

Edexcel A Level Maths

Core 1, Core 2, Core 3, Core 4 and two units from (Decision 1, Decision 2, Mechanics 1, Mechanics 2, Statistics 1 and Statistics 2). Each unit is out of 100 UMS, giving a total of 600 UMS. 480 and above is grade A. To obtain an A*, total score must be 480 or more, and total of C3 and C4 must be 180 or more.

Unit 1 Core 1

Chapter 1 Sequence and Series 1

Lesson 1 Sequence and Summation

Question 1 Experience: 10 Order: Level: Question-ID: 26

A sequence is defined by $a_n = 3n^2 - 4$, find the value of a_2 .

Solution 1

$$a_2 = 3(2)^2 - 4 = 8$$

Choice 1: $a_2 = -1$ false

Choice 2: $a_2 = 23$ false

Choice 3: $a_2 = 5$ false

Choice 4: $a_2 = 10$ false

Choice 5: $a_2 = 8$ true

Question 2 Experience: 25 Order: Level: Question-ID: 30

A sequence is defined by $x_n = 3n^2 - 5n + 2$, find the value of n such that $x_n = 14$.

Solution 2

$$x_n = 3n^2 - 5n + 2 = 14$$

$$3n^2 - 5n - 12 = 0 \quad S = -5 \quad P = -36$$

$$\left(n + \frac{4}{3}\right)(n - 3) = 0 \quad (4, -9) \quad \left(\frac{4}{3}, -3\right)$$

$$n = 3$$

Choice 1: $n = 4$ false

Choice 2: $n = 2$ false

Choice 3: $n = 5$ false

Choice 4: $n = 6$ false

Choice 5: $n = 3$ true

Question 3 Experience: 15 Order: Level: Question-ID: 27

A sequence is defined by $x_n = 6n - 3$, find the value of x_3 and x_5 .

Solution 3

$$x_3 = 6(3) - 3 = 15$$

$$x_5 = 6(5) - 3 = 27$$

- Choice 1: $x_3 = 3$ $x_5 = 15$ false
 Choice 2: $x_3 = 9$ $x_5 = 21$ false
 Choice 3: $x_3 = 15$ $x_5 = 21$ false
 Choice 4: $x_3 = 3$ $x_5 = 21$ false
 Choice 5: $x_3 = 15$ $x_5 = 27$ true

Question 4 Experience: 15 Order: Level: Question-ID: 31

A sequence is defined by $u_n = an - b$, find the sum of the first four terms in terms of a and b .

Solution 4

$$u_1 + u_2 + u_3 + u_4 = (a - b) + (2a - b) + (3a - b) + (4a - b) = 10a - 4b$$

- Choice 1: $10a - 6b$ false
 Choice 2: $6a - 4b$ false
 Choice 3: $6a - 6b$ false
 Choice 4: $10a - 8b$ false
 Choice 5: $10a - 4b$ true

Question 5 Experience: 15 Order: Level: Question-ID: 32

A sequence is defined by $x_n = an^2 - 4$, find the sum of the first three terms in terms of a .

Solution 5

$$x_1 + x_2 + x_3 = (a - 4) + (4a - 4) + (9a - 4) = 14a - 12$$

- Choice 1: $14a - 8$ false
 Choice 2: $5a - 12$ false
 Choice 3: $5a - 8$ false
 Choice 4: $5a - 14$ false
 Choice 5: $14a - 12$ true

Question 6 Experience: 25 Order: Level: Question-ID: 33

A sequence is defined by $y_n = an^2 + bn + c$, find the sum of the first three terms in terms of a, b and c .

Solution 6

$$y_1 + y_2 + y_3 = (a + b + c) + (4a + 2b + c) + (9a + 3b + c) = 14a + 6b + 3c$$

wrong choice

- Choice 1: $14a - 6b + 3c$ false
 Choice 2: $6a + 6b + 3c$ false
 Choice 3: $6a + 4b + 2c$ false
 Choice 4: $6a + 4b + 3c$ false
 Choice 5: $14a + 6b + 3c$ true

Question 7 Experience: 10 Order: Level: Question-ID: 34

A sequence is defined by $x_n = 4n - b$, find the third term in terms of b .

Solution 7

$$x_3 = 4(3) - b$$

$$x_3 = 12 - b$$

- Choice 1: $x_3 = 12 - 3b$ false
 Choice 2: $x_3 = 6 - b$ false
 Choice 3: $x_3 = 6 - 3b$ false
 Choice 4: $x_3 = 8 - 3b$ false
 Choice 5: $x_3 = 12 - b$ true

Question 8 Experience: 10 Order: Level: Question-ID: 35

A sequence is defined by $U_n = \frac{a}{n} + b$, find the fourth term in terms of a and b .

Solution 8

$$U_4 = \frac{a}{4} + b$$

- Choice 1: $U_4 = \frac{a+b}{4}$ false
 Choice 2: $U_4 = \frac{a}{4} + 4b$ false
 Choice 3: $U_4 = \frac{a}{4} + 2b$ false
 Choice 4: $U_4 = \frac{a}{4} + b$ true
 Choice 5: $U_4 = \frac{a+4b}{8}$ false

Question 9 Experience: 15 Order: Level: Question-ID: 36

A sequence is defined by $y_n = \frac{a-3b}{n^2}$, find the fifth term in terms of a and b .

Solution 9

$$y_5 = \frac{a-3b}{(5)^2}$$

$$y_5 = \frac{a-3b}{25}$$

- Choice 1: $y_5 = \frac{5a-3b}{25}$ false
 Choice 2: $y_5 = \frac{5a-3b}{16}$ false
 Choice 3: $y_5 = \frac{a-3b}{16}$ false
 Choice 4: $y_5 = \frac{5a-b}{25}$ false
 Choice 5: $y_5 = \frac{a-3b}{25}$ true

Question 10 Experience: 50 Order: Level: Question-ID: 37

A sequence is defined by $U_n = an + 2b$, given the Sum of the first four terms is 26 and the fifth term is 9, find the values of a and b .

Solution 10

$$S_4 = (a + 2b) + (2a + 2b) + (3a + 2b) + (4a + 2b)$$

$$S_4 = 10a + 8b \quad S_4 = 26$$

$$10a + 8b = 26$$

$$5a + 4b = 13 \quad (1)$$

$$U_5 = 5a + 2b \quad U_5 = 9$$

$$5a + 2b = 9 \quad (2)$$

$$(1) - (2) \quad 5a + 4b - (5a + 2b) = 13 - 9$$

$$2b = 4$$

$$b = 2$$

$$\text{sub into } (2) \quad 5a + 2(2) = 9$$

$$5a = 5$$

$$a = 1$$

$$\text{Choice 1:} \quad a = 1 \quad b = 3 \quad \text{false}$$

$$\text{Choice 2:} \quad a = 2 \quad b = 3 \quad \text{false}$$

$$\text{Choice 3:} \quad a = 2 \quad b = 2 \quad \text{false}$$

$$\text{Choice 4:} \quad a = 3 \quad b = 2 \quad \text{false}$$

$$\text{Choice 5:} \quad a = 1 \quad b = 2 \quad \text{true}$$

Question 11 Experience: 25 Order: Level: Question-ID: 40

A sequence is defined by $U_{n+1} = U_n - 4$, $U_1 = 20$, find the values of U_2, U_3 and U_4 .

Solution 11

$$U_2 = U_1 - 4 = 20 - 4 = 16$$

$$U_3 = U_2 - 4 = 16 - 4 = 12$$

$$U_4 = U_3 - 4 = 12 - 4 = 8$$

$$\text{Choice 1:} \quad U_2 = 16 \quad U_3 = 12 \quad U_4 = 4 \quad \text{false}$$

$$\text{Choice 2:} \quad U_2 = 12 \quad U_3 = 8 \quad U_4 = 4 \quad \text{false}$$

$$\text{Choice 3:} \quad U_2 = 12 \quad U_3 = 4 \quad U_4 = 0 \quad \text{false}$$

$$\text{Choice 4:} \quad U_2 = 16 \quad U_3 = 4 \quad U_4 = 4 \quad \text{false}$$

$$\text{Choice 5:} \quad U_2 = 16 \quad U_3 = 12 \quad U_4 = 8 \quad \text{true}$$

Question 12 Experience: 25 Order: Level: Question-ID: 41

A sequence is defined by $X_{n+1} = X_n + 5$, $X_4 = 17$, find the values of X_1, X_2 and X_3 .

Solution 12

$$X_4 = X_3 + 5$$

$$17 = X_3 + 5$$

$$X_3 = 12$$

$$X_3 = X_2 + 5$$

$$12 = X_2 + 5$$

$$X_2 = 7$$

$$X_2 = X_1 + 5$$

$$7 = X_1 + 5$$

$$X_1 = 2$$

Choice 1: $X_1 = 2$ $X_2 = 6$ $X_3 = 12$ false

Choice 2: $X_1 = 5$ $X_2 = 8$ $X_3 = 11$ false

Choice 3: $X_1 = 5$ $X_2 = 7$ $X_3 = 9$ false

Choice 4: $X_1 = 5$ $X_2 = 6$ $X_3 = 10$ false

Choice 5: $X_1 = 2$ $X_2 = 7$ $X_3 = 12$ true

Question 13 Experience: 30 Order: Level: Question-ID: 42

A sequence is defined by $a_{n+1} = (a_n)^2 - 4$, $a_1 = 2$, find the values of a_2 , a_3 and a_4 .

Solution 13

$$a_2 = (a_1)^2 - 4 = 4 - 4 = 0$$

$$a_3 = (a_2)^2 - 4 = 0 - 4 = -4$$

$$a_4 = (a_3)^2 - 4 = (-4)^2 - 4 = 16 - 4 = 12$$

Choice 1: $a_2 = 0$ $a_3 = 4$ $a_4 = 12$ false

Choice 2: $a_2 = 4$ $a_3 = -4$ $a_4 = 8$ false

Choice 3: $a_2 = 4$ $a_3 = 8$ $a_4 = -8$ false

Choice 4: $a_2 = 0$ $a_3 = 8$ $a_4 = 12$ false

Choice 5: $a_2 = 0$ $a_3 = -4$ $a_4 = 12$ true

Question 14 Experience: 25 Order: Level: Question-ID: 43

A sequence is defined by $y_{n+2} = 3y_{n+1} - y_n$, $y_1 = 3$, $y_2 = 2$, find the values of y_3 , y_4 and y_5 .

Solution 14

$$y_3 = 3(y_2) - y_1 = 3(2) - 3 = 3$$

$$y_4 = 3(y_3) - y_2 = 3(3) - 2 = 7$$

$$y_5 = 3(y_4) - y_3 = 3(7) - 3 = 18$$

Choice 1: $y_3 = 3$ $y_4 = 5$ $y_5 = 18$ false

Choice 2: $y_3 = 7$ $y_4 = 4$ $y_5 = 5$ false

Choice 3: $y_3 = 7$ $y_4 = 8$ $y_5 = 5$ false

Choice 4: $y_3 = 3$ $y_4 = 7$ $y_5 = 5$ false

Choice 5: $y_3 = 3$ $y_4 = 7$ $y_5 = 18$ true

Question 15 Experience: 15 Order: Level: Question-ID: 46

Calculate the following sum:

$$\sum_{r=2}^5 (r-1)$$

Solution 15

$$\begin{aligned}\sum_{r=2}^5 (r-1) &= (2-1) + (3-1) + (4-1) + (5-1) \\ &= 1 + 2 + 3 + 4 \\ &= 10\end{aligned}$$

Choice 1: 9 false

Choice 2: 8 false

Choice 3: 11 false

Choice 4: 12 false

Choice 5: 10 true

Question 16 Experience: 15 Order: Level: Question-ID: 47

Calculate the following sum:

$$\sum_{r=4}^8 (r^2 - 2r + 1)$$

Solution 16

$$\begin{aligned}\sum_{r=4}^8 (r^2 - 2r + 1) \\ &= \sum_{r=4}^8 (r-1)^2 \\ &= (4-1)^2 + (5-1)^2 + (6-1)^2 + (7-1)^2 + (8-1)^2 \\ &= 9 + 16 + 25 + 36 + 49 \\ &= 135\end{aligned}$$

Choice 1: 137 false

Choice 2: 128 false

Choice 3: 130 false

Choice 4: 136 false

Choice 5: 135 true

Question 17 Experience: 10 Order: Level: Question-ID: 28

A sequence is defined by $X_n = 2n - 1$, find the value of n such that $a_n = 15$.

