2:	$Y_1 = 4$	false
Choice 3:	$Y_1 = 1$	false
Choice 4:	$Y_1 = 2$	false
Choice 5:	$Y_1 = 3$	true

Question 9 Experience: 40 Order: Level: Question-ID: 69

A sequence is defined by the recurrence relation $a_{n+1} = a_n - \dfrac{2a_n + 6}{a_n + 3}$, given that $a_2 = 5$, find the value of a_1.

Solution 9

$$a_2 = a_1 - \frac{2a_1 + 6}{a_1 + 3}$$
$$5 = a_1 - \frac{2a_1 + 6}{a_1 + 3}$$
$$5(a_1 + 3) = a_1(a_1 + 3) - (2a_1 + 6)$$
$$5a_1 + 15 = (a_1)^2 + 3a_1 - 2a_1 - 6$$
$$0 = (a_1)^2 - 4a_1 - 21 \qquad S = -4 \quad P = -21$$
$$0 = (a_1 + 3)(a_1 - 7) \qquad (3, -7)$$
$$a_1 = 7$$

Choice 1:	$a_1 = 8$	false
Choice 2:	$a_1 = 4$	false
Choice 3:	$a_1 = 5$	false
Choice 4:	$a_1 = 6$	false
Choice 5:	$a_1 = 7$	true

Question 10 Experience: 25 Order: Level: Question-ID: 70

A sequence is defined by the recurrence relation $X_{n+1} = 3(X_n)^2 - 11$, given that $X_1 = 2$, find $\displaystyle\sum_{r=1}^{4} X_r$.

Solution 10

$$X_2 = 3(X_1)^2 - 11$$
$$X_2 = 3(2)^2 - 11$$
$$X_2 = 1$$

$$X_3 = 3(X_2)^2 - 11$$
$$X_3 = 3(1)^2 - 11$$
$$X_3 = -8$$

$$X_4 = 3(X_3)^2 - 11$$
$$X_4 = 3(-8)^2 - 11$$
$$X_4 = 181$$

$$\sum_{r=1}^{4} X_r = X_1 + X_2 + X_3 + X_4$$
$$\sum_{r=1}^{4} X_r = (2) + (1) + (-8) + (181)$$
$$\sum_{r=1}^{4} X_r = 176$$

Choice 1: $\sum_{r=1}^{4} X_r = 173$ false

Choice 2: $\sum_{r=1}^{4} X_r = 170$ false

Choice 3: $\sum_{r=1}^{4} X_r = 177$ false

Choice 4: $\sum_{r=1}^{4} X_r = 172$ false

Choice 5: $\sum_{r=1}^{4} X_r = 176$ true

Question 11 Experience: 25 Order: Level: Question-ID: 71

A sequence is defined by the recurrence relation $U_{n+2} = 3U_{n+1} - U_n + 5$, given that $U_1 = 4, U_2 = 2$, find $\sum_{r=1}^{4} U_r$.

Solution 11

$$U_3 = 3U_2 - U_1 + 5$$
$$U_3 = 3(2) - (4) + 5$$
$$U_3 = 7$$

$$U_4 = 3U_3 - U_2 + 5$$
$$U_4 = 3(7) - (2) + 5$$
$$U_4 = 24$$

$$\sum_{r=1}^{4} U_r = U_1 + U_2 + U_3 + U_4$$
$$\sum_{r=1}^{4} U_r = 4 + 2 + 7 + 24$$
$$\sum_{r=1}^{4} U_r = 37$$

Choice 1: $\sum_{r=1}^{4} U_r = 36$ false

Choice 2: $\sum_{r=1}^{4} U_r = 35$ false

Choice 3: $\sum_{r=1}^{4} U_r = 38$ false

Choice 4: $\sum_{r=1}^{4} U_r = 34$ false

Choice 5: $\sum_{r=1}^{4} U_r = 37$ true

Question 12 Experience: 25 Order: Level: Question-ID: 72

A sequence is defined by the recurrence relation $Y_{n+1} = 21 - 2Y_n$, given that $Y_1 = 5$, find $\sum_{r=2}^{4} Y_r$.

Solution 12

$$Y_2 = 21 - 2Y_1$$
$$Y_2 = 21 - 2(5)$$
$$Y_2 = 11$$

$$Y_3 = 21 - 2Y_2$$
$$Y_3 = 21 - 2(11)$$
$$Y_3 = -1$$

$$Y_4 = 21 - 2Y_3$$
$$Y_4 = 21 - 2(-1)$$
$$Y_4 = 23$$

$$\sum_{r=2}^{4} Y_r = Y_2 + Y_3 + Y_4$$
$$\sum_{r=2}^{4} Y_r = 11 + (-1) + 23$$
$$\sum_{r=2}^{4} Y_r = 33$$

Choice 1: $\sum_{r=2}^{4} Y_r = 32$ false

Choice 2: $\sum_{r=2}^{4} Y_r = 31$ false

Choice 3: $\sum_{r=2}^{4} Y_r = 30$ false

Choice 4: $\sum_{r=2}^{4} Y_r = 34$ false

Choice 5: $\sum_{r=2}^{4} Y_r = 33$ true

Question 13 Experience: 30 Order: Level: Question-ID: 74

A sequence is defined by the recurrence relation $X_{n+1} = 5 - X_n$, given that $X_1 = 7$, find $\sum_{r=1}^{20} X_r$.

Solution 13

$X_2 = 5 - X_1 = 5 - 7 = -2$

$X_3 = 5 - X_2 = 5 - (-2) = 7$

$X_4 = 5 - X_3 = 5 - 7 = -2$

$X_5 = 5 - X_4 = 5 - (-2) = 7$

$$\sum_{r=1}^{20} X_r = X_1 + X_2 + X_3 + X_4 + ... + X_{20}$$

$$\sum_{r=1}^{20} X_r = -2 + 7 + -2 + 7 + -2 + ... + 7$$

$$\sum_{r=1}^{20} X_r = 10(-2) + 10(7)$$

$$\sum_{r=1}^{20} X_r = 50$$

Choice 1: $\sum_{r=1}^{20} X_r = 20$ false

Choice 2: $\sum_{r=1}^{20} X_r = 60$ false

Choice 3: $\sum_{r=1}^{20} X_r = 30$ false

Choice 4: $\sum_{r=1}^{20} X_r = 40$ false

Choice 5: $\sum_{r=1}^{20} X_r = 50$ true

Question 14 Experience: 30 Order: Level: Question-ID: 77

A sequence is defined by the recurrence relation $Y_{n+1} = 5 + 5Y_n - 2(Y_n)^3$, given that $Y_1 = 2$, find Y_{1000}.

Solution 14

$Y_2 = 5 + 5Y_1 - 2(Y_1)^3 = 5 + 5(2) - 2(2)^3 = -1$

$Y_3 = 5 + 5Y_2 - 2(Y_2)^3 = 5 + 5(-1) - 2(-1)^3 = 2$

$Y_4 = 5 + 5Y_3 - 2(Y_3)^3 = 5 + 5(2) - 2(2)^3 = -1$

Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
-1	2	-1	2	-1	2

We can see that $Y_2 = Y_4 = Y_6 = Y_8 = ... = 2$

Every numbered term divisible by 2 is 2

Find a numbered term that is close to Y_{1000} that is divisible by 2

$Y_2 = 2 \quad Y_4 = 2 \quad Y_{100} = 2 \quad Y_{1000} = 2$

Choice 1:	$Y_{1000} = 1$	false
Choice 2:	$Y_{1000} = 0$	false
Choice 3:	$Y_{1000} = 3$	false
Choice 4:	$Y_{1000} = 4$	false
Choice 5:	$Y_{1000} = 2$	true

Question 15 Experience: 15 Order: Level: Question-ID: 78

Given $\sum_{r=1}^{n} x_r = 5n^2 - 3$, find $\sum_{r=1}^{7} x_r$

Solution 15

$$\sum_{r=1}^{7} x_r = 5(7)^2 - 3 = 242$$

Choice 1:	$\sum_{r=1}^{7} x_r = 239$	false
Choice 2:	$\sum_{r=1}^{7} x_r = 240$	false
Choice 3:	$\sum_{r=1}^{7} x_r = 243$	false
Choice 4:	$\sum_{r=1}^{7} x_r = 241$	false
Choice 5:	$\sum_{r=1}^{7} x_r = 242$	true

Question 16 Experience: 40 Order: Level: Question-ID: 76

A sequence is defined by the recurrence relation $U_{n+1} = \dfrac{13 - 5U_n}{7 - 3U_n}$, given that $U_1 = 1$, find U_{50}.

Solution 16

$$U_2 = \frac{13 - 5U_1}{7 - 3U_1} = \frac{13 - 5(1)}{7 - 3(1)} = \frac{8}{4} = 2$$

$$U_3 = \frac{13 - 5U_2}{7 - 3U_2} = \frac{13 - 5(2)}{7 - 3(2)} = \frac{3}{1} = 3$$

$$U_4 = \frac{13 - 5U_3}{7 - 3U_3} = \frac{13 - 5(3)}{7 - 3(3)} = \frac{-2}{-2} = 1$$

$$U_5 = \frac{13 - 5U_4}{7 - 3U_4} = \frac{13 - 5(1)}{7 - 3(1)} = \frac{8}{4} = 2$$

U_1	U_2	U_3	U_4	U_5	U_6
1	2	3	1	2	3

We can see that $U_3 = U_6 = U_9 = U_{12} = ... = 3$

Every numbered term divisible by 3 is 3

Find a numbered term that is close to U_{50} that is divisible by 3

$U_3 = 3 \quad U_9 = 3 \quad U_{30} = 3 \quad U_{51} = 3$

$U_{51} = 3 \quad \Rightarrow \quad U_{50} = 2$ since 2 is the term before 3 in the sequence

i.e. $1, 2, 3, 1, 2, 3, 1,$

Choice 1:	$U_{50} = 1$	false
Choice 2:	$U_{50} = 3$	false
Choice 3:	$U_{50} = 4$	false
Choice 4:	$U_{50} = 5$	false
Choice 5:	$U_{50} = 2$	true

Question 17 Experience: 30 Order: Level: Question-ID: 79

Given $\sum_{r=1}^{n} a_r = 2n^3 + 5$, find a_2

Solution 17

$$\sum_{r=1}^{n} a_r = 2n^3 + 5$$

$$a_2 = \sum_{r=1}^{2} a_r - \sum_{r=1}^{1} a_r$$

$$a_2 = 2(2)^3 + 5 - (2(1)^3 + 5)$$

$$a_2 = 21 - 7$$

$$a_2 = 14$$

Choice 1:	$a_2 = 13$	false
Choice 2:	$a_2 = 12$	false
Choice 3:	$a_2 = 11$	false
Choice 4:	$a_2 = 15$	false
Choice 5:	$a_2 = 14$	true

Question 18 Experience: 15 Order: Level: Question-ID: 80

Given $\sum_{r=1}^{n} U_r = 6n^2 + 11$, find U_1

Solution 18

$$\sum_{r=1}^{1} U_r = U_1 = 6(1)^2 + 11 = 17$$

Choice 1:	$U_1 = 16$	false
Choice 2:	$U_1 = 15$	false
Choice 3:	$U_1 = 14$	false
Choice 4:	$U_1 = 18$	false
Choice 5:	$U_1 = 17$	true

Question 19 Experience: 15 Order: Level: Question-ID: 81

Given $\sum_{r=1}^{n} u_r = n^3 + 4$, find $\sum_{r=1}^{5} u_r$

Solution 19

$$\sum_{r=1}^{5} u_r = (5)^3 + 4 = 129$$

Choice 1: $\sum_{r=1}^{5} u_r = 130$ false

Choice 2: $\sum_{r=1}^{5} u_r = 126$ false

Choice 3: $\sum_{r=1}^{5} u_r = 127$ false

Choice 4: $\sum_{r=1}^{5} u_r = 128$ false

Choice 5: $\sum_{r=1}^{5} u_r = 129$ true

Question 20 Experience: 30 Order: Level: Question-ID: 82

Given $\sum_{r=1}^{n} Y_r = 3n^3 - 2$, find Y_3

Solution 20

$$Y_3 = \sum_{r=1}^{3} - \sum_{r=1}^{2}$$

$$Y_3 = 3(3)^3 - 2 - (3(2)^3 - 2)$$

$$Y_3 = 57$$

Choice 1: $Y_3 = 54$ false

Choice 2: $Y_3 = 55$ false

Choice 3: $Y_3 = 52$ false

Choice 4: $Y_3 = 56$ false

Choice 5: $Y_3 = 57$ true

Question 21 Experience: 30 Order: Level: Question-ID: 83

Given $\sum_{r=1}^{n} U_r = 3n + 7$, find U_5

Solution 21

$$U_5 = \sum_{r=1}^{5} U_r - \sum_{r=1}^{4} U_r$$

$$U_5 = 3(5) + 7 - (3(4) + 7)$$

$$U_5 = 3$$

Choice 1: $U_5 = 2$ false

Choice 2: $U_5 = 1$ false

Choice 3: $U_5 = 4$ false

Choice 4: $U_5 = 5$ false
Choice 5: $U_5 = 3$ true

Question 22 Experience: 45 Order: Level: Question-ID: 55

A sequence is defined by the recurrence relation $U_{n+1} = kU_n - 4, U_1 = 3, k > 0$, given that $U_3 = 0$ find the value of k