Solution 17

$$\begin{aligned}15 &= 2n - 1 \\ n &= 8\end{aligned}$$

- Choice 1: $n = 6$ false
 Choice 2: $n = 7$ false
 Choice 3: $n = 3$ false
 Choice 4: $n = 9$ false
 Choice 5: $n = 8$ true

Question 18 Experience: 30 Order: Level: Question-ID: 53

Calculate the following sum:

$$\sum_{r=5}^9 U_r \quad U_r = 3r^2 + 4$$

Solution 18

$$\begin{aligned} & \sum_{r=5}^9 U_r \\ &= \sum_{r=5}^9 3r^2 + 4 \\ &= (3(5)^2 + 4) + (3(6)^2 + 4) + (3(7)^2 + 4) + (3(8)^2 + 4) + (3(9)^2 + 4) \\ &= 785 \end{aligned}$$

- Choice 1: 795 false
 Choice 2: 790 false
 Choice 3: 780 false
 Choice 4: 800 false
 Choice 5: 785 true

Question 19 Experience: 30 Order: Level: Question-ID: 52

Calculate the following sum:

$$\sum_{r=1}^3 a_r \quad a_r = 4r - 1$$

Solution 19

$$\begin{aligned} & \sum_{r=1}^3 a_r \\ &= \sum_{r=1}^3 4r - 1 \\ &= (4(1) - 1) + (4(2) - 1) + (4(3) - 1) \\ &= 21 \end{aligned}$$

- Choice 1: 22 false
 Choice 2: 19 false
 Choice 3: 20 false
 Choice 4: 18 false
 Choice 5: 21 true

Question 20 Experience: 45 Order: Level: Question-ID: 54

A sequence is defined by $U_{n+1} = 3(U_n - 1), U_1 = 2$, find the following sum: $\sum_2^4 (U_r + 2)^2$

Solution 20

$$U_2 = 3(2 - 1)$$

$$U_2 = 3$$

$$U_3 = 3(3 - 1)$$

$$U_3 = 6$$

$$U_4 = 3(6 - 1)$$

$$U_4 = 15$$

$$\begin{aligned}\sum_2^4 (U_r + 2)^2 &= (U_2 + 2)^2 + (U_3 + 2)^2 + (U_4 + 2)^2 \\ &= (3 + 2)^2 + (6 + 2)^2 + (15 + 2)^2 \\ &= 5^2 + 8^2 + 17^2 \\ &= 378\end{aligned}$$

Choice 1: 380 false

Choice 2: 377 false

Choice 3: 379 false

Choice 4: 381 false

Choice 5: 378 true

Question 21 Experience: 15 Order: Level: Question-ID: 48

Calculate the following sum:

$$\sum_{r=1}^4 (2r + 4)$$

Solution 21

$$\begin{aligned}&\sum_{r=1}^4 (2r + 4) \\ &= \sum_{r=1}^4 2(r + 2) \\ &= 2 \sum_{r=1}^4 (r + 2) \\ &= 2[(1 + 2) + (2 + 2) + (3 + 2) + (4 + 2)] \\ &= 2(3 + 4 + 5 + 6) \\ &= 36\end{aligned}$$

Choice 1: 37 false

Choice 2: 35 false

Choice 3: 34 false

Choice 4: 33 false

Choice 5: 36 true

Question 22 Experience: 25 Order: Level: Question-ID: 50

Calculate the following sum:

$$\sum_{r=3}^6 (r^2 - 1)$$

Solution 22

$$\begin{aligned} & \sum_{r=3}^6 (r^2 - 1) \\ &= (3^2 - 1) + (4^2 - 1) + (5^2 - 1) + (6^2 - 1) \\ &= 8 + 15 + 24 + 35 \\ &= 82 \end{aligned}$$

Choice 1: 81 false

Choice 2: 80 false

Choice 3: 83 false

Choice 4: 84 false

Choice 5: 82 true

Question 23 Experience: 15 Order: Level: Question-ID: 51

Calculate the following sum:

$$\sum_{r=1}^{45} 2$$

Solution 23

$$\begin{aligned} & \sum_{r=1}^{45} 2 \\ &= 2 + 2 + 2 + 2 + 2 + \dots + 2 \\ &= 2 \times 45 \\ &= 90 \end{aligned}$$

Choice 1: 94 false

Choice 2: 92 false

Choice 3: 88 false

Choice 4: 86 false

Choice 5: 90 true

Question 24 Experience: 15 Order: Level: Question-ID: 49

Calculate the following sum:

$$\sum_{r=1}^{100} 5$$

Solution 24

$$\begin{aligned}
& \sum_{r=1}^{100} 5 \\
&= 5 + 5 + 5 + 5 + 5 + 5 + \dots + 5 \\
&= 5 \times 100 \\
&= 500
\end{aligned}$$

- Choice 1: 495 false
 Choice 2: 490 false
 Choice 3: 480 false
 Choice 4: 500 true
 Choice 5: 485 false

Question 25 Experience: 25 Order: Level: Question-ID: 25

A sequence is defined by $U_n = 2n + 3$, find the value of U_2, U_4 and U_5 .

Solution 25

$$U_2 = 2(2) + 3 = 7$$

$$U_4 = 2(4) + 3 = 11$$

$$U_5 = 2(5) + 3 = 13$$

- Choice 1: $U_2 = 5$ $U_4 = 9$ $U_5 = 11$ false
 Choice 2: $U_2 = 7$ $U_4 = 9$ $U_5 = 13$ false
 Choice 3: $U_2 = 7$ $U_4 = 10$ $U_5 = 15$ false
 Choice 4: $U_2 = 7$ $U_4 = 11$ $U_5 = 13$ true
 Choice 5: $U_2 = 5$ $U_4 = 11$ $U_5 = 13$ false

Question 26 Experience: 25 Order: Level: Question-ID: 29

A sequence is defined by $u_n = 2n^2 - 5n - 3$, find the value of n such that $u_n = 9$.

Solution 26

$$u_n = 2n^2 - 5n - 3 = 9$$

$$2n^2 - 5n - 12 = 0 \quad S = -5 \quad P = -24$$

$$\left(n + \frac{3}{2}\right)(n - 4) = 0 \quad (3, -8) \quad \left(\frac{3}{2}, -4\right)$$

$$n = 4$$

- Choice 1: $n = 5$ false
 Choice 2: $n = 2$ false
 Choice 3: $n = 3$ false
 Choice 4: $n = 6$ false
 Choice 5: $n = 4$ true

Question 27 Experience: 50 Order: Level: Question-ID: 39

A sequence is defined by $a_n = an^2 + b$, given the Sum of the first five terms is -5 and the sixth term is 4, find the

values of a and b .

Solution 27

$$S_5 = (a + b) + (4a + b) + (9a + b) + (16a + b)$$

$$S_5 = 30a + 5b \quad S_5 = -5$$

$$30a + 5b = -5$$

$$6a + b = -1 \quad (1)$$

$$a_6 = 25a + b \quad a_6 = 4$$

$$36a + b = 4 \quad (2)$$

$$(2) - (1) \quad 36a + b - (6a + b) = 4 - (-1)$$

$$30a = 5$$

$$a = \frac{1}{6}$$

$$\text{sub into (1)} \quad 6\left(\frac{1}{6}\right) + b = -1$$

$$b = -2$$

$$\text{Choice 1:} \quad a = \frac{1}{6} \quad b = 2 \quad \text{false}$$

$$\text{Choice 2:} \quad a = 1 \quad b = -2 \quad \text{false}$$

$$\text{Choice 3:} \quad a = 1 \quad b = \frac{2}{1} \quad \text{false}$$

$$\text{Choice 4:} \quad a = 2 \quad b = \frac{1}{6} \quad \text{false}$$

$$\text{Choice 5:} \quad a = \frac{1}{6} \quad b = -2 \quad \text{true}$$

Lesson 2 Arithmetic Sequence 1

Question 1 Experience: 30 Order: Level: Question-ID: 114

$$\text{Evaluate } \sum_{r=1}^{15} (5r + 2)$$

Solution 1

$$\sum_{r=1}^{15} (5r + 2) = 7 + 12 + 17 + 22 + \dots + 77$$

$$a = 7 \quad l = 77 \quad n = 15$$

$$\sum_{r=1}^{15} (5r + 2) = \frac{15}{2} (7 + 77)$$

$$= 630$$

$$\text{Choice 1:} \quad \sum_{r=1}^{15} (5r + 2) = 625 \quad \text{false}$$

$$\text{Choice 2:} \quad \sum_{r=1}^{15} (5r + 2) = 620 \quad \text{false}$$

$$\text{Choice 3:} \quad \sum_{r=1}^{15} (5r + 2) = 615 \quad \text{false}$$

Choice 4: $\sum_{r=1}^{15} (5r + 2) = 635$ false

Choice 5: $\sum_{r=1}^{15} (5r + 2) = 630$ true

Question 2 Experience: 30 Order: Level: Question-ID: 100

How many terms are there in the arithmetic sequence 19,21,23,...,87

Solution 2

$$a = 19 \quad d = 2$$

$$U_n = a + (n - 1)d$$

$$87 = 19 + (n - 1)2$$

$$n - 1 = 34$$

$$n = 35$$

Choice 1: $n = 38$ false

Choice 2: $n = 37$ false

Choice 3: $n = 36$ false

Choice 4: $n = 34$ false

Choice 5: $n = 35$ true

Question 3 Experience: 30 Order: Level: Question-ID: 101

How many terms are there in the arithmetic sequence 21,26,31,...,256

Solution 3

$$a = 21 \quad d = 5$$

$$U_n = a + (n - 1)d$$

$$256 = 21 + (n - 1)5$$

$$n - 1 = 47$$

$$n = 48$$

Choice 1: $n = 51$ false

Choice 2: $n = 47$ false

Choice 3: $n = 50$ false

Choice 4: $n = 49$ false

Choice 5: $n = 48$ true

Question 4 Experience: 35 Order: Level: Question-ID: 102

How many terms are there in the arithmetic sequence 88,86,84,...,22

Solution 4

Reverse the order of the sequence 22,24,26,28...88

$$a = 88 \quad d = 2$$

Choice 1: $U_n = a + (n - 1)d$ false

Choice 2: $88 = 22 + (n - 1)2$ false

Choice 3: $n = 32$ false

$$n - 1 = 33$$

Choice 4: $n = 33$ false

Choice 5: $n = 34$ true

Question 5 Experience: 30 Order: Level: Question-ID: 103

Evaluate $S = 1 + 2 + 3 + 4 + \dots + 50$

Solution 5

$$S = 1 + 2 + 3 + 4 + \dots + 50$$

$$S = 50 + 49 + 48 + 47 + \dots + 1$$

$$2S = 51 \times 50$$

$$S = \frac{51 \times 50}{2}$$

$$S = 1275$$

Choice 1: $S = 1270$ false

Choice 2: $S = 1280$ false

Choice 3: $S = 1285$ false

Choice 4: $S = 1290$ false

Choice 5: $S = 1275$ true

Question 6 Experience: 30 Order: Level: Question-ID: 104

Evaluate $T = 2 + 4 + 6 + 8 + \dots + 100$

Solution 6

$$T = 2 + 4 + 6 + 8 + \dots + 100$$

$$T = 100 + 98 + 96 + 94 + \dots + 2$$

$$2T = 102 \times 50$$

$$T = \frac{102 \times 50}{2}$$

$$T = 2550$$

Choice 1: $T = 2565$ false

Choice 2: $T = 2560$ false

Choice 3: $T = 2555$ false

Choice 4: $T = 2545$ false

Choice 5: $T = 2550$ true

Question 7 Experience: 30 Order: Level: Question-ID: 105

Evaluate $R = 1 + 3 + 5 + 7 + \dots + 99$

Solution 7

$$R = 1 + 3 + 5 + 7 + \dots + 99$$

$$R = 99 + 97 + 95 + 93 + \dots + 1$$

$$2R = 100 \times 100$$

$$R = \frac{100 \times 100}{2}$$

$$R = 5000$$

- Choice 1: $R = 5015$ false
 Choice 2: $R = 5010$ false
 Choice 3: $R = 5005$ false
 Choice 4: $R = 4995$ false
 Choice 5: $R = 5000$ true

Question 8 Experience: 30 Order: Level: Question-ID: 106

Evaluate $S = 1 + 2 + 3 + 4 + \dots + 200$

Solution 8

$$S = 1 + 2 + 3 + 4 + \dots + 200$$

$$S = 200 + 199 + 198 + 197 + \dots + 1$$

$$2S = 201 \times 200$$

$$S = \frac{201 \times 200}{2}$$

$$S = 20100$$

- Choice 1: $S = 20115$ false
 Choice 2: $S = 20110$ false
 Choice 3: $S = 20105$ false
 Choice 4: $S = 20095$ false
 Choice 5: $S = 20100$ true