Solution 22

$U_2 = kU_1 - 4$

$U_2 = 3k - 4$

$U_3 = kU_2 - 4$

$U_3 = k(3k - 4) - 4$

$U_3 = 3k^2 - 4k - 4 \quad U_3 = 0$

$0 = 3k^2 - 4k - 4 \qquad S = -4 \quad P = -12$

$0 = \left(k + \frac{2}{3}\right)(k - 2) \quad (2, -6) \quad \Rightarrow \quad \left(\frac{2}{3}, -2\right)$

$k = 2$

Choice 1: $k = 3$ false
Choice 2: $k = 4$ false
Choice 3: $k = 1$ false
Choice 4: $k = 5$ false
Choice 5: $k = 2$ true

Question 23 Experience: 60 Order: Level: Question-ID: 56

A sequence is defined by the recurrence relation $a_{n+1} = \frac{a_n}{k} + 3, a_1 = 3, k > 0$, given that $a_3 = 9$ find the value of k

Solution 23

$$a_2 = \frac{a_1}{k} + 3$$
$$a_2 = \frac{3}{k} + 3$$

$$a_3 = \frac{a_2}{k} + 3$$
$$a_3 = \frac{\left(\frac{3}{k} + 3\right)}{k} + 3$$
$$a_3 = \frac{3}{k^2} + \frac{3}{k} + 3 \quad a_3 = 9$$

$$9 = \frac{3}{k^2} + \frac{3}{k} + 3$$
$$6 - \frac{3}{k^2} - \frac{3}{k} = 0$$
$$6k^2 - 3k - 3 = 0$$
$$2k^2 - k - 1 = 0 \qquad S = -1 \quad P = -2$$
$$\left(x + \frac{1}{2}\right)(k - 1) = 0 \qquad (1, -2) \quad \Rightarrow \quad \left(\frac{1}{2}, -1\right)$$
$$k = 1$$

Choice 1:	$k = 2$	false
Choice 2:	$k = 3$	false
Choice 3:	$k = 4$	false
Choice 4:	$k = 5$	false
Choice 5:	$k = 1$	true

Question 24 Experience: 60 Order: Level: Question-ID: 62

A sequence is defined by the recurrence relation $u_{n+1} = \sqrt{a}\left(u_n - \frac{1}{b}\right), 5u_1 = 4$, given that $u_2 = 7$ and $u_3 = 13$ find the value of a and b .

Solution 24

$$u_2 = \sqrt{a}\left(u_1 - \frac{1}{b}\right)$$

$$7 = \sqrt{a}\left(4 - \frac{1}{b}\right) \qquad (1)$$

$$7 = 4\sqrt{a} - \frac{\sqrt{a}}{b} \qquad (2)$$

$$u_3 = \sqrt{a}\left(u_2 - \frac{1}{b}\right)$$

$$13 = \sqrt{a}\left(7 - \frac{1}{b}\right)$$

$$13 = 7\sqrt{a} - \frac{\sqrt{a}}{b} \qquad (3)$$

(3) − (2) $13 - 7 = 7\sqrt{a} - \frac{\sqrt{a}}{b} - \left(4\sqrt{a} - \frac{\sqrt{a}}{b}\right)$

$$6 = 3\sqrt{a}$$

$$2 = \sqrt{a}$$

$$a = 4$$

Sub into (1) $7 = \sqrt{4}\left(4 - \frac{1}{b}\right)$

$$\frac{7}{2} = 4 - \frac{1}{b}$$

$$-\frac{1}{2} = -\frac{1}{b}$$

$$b = 2$$

Choice 1:	$a = 4$	$b = 2$	true
Choice 2:	$a = 3$	$b = 2$	false
Choice 3:	$a = 4$	$b = 3$	false
Choice 4:	$a = 3$	$b = 3$	false
Choice 5:	$a = 2$	$b = 3$	false

Question 25 Experience: 30 Order: Level: Question-ID: 73

A sequence is defined by the recurrence relation $a_{n+1} = 3 - a_n$, given that $a_1 = 1$, find $\sum_{r=1}^{100} a_r$.

Solution 25

$$a_2 = 3 - a_1 = 3 - 1 = 2$$

$$a_3 = 3 - a_2 = 3 - 2 = 1$$

$$a_4 = 3 - a_3 = 3 - 1 = 2$$

$$a_5 = 3 - a_4 = 3 - 2 = 1$$

$$\sum_{r=1}^{100} a_r = a_1 + a_2 + a_3 + a_4 + ... + a_{100}$$

$$\sum_{r=1}^{100} a_r = 1 + 2 + 1 + 2 + 1 + 2 + ... + 2$$

$$\sum_{r=1}^{100} a_r = 50(2) + 50(1)$$

$$\sum_{r=1}^{100} a_r = 150$$

Choice 1: $\sum_{r=1}^{100} a_r = 100$ false

Choice 2: $\sum_{r=1}^{100} a_r = 200$ false

Choice 3: $\sum_{r=1}^{100} a_r = 50$ false

Choice 4: $\sum_{r=1}^{100} a_r = 250$ false

Choice 5: $\sum_{r=1}^{100} a_r = 150$ true

Question 26 Experience: 40 Order: Level: Question-ID: 75

A sequence is defined by the recurrence relation $A_{n+1} = \dfrac{4A_n - 16}{3A_n - 8}$, given that $A_1 = 0$, find A_{100}.

Solution 26

$$A_2 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(0) - 16}{3(0) - 8} = \frac{-16}{-8} = 2$$

$$A_3 = \frac{4A_2 - 16}{3A_2 - 8} = \frac{4(2) - 16}{3(2) - 8} = \frac{-8}{-2} = 4$$

$$A_4 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(4) - 16}{3(4) - 8} = 0$$

$$A_5 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(0) - 16}{3(0) - 8} = \frac{-16}{-8} = 2$$

a_1	a_2	a_3	a_4	a_5	a_6
0	2	4	0	2	4

We can see that $a_3 = a_6 = a_9 = a_{12} = ... = 4$

Every numbered term divisible by 3 is 4

Find a numbered term that is close to a_{100} that is divisible by 3

$a_3 = 4 \quad a_9 = 4 \quad a_{30} = 4 \quad a_{99} = 4$

$a_{99} = 4 \quad \Rightarrow \quad a_{100} = 0$ since 0 is the next term after 4 in the sequence

i.e. $0, 2, 4, 0, 2, 4, 0$

Choice 1:	$a_{100} = 1$	false
Choice 2:	$a_{100} = 2$	false
Choice 3:	$a_{100} = 3$	false
Choice 4:	$a_{100} = 4$	false
Choice 5:	$a_{100} = 0$	true

Lesson 4 Arithmetic Sequence 2

Question 1 Experience: 50 Order: Level: Question-ID: 123

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 11$ and $U_7 = 23$, find S_{11}

Solution 1

$$U_n = a + (n-1)d$$

$$U_4 = a + (4-1)d = a + 3d = 11 \quad (1)$$

$$U_7 = a + (7-1)d = a + 6d = 23 \quad (2)$$

$$(2) - (1) \quad a + 6d - (a + 3d) = 23 - 11$$

$$3d = 12$$

$$d = 4$$

$$\text{Sub into} \quad (1) \quad a + 3(4) = 11$$

$$a = -1$$

$$S_n = \frac{n}{2}(2(a) + (n-1)d)$$

$$S_{11} = \frac{11}{2}(2(-1) + (11-1)4) = 209$$

Choice 1:	$S_{11} = 208$	false
Choice 2:	$S_{11} = 205$	false
Choice 3:	$S_{11} = 206$	false
Choice 4:	$S_{11} = 207$	false
Choice 5:	$S_{11} = 209$	true

Question 2 Experience: 50 Order: Level: Question-ID: 124

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_3 = -5$ and $U_5 = -11$, find S_7

Solution 2

$$U_n = a + (n-1)d$$

$$U_3 = a + (3-1)d = a + 2d = -5 \quad (1)$$

$$U_5 = a + (5-1)d = a + 4d = -11 \quad (2)$$

$(2) - (1)$ $a + 4d - (a + 2d) = -11 - (-5)$

$$2d = -6$$

$$d = -3$$

Sub into (1) $a + 2(-3) = -5$

$$a = 1$$

$$U_7 = 1 + (7-1)(-3) = -17$$

$$S_7 = \frac{7}{2}(1 + (-17)) = -56$$

Choice 1:	$S_7 = -52$	false
Choice 2:	$S_7 = -53$	false
Choice 3:	$S_7 = -54$	false
Choice 4:	$S_7 = -55$	false
Choice 5:	$S_7 = -56$	true

Question 3 Experience: 45 Order: Level: Question-ID: 140

The first three terms of an arithmetic sequence are $60, 58, 56...$, there exists a k^{th} term which $= 0$, find the value of k, hence of otherwise find the maximum value of S_n

Solution 3

$$U_n = a + (n-1)d$$

$$U_k = 60 + (k-1)(-2) = 0$$

$$k - 1 = 30$$

$$k = 31$$

maimum value of $S_n = S_k$ as any term after U_k is negative

$$S_n = \frac{n}{2}(a + d)$$

$$S_k = \frac{31}{2}(60 + 0)$$

$$S_k = 930$$

Choice 1:	$k = 28$	$S_k = 935$	false
Choice 2:	$k = 29$	$S_k = 915$	false
Choice 3:	$k = 30$	$S_k = 920$	false
Choice 4:	$k = 32$	$S_k = 925$	false
Choice 5:	$k = 31$	$S_k = 930$	true

Question 4 Experience: 50 Order: Level: Question-ID: 126

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $S_5 = 85$ and $S_8 = 184$, find U_6

Solution 4

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_5 = \frac{5}{2}(2a + (5-1)d) = 85$$

$$2a + 4d = 34 \quad (1)$$

$$S_8 = \frac{8}{2}(2a + (8-1)d) = 184$$

$$2a + 7d = 46 \quad (2)$$

$$(2) - (1) \quad 2a + 7d - (2a + 4d) = 46 - 34$$

$$3d = 12$$

$$d = 4$$

$$\text{Sub into} \quad (1) \quad 2a + 4(4) = 34$$

$$a = 9$$

$$U_6 = a + (6-1)d = 9 + 5(4) = 29$$

Choice 1:	$U_6 = 31$	false
Choice 2:	$U_6 = 30$	false
Choice 3:	$U_6 = 27$	false
Choice 4:	$U_6 = 28$	false
Choice 5:	$U_6 = 29$	true

Question 5 Experience: 50 Order: Level: Question-ID: 127

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_5 = 19$ and $S_{10} = 170$, find U_4

Solution 5

$$U_n = a + (n-1)d$$

$$U_5 = a + 4d = 19 \quad (1)$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2a + (10-1)d) = 170$$

$$2a + 9d = 34 \quad (2)$$

$$(2) - 2(1) \quad 2a + 9d - 2(a + 4d) = 34 - 38$$

$$d = -4$$

$$d = -4$$

$$\text{Sub into} \quad (1) \quad a + 4(-4) = 19$$

$$a = 35$$

$$U_4 = a + (4-1)d = 35 + (3)(-4) = 23$$

Choice 1:	$U_4 = 19$	false
Choice 2:	$U_4 = 20$	false
Choice 3:	$U_4 = 21$	false

Choice 4: $U_4 = 22$ false
Choice 5: $U_4 = 23$ true

Question 6 Experience: 50 Order: Level: Question-ID: 128

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 8$ and $S_{12} = 0$, find S_9

Solution 6

$$U_n = a + (n-1)d$$

$$U_4 = a + 3d = 8 \qquad (1)$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{12} = \frac{12}{2}(2a + (12-1)d) = 0$$

$$2a + 11d = 0 \qquad (2)$$

$(2) - 2(1)$ $2a + 11d - 2(a + 3d) = 0 - 16$

$$8d = -16$$

$$d = -2$$

Sub into (1) $a + 3(-2) = 8$

$$a = 14$$

$$S_9 = \frac{9}{2}(2(14) + (9-1)(-2)) = 54$$

Choice 1: $S_9 = 55$ false
Choice 2: $S_9 = 51$ false
Choice 3: $S_9 = 52$ false
Choice 4: $S_9 = 53$ false
Choice 5: $S_9 = 54$ true

Question 7 Experience: 50 Order: Level: Question-ID: 129

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_3 = 4$ and $U_7 = 0$, find S_{10}

Solution 7

$$U_n = a + (n-1)d$$
$$U_3 = a + 2d = 4 \quad (1)$$
$$U_7 = a + 6d = 0 \quad (2)$$
$$(2) - (1) \quad a + 6d - (a + 2d) = 0 - 4$$
$$4d = -4$$
$$d = -1$$
$$\text{Sub into} \quad (1) \quad a + 2(-1) = 4$$
$$a = 6$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_{10} = \frac{10}{2}(2(6) + (10-1)(-1)) = 15$$

Choice 1: $S_{10} = 14$ false
Choice 2: $S_{10} = 13$ false
Choice 3: $S_{10} = 12$ false
Choice 4: $S_{10} = 16$ false
Choice 5: $S_{10} = 15$ true

Question 8 Experience: 50 Order: Level: Question-ID: 130

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 10$ and $S_6 = 57$, find S_{11}

Solution 8

$$U_n = a + (n-1)d$$
$$U_4 = a + 3d = 10 \quad (1)$$
$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_6 = \frac{6}{2}(2a + (5-1)d) = 57$$
$$a + 2d = \frac{19}{2} \quad (2)$$
$$(1) - (2) \quad a + 3d - (a + 2d) = 10 - \frac{19}{2}$$
$$d = \frac{1}{2}$$
$$\text{Sub into} \quad (1) \quad a + 3\left(\frac{1}{2}\right) = 10$$
$$a = \frac{17}{2}$$

$$S_{11} = \frac{11}{2}\left(2\left(\frac{17}{2}\right) + (11-1)\left(\frac{1}{2}\right)\right) = 121$$

Choice 1: $S_{10} = 14$ false
Choice 2: $S_{10} = 13$ false
Choice 3: $S_{10} = 12$ false

Choice 4: $S_{10} = 16$ false
Choice 5: $S_{10} = 15$ true

Question 9 Experience: 30 Order: Level: Question-ID: 131

Three consecutive terms in an arithmetic sequence are $3k + 2, 2k + 5, 4k + 5$, find the value of k

Solution 9

$$2k + 5 - (3k + 2) = d = 4k + 5 - (2k + 5)$$

$$-k + 3 = 2k$$

$$3k = 3$$

$$k = 1$$

Choice 1: $k = 5$ false
Choice 2: $k = 4$ false
Choice 3: $k = 3$ false
Choice 4: $k = 2$ false
Choice 5: $k = 1$ true

Question 10 Experience: 30 Order: Level: Question-ID: 132

Three consecutive terms in an arithmetic sequence are $k^2 + 3, -k, k - 1$, find the possible values of k

Solution 10

$$-k - (k^2 + 3) = d = k - 1 - (-k)$$

$$-k - k^2 - 3 = 2k - 1$$

$$0 = k^2 + 3k + 2$$

$$0 = (k + 2)(k + 1)$$

Choice 1: $k = -2, -3$ false
Choice 2: $k = -1, -2$ false
Choice 3: $k = -3, -1$ false
Choice 4: $k = -1, -2$ false
Choice 5: $k = -2, -1$ true

Question 11 Experience: 30 Order: Level: Question-ID: 133

Three consecutive terms in an arithmetic sequence are $k + 16, 3k + 12, 7k - 2$, find the value of k

Solution 11

$$3k + 12 - (k + 16) = d = 7k - 2 - (3k + 12)$$

$$2k - 4 = 4k - 14$$

$$2k = 10$$

$$k = 5$$

Choice 1: $k = 6$ false
Choice 2: $k = 2$ false
Choice 3: $k = 3$ false
Choice 4: $k = 4$ false
Choice 5: $k = 5$ true

Question 12 Experience: 45 Order: Level: Question-ID: 134

The first three terms in an arithmetic sequence are $2k, k + 9, 3k$, find the smallest n such that $S_n > 117$

Solution 12

$$k+9-2k = d = 3k-(k+9)$$
$$-k+9 = 2k-9$$
$$3k = 18$$
$$k = 6$$

$$\Rightarrow \quad U_1 = 12 \quad U_2 = 15 \quad U_3 = 18$$
$$S_n = \frac{n}{2}(2a+(n-1)d)$$
$$\frac{n}{2}(2(12)+(n-1)3) > 117$$
$$n(24+3n-3) > 234$$
$$3n^2+21n-234 > 0$$
$$n^2+7n-78 > 0 \quad P=-78 \quad S=7$$
$$(n+13)(n-6) > 0 \quad (13,-6)$$
$$n = 6$$

Choice 1:	$n=7$	false
Choice 2:	$n=3$	false
Choice 3:	$n=4$	false
Choice 4:	$n=5$	false
Choice 5:	$n=6$	true

Question 13 Experience: 40 Order: Level: Question-ID: 135

The first three terms of an arithmetic sequence are $99, 96, 93...$, there exists a k^{th} term which $= 0$, find the value of k, hence of otherwise find the maximum value of S_n

Solution 13

$$U_n = a+(n-1)d$$
$$U_k = 99+(k-1)(-3) = 0$$
$$k-1 = 33$$
$$k = 34$$

maimum value of $S_n = S_k$ as any term after U_k is negative

$$S_n = \frac{n}{2}(a+l)$$
$$S_k = \frac{34}{2}(99+0)$$
$$S_k = 1683$$

Choice 1:	$k=33$	$S_k=1689$	false
Choice 2:	$k=32$	$S_k=1686$	false
Choice 3:	$k=35$	$S_k=1677$	false
Choice 4:	$k=36$	$S_k=1680$	false
Choice 5:	$k=34$	$S_k=1683$	true

Question 14 Experience: 35 Order: Level: Question-ID: 136

The first three terms in an arithmetic sequence are $5, 7, 9$, find the smallest n such that $S_n > 252$

Solution 14

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(2(5) + (n-1)2) > 252$$

$$n(5 + n - 1) > 252$$

$$n^2 + 4n - 252 > 0 \quad P = -252 \quad S = 4$$

$$(n + 18)(n - 14) > 0 \quad (18, -14)$$

$$n = 14$$

Choice 1: $n = 15$ false
Choice 2: $n = 11$ false
Choice 3: $n = 12$ false
Choice 4: $n = 13$ false
Choice 5: $n = 14$ true

Question 15 Experience: 35 Order: Level: Question-ID: 137

The first three terms in an arithmetic sequence are $9, 12, 15$, find the smallest n such that $S_n > 750$

Solution 15

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(2(9) + (n-1)3) > 750$$

$$n(18 + 3n - 3) > 1500$$

$$3n(5 + n) > 1500$$

$$n(5 + n) > 500$$

$$n^2 + 5n - 500 > 0 \quad P = -500 \quad S = 5$$

$$(n + 25)(n - 20) > 0 \quad (25, -20)$$

$$n = 20$$

Choice 1: $n = 21$ false
Choice 2: $n = 17$ false
Choice 3: $n = 18$ false
Choice 4: $n = 19$ false
Choice 5: $n = 20$ true

Question 16 Experience: 35 Order: Level: Question-ID: 138

The first three terms in an arithmetic sequence are $12, 16, 20, 24$, find the smallest n such that $S_n > 672$

Solution 16

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(2(12) + (n-1)4) > 672$$

$$n(12 + 2n - 2) > 672$$

$$2n^2 + 10n - 672 > 0$$

$$n^2 + 5n - 336 > 0 \quad P = -336 \quad S = 5$$

$$(n + 21)(n - 16) > 0 \quad (21, -16)$$

$$n = 16$$

Choice 1: $n = 15$ false
Choice 2: $n = 19$ false
Choice 3: $n = 18$ false
Choice 4: $n = 17$ false
Choice 5: $n = 16$ true

Question 17 Experience: 50 Order: Level: Question-ID: 142

Judith is playing with 294 sticks, she puts them in rows. The first row has 8 sticks, next row has 10 sticks, subsequent rows have 2 more sticks then the previous row. She has enough for k rows but not enough for $k + 1$ rows. Find k.

Solution 17

Sequence goes: 8,10,12,14,18,20....

Not having enough for k+1 rows means that $S_k \leq 294$

$$S_n = \frac{n}{2}(2a + (k-1)d)$$

$$S_k = \frac{k}{2}(2(8) + (k-1)2)$$

$$S_k = k(8 + k - 1)$$

$$S_k = k(k + 7)$$

$$S_k = k^2 + 7k \qquad (1)$$

$$S_k \leq 294$$

$$(1) \qquad k^2 + 7k \leq 294$$

$$k^2 + 7k - 294 \leq 0 \qquad P = 294 \quad S = 7$$

$$(k + 21)(k - 14) \leq 0 \qquad (21, -14)$$

Choice 1: $k = 11$ false
Choice 2: $k = 15$ false
Choice 3: $k = 12$ false
Choice 4: $k = 13$ false
Choice 5: $k = 14$ true

Question 18 Experience: 50 Order: Level: Question-ID: 125

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $S_{11} = 0$ and $U_2 = 8$, find U_6

Solution 18

$$U_n = a + (n-1)d$$

$$U_2 = a + (2-1)d = a + d = 8 \qquad (1)$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{11} = \frac{11}{2}(2a + (11-1)d) = 0$$

$$S_{11} = a + 5d = 0 \qquad (2)$$

(2) − (1) $a + 5d - (a + d) = 0 - 8$

$$4d = -8$$

$$d = -2$$

Sub into (1) $a + (-2) = 8$

$$a = 10$$

$$U_6 = 1 + (7-1)(-2) = -11$$

Choice 1: $U_6 = -13$ false
Choice 2: $U_6 = -12$ false
Choice 3: $U_6 = -9$ false
Choice 4: $U_6 = -10$ false
Choice 5: $U_6 = -11$ true

Question 19 Experience: 45 Order: Level: Question-ID: 139

The first three terms of an arithmetic sequence are $44, 41, 38...$, there exists a k^{th} term which is the smallest positive term in the sequence, find the value of k, hence of otherwise find the maximum value of S_n

Solution 19

$$U_n = a + (n-1)d$$

$$U_k = 44 + (k-1)(-3) = 0$$

$$k - 1 = \frac{44}{3}$$

$$k = \frac{44}{3} + 1 = 15.6$$

$$k = 15$$

maimum value of $S_n = S_k$ as any term after U_k is negative

$$S_k = \frac{n}{2}(2a + (n-1)d)$$

$$S_k = \frac{34}{2}(2(99) + (15-1)(-3))$$

$$S_k = 2652$$

Choice 1: $k = 14$ $S_k = 2658$ false
Choice 2: $k = 13$ $S_k = 2656$ false
Choice 3: $k = 12$ $S_k = 2654$ false
Choice 4: $k = 16$ $S_k = 2650$ false
Choice 5: $k = 15$ $S_k = 2652$ true

Question 20 Experience: 50 Order: Level: Question-ID: 141

At the start of the year 2000, Tony the farmer has $50m^2$ of land, he buys $7m^2$ of land at the end of each year. At the beginning of this year, Tony owns $141m^2$ of land. What year is it?