Question 9 Experience: 30 Order: Level: Question-ID: 107

Evaluate $T = 102 + 104 + 106 + 108 + \dots + 200$

Solution 9

$$T = 102 + 104 + 106 + 108 + \dots + 200$$

$$T = 200 + 198 + 196 + 194 + \dots + 102$$

$$2T = 302 \times 50$$

$$T = \frac{302 \times 50}{2}$$

$$T = 7550$$

- Choice 1: $T = 7565$ false
 Choice 2: $T = 7560$ false
 Choice 3: $T = 7555$ false
 Choice 4: $T = 7545$ false
 Choice 5: $T = 7550$ true

Question 10 Experience: 40 Order: Level: Question-ID: 109

Find the sum of all numbers divisible by 5 between 1 and 300

Solution 10

$$300 \div 5 = 60$$

$$\Rightarrow \text{last term} = 300$$

$$S = 5 + 10 + 15 + \dots + 300$$

$$a = 5 \quad d = 5 \quad U_n = 300$$

$$U_n = a + (n - 1)d$$

$$300 = 5 + (n - 1)5$$

$$n = 60$$

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{60}{2}(5 + 300)$$

$$\text{Choice 1: } S = 9165 \quad \text{false}$$

$$\text{Choice 2: } S = 9150 \quad \text{false}$$

$$\text{Choice 3: } S = 9155 \quad \text{false}$$

$$\text{Choice 4: } S = 9145 \quad \text{false}$$

$$\text{Choice 5: } S = 9150 \quad \text{true}$$

Question 11 Experience: 40 Order: Level: Question-ID: 110

Find the sum of all numbers divisible by 7 between 1 and 200

Solution 11

$$200 \div 7 = 28 \text{ remainder } 4$$

$$\Rightarrow \text{last term} = 7 \times 28 = 196$$

$$S = 7 + 14 + 21 + \dots + 196$$

$$a = 7 \quad d = 7 \quad U_n = 196$$

$$U_n = a + (n - 1)d$$

$$\text{Choice 1: } n = 28 \quad S = 2863 \quad \text{false}$$

$$\text{Choice 2: } n = 28 \quad S = 2856 \quad \text{false}$$

$$\text{Choice 3: } S = 2849 \quad \text{false}$$

$$\text{Choice 4: } S = \frac{n}{2}(a + l) \quad S = 2835 \quad \text{false}$$

$$\text{Choice 5: } S = \frac{n}{2}(a + l) \quad S = 2842 \quad \text{true}$$

$$S = \frac{28}{2}(7 + 196)$$

Question 12 Experience: 45 Order: Level: Question-ID: 111

Evaluate $S = 27 + 31 + 35 + 39 + \dots + 107$

Solution 12

$$S = 27 + 31 + 35 + 39 + \dots + 107$$

$$a = 27 \quad d = 4 \quad U_n = 107$$

$$U_n = a + (n - 1)d$$

$$\text{Choice 1: } n = 27 \quad S = 1414 \quad \text{false}$$

$$\text{Choice 2: } n = 21 \quad S = 1386 \quad \text{false}$$

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$$S = \frac{21}{2}(27 + 107)$$

- Choice 3: $S = 1393$ false
 Choice 4: $S = 1400$ false
 Choice 5: $S = 1407$ true

Question 13 Experience: 45 Order: Level: Question-ID: 112

Evaluate $T = 31 + 33 + 35 + 37 + \dots + 81$

Solution 13

$$T = 31 + 33 + 35 + 37 + \dots + 81$$

$$a = 31 \quad d = 2 \quad U_n = 81$$

$$U_n = a + (n - 1)d$$

Choice 1: $S = 1402$ false

Choice 2: $S = 1460$ false

Choice 3: $S = 1458$ false

Choice 4: $S = 1454$ false

Choice 5: $S = 1456$ true

$$S = \frac{26}{2}(31 + 81)$$

Question 14 Experience: 45 Order: Level: Question-ID: 113

Evaluate $R = 97 + 92 + 87 + 82 + \dots + 22$

Solution 14

Reverse the sequence: $R = 22 + 27 + 32 + 27 + \dots + 97$

$$R = 22 + 27 + 32 + 27 + \dots + 97$$

$$a = 22 \quad d = 5 \quad U_n = 97$$

$$U_n = a + (n - 1)d$$

$$97 = 22 + (n - 1)5$$

$$n = 16$$

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{16}{2}(22 + 97)$$

$$S = 952$$

- Choice 1: $S = 950$ false
 Choice 2: $S = 954$ false
 Choice 3: $S = 948$ false
 Choice 4: $S = 950$ false
 Choice 5: $S = 952$ true

Question 15 Experience: 30 Order: Level: Question-ID: 115

Evaluate $\sum_{r=9}^{35} (3r - 1)$

Solution 15

$$\sum_{r=9}^{35} (3r - 1) = 26 + 29 + 32 + 35 + \dots + 104$$

$$a = 26 \quad l = 104 \quad n = 27$$

$$\begin{aligned} \sum_{r=9}^{35} (3r - 1) &= \frac{27}{2} (26 + 104) \\ &= 1755 \end{aligned}$$

Choice 1: $\sum_{r=9}^{35} (3r - 1) = 1760$ false

Choice 2: $\sum_{r=9}^{35} (3r - 1) = 1740$ false

Choice 3: $\sum_{r=9}^{35} (3r - 1) = 1745$ false

Choice 4: $\sum_{r=9}^{35} (3r - 1) = 1750$ false

Choice 5: $\sum_{r=9}^{35} (3r - 1) = 1755$ true

Question 16 Experience: 30 Order: Level: Question-ID: 116

Evaluate $\sum_{r=1}^{20} (3r - 1)$

Solution 16

$$\sum_{r=1}^{20} (3r - 1) = 2 + 5 + 8 + 11 + \dots + 59$$

$$a = 2 \quad l = 59 \quad n = 20$$

$$\begin{aligned} \sum_{r=1}^{20} (3r - 1) &= \frac{20}{2} (2 + 59) \\ &= 610 \end{aligned}$$

Choice 1: $\sum_{r=1}^{20} (3r - 1) = 625$ false

Choice 2: $\sum_{r=1}^{20} (3r - 1) = 605$ false

Choice 3: $\sum_{r=1}^{20} (3r - 1) = 615$ false

Choice 4: $\sum_{r=1}^{20} (3r - 1) = 620$ false

Choice 5: $\sum_{r=1}^{20} (3r - 1) = 610$ true

Question 17 Experience: 30 Order: Level: Question-ID: 117

Evaluate $\sum_{r=21}^{45} (2r - 25)$

Solution 17

$$\sum_{r=21}^{45} (2r - 25) = 17 + 19 + 21 + 23 + \dots + 65$$

$$a = 17 \quad l = 65 \quad n = 25$$

$$\begin{aligned} \sum_{r=21}^{45} (2r - 25) &= \frac{25}{2}(17 + 65) \\ &= 1025 \end{aligned}$$

Choice 1: $\sum_{r=21}^{45} (2r - 25) = 1020$ false

Choice 2: $\sum_{r=21}^{45} (2r - 25) = 1015$ false

Choice 3: $\sum_{r=21}^{45} (2r - 25) = 1010$ false

Choice 4: $\sum_{r=21}^{45} (2r - 25) = 1030$ false

Choice 5: $\sum_{r=21}^{45} (2r - 25) = 1025$ true

Question 18 Experience: 20 Order: Level: Question-ID: 84

The first three terms of an arithmetic sequence are 3,5,7, find U_{10}

Solution 18

$$a = 3 \quad n = 10 \quad d = 5 - 3 = 2$$

$$U_n = a + (n - 1)d$$

$$U_{10} = 3 + (10 - 1)2 = 21$$

Choice 1: $U_{10} = 20$ false

Choice 2: $U_{10} = 17$ false

Choice 3: $U_{10} = 18$ false

Choice 4: $U_{10} = 19$ false

Choice 5: $U_{10} = 21$ true

Question 19 Experience: 20 Order: Level: Question-ID: 85

The first four terms of an arithmetic sequence are 5,9,13,17, find A_7

Solution 19

$$a = 5 \quad n = 7 \quad d = 9 - 5 = 4$$

$$A_n = a + (n - 1)d$$

$$A_7 = 5 + (7 - 1)4 = 29$$

Choice 1: $A_7 = 28$ false

Choice 2: $A_7 = 27$ false

Choice 3: $A_7 = 30$ false

Choice 4: $A_7 = 26$ false

Choice 5: $A_7 = 29$ true

Question 20 Experience: 20 Order: Level: Question-ID: 89

The first three terms of an arithmetic sequence are 22,19,16, find X_6

Solution 20

$$a = 22 \quad n = 6 \quad d = 22 - 19 = 3$$

$$X_n = a + (n - 1)d$$

$$X_7 = 22 + (6 - 1)3 = 37$$

Choice 1: $A_7 = 28$ false

Choice 2: $A_7 = 27$ false

Choice 3: $A_7 = 30$ false

Choice 4: $A_7 = 26$ false

Choice 5: $A_7 = 29$ true

Question 21 Experience: 40 Order: Level: Question-ID: 90

a_n is an arithmetic sequence, given that $a_3 = 13$ and $a_6 = 19$, find a_{11}

Solution 21

$$a_n = a + (n - 1)d$$

$$a_3 = a + (3 - 1)d = a + 2d = 13 \quad (1)$$

$$a_6 = a + (6 - 1)d = a + 5d = 19 \quad (2)$$

$$(2) - (1) \quad a + 5d - (a + 2d) = 19 - 13$$

$$3d = 6$$

$$d = 2$$

$$\text{Sub into(1)} \quad a + 2(2) = 13$$

$$a = 9$$

$$a_{11} = 9 + (11 - 1)2 = 29$$

Choice 1: $a_{11} = 25$ false

Choice 2: $a_{11} = 26$ false

Choice 3: $a_{11} = 27$ false

Choice 4: $a_{11} = 28$ false

Choice 5: $a_{11} = 29$ true

Question 22 Experience: 40 Order: Level: Question-ID: 91

U_n is an arithmetic sequence, given that $U_4 = 25$ and $U_9 = 40$, find U_{13}

Solution 22

$$U_n = a + (n - 1)d$$

$$U_4 = a + (4 - 1)d = a + 3d = 25 \quad (1)$$

$$U_9 = a + (9 - 1)d = a + 8d = 40 \quad (2)$$

$$(2) - (1) \quad a + 8d - (a + 3d) = 40 - 25$$

$$5d = 15$$

$$d = 3$$

$$\text{Sub into(1)} \quad a + 3(3) = 25$$

$$a = 16$$

$$\text{Choice 1:} \quad U_{13} = 51 \quad \text{false} \quad U_{13} = 16 + (13 - 1)3 = 52$$

$$\text{Choice 2:} \quad U_{13} = 50 \quad \text{false}$$

$$\text{Choice 3:} \quad U_{13} = 49 \quad \text{false}$$

$$\text{Choice 4:} \quad U_{13} = 53 \quad \text{false}$$

$$\text{Choice 5:} \quad U_{13} = 52 \quad \text{true}$$

Question 23 Experience: 45 Order: Level: Question-ID: 96

X_n is an arithmetic sequence, given that $X_{13} = 51$ and $X_{19} = 33$, find X_{10}

Solution 23

$$X_n = a + (n - 1)d$$

$$X_{13} = a + (13 - 1)d = a + 12d = 51 \quad (1)$$

$$X_{19} = a + (19 - 1)d = a + 18d = 33 \quad (2)$$

$$(2) - (1) \quad a + 18d - (a + 12d) = 33 - 51$$

$$6d = -18$$

$$d = -3$$

$$\text{Sub into(1)} \quad a + 12(-3) = 51$$

$$a = 87$$

$$\text{Choice 1:} \quad X_{10} = 63 \quad \text{false} \quad X_{10} = 87 + (10 - 1)(-3) = 60$$

$$\text{Choice 2:} \quad X_{10} = 62 \quad \text{false}$$

$$\text{Choice 3:} \quad X_{10} = 61 \quad \text{false}$$

$$\text{Choice 4:} \quad X_{10} = 59 \quad \text{false}$$

$$\text{Choice 5:} \quad X_{10} = 60 \quad \text{true}$$

Question 24 Experience: 45 Order: Level: Question-ID: 97

u_n is an arithmetic sequence, given that $u_3 = 5$ and $u_7 = 13$, for what value of n is $u_n = 71$

Solution 24

$$u_n = a + (n - 1)d$$

$$u_3 = a + (3 - 1)d = a + 2d = 5 \quad (1)$$

$$u_7 = a + (7 - 1)d = a + 6d = 13 \quad (2)$$

$$(2) - (1) \quad a + 6d - (a + 2d) = 13 - 5$$

$$4d = 8$$

$$d = 2$$

$$\text{Sub into (1)} \quad a + 2(2) = 5$$

$$a = 1$$

$$u_n = 1 + (n - 1)2 = 71$$

Choice 1: $n = 35$ false $n - 1 = 35$

Choice 2: $n = 32$ false $n = 36$

Choice 3: $n = 33$ false

Choice 4: $n = 34$ false

Choice 5: $n = 36$ true

Question 25 Experience: 30 Order: Level: Question-ID: 98

The first three terms of an arithmetic sequence are 11,14,17, find a n for which $U_n = 83$

Solution 25

$$u_n = 83 \quad a = 11 \quad d = 3$$

$$u_n = a + (n - 1)d$$

$$83 = 11 + (n - 1)3$$

$$n - 1 = 24$$

$$n = 25$$

Choice 1: $n = 24$ false

Choice 2: $n = 23$ false

Choice 3: $n = 22$ false

Choice 4: $n = 26$ false

Choice 5: $n = 25$ true

Question 26 Experience: 45 Order: Level: Question-ID: 99

Y_n is an arithmetic sequence, given that $Y_{15} = 51$ and $X_{19} = 71$, find Y_{26}

Solution 26

$$Y_n = a + (n - 1)d$$

$$Y_{15} = a + (15 - 1)d = a + 14d = 51 \quad (1)$$

$$Y_{19} = a + (19 - 1)d = a + 18d = 71 \quad (2)$$

$$(2) - (1) \quad a + 18d - (a + 14d) = 71 - 51$$

$$4d = 20$$

$$d = 5$$

$$\text{Sub into (1)} \quad a + 14(5) = 51$$

Choice 1:	$Y_{26} = 102$	false	$a = -19$
Choice 2:	$Y_{26} = 103$	false	
Choice 3:	$Y_{26} = 104$	false	$Y_{26} = -19 + (26 - 1)5 = 106$
Choice 4:	$Y_{26} = 105$	false	
Choice 5:	$Y_{26} = 106$	true	

Question 27 Experience: 50 Order: Level: Question-ID: 147

Kendrick decides to open up a savings account. He puts in 100 for the first month, 120 for the second month and an extra 20 for subsequent months till he's putting in 300 a month. Find the total amount he's saved in 2 years.

Solution 27

Sequence goes: 100,120,140,160,180,200...300,300,300,300...

$$U_n = a + (n - 1)d$$

$$U_n = 300 \quad a = 100 \quad d = 20$$

$$300 = 100 + (n - 1)20$$

$$n = 11$$

$$S_n = \frac{n}{2}(a + l)$$

$$n = 11 \quad a = 100 \quad l = 300$$

$$S_{11} = \frac{11}{2}(100 + 300)$$

$$S_{11} = 2200$$

Every term after is 300

$$\sum_{r=12}^{24} 300 = 13 \times 300$$

$$= 3900$$

$$\Rightarrow \text{Total days} = 2200 + 3900 = 6100$$

Choice 1:	6105	false
Choice 2:	6085	false

Choice 3: 6090 false
 Choice 4: 6095 false
 Choice 5: 6100 true

Question 28 Experience: 50 Order: Level: Question-ID: 144

Avery is playing with 340 sticks, she puts them in rows. The first row has 7 sticks, next row has 13 sticks, subsequent rows have 6 more sticks than the previous row. She has enough for k rows but not enough for $k + 1$ rows. Find k .

Solution 28

Sequence goes: 7,13,19,25,31,37....

Not having enough for $k+1$ rows means that $S_k \leq 340$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_k = \frac{k}{2}(2(7) + (k-1)6)$$

$$S_k = k(7 + 3(k-1))$$

$$S_k = k(3k + 4)$$

$$S_k = 3k^2 + 4k \quad (1)$$

$$S_k \leq 340$$

$$(1) \quad 3k^2 + 4k \leq 340$$

$$3k^2 + 4k - 340 \leq 0 \quad P = -1020 \quad S = 4$$

Choice 1: $k = 9$ false
 Choice 2: $k = 10$ false $(34, -30) \quad \left(\frac{34}{3}, -10\right)$

Choice 3: $k = 7$ false

Choice 4: $k = 8$ false

Choice 5: $k = 10$ true

Question 29 Experience: 50 Order: Level: Question-ID: 146

Griffin is training daily for a cycling marathon in 100 days. He cycles 10km on the first day, 11km on the second day and 1 more km than the previous day till he's cycling 40km a day. Calculate the total number of km he's cycled as training for the marathon.

Solution 29

Sequence goes: 10,11,12,13,14,15...40,40,40,40...

$$U_n = a + (n - 1)d$$

$$U_n = 40 \quad a = 10 \quad d = 1$$

$$40 = 10 + (n - 1)1$$

$$n = 31$$

$$S_n = \frac{n}{2}(a + l)$$

$$n = 31 \quad a = 10 \quad l = 40$$

$$S_{31} = \frac{31}{2}(10 + 40)$$

$$S_{31} = 775$$

Every term after is 40

$$\sum_{r=32}^{100} 40 = 69 \times 40$$

$$= 2760$$

$$\Rightarrow \text{Total days} = 775 + 2760 = 3535$$

Choice 1: 3540 false

Choice 2: 3520 false

Choice 3: 3525 false

Choice 4: 3530 false

Choice 5: 3535 true

Question 30 Experience: 50 Order: Level: Question-ID: 145

Heidi is training daily for a swimming competition in 60 days. She swims 10 laps on the first day, 12 laps on the second day and 2 more laps than the previous day till she's swimming 30 laps a day. Calculate the total number of laps she's swum as training for the competition.

Solution 30

Sequence goes: 10,12,14,16,18,20...30,30,30,30...

$$U_n = a + (n - 1)d$$

$$U_n = 30 \quad a = 10 \quad d = 2$$

$$30 = 10 + (n - 1)2$$

$$n = 11$$

$$S_n = \frac{n}{2}(a + l)$$

$$n = 11 \quad a = 10 \quad l = 30$$

$$S_{11} = \frac{11}{2}(10 + 30)$$

$$S_{11} = 220$$

Every term after is 30

$$\begin{aligned} \sum_{r=12}^{60} 30 &= 49 \times 30 \\ &= 1470 \end{aligned}$$

$$\Rightarrow \text{Total days} = 220 + 1470 = 1690$$

Choice 1: 1685 false

Choice 2: 1705 false

Choice 3: 1700 false

Choice 4: 1695 false

Choice 5: 1690 true

Question 31 Experience: 40 Order: Level: Question-ID: 108

Find the sum of all numbers divisible by 3 between 2 and 200

Solution 31

$$200 \div 3 = 66 \text{ remainder } 2$$

$$\Rightarrow \text{last term} = 3 \times 66 = 198$$

$$S = 3 + 6 + 9 + \dots + 198$$

$$a = 3 \quad d = 3 \quad U_n = 198$$

$$U_n = a + (n - 1)d$$

$$198 = 3 + (n - 1)3$$

$$n = 66$$

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{66}{2}(3 + 198)$$

$$S = 6633$$

- Choice 1: $S = 6642$ false
 Choice 2: $S = 6639$ false
 Choice 3: $S = 6636$ false
 Choice 4: $S = 6630$ false
 Choice 5: $S = 6633$ true

Question 32 Experience: 50 Order: Level: Question-ID: 143

James is playing with 324 sticks, she puts them in rows. The first row has 5 sticks, next row has 9 sticks, subsequent rows have 4 more sticks than the previous row. She has enough for k rows but not enough for $k + 1$ rows. Find k .

Solution 32

Sequence goes: 5,9,13,17,21,25....

Not having enough for $k+1$ rows means that $S_k \leq 324$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_k = \frac{k}{2}(2(5) + (k-1)4)$$

$$S_k = k(5 + 2k - 2)$$

$$S_k = k(2k + 3)$$

$$S_k = 2k^2 + 3k \quad (1)$$

$$S_k \leq 324$$

$$(1) \quad 2k^2 + 3k \leq 324$$

$$2k^2 + 3k - 324 \leq 0 \quad P = -648 \quad S = 3$$

$$\left(k + \frac{27}{2}\right)(k - 12) \leq 0 \quad (27, -24) \quad \left(\frac{27}{2}, -12\right)$$

$$k = 12$$

- Choice 1: $k = 11$ false
 Choice 2: $k = 15$ false
 Choice 3: $k = 14$ false
 Choice 4: $k = 13$ false
 Choice 5: $k = 12$ true

Lesson 3 Recurrence Relations

Question 1 Experience: 50 Order: Level: Question-ID: 57

A sequence is defined by the recurrence relation $X_{n+1} = \sqrt{k}X_n - 2$, $X_1 = 2$, $k > 0$, given that $X_3 = 2$ find the value of k .

Solution 1

$$X_2 = \sqrt{k}X_1 - 2$$

$$X_2 = 2\sqrt{k} - 2$$

$$X_3 = \sqrt{k}X_2 - 2$$

$$X_3 = \sqrt{k}(2\sqrt{k} - 2) - 2$$

$$X_3 = 2k - 2\sqrt{k} - 2 \quad \text{set } x = \sqrt{k}$$

$$X_3 = 2x^2 - 2x - 2 \quad X_3 = 2$$

$$2 = 2x^2 - 2x - 2$$

$$1 = x^2 - x - 1$$

$$0 = x^2 - x - 2 \quad S = -1 \quad P = -2$$

$$0 = (x - 2)(x + 1) \quad (-2, 1)$$

$$\sqrt{k} = 2$$

$$k = 4$$

Choice 1: $k = 5$ false

Choice 2: $k = 3$ false

Choice 3: $k = 6$ false

Choice 4: $k = 7$ false

Choice 5: $k = 4$ true

Question 2 Experience: 50 Order: Level: Question-ID: 58

A sequence is defined by the recurrence relation $U_{n+1} = aU_n + \frac{1}{b}$, $U_1 = 3$, given that $U_2 = 7$ and $U_3 = 15$ find the value of a and b .

Solution 2

$$U_2 = aU_1 + \frac{1}{b} \quad U_2 = 7$$

$$7 = 3a + \frac{1}{b} \quad (1)$$

$$U_3 = aU_2 + \frac{1}{b} \quad U_2 = 7, U_3 = 15$$

$$15 = 7a + \frac{1}{b} \quad (2)$$

$$(2) - (1) \quad 15 - 7 = 7a + \frac{1}{b} - \left(3a + \frac{1}{b}\right)$$

$$8 = 4a$$

$$a = 2$$

$$\text{Sub into (1)} \quad 7 = 3(2) + \frac{1}{b}$$

$$\frac{1}{b} = 1$$

$$b = 1$$

- Choice 1: $a = 2$ $b = 3$ false
 Choice 2: $a = 3$ $b = 3$ false
 Choice 3: $a = 3$ $b = 1$ false
 Choice 4: $a = 1$ $b = 1$ false
 Choice 5: $a = 2$ $b = 1$ true

Question 3 Experience: 70 Order: Level: Question-ID: 63

A sequence is defined by the recurrence relation $a_{n+1} = ka_n - 4, k > 0, a_1 = 5$, given that $\sum_{r=1}^3 a_r = 19$, find the value of k .

Solution 3

$$a_2 = ka_1 - 4$$

$$a_2 = 5k - 4$$

$$a_3 = ka_2 - 4$$

$$a_3 = k(5k - 4) - 4$$

$$a_3 = 5k^2 - 4k - 4$$

$$\sum_{r=1}^3 a_r = a_1 + a_2 + a_3$$

$$\sum_{r=1}^3 a_r = (5) + (5k - 4) + (5k^2 - 4k - 4)$$

$$\sum_{r=1}^3 a_r = 5k^2 + k - 3 \quad \sum_{r=1}^3 a_r = 19$$

$$19 = 5k^2 + k - 3$$

$$0 = 5k^2 + k - 22 \quad S = 1 \quad P = -110$$

$$0 = \left(k + \frac{11}{5}\right)(k - 2) \quad (11, -10) \Rightarrow \left(\frac{11}{5}, -2\right)$$

$$k = 2$$

- Choice 1: $k = 3$ false
 Choice 2: $k = 4$ false
 Choice 3: $k = 1$ false
 Choice 4: $k = 5$ false
 Choice 5: $k = 2$ true

Question 4 Experience: 60 Order: Level: Question-ID: 64

A sequence is defined by the recurrence relation $U_{n+1} = 5U_n - \frac{1}{k}, k > 0, U_1 = 2$, given that $\sum_{r=1}^4 U_r = 293$, find the value of k .