Solution 20

Sequence goes from the start of every year: 50,57,64,71,78,85....

$$U_n = a + (n-1)d$$

$$U_n = 141 \quad a = 50 \quad d = 7$$

$$141 = 50 + (n-1)7$$

$$n - 1 = 13$$

$$n = 14$$

$$\text{Year} = 2000 + 14 = 2014$$

Choice 1: Year = 2015 false
Choice 2: Year = 2011 false
Choice 3: Year = 2012 false
Choice 4: Year = 2013 false
Choice 5: Year = 2014 true

End of Chapter Questions

Unit 2 Core 2

Chapter 1 Logarithms

Lesson 1 Basic logarithms

Question 1 Experience: 20 Order: f2 Level: f2 Question-ID: 196

Express $\log_9 30$ in terms of ln

Solution 1

$$\log_9 30 = \frac{\log_e 30}{\log_e 9} = \frac{\ln 30}{\ln 9}$$

Choice 1: $\frac{\ln 10}{\ln 3}$ false
Choice 2: $\ln 21$ false
Choice 3: $\ln 39$ false
Choice 4: $\ln \frac{10}{3}$ false
Choice 5: $\frac{\ln 30}{\ln 9}$ true

Question 2 Experience: 25 Order: g1 Level: g1 Question-ID: 197

Solve $4^x = 16$ for x

Solution 2

$$4^x = 16$$

$$x = \log_4 16$$

$$x = \frac{\log 16}{\log 4}$$

$$x = 2$$

Answer part 1: Label $x =$ Solution 2
Answer part 1 hint: x is an integer value

Question 3 Experience: 10 Order: a2 Level: a2 Question-ID: 149

Express $\log_{x+5} 10 = 4$ in power form

Solution 3

$$\log_{x+5} 10 = 4$$

$$(x+5)^4 = 10$$

Choice 1: $4^{x+5} = 10$ false
Choice 2: $(x+5)^{10} = 4$ false

Choice 3: $10^{x+5} = 4$ false
Choice 4: $(x+5)^{10} = 4$ false
Choice 5: $(x+5)^4 = 10$ true

Question 4 Experience: 10 Order: a2 Level: a2 Question-ID: 150

Express $\log_{a+b} 6 = c$ in power form

Solution 4

$$\log_{a+b} 6 = c$$

$$(a+b)^c = 6$$

Choice 1: $(a+b)^6 = c$ false
Choice 2: $6^c = a+b$ false
Choice 3: $(a+b)^c = 6$ false
Choice 4: $6^{a+b} = 6$ false
Choice 5: $(a+b)^c = 6$ true

Question 5 Experience: 10 Order: a2 Level: a2 Question-ID: 152

Express $\log_{xy} 3 = 2$ in power form

Solution 5

$$\log_{xy} 3 = 2$$

$$(xy)^2 = 3$$

Choice 1: $2^{xy} = 3$ false
Choice 2: $3^2 = xy$ false
Choice 3: $xy^3 = 2$ false
Choice 4: $(3)^{xy} = 2$ false
Choice 5: $(xy)^2 = 3$ true

Question 6 Experience: 10 Order: b1 Level: b1 Question-ID: 154

Express $a^b = c$ in log form

Solution 6

$$a^b = c$$

$$\log_a c = b$$

Choice 1: $\log_c a = b$ false
Choice 2: $\log_b c = a$ false
Choice 3: $\log_b a = c$ false
Choice 4: $\log_a b = c$ false
Choice 5: $\log_a c = b$ true

Question 7 Experience: 10 Order: b1 Level: b1 Question-ID: 157

Express $5^2 = 25$ in log form

Solution 7

$$5^2 = 25$$

$$\log_5 25 = 2$$

Choice 1: $\log_5 2 = 25$ false
Choice 2: $\log_{25} 2 = 5$ false
Choice 3: $\log_{25} 5 = 2$ false
Choice 4: $\log_2 25 = 5$ false
Choice 5: $\log_5 25 = 2$ true

Question 8 Experience: 10 Order: b2 Level: b2 Question-ID: 156

Express $(xy)^5 = 20$ in log form

Solution 8

$$(xy)^5 = 20$$

$$\log_{xy} 20 = 5$$

Choice 1: $\log_5 20 = xy$ false
Choice 2: $\log_{xy} 5 = 20$ false
Choice 3: $\log_{20} 5 = xy$ false
Choice 4: $\log_{20} xy = 5$ false
Choice 5: $\log_{xy} 20 = 5$ true

Question 9 Experience: 15 Order: c2 Level: c2 Question-ID: 162

Express $\log_2(x^2y) - \log_2 x$ as a single logarithm

Solution 9

$$\log_2(x^2y) - \log_2 x$$

$$= \log_2((x^2y) \div x)$$

$$= \log_2 xy$$

Choice 1: $2\log_{x^2y} 1$ false
Choice 2: $\log_{x^2y} 2$ false
Choice 3: $\log_2 x^2y$ false
Choice 4: $2x \log_2 y$ false
Choice 5: $\log_2 xy$ true

Question 10 Experience: 10 Order: b1 Level: b1 Question-ID: 159

Express $a^{bc} = 6$ in log form

Solution 10

$$a^{bc} = 6$$

$$\log_a 6 = bc$$

Choice 1: $\log_6 ab = c$ false
Choice 2: $\log_{bc} a = 6$ false
Choice 3: $\log_{bc} 6 = a$ false
Choice 4: $\log_a bc = 6$ false
Choice 5: $\log_a 6 = bc$ true

Question 11 Experience: 10 Order: b2 Level: b2 Question-ID: 155

Express $(a + b)^4 = 15$ in log form

Solution 11

$$(a+b)^4 = 15$$

$$\log_{(a+b)} 15 = 4$$

Choice 1: $\log_4 15 = a+b$ false
Choice 2: $\log_{15}(a+b) = 4$ false
Choice 3: $\log_{15} 4 = a+b$ false
Choice 4: $\log_4(a+b) = 15$ false
Choice 5: $\log_{(a+b)} 15 = 4$ true

Question 12 Experience: 10 Order: b2 Level: b2 Question-ID: 158

Express $(x+4)^4 = 5$ in log form

Solution 12

$$(x+4)^4 = 5$$

$$\log_{(x+4)} 5 = 4$$

Choice 1: $\log_4(x+4) = 5$ false
Choice 2: $\log_5 4 = x+4$ false
Choice 3: $\log_{(x+4)} 4 = 5$ false
Choice 4: $\log_5(x+4) = 5$ false
Choice 5: $\log_{(x+4)} 5 = 4$ true

Question 13 Experience: 15 Order: c1 Level: c1 Question-ID: 161

Express $\log_4(x+y) + \log_4 6$ as a single logarithm

Solution 13

$$\log_4(x+y) + \log_4 6$$

$$= \log_4((x+y) \text{ x } 6)$$

$$= \log_4 6(x+y)$$

Choice 1: $4\log_{(x+y)} 6$ false
Choice 2: $\log_{(x+y)} 24$ false
Choice 3: $4\log_6 x+y$ false
Choice 4: $6\log_4 x+y$ false
Choice 5: $\log_4 6(x+y)$ true

Question 14 Experience: 10 Order: a1 Level: a1 Question-ID: 148

Express $\log_x 9 = 2$ in power form

Solution 14

$$\log_x 9 = 2$$

$$x^2 = 9$$

Choice 1: $x^9 = 2$ false
Choice 2: $x^2 = 2$ false
Choice 3: $x^9 = 9$ false
Choice 4: $x^2 = 7$ false

Choice 5: $x^2 = 9$ true

Question 15 Experience: 15 Order: c1 Level: c1 Question-ID: 163

Express $3\log_3(a+b) + \log_3 4$ as a single logarithm

Solution 15

$$3\log_3(a+b) + \log_3 4$$

Choice 1: $3\log_3(4)$ false
$= \log_3(a+b)^3 + \log_3 4$
Choice 2: $\log_{a+b}(12)$ false
$= \log_3((a+b)^3 \times 4)$
Choice 3: $12\log_3 a + b$ false
$= 4\log_3 4(a+b)^3$
Choice 4: $4\log_3(a+b)^3$ false
$= \log_3 4(a+b)^3$
Choice 5: $\log_3 4(a+b)^3$ true

Question 16 Experience: 15 Order: c2 Level: c2 Question-ID: 164

Express $\log_4(a^2-b^2) - 2\log_4(a+b)$ as a single logarithm

Solution 16

$$\log_4(a^2-b^2) - 2\log_4 a+b$$

Choice 1: $\log_3 \frac{a-b}{(a+b)^2}$ false
$= \log_4(a^2-b^2) - \log_4(a+b)^2$
Choice 2: $\log_3 \frac{a+b}{(a+b)^2}$ false
$= \log_3((a^2-b^2) \div (a+b)^2)$
Choice 3: $\log_3 \frac{a^2+b^2}{(a-b)^2}$ false
$= \log_3 \left(\frac{(a-b)(a-b)}{(a+b)^2} \right)$
Choice 4: $\log_3 \frac{a^2-b^2}{a^2+b^2}$ false
$= \log_3 \frac{a-b}{a+b}$
Choice 5: $\log_3 \frac{a-b}{a+b}$ true

Question 17 Experience: 15 Order: c2 Level: c2 Question-ID: 165

Express $\log_x(4a-6b) + \log_x \frac{1}{2}$ as a single logarithm

Solution 17

$$\log_x(4a-6b) + \log_x \frac{1}{2}$$

$$= \log_x \frac{1}{2}(4a-6b)$$

$$= \log_x(2a-3b)$$

Choice 1: $\log_{(4a-6b)} \frac{1}{2}x$ false
Choice 2: $\log_x(4a-6b)$ false
Choice 3: $\frac{1}{2}\log_{(4a-6b)} x$ false
Choice 4: $\frac{1}{2}\log_x(2a-3b)$ false
Choice 5: $\log_x(2a-3b)$ true

Question 18 Experience: 15 Order: d1 Level: d1 Question-ID: 166

Express $\log_4(6a) - \log_4(2a)$ as a single logarithm

Solution 18

$$\log_4(6a) - \log_4(2a)$$

$$= \log_4(6a \div 2a)$$

$$= \log_4 3$$

Choice 1: $\log_a 2$ false
Choice 2: $\log_a 3$ false
Choice 3: $\log_4 3a$ false
Choice 4: $\log_4 12a^2$ false
Choice 5: $\log_4 3$ true

Question 19 Experience: 15 Order: d1 Level: d1 Question-ID: 167

Express $\log_{10}(15) - \log_{10}(3)$ as a single logarithm

Solution 19

$$\begin{aligned} & \log_{10}(15) - \log_{10}(3) \\ = & \log_{10}(15 \div 3) \\ = & \log_{10} 5 \end{aligned}$$

Choice 1: $\log_{10} 3$ false
Choice 2: $\log_5 45$ false
Choice 3: $\log_{10} 45$ false
Choice 4: $\log_5 10$ false
Choice 5: $\log_{10} 5$ true

Question 20 Experience: 15 Order: d2 Level: d2 Question-ID: 168

Express $3\log_y(5) + \log_y(4)$ as a single logarithm

Solution 20

$$\begin{aligned} & 3\log_y(5) + \log_y(4) \\ = & \log_y 5^3 + \log_y 4 \\ = & \log_y(5^3 \text{ x } 4) \\ = & \log_y 500 \end{aligned}$$

Choice 1: $\log_y 100$ false
Choice 2: $\log_y 8000$ false
Choice 3: $4\log_y 125$ false
Choice 4: $4\log_y 50$ false
Choice 5: $\log_y 500$ true

Question 21 Experience: 15 Order: d2 Level: d2 Question-ID: 169

Express $3\log_a(4) - 4\log_a(2)$ as a single logarithm

Solution 21

$$\begin{aligned} & \log_a(4^3) - \log_a(2^4) \\ = & \log_a(64) - \log_a(16) \\ = & \log_y(64 \div 16) \\ = & \log_y 4 \end{aligned}$$

Choice 1: $\log_4 a^2$ false
Choice 2: $\log_4 16$ false
Choice 3: $\log_4 64$ false

Choice 4: $\log_y 16$ false
Choice 5: $\log_y 4$ true

Question 22 Experience: 10 Order: a1 Level: a1 Question-ID: 153

Express $\log_3 7 = a + b^2$ in power form

Solution 22

$$\log_3 7 = a + b^2$$

$$3^{a+b^2} = 7$$

Choice 1: $7^{a+b^2} = 3$ false
Choice 2: $3^7 = 7a + b^2$ false
Choice 3: $(a + b^2)^3 = 7$ false
Choice 4: $3^7 = a + b^2$ false
Choice 5: $3^{a+b^2} = 7$ true

Question 23 Experience: 30 Order: d3 Level: d3 Question-ID: 171

Express $4\log_9 5 - 2\log_3(15)$ as a single logarithm

Solution 23

$$\begin{aligned}
&4\log_9 5 - 2\log_3(9) \\
=&4\left(\frac{\log_3 5}{\log_3 9}\right) - 2\log_3(15) \\
=&\left(\frac{4\log_3 5}{2}\right) - \log_3(15^2) \\
=&2\log_3 5 - \log_3(15^2) \\
=&\log_3(5^2 \div 15^2) \\
=&\log_3 \frac{25}{225} \\
=&\log_3 \frac{1}{9}
\end{aligned}$$

Choice 1: $\log_9 225$ false
Choice 2: $\log_9 25$ false
Choice 3: $\log_9 \frac{1}{3}$ false
Choice 4: $\log_3 9$ false
Choice 5: $\log_3 \frac{1}{9}$ true

Question 24 Experience: 15 Order: c1 Level: c1 Question-ID: 160

Express $\log_a 4 + \log_a 5$ as a single logarithm

Solution 24

$$\begin{aligned}
&\log_a 4 + \log_a 5 \\
=&\log_a(4 \text{ x } 5) \\
=&\log_a 20
\end{aligned}$$