Solution 4

$$U_2 = 5U_1 - \frac{1}{k}$$

$$U_2 = 5(2) - \frac{1}{k}$$

$$U_2 = 10 - \frac{1}{k}$$

$$U_3 = 5U_2 - \frac{1}{k}$$

$$U_3 = 5\left(10 - \frac{1}{k}\right) - \frac{1}{k}$$

$$U_3 = 50 - \frac{6}{k}$$

$$U_4 = 5U_3 - \frac{1}{k}$$

$$U_4 = 5\left(50 - \frac{6}{k}\right) - \frac{1}{k}$$

$$U_4 = 250 - \frac{31}{k}$$

$$\sum_{r=1}^4 U_r = U_1 + U_2 + U_3 + U_4$$

$$\sum_{r=1}^4 U_r = (2) + \left(10 - \frac{1}{k}\right) + \left(50 - \frac{6}{k}\right) + \left(250 - \frac{31}{k}\right)$$

$$\sum_{r=1}^4 U_r = 312 - \frac{38}{k} \quad \sum_{r=1}^4 U_r = 293$$

$$312 - \frac{38}{k} = 293$$

$$19 = \frac{38}{k}$$

$$k = 2$$

Choice 1: $k = 5$ false

Choice 2: $k = 4$ false

Choice 3: $k = 3$ false

Choice 4: $k = 1$ false

Choice 5: $k = 2$ false

Question 5 Experience: 100 Order: Level: Question-ID: 65

A sequence is defined by the recurrence relation $X_{n+1} = \frac{k}{X_n} + 3$, $X_1 = 1$, given that $2 \sum_{r=1}^3 X_r = 21$, find the value of k .

Solution 5

$$X_2 = \frac{k}{X_1} + 3$$

$$X_2 = \frac{k}{1} + 3$$

$$X_2 = k + 3$$

$$X_3 = \frac{k}{X_2} + 3$$

$$X_3 = \frac{k}{k+3} + 3$$

$$\sum_{r=1}^3 X_r = X_1 + X_2 + X_3$$

$$\sum_{r=1}^3 X_r = (1) + (k+3) + \left(\frac{k}{k+3} + 3\right)$$

$$\sum_{r=1}^3 X_r = k + 7 + \frac{k}{k+3} \quad 2 \sum_{r=1}^3 X_r = 21$$

$$21 = 2 \left(k + 7 + \frac{k}{k+3} \right)$$

$$21 = 2k + 14 + \frac{2k}{k+3}$$

$$7 = 2k + \frac{2k}{k+3}$$

$$7(k+3) = 2k(k+3) + 2k$$

$$7k + 21 = 2k^2 + 6k + 2k$$

$$0 = 2k^2 - k - 21 \quad S = -1 \quad P = -42$$

$$0 = \left(k + \frac{7}{2}\right)(k-3) \quad (7, -6) \Rightarrow \left(\frac{7}{2}, -3\right)$$

$$k = 3$$

Choice 1: $k = 5$ false

Choice 2: $k = 2$ false

Choice 3: $k = 4$ false

Choice 4: $k = 1$ false

Choice 5: $k = 3$ true

Question 6 Experience: 35 Order: Level: Question-ID: 66

A sequence is defined by the recurrence relation $a_{n+1} = a_n^2 - a_n$, given that a_n is a positive sequence and that $a_3 = 132$ find the value of a_1 .

Solution 6

$$a_3 = a_2^2 - a_2$$

$$132 = a_2^2 - a_2$$

$$0 = a_2^2 - a_2 - 132 \quad S = 1 \quad P = -132$$

$$0 = (a_2 + 11)(a_2 - 12) \quad (11, -12)$$

$$a_2 = 12$$

$$a_2 = a_1^2 - a_1$$

$$12 = a_1^2 - a_1$$

$$0 = a_1^2 - a_1 - 12$$

$$0 = (a_1 - 4)(a_1 + 3)$$

$$a_1 = 4$$

Choice 1: $a_1 = 6$ false

Choice 2: $a_1 = 5$ false

Choice 3: $a_1 = 3$ false

Choice 4: $a_1 = 7$ false

Choice 5: $a_1 = 4$ true

Question 7 Experience: 35 Order: Level: Question-ID: 67

A sequence is defined by the recurrence relation $U_{n+1} = 5U_n - \frac{6}{U_n}$, given that $U_3 = 13, U_2 > 0$, find the value of U_2 .

Solution 7

$$U_3 = 5U_2 - \frac{6}{U_2}$$

$$13 = 5U_2 - \frac{6}{U_2}$$

$$0 = 5U_2 - 13 - \frac{6}{U_2}$$

$$0 = 5(U_2)^2 - 13U_2 - 6 \quad S = -13 \quad P = -30$$

$$0 = \left(U_2 + \frac{2}{5}\right)(U_2 - 3) \quad (2, -15) \quad \left(\frac{2}{5}, -3\right)$$

$$U_2 = 3$$

Choice 1: $U_2 = 4$ false

Choice 2: $U_2 = 5$ false

Choice 3: $U_2 = 2$ false

Choice 4: $U_2 = 1$ false

Choice 5: $U_2 = 3$ true

Question 8 Experience: 15 Order: Level: Question-ID: 68

A sequence is defined by the recurrence relation $Y_{n+1} = 3Y_n - 5$, given that $Y_3 = 7$, find the value of Y_1 .

Solution 8

$$Y_3 = 3Y_2 - 5$$

$$7 = 3Y_2 - 5$$

$$Y_2 = 4$$

$$Y_2 = 3Y_1 - 5$$

$$4 = 3Y_1 - 5$$

$$Y_1 = 3$$

Choice 1: $Y_1 = 5$ false

Choice 2: $Y_1 = 4$ false

Choice 3: $Y_1 = 1$ false

Choice 4: $Y_1 = 2$ false

Choice 5: $Y_1 = 3$ true

Question 9 Experience: 40 Order: Level: Question-ID: 69

A sequence is defined by the recurrence relation $a_{n+1} = a_n - \frac{2a_n + 6}{a_n + 3}$, given that $a_2 = 5$, find the value of a_1 .

Solution 9

$$a_2 = a_1 - \frac{2a_1 + 6}{a_1 + 3}$$

$$5 = a_1 - \frac{2a_1 + 6}{a_1 + 3}$$

$$5(a_1 + 3) = a_1(a_1 + 3) - (2a_1 + 6)$$

$$5a_1 + 15 = (a_1)^2 + 3a_1 - 2a_1 - 6$$

$$0 = (a_1)^2 - 4a_1 - 21 \quad S = -4 \quad P = -21$$

$$0 = (a_1 + 3)(a_1 - 7) \quad (3, -7)$$

$$a_1 = 7$$

Choice 1: $a_1 = 8$ false

Choice 2: $a_1 = 4$ false

Choice 3: $a_1 = 5$ false

Choice 4: $a_1 = 6$ false

Choice 5: $a_1 = 7$ true

Question 10 Experience: 25 Order: Level: Question-ID: 70

A sequence is defined by the recurrence relation $X_{n+1} = 3(X_n)^2 - 11$, given that $X_1 = 2$, find $\sum_{r=1}^4 X_r$.

Solution 10

$$X_2 = 3(X_1)^2 - 11$$

$$X_2 = 3(2)^2 - 11$$

$$X_2 = 1$$

$$X_3 = 3(X_2)^2 - 11$$

$$X_3 = 3(1)^2 - 11$$

$$X_3 = -8$$

$$X_4 = 3(X_3)^2 - 11$$

$$X_4 = 3(-8)^2 - 11$$

$$X_4 = 181$$

$$\sum_{r=1}^4 X_r = X_1 + X_2 + X_3 + X_4$$

$$\sum_{r=1}^4 X_r = (2) + (1) + (-8) + (181)$$

$$\sum_{r=1}^4 X_r = 176$$

Choice 1: $\sum_{r=1}^4 X_r = 173$ false

Choice 2: $\sum_{r=1}^4 X_r = 170$ false

Choice 3: $\sum_{r=1}^4 X_r = 177$ false

Choice 4: $\sum_{r=1}^4 X_r = 172$ false

Choice 5: $\sum_{r=1}^4 X_r = 176$ true

Question 11 Experience: 25 Order: Level: Question-ID: 71

A sequence is defined by the recurrence relation $U_{n+2} = 3U_{n+1} - U_n + 5$, given that $U_1 = 4, U_2 = 2$, find $\sum_{r=1}^4 U_r$.

Solution 11

$$U_3 = 3U_2 - U_1 + 5$$

$$U_3 = 3(2) - (4) + 5$$

$$U_3 = 7$$

$$U_4 = 3U_3 - U_2 + 5$$

$$U_4 = 3(7) - (2) + 5$$

$$U_4 = 24$$

$$\sum_{r=1}^4 U_r = U_1 + U_2 + U_3 + U_4$$

$$\sum_{r=1}^4 U_r = 4 + 2 + 7 + 24$$

$$\sum_{r=1}^4 U_r = 37$$

Choice 1: $\sum_{r=1}^4 U_r = 36$ false

Choice 2: $\sum_{r=1}^4 U_r = 35$ false

Choice 3: $\sum_{r=1}^4 U_r = 38$ false

Choice 4: $\sum_{r=1}^4 U_r = 34$ false

Choice 5: $\sum_{r=1}^4 U_r = 37$ true

Question 12 Experience: 25 Order: Level: Question-ID: 72

A sequence is defined by the recurrence relation $Y_{n+1} = 21 - 2Y_n$, given that $Y_1 = 5$, find $\sum_{r=2}^4 Y_r$.

Solution 12

$$Y_2 = 21 - 2Y_1$$

$$Y_2 = 21 - 2(5)$$

$$Y_2 = 11$$

$$Y_3 = 21 - 2Y_2$$

$$Y_3 = 21 - 2(11)$$

$$Y_3 = -1$$

$$Y_4 = 21 - 2Y_3$$

$$Y_4 = 21 - 2(-1)$$

$$Y_4 = 23$$

$$\sum_{r=2}^4 Y_r = Y_2 + Y_3 + Y_4$$

$$\sum_{r=2}^4 Y_r = 11 + (-1) + 23$$

$$\sum_{r=2}^4 Y_r = 33$$

Choice 1: $\sum_{r=2}^4 Y_r = 32$ false

Choice 2: $\sum_{r=2}^4 Y_r = 31$ false

Choice 3: $\sum_{r=2}^4 Y_r = 30$ false

Choice 4: $\sum_{r=2}^4 Y_r = 34$ false

Choice 5: $\sum_{r=2}^4 Y_r = 33$ true

Question 13 Experience: 30 Order: Level: Question-ID: 74

A sequence is defined by the recurrence relation $X_{n+1} = 5 - X_n$, given that $X_1 = 7$, find $\sum_{r=1}^{20} X_r$.

Solution 13

$$X_2 = 5 - X_1 = 5 - 7 = -2$$

$$X_3 = 5 - X_2 = 5 - (-2) = 7$$

$$X_4 = 5 - X_3 = 5 - 7 = -2$$

$$X_5 = 5 - X_4 = 5 - (-2) = 7$$

$$\sum_{r=1}^{20} X_r = X_1 + X_2 + X_3 + X_4 + \dots + X_{20}$$

$$\sum_{r=1}^{20} X_r = -2 + 7 + -2 + 7 + -2 + \dots + 7$$

$$\sum_{r=1}^{20} X_r = 10(-2) + 10(7)$$

$$\sum_{r=1}^{20} X_r = 50$$

Choice 1: $\sum_{r=1}^{20} X_r = 20$ false

Choice 2: $\sum_{r=1}^{20} X_r = 60$ false

Choice 3: $\sum_{r=1}^{20} X_r = 30$ false

Choice 4: $\sum_{r=1}^{20} X_r = 40$ false

Choice 5: $\sum_{r=1}^{20} X_r = 50$ true

Question 14 Experience: 30 Order: Level: Question-ID: 77

A sequence is defined by the recurrence relation $Y_{n+1} = 5 + 5Y_n - 2(Y_n)^3$, given that $Y_1 = 2$, find Y_{1000} .