Choice 1: $\log_4 5a$ false
Choice 2: $a\log_4 5$ false

Choice 3: $4\log_a 5$ false
Choice 4: $5\log_a 4$ false
Choice 5: $\log_a 20$ true

Question 25 Experience: 20 Order: e2 Level: e2 Question-ID: 188

Express $\log_a \frac{a^2b^5}{c^3}$ as a linear combination

Solution 25

$$\log_a \frac{a^2b^5}{c^3}$$

$$=\log_a a^2 + \log_a b^5 - \log_a c^3$$

$$=2 + 5\log_a b - 3\log_a c$$

Choice 1: $3 + 5\log_a b - 2\log_a c$ false
Choice 2: $5 + 2\log_a b - 3\log_a c$ false
Choice 3: $5 + 3\log_a b - 2\log_a c$ false
Choice 4: $2 + 3\log_a b - 5\log_a c$ false
Choice 5: $2 + 5\log_a b - 3\log_a c$ true

Question 26 Experience: 20 Order: e2 Level: e2 Question-ID: 189

Express $\log_c \frac{a^4}{b^2c^6}$ as a linear combination

Solution 26

$$\log_c \frac{a^4}{b^2c^6}$$

$$=\log_c a^4 - \log_c b^2 - \log_c c^6$$

$$=4\log_c a - 2\log_c b - 6$$

Choice 1: $6\log_c a - 4\log_c b - 2$ false
Choice 2: $2\log_c a - 6\log_c b - 4$ false
Choice 3: $2\log_c a - 4\log_c b - 6$ false
Choice 4: $4\log_c a - 6\log_c b - 2$ false
Choice 5: $4\log_c a - 2\log_c b - 6$ true

Question 27 Experience: 10 Order: a1 Level: a1 Question-ID: 180

Express $\log_a b - 4 = 7$ in power form

Solution 27

$$\log_a b - 4 = 7$$

$$a^7 = b - 4$$

Choice 1: $a^{b-4} = 7$ false
Choice 2: $(b-4)^7 = a$ false
Choice 3: $7^a = b - 4$ false
Choice 4: $(b-4)^a = 7$ false
Choice 5: $a^7 = b - 4$ true

Question 28 Experience: 25 Order: g1 Level: g1 Question-ID: 198

Solve $3^x = 27$ for x

Solution 28

$$3^x = 27$$

$$x = \log_3 27$$

$$x = \frac{\log 27}{\log 3}$$

$$x = 3$$

Answer part 1: Label $x =$ Solution 3
Answer part 1 hint: x is an integer value

Question 29 Experience: 30 Order: d3 Level: d3 Question-ID: 170

Express $2\log_{16} 8 - 4\log_4(2)$ as a single logarithm

Solution 29

$$\begin{aligned} & 2\log_{16} 8 - 4\log_4(2) \\ =& 2\left(\frac{\log_4 8}{\log_4 16}\right) - 4\log_4(2) \\ =& \left(\frac{2\log_4 8}{2}\right) - \log_4(16) \\ =& \log_4(8 \div 16) \\ =& \log_4 \frac{1}{2} \end{aligned}$$

Choice 1: $\log_8 4$ false
Choice 2: $\log_8 2$ false
Choice 3: $\log_8 \frac{1}{2}$ false
Choice 4: $\log_4 2$ false
Choice 5: $\log_4 \frac{1}{2}$ true

Question 30 Experience: 30 Order: d4 Level: d4 Question-ID: 181

Express $3\log_4 5 + 4\log_{16}(3)$ as a single logarithm.

Solution 30

$$\begin{aligned} & 3\log_4 5 + 4\log_{16}(3) \\ =& 3\log_4 5 + 4\left(\frac{\log_4 3}{\log_4 16}\right) \\ =& \log_4 5^3 + 4\left(\frac{\log_4 3}{2}\right) \\ =& \log_4 5^3 + \log_4 3^2 \\ =& \log_4 1125 \end{aligned}$$

Choice 1: $\log_4 225$ false
Choice 2: $\log_4 25$ false
Choice 3: $\log_{16} 25$ false
Choice 4: $\log_{16}(225)$ false
Choice 5: $\log_4 1125$ true

Question 31 Experience: 30 Order: d4 Level: d4 Question-ID: 184

Express $8\log_4 3 - 2\log_2(4)$ as a single logarithm

Solution 31

$$\begin{aligned} & 8\log_4 3 - 2\log_2 4 \\ =& 8\left(\frac{\log_2 3}{\log_2 4}\right) - 2\log_2 4 \\ =& 4\log_2 3 - 2\log_2 4 \\ =& \log_2 3^4 - \log_2 4^2 \\ =& \log_2 \frac{3^4}{4^2} \\ =& \log_2 \frac{81}{16} \end{aligned}$$

Choice 1: $\log_4 65$ false
Choice 2: $\log_4 1296$ false
Choice 3: $\log_2 \frac{27}{4}$ false
Choice 4: $\log_2 65$ false
Choice 5: $\log_2 \frac{81}{16}$ true

Question 32 Experience: 20 Order: e1 Level: e1 Question-ID: 185

Express $\log_x a^2b^3c^4$ as a linear combination

Solution 32

$$\log_x a^2b^3c^4$$
$$= \log_x a^2 + \log_x b^3 + \log_x c^4$$
$$= 2\log_x a + 3\log_x b + 4\log_x c$$

Choice 1: $4\log_x a + 2\log_x b + 3\log_x c$ false
Choice 2: $4\log_x a + 3\log_x b + 2\log_x c$ false
Choice 3: $2\log_x a + 4\log_x b + 3\log_x c$ false
Choice 4: $3\log_x a + 2\log_x b + 4\log_x c$ false
Choice 5: $2\log_x a + 3\log_x b + 4\log_x c$ true

Question 33 Experience: 20 Order: e1 Level: e1 Question-ID: 186

Express $\log_a x^5y^4z^6$ as a linear combination

Solution 33

$$\log_a x^5y^4z^6$$
$$= \log_a x^5 + \log_a y^4 + \log_a z^6$$
$$= 5\log_a x + 4\log_a y + 6\log_a z$$

Choice 1: $5\log_a x + 6\log_a y + 4\log_a z$ false
Choice 2: $6\log_a x + 4\log_a y + 5\log_a z$ false
Choice 3: $4\log_a x + 5\log_a y + 6\log_a z$ false
Choice 4: $6\log_a x + 5\log_a y + 4\log_a z$ false
Choice 5: $5\log_a x + 4\log_a y + 6\log_a z$ true

Question 34 Experience: 20 Order: e1 Level: e1 Question-ID: 187

Express $\log_b a^7b^2c^5$ as a linear combination

Solution 34

$$\log_b a^7b^2c^5$$
$$= \log_b a^7 + \log_b b^2 + \log_b c^5$$
$$= 7\log_b a + 2 + 5\log_b c$$

Choice 1: $2\log_b a + 5 + 7\log_b c$ false
Choice 2: $2\log_b a + 7 + 5\log_b c$ false
Choice 3: $7\log_b a + 5 + 2\log_b c$ false
Choice 4: $5\log_b a + 2 + 7\log_b c$ false
Choice 5: $7\log_b a + 2 + 5\log_b c$ true

Question 35 Experience: 20 Order: e2 Level: e2 Question-ID: 190

Express $\log_x \frac{x^3y^8}{z^2}$ as a linear combination

Solution 35

$$\log_x \frac{x^3 y^8}{z^2}$$

$$= \log_x x^3 + \log_x y^8 - \log_x z^2$$

$$= 3 + 8\log_x y - 2\log_x z$$

Choice 1: $8 + 2\log_x y - 3\log_x z$ false
Choice 2: $2 + 8\log_x y - 3\log_x z$ false
Choice 3: $2 + 3\log_x y - 8\log_x z$ false
Choice 4: $3 + 2\log_x y - 8\log_x z$ false
Choice 5: $3 + 8\log_x y - 2\log_x z$ true

Question 36 Experience: 20 Order: f1 Level: f1 Question-ID: 191

Express $\log_4 20$ in terms of ln

Solution 36

Choice 1: $\log_4 20$ $\frac{\ln 20}{\ln 80}$ false
Choice 2: $= \frac{\log_e 20}{\log_e 4}$ $\ln 24$ false
Choice 3: $\ln 80$ false
Choice 4: $\ln 20$ $\ln 5$ false
Choice 5: $= \frac{\ln 20}{\ln 4}$ $\frac{\ln 20}{\ln 4}$ false

Question 37 Experience: 20 Order: f1 Level: f1 Question-ID: 192

Express $\log_8 24$ in terms of ln

Solution 37

Choice 1: $\log_8 24$ $\frac{\ln 24}{\ln 192}$ false
Choice 2: $= \frac{\log_e 24}{\log_e 8}$ $\ln 192$ false
Choice 3: $\ln 32$ false
Choice 4: $\ln 24$ $\ln 3$ false
Choice 5: $= \frac{\ln 24}{\ln 8}$ $\frac{\ln 24}{\ln 8}$ false

Question 38 Experience: 20 Order: f1 Level: f1 Question-ID: 193

Express $\log_6 18$ in terms of ln

Solution 38

Choice 1: $\log_6 18$ $\frac{\ln 6}{\ln 2}$ false
Choice 2: $= \frac{\log_e 18}{\log_e 6}$ $\ln 108$ false
Choice 3: $\ln 24$ false
Choice 4: $\ln 18$ $\ln 3$ false
Choice 5: $= \frac{\ln 18}{\ln 6}$ $\frac{\ln 18}{\ln 6}$ true

Question 39 Experience: 20 Order: f2 Level: f2 Question-ID: 194

Express $\log_2 8$ in terms of ln

Solution 39

Choice 1: $\log_2 8$ $\frac{\ln 8}{\ln 16}$ false
Choice 2: $= \frac{\log_e 8}{\log_e 2}$ $\ln 10$ false
Choice 3: $\ln 16$ false
$= \frac{\ln 8}{\ln 2}$

Choice 4: $\ln 4$ false
Choice 5: $\dfrac{\ln 8}{\ln 2}$ true

Question 40 Experience: 20 Order: f2 Level: f2 Question-ID: 195

Express $\log_6 15$ in terms of ln

Solution 40

Choice 1: $\log_6 15$ $\dfrac{\ln 5}{\ln 2}$ false
Choice 2: $\dfrac{\log_e 15}{\log_e 6}$ $\ln 9$ false
Choice 3: $\dfrac{\ln 15}{\ln 6}$ $\ln 21$ false
Choice 4: $\ln \dfrac{5}{2}$ false
Choice 5: $\dfrac{\ln 15}{\ln 6}$ true

Question 41 Experience: 25 Order: g1 Level: g1 Question-ID: 199

Solve $4^x = 64$ for x

Solution 41

$$4^x = 64$$

$$x = \log_4 64$$

$$x = \frac{\log 64}{\log 4}$$

$$x = 3$$

Answer part 1: Label $x =$ Solution 3
Answer part 1 hint: x is an integer value

Question 42 Experience: 25 Order: h1 Level: h1 Question-ID: 200

Solve $5^x = 35$ for x

Solution 42

$$5^x = 35$$

$$x = \log_5 35$$

$$x = \frac{\log 35}{\log 5}$$

$$x = 2.21$$

Answer part 1: Label $x =$ Solution 2.21
Answer part 1 hint: Write the answer to 2 d.p.

Question 43 Experience: 25 Order: h1 Level: h1 Question-ID: 201

Solve $4^x = 20$ for x

Solution 43

$$4^x = 20$$

$$x = \log_4 20$$

$$x = \frac{\log 20}{\log 4}$$

$$x = 2.16$$

Answer part 1: Label $x =$ Solution 2.16
Answer part 1 hint: Write the answer to 2 d.p.

Question 44 Experience: 25 Order: h1 Level: h1 Question-ID: 202

Solve $5^x = 75$ for x

Solution 44

$$5^x = 75$$

$$x = \log_5 75$$

$$x = \frac{\log 75}{\log 5}$$

$$x = 2.68$$

Answer part 1: Label $x =$ Solution 2.68
Answer part 1 hint: Write the answer to 2 d.p.

Question 45 Experience: 40 Order: i1 Level: i1 Question-ID: 203

Solve $3^{5x+3} = 5^{3x-1}$ for x

Solution 45

$$3^{5x+3} = 5^{3x-1}$$

$$\log(3^{5x+3}) \log(5^{3x-1})$$

$$(5x+3)\log 3 = (3x-1)\log 5$$

$$5x\log 3 + 3\log 3 = 3x\log 5 - \log 5$$

$$5x\log 3 - 3x\log 5 = -\log 5 - 3\log 3$$

$$x(5\log 3 - 3\log 5) = -(\log 5 + 3\log 3)$$

$$x \log \frac{3^5}{5^3} = -\log(5 \text{ x } 3^3)$$

$$0.289x = -2.13$$

$$x = -7.38$$

Answer part 1: Label $x =$ Solution -7.38
Answer part 1 hint: Write the answer to 2 d.p.