Solution 14

$$Y_2 = 5 + 5Y_1 - 2(Y_1)^3 = 5 + 5(2) - 2(2)^3 = -1$$

$$Y_3 = 5 + 5Y_2 - 2(Y_2)^3 = 5 + 5(-1) - 2(-1)^3 = 2$$

$$Y_4 = 5 + 5Y_3 - 2(Y_3)^3 = 5 + 5(2) - 2(2)^3 = -1$$

Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
- 1	2	- 1	2	- 1	2

We can see that $Y_2 = Y_4 = Y_6 = Y_8 = \dots = 2$

Every numbered term divisible by 2 is 2

Find a numbered term that is close to Y_{1000} that is divisible by 2

$$Y_2 = 2 \quad Y_4 = 2 \quad Y_{100} = 2 \quad Y_{1000} = 2$$

Choice 1: $Y_{1000} = 1$ false

Choice 2: $Y_{1000} = 0$ false

Choice 3: $Y_{1000} = 3$ false

Choice 4: $Y_{1000} = 4$ false

Choice 5: $Y_{1000} = 2$ true

Question 15 Experience: 15 Order: Level: Question-ID: 78

Given $\sum_{r=1}^n x_r = 5n^2 - 3$, find $\sum_{r=1}^7 x_r$

Solution 15

$$\sum_{r=1}^7 x_r = 5(7)^2 - 3 = 242$$

Choice 1: $\sum_{r=1}^7 x_r = 239$ false

Choice 2: $\sum_{r=1}^7 x_r = 240$ false

Choice 3: $\sum_{r=1}^7 x_r = 243$ false

Choice 4: $\sum_{r=1}^7 x_r = 241$ false

Choice 5: $\sum_{r=1}^7 x_r = 242$ true

Question 16 Experience: 40 Order: Level: Question-ID: 76

A sequence is defined by the recurrence relation $U_{n+1} = \frac{13 - 5U_n}{7 - 3U_n}$, given that $U_1 = 1$, find U_{50} .

Solution 16

$$U_2 = \frac{13 - 5U_1}{7 - 3U_1} = \frac{13 - 5(1)}{7 - 3(1)} = \frac{8}{4} = 2$$

$$U_3 = \frac{13 - 5U_2}{7 - 3U_2} = \frac{13 - 5(2)}{7 - 3(2)} = \frac{3}{1} = 3$$

$$U_4 = \frac{13 - 5U_3}{7 - 3U_3} = \frac{13 - 5(3)}{7 - 3(3)} = \frac{-2}{-2} = 1$$

$$U_5 = \frac{13 - 5U_4}{7 - 3U_4} = \frac{13 - 5(1)}{7 - 3(1)} = \frac{8}{4} = 2$$

U_1	U_2	U_3	U_4	U_5	U_6
1	2	3	1	2	3

We can see that $U_3 = U_6 = U_9 = U_{12} = \dots = 3$

Every numbered term divisible by 3 is 3

Find a numbered term that is close to U_{50} that is divisible by 3

$$U_3 = 3 \quad U_9 = 3 \quad U_{30} = 3 \quad U_{51} = 3$$

$U_{51} = 3 \Rightarrow U_{50} = 2$ since 2 is the term before 3 in the sequence

i.e. 1, 2, 3, 1, 2, 3, 1,

Choice 1: $U_{50} = 1$ false

Choice 2: $U_{50} = 3$ false

Choice 3: $U_{50} = 4$ false

Choice 4: $U_{50} = 5$ false

Choice 5: $U_{50} = 2$ true

Question 17 Experience: 30 Order: Level: Question-ID: 79

Given $\sum_{r=1}^n a_r = 2n^3 + 5$, find a_2

Solution 17

$$\sum_{r=1}^n a_r = 2n^3 + 5$$

$$a_2 = \sum_{r=1}^2 a_r - \sum_{r=1}^1 a_r$$

$$a_2 = 2(2)^3 + 5 - (2(1)^3 + 5)$$

$$a_2 = 21 - 7$$

$$a_2 = 14$$

Choice 1: $a_2 = 13$ false

Choice 2: $a_2 = 12$ false

Choice 3: $a_2 = 11$ false

Choice 4: $a_2 = 15$ false

Choice 5: $a_2 = 14$ true

Question 18 Experience: 15 Order: Level: Question-ID: 80

Given $\sum_{r=1}^n U_r = 6n^2 + 11$, find U_1

Solution 18

$$\sum_{r=1}^1 U_r = U_1 = 6(1)^2 + 11 = 17$$

Choice 1: $U_1 = 16$ false

Choice 2: $U_1 = 15$ false

Choice 3: $U_1 = 14$ false

Choice 4: $U_1 = 18$ false

Choice 5: $U_1 = 17$ true

Question 19 Experience: 15 Order: Level: Question-ID: 81

Given $\sum_{r=1}^n u_r = n^3 + 4$, find $\sum_{r=1}^5 u_r$

Solution 19

$$\sum_{r=1}^5 u_r = (5)^3 + 4 = 129$$

Choice 1: $\sum_{r=1}^5 u_r = 130$ false

Choice 2: $\sum_{r=1}^5 u_r = 126$ false

Choice 3: $\sum_{r=1}^5 u_r = 127$ false

Choice 4: $\sum_{r=1}^5 u_r = 128$ false

Choice 5: $\sum_{r=1}^5 u_r = 129$ true

Question 20 Experience: 30 Order: Level: Question-ID: 82

Given $\sum_{r=1}^n Y_r = 3n^3 - 2$, find Y_3

Solution 20

$$Y_3 = \sum_{r=1}^3 - \sum_{r=1}^2$$

$$Y_3 = 3(3)^3 - 2 - (3(2)^3 - 2)$$

$$Y_3 = 57$$

Choice 1: $Y_3 = 54$ false

Choice 2: $Y_3 = 55$ false

Choice 3: $Y_3 = 52$ false

Choice 4: $Y_3 = 56$ false

Choice 5: $Y_3 = 57$ true

Question 21 Experience: 30 Order: Level: Question-ID: 83

Given $\sum_{r=1}^n U_r = 3n + 7$, find U_5

Solution 21

$$U_5 = \sum_{r=1}^5 U_r - \sum_{r=1}^4 U_r$$

$$U_5 = 3(5) + 7 - (3(4) + 7)$$

$$U_5 = 3$$

Choice 1: $U_5 = 2$ false

Choice 2: $U_5 = 1$ false

Choice 3: $U_5 = 4$ false

Choice 4: $U_5 = 5$ false

Choice 5: $U_5 = 3$ true

Question 22 Experience: 45 Order: Level: Question-ID: 55

A sequence is defined by the recurrence relation $U_{n+1} = kU_n - 4, U_1 = 3, k > 0$, given that $U_3 = 0$ find the value of k

Solution 22

$$U_2 = kU_1 - 4$$

$$U_2 = 3k - 4$$

$$U_3 = kU_2 - 4$$

$$U_3 = k(3k - 4) - 4$$

$$U_3 = 3k^2 - 4k - 4 \quad U_3 = 0$$

$$0 = 3k^2 - 4k - 4 \quad S = -4 \quad P = -12$$

$$0 = \left(k + \frac{2}{3}\right)(k - 2) \quad (2, -6) \Rightarrow \left(\frac{2}{3}, -2\right)$$

$$k = 2$$

Choice 1: $k = 3$ false

Choice 2: $k = 4$ false

Choice 3: $k = 1$ false

Choice 4: $k = 5$ false

Choice 5: $k = 2$ true

Question 23 Experience: 60 Order: Level: Question-ID: 56

A sequence is defined by the recurrence relation $a_{n+1} = \frac{a_n}{k} + 3, a_1 = 3, k > 0$, given that $a_3 = 9$ find the value of k

Solution 23

$$a_2 = \frac{a_1}{k} + 3$$

$$a_2 = \frac{3}{k} + 3$$

$$a_3 = \frac{a_2}{k} + 3$$

$$a_3 = \frac{\left(\frac{3}{k} + 3\right)}{k} + 3$$

$$a_3 = \frac{3}{k^2} + \frac{3}{k} + 3 \quad a_3 = 9$$

$$9 = \frac{3}{k^2} + \frac{3}{k} + 3$$

$$6 - \frac{3}{k^2} - \frac{3}{k} = 0$$

$$6k^2 - 3k - 3 = 0$$

$$2k^2 - k - 1 = 0 \quad S = -1 \quad P = -2$$

$$\left(x + \frac{1}{2}\right)(k - 1) = 0 \quad (1, -2) \Rightarrow \left(\frac{1}{2}, -1\right)$$

$$k = 1$$

Choice 1: $k = 2$ false

Choice 2: $k = 3$ false

Choice 3: $k = 4$ false

Choice 4: $k = 5$ false

Choice 5: $k = 1$ true

Question 24 Experience: 60 Order: Level: Question-ID: 62

A sequence is defined by the recurrence relation $u_{n+1} = \sqrt{a} \left(u_n - \frac{1}{b}\right)$, $5u_1 = 4$, given that $u_2 = 7$ and $u_3 = 13$ find the value of a and b .

Solution 24

$$u_2 = \sqrt{a} \left(u_1 - \frac{1}{b} \right)$$

$$7 = \sqrt{a} \left(4 - \frac{1}{b} \right) \quad (1)$$

$$7 = 4\sqrt{a} - \frac{\sqrt{a}}{b} \quad (2)$$

$$u_3 = \sqrt{a} \left(u_2 - \frac{1}{b} \right)$$

$$13 = \sqrt{a} \left(7 - \frac{1}{b} \right)$$

$$13 = 7\sqrt{a} - \frac{\sqrt{a}}{b} \quad (3)$$

$$(3) - (2) \quad 13 - 7 = 7\sqrt{a} - \frac{\sqrt{a}}{b} - \left(4\sqrt{a} - \frac{\sqrt{a}}{b} \right)$$

$$6 = 3\sqrt{a}$$

$$2 = \sqrt{a}$$

$$a = 4$$

$$\text{Sub into (1)} \quad 7 = \sqrt{4} \left(4 - \frac{1}{b} \right)$$

$$\frac{7}{2} = 4 - \frac{1}{b}$$

$$-\frac{1}{2} = -\frac{1}{b}$$

$$b = 2$$

Choice 1: $a = 4$ $b = 2$ true

Choice 2: $a = 3$ $b = 2$ false

Choice 3: $a = 4$ $b = 3$ false

Choice 4: $a = 3$ $b = 3$ false

Choice 5: $a = 2$ $b = 3$ false

Question 25 Experience: 30 Order: Level: Question-ID: 73

A sequence is defined by the recurrence relation $a_{n+1} = 3 - a_n$, given that $a_1 = 1$, find $\sum_{r=1}^{100} a_r$.

Solution 25

$$a_2 = 3 - a_1 = 3 - 1 = 2$$

$$a_3 = 3 - a_2 = 3 - 2 = 1$$

$$a_4 = 3 - a_3 = 3 - 1 = 2$$

$$a_5 = 3 - a_4 = 3 - 2 = 1$$

$$\sum_{r=1}^{100} a_r = a_1 + a_2 + a_3 + a_4 + \dots + a_{100}$$

$$\sum_{r=1}^{100} a_r = 1 + 2 + 1 + 2 + 1 + 2 + \dots + 2$$

$$\sum_{r=1}^{100} a_r = 50(2) + 50(1)$$

$$\sum_{r=1}^{100} a_r = 150$$

Choice 1: $\sum_{r=1}^{100} a_r = 100$ false

Choice 2: $\sum_{r=1}^{100} a_r = 200$ false

Choice 3: $\sum_{r=1}^{100} a_r = 50$ false

Choice 4: $\sum_{r=1}^{100} a_r = 250$ false

Choice 5: $\sum_{r=1}^{100} a_r = 150$ true

Question 26 Experience: 40 Order: Level: Question-ID: 75

A sequence is defined by the recurrence relation $A_{n+1} = \frac{4A_n - 16}{3A_n - 8}$, given that $A_1 = 0$, find A_{100} .