Question 46 Experience: 40 Order: i1 Level: i1 Question-ID: 204

Solve $5^{2x-5} = 3^{6x+7}$ for x

Solution 46

$$
\begin{aligned}
5^{2x-5} &= 3^{6x+7} \\
\log(5^{2x-5}) &= \log(3^{6x+7}) \\
(2x-5)\log 5 &= (6x+7)\log 3 \\
2x\log 5 - 5\log 5 &= 6x\log 3 + 7\log 3 \\
2x\log 5 - 6x\log 3 &= 5\log 5 + 7\log 3 \\
x(\log 5^2 - \log 3^6) &= \log 5^5 + \log 3^7 \\
x\left(\log \frac{5^2}{3^6}\right) &= \log(5^5 \text{ x } 3^7) \\
-1.46x &= 6.83 \\
x &= -4.67
\end{aligned}
$$

Answer part 1: Label $x =$ Solution -4.67
Answer part 1 hint: Write the answer to 2 d.p.

Question 47 Experience: 40 Order: i2 Level: i2 Question-ID: 205
Solve $2^{4x-3} = 5^{7x+4}$ for x

Solution 47

$$
\begin{aligned}
2^{4x-3} &= 5^{7x+4} \\
\log(2^{4x-3}) &= \log(5^{7x+4}) \\
(4x-3)\log 2 &= (7x+4)\log 5 \\
4x\log 2 - 3\log 2 &= 7x\log 5 + 4\log 5 \\
4x\log 2 - 7x\log 5 &= 3\log 2 + 4\log 5 \\
x(\log 2^4 - \log 5^7) &= \log 2^3 + \log 5^4 \\
x\left(\log \frac{2^4}{5^7}\right) &= \log(2^3 \text{ x } 5^4) \\
-3.69x &= 3.70 \\
x &= -1.00
\end{aligned}
$$

Answer part 1: Label $x =$ Solution -1.00
Answer part 1 hint: Write the answer to 2 d.p.

Question 48 Experience: 40 Order: i2 Level: i2 Question-ID: 206
Solve $7^{6x+2} = 4^{4x-5}$ for x

Solution 48

$$7^{6x+2} = 4^{4x-5}$$
$$\log(7^{6x+2}) = \log(4^{4x-5})$$
$$(6x+2)\log 7 = (4x-5)\log 4$$
$$6x\log 7 + 2\log 7 = 4x\log 4 - 5\log 4$$
$$6x\log 7 - 4x\log 4 = -2\log 7 - 5\log 4$$
$$x(6\log 7 - 4\log 4) = -(2\log 7 + 5\log 4)$$
$$x(\log 7^6 - \log 4^4) = -(\log 7^2 + \log 4^5)$$
$$x\left(\log \frac{7^6}{4^4}\right) = -\log(7^2 \text{ x } 4^5)$$
$$2.66x = -4.70$$
$$x = -1.77$$

Answer part 1: Label $x =$ Solution -1.77
Answer part 1 hint: Write the answer to 2 d.p.

Question 49 Experience: 40 Order: i1 Level: i1 Question-ID: 234
Solve $5^{2-3x} = 4^{3+2x}$ for x

Solution 49

$$5^{2-3x} = 4^{3+2x}$$
$$\log(5^{2-3x}) = \log(4^{3+2x})$$
$$(2-3x)\log 5 = (3+2x)\log 4$$
$$2\log 5 - 3x\log 5 = 3\log 4 + 2x\log 4$$
$$-3x\log 5 - 2x\log 4 = 3\log 4 - 2\log 5$$
$$3x\log 5 + 2x\log 4 = -3\log 4 + 2\log 5$$
$$x(\log 5^3 + \log 4^2) = 2\log 5 - 3\log 4$$
$$x\log(5^3 \text{ x } 4^2) = \log\left(\frac{5^2}{4^3}\right)$$
$$3.30x = -0.408$$
$$x = -0.12$$

Answer part 1: Label $x =$ Solution -0.12
Answer part 1 hint: Write the answer to 2 d.p.

Question 50 Experience: 40 Order: i2 Level: i2 Question-ID: 235
Solve $3^{2x+1} = 7^{2x-5}$ for x

Solution 50

$$
\begin{aligned}
3^{2x+1} &= 7^{2x-5} \\
\log(3^{2x+1}) &= \log(7^{2x-5}) \\
(2x+1)\log 3 &= (2x-5)\log 7 \\
2x\log 3 + \log 3 &= 2x\log 7 - 5\log 7 \\
2x\log 3 - 2x\log 7 &= -5\log 7 - \log 3 \\
x(2\log 3 - 2\log 7) &= -(5\log 7 + \log 3) \\
x(\log 3^2 - \log 7^2) &= -(\log 7^5 + \log 3) \\
x\left(\log \frac{3^2}{7^2}\right) &= -\log(7^5 \text{ x } 3) \\
-0.736x &= -4.70 \\
x &= 6.39
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 6.39
Answer part 1 hint: Write the answer to 2 d.p.

Lesson 2 Applications of logarithms

Question 1 Experience: 50 Order: j1 Level: j1 Question-ID: 207
Solve $(5^x)^2 - 8(5^x) + 15 = 0$ for x

Solution 1

$$
\begin{aligned}
(5^x)^2 - 8(5^x) + 15 = 0 \qquad & y = 5^x \\
y^2 - 8y + 15 = 0 \qquad & P = 15 \quad S = -8 \\
(y-5)(y-3) = 0 \qquad & (-5,-3) \\
y = 5 \qquad y = 3 & \\
5^x = 5 \qquad 5^x = 3 & \\
x = 1 \qquad x = 0.68 &
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 0.68,1
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 2 Experience: 50 Order: j1 Level: j1 Question-ID: 208
Solve $(2^x)^2 - 9(2^x) + 14 = 0$ for x

Solution 2

$$
\begin{aligned}
(2^x)^2 - 9(2^x) + 14 = 0 \qquad & y = 2^x \\
y^2 - 9y + 14 = 0 \qquad & P = 14 \quad S = -9 \\
(y-7)(y-2) = 0 \qquad & (-7,-2) \\
y = 7 \qquad y = 2 & \\
2^x = 7 \qquad 2^x = 2 & \\
x = 2.81 \quad x = 1 &
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 1,2.81
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 3 Experience: 50 Order: j2 Level: j2 Question-ID: 209

Solve $2^{2x+1} - 10(2^x) + 12 = 0$ for x

Solution 3

$$2^{2x+1} - 10(2^x) + 12 = 0$$
$$2(2^x)^2 - 10(2^x) + 12 = 0 \qquad y = 2^x$$
$$2y^2 - 10y + 12 = 0 \qquad P = 24 \quad S = -10$$
$$(y-2)(y-3) = 0 \qquad (-4,-6) \quad (-2,-3)$$
$$y = 2 \qquad y = 3$$
$$2^x = 2 \qquad 2^x = 3$$
$$x = 1 \qquad x = 1.58$$

Answer part 1: Label $x =$ Solution 1,1.58
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 4 Experience: 50 Order: j2 Level: j2 Question-ID: 210

Solve $3^{2x+1} - 17(3^x) + 20 = 0$ for x

Solution 4

$$3^{2x+1} - 17(3^x) + 20 = 0$$
$$3(3^x)^2 - 17(3^x) + 20 = 0 \qquad y = 3^x$$
$$3y^2 - 17y + 20 = 0 \qquad P = 60 \quad S = -17$$
$$(y-4)\left(y - \frac{5}{3}\right) = 0 \qquad (-12,-5) \quad \left(-4, -\frac{5}{3}\right)$$
$$y = 4 \qquad y = \frac{5}{3}$$
$$3^x = 4 \qquad 3^x = \frac{5}{3}$$
$$x = 1.26 \quad x = 0.46$$

Answer part 1: Label $x =$ Solution 0.46,1.26
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 5 Experience: 50 Order: k1 Level: k1 Question-ID: 212

$3\log_4 x - 4\log_{16} x = 2$ for x

Solution 5

$$
\begin{aligned}
3\log_4 x - 4\log_{16} x &= 2 \\
3\log_4 x - 4\left(\frac{\log_4 x}{\log_4 16}\right) &= 2 \\
3\log_4 x - 4\left(\frac{\log_4 x}{2}\right) &= 2 \\
3\log_4 x - 2\log_4 x &= 2 \\
\log_4 x^3 - \log_4 x^2 &= 2 \\
\log_4 \frac{x^3}{x^2} &= 2 \\
\log_4 x &= 2 \\
x &= 4^2 \\
x &= 16
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 16
Answer part 1 hint:

Question 6 Experience: 50 Order: k2 Level: k2 Question-ID: 213
Solve $5\log_2 x - 9\log_8 x = 10$ for x

Solution 6

$$
\begin{aligned}
5\log_2 x - 9\log_8 x &= 10 \\
5\log_2 x - 9\left(\frac{\log_2 x}{\log_2 8}\right) &= 10 \\
5\log_2 x - 9\left(\frac{\log_4 x}{3}\right) &= 10 \\
\log_2 x^5 - \log_2 x^3 &= 10 \\
\log_2 \frac{x^5}{x^3} &= 10 \\
x^2 &= 2^{10} \\
x &= 32
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 32
Answer part 1 hint:

Question 7 Experience: 50 Order: k2 Level: k2 Question-ID: 214
Solve $2\log_2 x + 5\log_4 x = 18$ for x

Solution 7

$$
\begin{aligned}
2\log_2 x + 5\log_4 x &= 16 \\
2\log_2 x + 5\left(\frac{\log_2 x}{\log_2 4}\right) &= 18 \\
2\log_2 x + 5\left(\frac{\log_2 x}{2}\right) &= 18 \\
\log_2 x^2 + \log_2 x^{\frac{5}{2}} &= 18 \\
\log_2 x^{\frac{9}{2}} &= 18 \\
\frac{9}{2}\log_2 x &= 18 \\
\log_2 x &= 4 \\
x &= 2^4 \\
x &= 16
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 16
Answer part 1 hint:

Question 8 Experience: 35 Order: k3 Level: k3 Question-ID: 216

Solve $\log_2(3x+2) - \log_2(x-8) = 4$ for x

Solution 8

$$
\begin{aligned}
\log_2(3x+2) - \log_2(x-8) &= 4 \\
\log_2 \frac{3x+2}{x-8} &= 4 \\
\frac{3x+2}{x-8} &= 16 \\
3x+2 &= 16(x-8) \\
3x+2 &= 16x - 128 \\
13x &= 130 \\
x &= 10
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 10
Answer part 1 hint:

Question 9 Experience: 50 Order: k3 Level: k3 Question-ID: 217

Solve $\log_4(6x+2) - \log_4(x-3) = 2$ for x

Solution 9

$$
\begin{aligned}
\log_4(6x+2) - \log_4(x-3) &= 2 \\
\log_4 \frac{6x+2}{x-3} &= 2 \\
\frac{6x+2}{x-3} &= 16 \\
6x+2 &= 16(x-3) \\
6x+2 &= 16x - 48 \\
10x &= 50 \\
x &= 5
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 5
Answer part 1 hint:

Question 10 Experience: 45 Order: l2 Level: l2 Question-ID: 220

Solve $\log_5 x = 8 - \dfrac{15}{\log_5 x}$ for all values of$x$

Solution 10

$$\log_5 x = 8 - \frac{15}{\log_5 x}$$

$$(\log_5 x)^2 = 8\log_5 x - 15$$

$$(\log_5 x)^2 - 8\log_5 x + 15 = 0 \qquad P = 15 \qquad S = -8$$

$$(\log_5 x - 5)(\log_5 x - 3) = 0 \qquad (-5, -3)$$

$$\log_5 x = 5 \qquad \log_5 x = 3$$

$$x = 5^5 \qquad x = 5^3$$

$$x = 3125 \qquad x = 125$$

Answer part 1: Label $x =$ Solution 125,3125
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 11 Experience: 45 Order: l4 Level: l4 Question-ID: 225

Solve $\log_2(x-6) = \log_4(5x) + \log_4(2x+3)$ for all values of x

Solution 11

$$\log_2(x-6) = \log_4(5x) + \log_4(2x+3)$$

$$\log_2(x-6) = \log_4 5x(2x+3)$$

$$\log_2(x-6) = \frac{\log_2 5x(2x+3)}{\log_2 4}$$

$$\log_2(x-6) = \frac{\log_2 5x(2x+3)}{2}$$

$$2\log_2(x-6) = \log_2 5x(2x+3)$$

$$\log_2(x-6)^2 = \log_2 5x(2x+3)$$

$$(x-6)^2 = 5x(2x+3)$$

$$x^2 - 12x + 36 = 10x^2 + 15x$$

$$9x^2 + 27x - 36 = 0$$

$$x^2 + 3x - 4 = 0 \qquad P = -4 \qquad S = 3$$

$$(x+4)(x-1) = 0 \qquad (4, -1)$$

$$x = -4 \qquad x = 1$$

Answer part 1: Label $x =$ Solution -4,1
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 12 Experience: 45 Order: l3 Level: l3 Question-ID: 223

Solve $\log_2 x + \log_2(x+1) = \log_2(x+28) - \log_2 2$ for all values of x.