Solution 26

$$A_2 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(0) - 16}{3(0) - 8} = \frac{-16}{-8} = 2$$

$$A_3 = \frac{4A_2 - 16}{3A_2 - 8} = \frac{4(2) - 16}{3(2) - 8} = \frac{-8}{-2} = 4$$

$$A_4 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(4) - 16}{3(4) - 8} = 0$$

$$A_5 = \frac{4A_1 - 16}{3A_1 - 8} = \frac{4(0) - 16}{3(0) - 8} = \frac{-16}{-8} = 2$$

a_1	a_2	a_3	a_4	a_5	a_6
0	2	4	0	2	4

We can see that $a_3 = a_6 = a_9 = a_{12} = \dots = 4$

Every numbered term divisible by 3 is 4

Find a numbered term that is close to a_{100} that is divisible by 3

$$a_3 = 4 \quad a_9 = 4 \quad a_{30} = 4 \quad a_{99} = 4$$

$$a_{99} = 4 \Rightarrow a_{100} = 0 \text{ since 0 is the next term after 4 in the sequence}$$

i.e. 0, 2, 4, 0, 2, 4, 0

Choice 1: $a_{100} = 1$ false

Choice 2: $a_{100} = 2$ false

Choice 3: $a_{100} = 3$ false

Choice 4: $a_{100} = 4$ false

Choice 5: $a_{100} = 0$ true

Lesson 4 Arithmetic Sequence 2

Question 1 Experience: 50 Order: Level: Question-ID: 123

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 11$ and $U_7 = 23$, find S_{11}

Solution 1

$$U_n = a + (n - 1)d$$

$$U_4 = a + (4 - 1)d = a + 3d = 11 \quad (1)$$

$$U_7 = a + (7 - 1)d = a + 6d = 23 \quad (2)$$

$$(2) - (1) \quad a + 6d - (a + 3d) = 23 - 11$$

$$3d = 12$$

$$d = 4$$

$$\text{Sub into (1)} \quad a + 3(4) = 11$$

$$a = -1$$

$$S_n = \frac{n}{2}(2(a) + (n - 1)d)$$

$$S_{11} = \frac{11}{2}(2(-1) + (11 - 1)4) = 209$$

Choice 1: $S_{11} = 208$ false

Choice 2: $S_{11} = 205$ false

Choice 3: $S_{11} = 206$ false

Choice 4: $S_{11} = 207$ false

Choice 5: $S_{11} = 209$ true

Question 2 Experience: 50 Order: Level: Question-ID: 124

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_3 = -5$ and $U_5 = -11$, find S_7

Solution 2

$$U_n = a + (n - 1)d$$

$$U_3 = a + (3 - 1)d = a + 2d = -5 \quad (1)$$

$$U_5 = a + (5 - 1)d = a + 4d = -11 \quad (2)$$

$$(2) - (1) \quad a + 4d - (a + 2d) = -11 - (-5)$$

$$2d = -6$$

$$d = -3$$

$$\text{Sub into (1)} \quad a + 2(-3) = -5$$

$$a = 1$$

$$U_7 = 1 + (7 - 1)(-3) = -17$$

$$S_7 = \frac{7}{2}(1 + (-17)) = -56$$

Choice 1: $S_7 = -52$ false

Choice 2: $S_7 = -53$ false

Choice 3: $S_7 = -54$ false

Choice 4: $S_7 = -55$ false

Choice 5: $S_7 = -56$ true

Question 3 Experience: 45 Order: Level: Question-ID: 140

The first three terms of an arithmetic sequence are 60, 58, 56..., there exists a k^{th} term which = 0, find the value of k , hence of otherwise find the maximum value of S_n

Solution 3

$$U_n = a + (n - 1)d$$

$$U_k = 60 + (k - 1)(-2) = 0$$

$$k - 1 = 30$$

$$k = 31$$

maximum value of $S_n = S_k$ as any term after U_k is negative

Choice 1: $k = 28$ $S_n = \frac{n}{2}(a + d)$ $S_k = 935$ false

Choice 2: $k = 29$ $S_k = 915$ false

Choice 3: $k = 30$ $S_k = 920$ false

Choice 4: $k = 32$ $S_k = 929$ false

Choice 5: $k = 31$ $S_k = 930$ true

Question 4 Experience: 50 Order: Level: Question-ID: 126

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $S_5 = 85$ and $S_8 = 184$, find U_6

Solution 4

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_5 = \frac{5}{2}(2a + (5-1)d) = 85$$

$$2a + 4d = 34 \quad (1)$$

$$S_8 = \frac{8}{2}(2a + (8-1)d) = 184$$

$$2a + 7d = 46 \quad (2)$$

$$(2) - (1) \quad 2a + 7d - (2a + 4d) = 46 - 34$$

$$3d = 12$$

$$d = 4$$

$$\text{Sub into (1)} \quad 2a + 4(4) = 34$$

$$a = 9$$

$$U_6 = a + (6-1)d = 9 + 5(4) = 29$$

Choice 1: $U_6 = 31$ false

Choice 2: $U_6 = 30$ false

Choice 3: $U_6 = 27$ false

Choice 4: $U_6 = 28$ false

Choice 5: $U_6 = 29$ true

Question 5 Experience: 50 Order: Level: Question-ID: 127

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_5 = 19$ and $S_{10} = 170$, find U_4

Solution 5

$$U_n = a + (n-1)d$$

$$U_5 = a + 4d = 19 \quad (1)$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2a + (10-1)d) = 170$$

$$2a + 9d = 34 \quad (2)$$

$$(2) - 2(1) \quad 2a + 9d - 2(a + 4d) = 34 - 38$$

$$d = -4$$

$$d = -4$$

$$\text{Sub into (1)} \quad a + 4(-4) = 19$$

$$a = 35$$

$$U_4 = a + (4-1)d = 35 + (3)(-4) = 23$$

Choice 1: $U_4 = 19$ false

Choice 2: $U_4 = 20$ false

Choice 3: $U_4 = 21$ false

Choice 4: $U_4 = 22$ false

Choice 5: $U_4 = 23$ true

Question 6 Experience: 50 Order: Level: Question-ID: 128

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 8$ and $S_{12} = 0$, find S_9

Solution 6

$$U_n = a + (n - 1)d$$

$$U_4 = a + 3d = 8 \quad (1)$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_{12} = \frac{12}{2}(2a + (12 - 1)d) = 0$$

$$2a + 11d = 0 \quad (2)$$

$$(2) - 2(1) \quad 2a + 11d - 2(a + 3d) = 0 - 16$$

$$8d = -16$$

$$d = -2$$

$$\text{Sub into (1)} \quad a + 3(-2) = 8$$

$$a = 14$$

$$S_9 = \frac{9}{2}(2(14) + (9 - 1)(-2)) = 54$$

Choice 1: $S_9 = 55$ false

Choice 2: $S_9 = 51$ false

Choice 3: $S_9 = 52$ false

Choice 4: $S_9 = 53$ false

Choice 5: $S_9 = 54$ true

Question 7 Experience: 50 Order: Level: Question-ID: 129

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_3 = 4$ and $U_7 = 0$, find S_{10}

Solution 7

$$U_n = a + (n - 1)d$$

$$U_3 = a + 2d = 4 \quad (1)$$

$$U_7 = a + 6d = 0 \quad (2)$$

$$(2) - (1) \quad a + 6d - (a + 2d) = 0 - 4$$

$$4d = -4$$

$$d = -1$$

$$\text{Sub into (1)} \quad a + 2(-1) = 4$$

$$a = 6$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_{10} = \frac{10}{2}(2(6) + (10 - 1)(-1)) = 15$$

$$\text{Choice 1:} \quad S_{10} = 14 \quad \text{false}$$

$$\text{Choice 2:} \quad S_{10} = 13 \quad \text{false}$$

$$\text{Choice 3:} \quad S_{10} = 12 \quad \text{false}$$

$$\text{Choice 4:} \quad S_{10} = 16 \quad \text{false}$$

$$\text{Choice 5:} \quad S_{10} = 15 \quad \text{true}$$

Question 8 Experience: 50 Order: Level: Question-ID: 130

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $U_4 = 10$ and $S_6 = 57$, find S_{11}

Solution 8

$$U_n = a + (n - 1)d$$

$$U_4 = a + 3d = 10 \quad (1)$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_6 = \frac{6}{2}(2a + (5 - 1)d) = 57$$

$$a + 2d = \frac{19}{2} \quad (2)$$

$$(1) - (2) \quad a + 3d - (a + 2d) = 10 - \frac{19}{2}$$

$$d = \frac{1}{2}$$

$$\text{Sub into (1)} \quad a + 3\left(\frac{1}{2}\right) = 10$$

$$a = \frac{17}{2}$$

$$S_{11} = \frac{11}{2} \left(2\left(\frac{17}{2}\right) + (11 - 1)\left(\frac{1}{2}\right) \right) = 121$$

$$\text{Choice 1:} \quad S_{10} = 14 \quad \text{false}$$

$$\text{Choice 2:} \quad S_{10} = 13 \quad \text{false}$$

$$\text{Choice 3:} \quad S_{10} = 12 \quad \text{false}$$

- Choice 4: $S_{10} = 16$ false
 Choice 5: $S_{10} = 15$ true

Question 9 Experience: 30 Order: Level: Question-ID: 131

Three consecutive terms in an arithmetic sequence are $3k + 2, 2k + 5, 4k + 5$, find the value of k

Solution 9

$$2k + 5 - (3k + 2) = d = 4k + 5 - (2k + 5)$$

- Choice 1: $k = 5$ false
 Choice 2: $k = 4$ false
 Choice 3: $k = 3$ false
 Choice 4: $k = 2$ false
 Choice 5: $k = 1$ true

Question 10 Experience: 30 Order: Level: Question-ID: 132

Three consecutive terms in an arithmetic sequence are $k^2 + 3, -k, k - 1$, find the possible values of k

Solution 10

$$-k - (k^2 + 3) = d = k - 1 - (-k)$$

- Choice 1: $k = -2, -3$ false
 Choice 2: $k = -1, -2$ false
 Choice 3: $k = -3, 0$ false
 Choice 4: $k = -1, -2$ false
 Choice 5: $k = -2, -1$ true

Question 11 Experience: 30 Order: Level: Question-ID: 133

Three consecutive terms in an arithmetic sequence are $k + 16, 3k + 12, 7k - 2$, find the value of k

Solution 11

$$3k + 12 - (k + 16) = d = 7k - 2 - (3k + 12)$$

- Choice 1: $k = 6$ false
 Choice 2: $k = 2$ false
 Choice 3: $k = 3$ false
 Choice 4: $k = 4$ false
 Choice 5: $k = 5$ true

Question 12 Experience: 45 Order: Level: Question-ID: 134

The first three terms in an arithmetic sequence are $2k, k + 9, 3k$, find the smallest n such that $S_n > 117$

Solution 12

$$k + 9 - 2k = d = 3k - (k + 9)$$

$$-k + 9 = 2k - 9$$

$$3k = 18$$

$$k = 6$$

$$\Rightarrow U_1 = 12 \quad U_2 = 15 \quad U_3 = 18$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$\frac{n}{2}(2(12) + (n - 1)3) > 117$$

$$n(24 + 3n - 3) > 234$$

$$3n^2 + 21n - 234 > 0$$

$$n^2 + 7n - 78 > 0 \quad P = -78 \quad S = 7$$

$$(n + 13)(n - 6) > 0 \quad (13, -6)$$

$$n = 6$$

Choice 1: $n = 7$ false

Choice 2: $n = 3$ false

Choice 3: $n = 4$ false

Choice 4: $n = 5$ false

Choice 5: $n = 6$ true

Question 13 Experience: 40 Order: Level: Question-ID: 135

The first three terms of an arithmetic sequence are 99, 96, 93..., there exists a k^{th} term which = 0, find the value of k , hence of otherwise find the maximum value of S_n

Solution 13

$$U_n = a + (n - 1)d$$

$$U_k = 99 + (k - 1)(-3) = 0$$

$$k - 1 = 33$$

$$k = 34$$

maximum value of $S_n = S_k$ as any term after U_k is negative

$$S_n = \frac{n}{2}(a + l)$$

$$S_k = \frac{34}{2}(99 + 0)$$

$$S_k = 1683$$

Choice 1: $k = 33$ $S_k = 1689$ false

Choice 2: $k = 32$ $S_k = 1686$ false

Choice 3: $k = 35$ $S_k = 1677$ false

Choice 4: $k = 36$ $S_k = 1680$ false

Choice 5: $k = 34$ $S_k = 1683$ true

Question 14 Experience: 35 Order: Level: Question-ID: 136

The first three terms in an arithmetic sequence are 5, 7, 9, find the smallest n such that $S_n > 252$

Solution 14

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(2(5) + (n-1)2) > 252$$

$$n(5 + n - 1) > 252$$

$$n^2 + 4n - 252 > 0 \quad P = -252 \quad S = 4$$

$$(n + 18)(n - 14) > 0 \quad (18, -14)$$

$$n = 14$$

Choice 1: $n = 15$ false

Choice 2: $n = 11$ false

Choice 3: $n = 12$ false

Choice 4: $n = 13$ false

Choice 5: $n = 14$ true

Question 15 Experience: 35 Order: Level: Question-ID: 137

The first three terms in an arithmetic sequence are 9, 12, 15, find the smallest n such that $S_n > 750$

Solution 15

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(2(9) + (n-1)3) > 750$$

$$n(18 + 3n - 3) > 1500$$

$$3n(5 + n) > 1500$$

$$n(5 + n) > 500$$

$$n^2 + 5n - 500 > 0 \quad P = -500 \quad S = 5$$

$$(n + 25)(n - 20) > 0 \quad (25, -20)$$

$$n = 20$$

Choice 1: $n = 21$ false

Choice 2: $n = 17$ false

Choice 3: $n = 18$ false

Choice 4: $n = 19$ false

Choice 5: $n = 20$ true

Question 16 Experience: 35 Order: Level: Question-ID: 138

The first three terms in an arithmetic sequence are 12, 16, 20, 24, find the smallest n such that $S_n > 672$

Solution 16

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(2(12) + (n-1)4) > 672$$

$$n(12 + 2n - 2) > 672$$

$$2n^2 + 10n - 672 > 0$$

$$n^2 + 5n - 336 > 0 \quad P = -336 \quad S = 5$$

$$(n+21)(n-16) > 0 \quad (21, -16)$$

$$n = 16$$

Choice 1: $n = 15$ false

Choice 2: $n = 19$ false

Choice 3: $n = 18$ false

Choice 4: $n = 17$ false

Choice 5: $n = 16$ true

Question 17 Experience: 50 Order: Level: Question-ID: 142

Judith is playing with 294 sticks, she puts them in rows. The first row has 8 sticks, next row has 10 sticks, subsequent rows have 2 more sticks than the previous row. She has enough for k rows but not enough for $k+1$ rows. Find k .

Solution 17

Sequence goes: 8,10,12,14,16,18,20....

Not having enough for $k+1$ rows means that $S_k \leq 294$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_k = \frac{k}{2}(2(8) + (k-1)2)$$

$$S_k = k(8 + k - 1)$$

$$S_k = k(k + 7)$$

$$S_k = k^2 + 7k \quad (1)$$

$$S_k \leq 294$$

$$(1) \quad k^2 + 7k \leq 294$$

$$k^2 + 7k - 294 \leq 0 \quad P = 294 \quad S = 7$$

Choice 1: $k = 11$ false

Choice 2: $(k+21)(k-14) \leq 0$ false $(21, -14)$

Choice 3: $k = 12$ false

Choice 4: $k = 13$ false

Choice 5: $k = 14$ true

Question 18 Experience: 50 Order: Level: Question-ID: 125

U_n is an arithmetic sequence with S_n being the sum of the first n terms of the sequence. Given that $S_{11} = 0$ and $U_2 = 8$, find U_6

Solution 18

$$U_n = a + (n - 1)d$$

$$U_2 = a + (2 - 1)d = a + d = 8 \quad (1)$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_{11} = \frac{11}{2}(2a + (11 - 1)d) = 0$$

$$S_{11} = a + 5d = 0 \quad (2)$$

$$(2) - (1) \quad a + 5d - (a + d) = 0 - 8$$

$$4d = -8$$

$$d = -2$$

$$\text{Sub into (1)} \quad a + (-2) = 8$$

$$a = 10$$

$$U_6 = 1 + (7 - 1)(-2) = -11$$

$$\text{Choice 1:} \quad U_6 = -13 \quad \text{false}$$

$$\text{Choice 2:} \quad U_6 = -12 \quad \text{false}$$

$$\text{Choice 3:} \quad U_6 = -9 \quad \text{false}$$

$$\text{Choice 4:} \quad U_6 = -10 \quad \text{false}$$

$$\text{Choice 5:} \quad U_6 = -11 \quad \text{true}$$

Question 19 Experience: 45 Order: Level: Question-ID: 139

The first three terms of an arithmetic sequence are 44, 41, 38..., there exists a k^{th} term which is the smallest positive term in the sequence, find the value of k , hence of otherwise find the maximum value of S_n

Solution 19

$$U_n = a + (n - 1)d$$

$$U_k = 44 + (k - 1)(-3) = 0$$

$$k - 1 = \frac{44}{3}$$

$$k = \frac{44}{3} + 1 = 15.6$$

$$k = 15$$

maximum value of $S_n = S_k$ as any term after U_k is negative

$$\text{Choice 1:} \quad k = 14 \quad S_k = \frac{n}{2}(2a + (n - 1)d) \quad \text{false}$$

$$\text{Choice 2:} \quad k = 13 \quad S_k = \frac{n}{2}(2a + (n - 1)d) \quad \text{false}$$

$$\text{Choice 3:} \quad k = 12 \quad S_k = \frac{n}{2}(2a + (n - 1)d) \quad \text{false}$$

$$\text{Choice 4:} \quad k = 16 \quad S_k = \frac{n}{2}(2a + (n - 1)d) \quad \text{false}$$

$$\text{Choice 5:} \quad k = 15 \quad S_k = 2652 \quad \text{true}$$

Question 20 Experience: 50 Order: Level: Question-ID: 141

At the start of the year 2000, Tony the farmer has $50m^2$ of land, he buys $7m^2$ of land at the end of each year. At the beginning of this year, Tony owns $141m^2$ of land. What year is it?

Solution 20

Sequence goes from the start of every year: 50,57,64,71,78,85....

$$U_n = a + (n - 1)d$$

$$U_n = 141 \quad a = 50 \quad d = 7$$

$$141 = 50 + (n - 1)7$$

$$n - 1 = 13$$

Choice 1: Year = 2015 false

Choice 2: Year = 2011 false

Choice 3: Year = 2012 false

Choice 4: Year = 2013 false

Choice 5: Year = 2014 true

End of Chapter Questions**Unit 2 Core 2****Chapter 1 Logarithms****Lesson 1 Basic logarithms**

Question 1 Experience: 20 Order: f2 Level: f2 Question-ID: 196

Express $\log_9 30$ in terms of \ln

Solution 1

Choice 1: $\log_9 30 \frac{\ln 10}{\ln 3}$ false

Choice 2: $\log_9 30 \frac{\ln 21}{\ln 3}$ false

Choice 3: $\log_9 30 \frac{\ln 39}{\ln 9}$ false

Choice 4: $\log_9 30 \frac{\ln \frac{10}{3}}{\ln 3}$ false

Choice 5: $\log_9 30 \frac{\ln 30}{\ln 9}$ true

Question 2 Experience: 25 Order: g1 Level: g1 Question-ID: 197

Solve $4^x = 16$ for x

Solution 2

Answer part 1: Label $x =$ Solution 2

Answer part 1 hint: x is an integer value

Question 3 Experience: 10 Order: a2 Level: a2 Question-ID: 149

Express $\log_{x+5} 10 = 4$ in power form

Solution 3

$$\log_{x+5} 10 = 4$$

$$(x + 5)^4 = 10$$

Choice 1: $4^{x+5} = 10$ false

Choice 2: $(x + 5)^{10} = 4$ false

- Choice 3: $10^{x+5} = 4$ false
 Choice 4: $(x+5)^{10} = 4$ false
 Choice 5: $(x+5)^4 = 10$ true

Question 4 Experience: 10 Order: a2 Level: a2 Question-ID: 150

Express $\log_{a+b} 6 = c$ in power form

Solution 4

$$\log_{a+b} 6 = c$$

$$(a+b)^c = 6$$

- Choice 1: $(a+b)^6 = c$ false
 Choice 2: $6^c = a+b$ false
 Choice 3: $(a+b)^c = 6$ false
 Choice 4: $6^{a+b} = 6$ false
 Choice 5: $(a+b)^c = 6$ true

Question 5 Experience: 10 Order: a2 Level: a2 Question-ID: 152

Express $\log_{xy} 3 = 2$ in power form

Solution 5

$$\log_{xy} 3 = 2$$

$$(xy)^2 = 3$$

- Choice 1: $2^{xy} = 3$ false
 Choice 2: $3^2 = xy$ false
 Choice 3: $xy^3 = 2$ false
 Choice 4: $(3)^{xy} = 2$ false
 Choice 5: $(xy)^2 = 3$ true

Question 6 Experience: 10 Order: b1 Level: b1 Question-ID: 154

Express $a^b = c$ in log form

Solution 6

$$a^b = c$$

$$\log_a c = b$$

- Choice 1: $\log_c a = b$ false
 Choice 2: $\log_b c = a$ false
 Choice 3: $\log_b a = c$ false
 Choice 4: $\log_a b = c$ false
 Choice 5: $\log_a c = b$ true

Question 7 Experience: 10 Order: b1 Level: b1 Question-ID: 157

Express $5^2 = 25$ in log form

Solution 7

$$5^2 = 25$$

$$\log_5 25 = 2$$

- Choice 1: $\log_5 2 = 25$ false
 Choice 2: $\log_{25} 2 = 5$ false
 Choice 3: $\log_{25} 5 = 2$ false
 Choice 4: $\log_2 25 = 5$ false
 Choice 5: $\log_5 25 = 2$ true

Question 8 Experience: 10 Order: b2 Level: b2 Question-ID: 156

Express $(xy)^5 = 20$ in log form

Solution 8

$$(xy)^5 = 20$$

$$\log_{xy} 20 = 5$$

- Choice 1: $\log_5 20 = xy$ false
 Choice 2: $\log_{xy} 5 = 20$ false
 Choice 3: $\log_{20} 5 = xy$ false
 Choice 4: $\log_{20} xy = 5$ false
 Choice 5: $\log_{xy} 20 = 5$ true

Question 9 Experience: 15 Order: c2 Level: c2 Question-ID: 162

Express $\log_2(x^2y) - \log_2 x$ as a single logarithm

Solution 9

$$\log_2(x^2y) - \log_2 x$$

$$= \log_2((x^2y) \div x)$$

$$= \log_2 xy$$

- Choice 1: $2 \log_{x^2y} 1$ false
 Choice 2: $\log_{x^2y} 2$ false
 Choice 3: $\log_2 x^2y$ false
 Choice 4: $2x \log_2 y$ false
 Choice 5: $\log_2 xy$ true

Question 10 Experience: 10 Order: b1 Level: b1 Question-ID: 159

Express $a^{bc} = 6$ in log form

Solution 10

$$a^{bc} = 6$$

$$\log_a 6 = bc$$

- Choice 1: $\log_6 ab = c$ false
 Choice 2: $\log_{bc} a = 6$ false
 Choice 3: $\log_{bc} 6 = a$ false
 Choice 4: $\log_a bc = 6$ false
 Choice 5: $\log_a 6 = bc$ true

Question 11 Experience: 10 Order: b2 Level: b2 Question-ID: 155

Express $(a + b)^4 = 15$ in log form

Solution 11

$$(a + b)^4 = 15$$

$$\log_{(a+b)} 15 = 4$$

- Choice 1: $\log_4 15 = a + b$ false
 Choice 2: $\log_{15}(a + b) = 4$ false
 Choice 3: $\log_{15} 4 = a + b$ false
 Choice 4: $\log_4(a + b) = 15$ false
 Choice 5: $\log_{(a+b)} 15 = 4$ true

Question 12 Experience: 10 Order: b2 Level: b2 Question-ID: 158

Express $(x + 4)^4 = 5$ in log form

Solution 12

$$(x + 4)^4 = 5$$

$$\log_{(x+4)} 5 = 4$$

- Choice 1: $\log_4(x + 4) = 5$ false
 Choice 2: $\log_5 4 = x + 4$ false
 Choice 3: $\log_{(x+4)} 4 = 5$ false
 Choice 4: $\log_5(x + 4) = 5$ false
 Choice 5: $\log_{(x+4)} 5 = 4$ true

Question 13 Experience: 15 Order: c1 Level: c1 Question-ID: 161

Express $\log_4(x + y) + \log_4 6$ as a single logarithm

Solution 13

$$\begin{aligned} &\log_4(x + y) + \log_4 6 \\ &= \log_4((x + y) \times 6) \\ &= \log_4 6(x + y) \end{aligned}$$

- Choice 1: $4 \log_{(x+y)} 6$ false
 Choice 2: $\log_{(x+y)} 24$ false
 Choice 3: $4 \log_6 x + y$ false
 Choice 4: $6 \log_4 x + y$ false
 Choice 5: $\log_4 6(x + y)$ true

Question 14 Experience: 10 Order: a1 Level: a1 Question-ID: 148

Express $\log_x 9 = 2$ in power form

Solution 14

$$\begin{aligned} \log_x 9 &= 2 \\ x^2 &= 