Solution 12

$$\log_2 x + \log_2(x+1) = \log_2(x+28) - \log_2 2$$

$$\log_2 x(x+1) = \log_2 \frac{x+28}{2}$$

$$x(x+1) = \frac{x+28}{2}$$

$$2x^2 + 2x = x + 28$$

$$2x^2 + x - 28 = 0 \qquad P = -56 \qquad S = 1$$

$$\left(x - \frac{7}{2}\right)(x-4) = 0 \qquad (-7, 8) \qquad \left(-\frac{7}{2}, 4\right)$$

$$x = \frac{7}{2} \qquad x = 4$$

$$x = 3.5$$

Answer part 1: Label $x =$ Solution 3.5,4
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 13 Experience: 45 Order: l3 Level: l3 Question-ID: 222
Solve $\log_4 3 - \log_4(x+4) = \log_4 2 - 2\log_4 x$ for all values of x.

Solution 13

$$\log_4 3 - \log_4(x+4) = \log_4 2 - 2\log_4 x$$

$$\log_4 \frac{3}{x+4} = \log_4 \frac{2}{x^2}$$

$$\frac{3}{x+4} = \frac{2}{x^2}$$

$$3x^2 = 2(x+4)$$

$$3x^2 - 2x - 8 = 0 \qquad P = -24 \qquad S = -2$$

$$(x-2)\left(x + \frac{4}{3}\right) = 0 \qquad (-6, 4) \qquad \left(-2, \frac{4}{3}\right)$$

$$x = 2 \qquad x = -\frac{4}{3}$$

$$x = -1.33$$

Answer part 1: Label $x =$ Solution -1.33,2
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 14 Experience: 45 Order: l4 Level: l4 Question-ID: 224
Solve $\log_9(6x^2 - 12x + 1) = \log_3(x-5)$ for all values of x

Solution 14

$$\log_9(6x^2-12x+1)=\log_3(x-5)$$
$$\frac{\log_3(6x^2-12x+1)}{\log_3 9}=\log_3(x-5)$$
$$\log_3 6x^2-12x+1=2\log_3(x-5)$$
$$\log_3 6x^2-12x+1=\log_3(x-5)^2$$
$$6x^2-12x+1=(x-5)^2$$
$$6x^2-12x+1=x^2-10x+25$$
$$5x^2-2x-24=0 \qquad P=-120 \qquad S=-2$$
$$(x+2)\left(x-\frac{12}{5}\right)=0 \qquad (10,-12) \qquad \left(2,-\frac{12}{5}\right)$$
$$x=-2 \qquad x=-\frac{12}{5}$$
$$x=-2.4$$

Answer part 1: Label $x=$ Solution -2.4,-2
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 15 Experience: 45 Order: l1 Level: l1 Question-ID: 218
Solve $\log_4(x+9)+\log_4(x+3)=2$ for all values of x

Solution 15

$$\log_4(x+9)+\log_4(x+3)=2$$
$$\log_4(x+9)(x+3)=2$$
$$(x+9)(x+3)=4^2$$
$$x^2+12x+27=16$$
$$x^2+12x+11=0 \qquad P=11 \qquad S=12$$
$$(x+11)(x+1)=0 \qquad (11,1)$$
$$x=-1$$

Answer part 1: Label $x=$ Solution -1
Answer part 1 hint:

Question 16 Experience: 45 Order: l1 Level: l1 Question-ID: 219
Solve $\log_3(x+12)+\log_3(x+4)=2$ for all values of x

Solution 16

$$\log_3(x+12)+\log_3(x+4)=2$$
$$\log_3(x+12)(x+4)=2$$
$$(x+12)(x+4)=3^2$$
$$x^2+16x+48=9$$
$$x^2+16x+39=0 \qquad P=39 \qquad S=16$$
$$(x+13)(x+3)=0 \qquad (13,3)$$
$$x=-3$$

Answer part 1: Label $x=$ Solution -3
Answer part 1 hint:

Question 17 Experience: 45 Order: l1 Level: l1 Question-ID: 221
Solve $\log_{10}(x+10)+\log_{10}(x+4)=2$ for all values of x

Solution 17

$$\log_{10}(x+10)+\log_{10}(x+4)=2$$
$$\log_{10}(x+10)(x+4)=2$$
$$(x+10)(x+4)=10^2$$
$$x^2+14x+140=100$$
$$x^2+14x+40=0 \qquad P=40 \qquad S=14$$
$$(x+10)(x+4)=0 \qquad (10,4)$$
$$x=-4$$

Answer part 1: Label $x=$ Solution -4
Answer part 1 hint:

Question 18 Experience: 50 Order: j1 Level: j1 Question-ID: 236
Solve $3^{2x}-12(3^x)+27=0$ for x

Solution 18

$$3^{2x}-12(3^x)+27=0$$
$$(3^x)^2-12(3^x)+27=0 \qquad y=3^x$$
$$y^2-12y+27=0 \qquad P=27 \quad S=-12$$
$$(y-9)(y-3)=0 \qquad (-9,-3)$$
$$y=9 \qquad y=3$$
$$3^x=9 \qquad 3^x=3$$
$$x=2 \qquad x=1$$

Answer part 1: Label $x=$ Solution 1,2
Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 19 Experience: 50 Order: j2 Level: j2 Question-ID: 237

Solve $2(5^{2x+1}) - 17(5^x) + 3 = 0$ for x

Solution 19

$$
\begin{aligned}
2(5^{2x+1}) - 17(5^x) + 3 &= 0 \\
2 \cdot 5 \cdot (5^{2x}) - 17(5^x) + 3 &= 0 \\
10(5^x)^2 - 17(5^x) + 3 &= 0 \qquad y = 5^x \\
10y^2 - 17y + 3 &= 0 \qquad P = 30 \quad S = -17 \\
\left(y - \frac{3}{2}\right)\left(y - \frac{1}{5}\right) &= 0 \qquad (-15, -2) \quad \left(-\frac{3}{2}, -\frac{1}{5}\right) \\
y = \frac{3}{2} \qquad y &= \frac{1}{5} \\
5^x = \frac{3}{2} \qquad 5^x &= \frac{1}{5} \\
x = 0.25 \quad x &= -1
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 0.25,-1

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 20 Experience: 50 Order: k1 Level: k1 Question-ID: 211

Solve $2\log_3 x - 12\log_9 x = 4$ for x

Solution 20

$$
\begin{aligned}
\log_3 x - 6\log_9 x &= 4 \\
\log_3 x - 6\left(\frac{\log_3 x}{\log_3 9}\right) &= 4 \\
\log_3 x - 6\left(\frac{\log_3 x}{2}\right) &= 4 \\
\log_3 x - 3\log_3 x &= 4 \\
\log_3 x - \log_3 x^3 &= 4 \\
\log_3 \frac{x}{x^3} &= 4 \\
\frac{1}{x^2} &= 3^4 \\
\frac{1}{x} &= 3^2 \\
x &= \frac{1}{9} \\
x &= 0.11
\end{aligned}
$$

Answer part 1: Label $x =$ Solution 0.11

Answer part 1 hint: Write the answer to 2 d.p.

End of Chapter Questions

OCR A Level Maths

To be added .....

AQA A Level Maths

To be added .....

MEI A Level Maths

To be added .....

Unit 1 C 1

Chapter 1 asdfasfd

End of Chapter Questions

Edexcel A Level Further Maths

To be added .....

MEI A Level Further Maths

To be added .....

Unused Questions

Question 1 Experience: 50 Order: Level: Question-ID: 44

A sequence is defined by $2U_{n+4} = 3U_{n+3} - U_n, \quad U_1 = 2, U_5 = 5, U_2 = 2U_4$, find the values of U_2, U_4 and U_6.

Solution 1

$$2U_5 = 3U_4 - U_1$$
$$2(5) = 3U_4 - 2$$
$$3U_4 = 12$$
$$U_4 = 4$$

$$U_2 = 2U_4$$
$$U_2 = 2(4)$$
$$U_2 = 8$$

$$2U_6 = 3U_5 - U_2$$
$$2U_6 = 3(5) - 8$$
$$U_6 = \frac{7}{2}$$

Choice 1: $U_2 = 10\ U_4 = 3\ U_6 = 2$ false

Choice 2: $U_2 = 8\ U_4 = 3\ U_6 = \frac{7}{2}$ false
Choice 3: $U_2 = 6\ U_4 = 4\ U_6 = 2$ false
Choice 4: $U_2 = 6\ U_4 = 3\ U_6 = \frac{7}{2}$ false
Choice 5: $U_2 = 8\ U_4 = 4\ U_6 = \frac{7}{2}$ true

Question 2 Experience: 50 Order: Level: Question-ID: 45

A sequence is defined by $3x_{n+4} = \frac{2x_{n+3}}{x_n}, \quad x_1 = 3, x_4 = 9, \frac{x_6}{x_3} = 6$, find the value of x_5 and x_7.

Solution 2

$$3x_5 = \frac{2x_4}{x_1}$$
$$3x_5 = \frac{2(9)}{3}$$
$$x_5 = 2$$

$$3x_7 = \frac{2x_6}{x_3}$$
$$3x_7 = 2(6)$$
$$x_7 = 4$$

Choice 1: $x_5 = 2\ x_7 = 8$ false
Choice 2: $x_5 = 6\ x_7 = 4$ false
Choice 3: $x_5 = 6\ x_7 = 8$ false
Choice 4: $x_5 = 6\ x_7 = 2$ false
Choice 5: $x_5 = 2\ x_7 = 4$ true

Question 3 Experience: 60 Order: Level: Question-ID: 61

A sequence is defined by the recurrence relation $Y_{n+1} = \frac{a^2}{Y_n} + b, Y_1 = 3, a, b \in \mathbb{N}$, given that $Y_2 = 7$ and $Y_3 = \frac{37}{7}$ find the value of a and b.

Solution 3

$$Y_2 = \frac{a^2}{Y_1} + b$$

$$7 = \frac{a^2}{3} + b \qquad (1)$$

$$Y_3 = \frac{a^2}{Y_2} + b$$

$$\frac{37}{7} = \frac{a^2}{7} + b \qquad (2)$$

(1) − (2) $$7 - \frac{37}{7} = \frac{a^2}{3} + b - \left(\frac{a^2}{7} + b\right)$$

$$\frac{12}{7} = \frac{4a^2}{21}$$

$$a^2 = 9$$

$$a = 3$$

Sub into(1) $$7 = \frac{3^2}{3} + b$$

$$b = 4$$

Choice 1:	$a = 3$	$b = 3$	false
Choice 2:	$a = 2$	$b = 3$	false
Choice 3:	$a = 2$	$b = 4$	false
Choice 4:	$a = 4$	$b = 2$	false
Choice 5:	$a = 3$	$b = 4$	true

Question 4 Experience: 60 Order: Level: Question-ID: 60

A sequence is defined by the recurrence relation $u_{n+1} = \sqrt{a}\left(u_n - \frac{1}{b}\right), 5u_1 = 4$, given that $u_2 = 7$ and $u_3 = 13$ find the value of a and b .

Solution 4

$$u_2 = \sqrt{a}\left(u_1 - \frac{1}{b}\right)$$

$$7 = \sqrt{a}\left(4 - \frac{1}{b}\right) \qquad (1)$$

$$7 = 4\sqrt{a} - \frac{\sqrt{a}}{b} \qquad (2)$$

$$u_3 = \sqrt{a}\left(u_2 - \frac{1}{b}\right)$$

$$13 = \sqrt{a}\left(7 - \frac{1}{b}\right)$$

$$13 = 7\sqrt{a} - \frac{\sqrt{a}}{b} \qquad (3)$$

(3) − (2) $13 - 7 = 7\sqrt{a} - \frac{\sqrt{a}}{b} - \left(4\sqrt{a} - \frac{\sqrt{a}}{b}\right)$

$$6 = 3\sqrt{a}$$

$$2 = \sqrt{a}$$

$$a = 4$$

Sub into (1) $7 = \sqrt{4}\left(4 - \frac{1}{b}\right)$

$$\frac{7}{2} = 4 - \frac{1}{b}$$

$$-\frac{1}{2} = -\frac{1}{b}$$

$$b = 2$$

Choice 1:	$a = 3$	$b = 2$	false
Choice 2:	$a = 4$	$b = 3$	false
Choice 3:	$a = 3$	$b = 3$	false
Choice 4:	$a = 2$	$b = 3$	false
Choice 5:	$a = 4$	$b = 2$	true

Question 5 Experience: 50 Order: Level: Question-ID: 59

A sequence is defined by the recurrence relation $Y_{n+1} = \frac{a^2}{Y_n} + b, Y_1 = 3, a, b > 0$, given that $Y_2 = 7$ and $Y_3 = \frac{37}{7}$ find the value of a and b.

Solution 5

$$Y_2 = \frac{a^2}{Y_1} + b$$

$$7 = \frac{a^2}{3} + b \qquad (1)$$

$$Y_3 = \frac{a^2}{Y_2} + b$$

$$\frac{37}{7} = \frac{a^2}{7} + b \qquad (2)$$

(1) − (2) $$7 - \frac{37}{7} = \frac{a^2}{3} + b - \left(\frac{a^2}{7} + b\right)$$

$$\frac{12}{7} = \frac{4a^2}{21}$$

$$a^2 = 9$$

$$a = 3$$

Sub into(1) $$7 = \frac{3^2}{3} + b$$

$$b = 4$$

Choice 1:	$a = 3$	$b = 3$	false
Choice 2:	$a = 2$	$b = 3$	false
Choice 3:	$a = 2$	$b = 4$	false
Choice 4:	$a = 4$	$b = 2$	false
Choice 5:	$a = 3$	$b = 4$	true

Question 6 Experience: 100 Order: Level: Question-ID: 38

A sequence is defined by $X_n = \frac{a+1}{n} + b$, given the Sum of the first three terms is $\frac{2}{3}$ and the fifth term is $-\frac{3}{5}$, find the values of a and b.

Solution 6

$$S_3 = \left(\frac{a+1}{(1)} + b\right) + \left(\frac{a+1}{(2)} + b\right) + \left(\frac{a+1}{(3)} + b\right)$$

$$S_3 = a + \frac{a}{2} + \frac{a}{3} + 3b + 1 + \frac{1}{2} + \frac{1}{3}$$

$$S_3 = \frac{11}{6}a + 3b + \frac{11}{6} \quad S_3 = \frac{2}{3}$$

$$\frac{11}{6}a + 3b + \frac{11}{6} = \frac{2}{3}$$

$$11a + 18b + 11 = 4 \quad (1)$$

$$X_5 = \frac{a+1}{5} + b \qquad X_5 = -\frac{3}{5}$$

$$\frac{a+1}{5} + b = -\frac{3}{5}$$

$$a + 1 + 5b = -3 \quad (2)$$

$$11a + 11 + 55b = -33 \quad (3)$$

$$(3) - (1) \quad 11a + 11 + 55b - (11a + 18b + 11) = -33 - 4$$

$$37b = -37$$

$$b = -1$$

$$\text{sub into} \quad (2) \quad a + 1 + 5(-1) = -3$$

$$a - 4 = -3$$

$$a = 1$$

Choice 1: $a = 1 \quad b = 2$ false
Choice 2: $a = 3 \quad b = -1$ false
Choice 3: $a = 3 \quad b = 2$ false
Choice 4: $a = 2 \quad b = -1$ false
Choice 5: $a = 1 \quad b = -1$ true

Question 7 Experience: 35 Order: z Level: z Question-ID: 215

Solve $\log_3(x+6) - \log_3\left(\frac{x}{2} - 4\right) = 2$ for x

Solution 7

$$\log_3(x+6) - \log_3\left(\frac{x}{2} - 4\right) = 2$$

$$\log_3 \frac{x+6}{\left(\frac{x}{2} - 4\right)} = 2$$

$$\frac{2(x+6)}{x-8} = 9$$

$$2(x+6) = 9(x-8)$$

$$2x + 12 = 9x - 72$$

$$7x = 84$$

$$x = 12$$

Answer part 1: Label $x =$ Solution 12
Answer part 1 hint:

Question 8 Experience: 45 Order: l1 Level: l1 Question-ID: 241

Solve $\log_2(x-3) + \log_2(3x-7) = 4$ for all values of x

Solution 8

$$\log_2(x-3) + \log_2(3x-7) = 4$$
$$\log_2(x-3)(3x-7) = 4$$
$$(x-3)(3x-7) = 2^4$$
$$3x^2 - 16x + 21 = 16$$
$$3x^2 - 16x + 5 = 0 \qquad P = 15 \qquad S = -16$$
$$(x-5)\left(x - \frac{1}{5}\right) = 0 \qquad (-15, -1) \qquad \left(-5, -\frac{1}{5}\right)$$
$$x = 5$$

Answer part 1: Label $x =$ Solution 5
Answer part 1 hint:

Question 9 Experience: 50 Order: k1 Level: k1 Question-ID: 238

Solve $4\log_2 x + 3\log_8 x = 10$ for x

Solution 9

$$4\log_2 x + 3\log_8 x = 10$$
$$4\log_2 x + 3\left(\frac{\log_2 x}{\log_2 8}\right) = 10$$
$$4\log_2 x + 3\left(\frac{\log_2 x}{3}\right) = 10$$
$$4\log_2 x + \log_2 x = 10$$
$$\log_2 x^4 + \log_2 x = 10$$
$$\log_2 x^4 \text{ x } x = 10$$
$$x^5 = 2^{10}$$
$$x = 2^2$$
$$x = 4$$

Answer part 1: Label $x =$ Solution 4
Answer part 1 hint:

Question 10 Experience: 50 Order: k2 Level: k2 Question-ID: 239

Solve $\log_3 x - 8\log_9 x = 6$ for x

Solution 10

$$\log_3 x - 8\log_9 x = 6$$
$$\log_3 x - 8\left(\frac{\log_3 x}{\log_3 9}\right) = 6$$
$$\log_3 x - 8\left(\frac{\log_3 x}{2}\right) = 6$$
$$\log_3 x - 4\log_3 x = 6$$
$$\log_3 x - \log_3 x^4 = 6$$
$$\log_3 \frac{x}{x^4} = 6$$
$$\frac{1}{x^3} = 3^6$$
$$\frac{1}{x} = 3^2$$
$$x = \frac{1}{9}$$
$$x = 0.11$$

Answer part 1: Label $x =$ Solution 0.11
Answer part 1 hint: Write the answer to 2 d.p.

Question 11 Experience: 50 Order: z Level: z Question-ID: 229

Find values for x and y from the following, $x > 0, y > 0$:

$$3\log_y x^2 = 4$$
$$y^2 = x^2 + 12x$$

Solution 11

$$3\log_y x^2 = 4 \qquad (1)$$
$$y^2 = x^2 + 12x \qquad (2)$$

Simplifying (1) $\log_y x^2 = \frac{4}{3}$

$$x^2 = y^{\frac{4}{3}}$$
$$x^6 = y^4$$
$$x^3 = y^2 \qquad (3)$$

Sub (3) into (2) $x^3 = x^2 + 12x$

$$x^3 - x^2 - 12x = 0$$
$$x(x^2 - x - 12) = 0 \qquad P = -12 \qquad S = -1$$
$$x(x-4)(x+3) = 0 \qquad (-4, 3)$$
$$x = 4$$

Sub $x = 4$ into (3) $4^3 = y^2$

$$y^2 = 64$$
$$y = 8$$

Answer part 1: Label $x =$ Solution 4
Answer part 1 hint:
Answer part 2: Label $y =$ Solution 8
Answer part 2 hint:

Question 12 Experience: 50 Order: z Level: z Question-ID: 228

Find values for x and y from the following, $x > 0$:

$$3\log_x y - \log_x 125 = 2$$
$$125y = x^2$$

Solution 12

$$3\log_x y - \log_x 125 = 2 \qquad (1)$$
$$125y = x^2 \qquad (2)$$

Simplifying (1)
$$3\log_x y - 3\log_x 5 = 2$$
$$\log_x y - \log_x 5 = \frac{2}{3}$$
$$\log_x \frac{y}{5} = \frac{2}{3}$$
$$\frac{y}{5} = x^{\frac{2}{3}}$$
$$y = 5x^{\frac{2}{3}} \qquad (3)$$

Simplifying (2)
$$y = \frac{x^2}{125} \qquad (4)$$

(3) = (4)
$$5x^{\frac{2}{3}} = \frac{x^2}{125}$$
$$x^{\frac{4}{3}} = 625$$
$$x = 625^{\frac{3}{4}}$$
$$x = 125$$

Sub $x = 125$ into (2)
$$125y = (125)^2$$
$$y = 125$$

Answer part 1: Label $x =$ Solution 125
Answer part 1 hint:
Answer part 2: Label $y =$ Solution 125
Answer part 2 hint:

Question 13 Experience: 50 Order: z Level: z Question-ID: 227

Find values for x and y from the following:

$$\log_4 4x + \log_5 5y = 4$$
$$\log_2 2x + 6\log_{25} y = 6$$

Solution 13

$$\log_4 4x + \log_5 5y = 4 \qquad (1)$$

$$\log_2 2x + 6\log_{25} y = 6 \qquad (2)$$

Simplifying (1) $\log_4 4 + \log_4 x + \log_5 5 + \log_5 y = 4$

$$1 + \frac{\log_2 x}{\log_2 4} + 1 + \log_5 y = 4$$

$$\frac{\log_2 x}{2} + \log_5 y = 2$$

$$\log_2 x + 2\log_5 y = 4 \qquad (3)$$

Simplifying (2) $\log_2 2 + \log_2 x + \frac{6\log_5 y}{\log_5 25} = 6$

$$1 + \log_2 x + \frac{6\log_5 y}{2} = 6$$

$$\log_2 x + 3\log_5 y = 5 \qquad (4)$$

(4) − (3) $\log_2 x + 3\log_5 y - (\log_2 x + 2\log_5 y) = 5 - 4$

$$\log_5 y = 1$$

$$y = 5$$

Sub $y = 5$ into (3) $\log_2 x + 2\log_5 5 = 4$

$$\log_2 x + 2 = 4$$

$$\log_2 x = 2$$

$$x = 2^2$$

$$x = 4$$

Answer part 1: Label $x =$ Solution 4
Answer part 1 hint:
Answer part 2: Label $y =$ Solution 5
Answer part 2 hint:

Question 14 Experience: 50 Order: z Level: z Question-ID: 226

Find values for x and y from the following:

$$5\log_3 x - 2\log_5 y = 8$$

$$3\log_{27} x = 4\log_{25} y$$

Solution 14

$$5\log_3 x - 2\log_5 y = 8 \qquad (1)$$

$$3\log_{27} x = 4\log_{25} y \qquad (2)$$

Simplifying (2)

$$\frac{3\log_3 x}{\log_3 27} = \frac{4\log_5 y}{\log_5 25}$$

$$\frac{3\log_3 x}{3} = \frac{4\log_5 y}{2}$$

$$\log_3 x = 2\log_5 y \qquad (3)$$

Sub (3) into (1)

$$5(2\log_5 y) - 2\log_5 y = 8$$

$$10\log_5 y - 2\log_5 y = 8$$

$$8\log_5 y = 8$$

$$\log_5 y = 1$$

$$y = 5$$

Sub $y = 5$ into (1)

$$5\log_3 x - 2\log_5 5 = 8$$

$$5\log_3 x - 2 = 8$$

$$\log_3 x = 2$$

$$x = 3^2$$

$$x = 9$$

Answer part 1: Label $x =$ Solution 9
Answer part 1 hint:
Answer part 2: Label $y =$ Solution 5
Answer part 2 hint:

Question 15 Experience: 35 Order: k3 Level: k3 Question-ID: 240

Solve $\log_3(4x-5) - \log_3(x-5) = 2$ for x

Solution 15

$$\log_3(4x-5) - \log_3(x-5) = 2$$

$$\log_3 \frac{4x-5}{x-5} = 2$$

$$\frac{4x-5}{x-5} = 9$$

$$4x - 5 = 9(x-5)$$

$$4x - 5 = 9x - 45$$

$$5x = 40$$

$$x = 8$$

Answer part 1: Label $x =$ Solution 8
Answer part 1 hint: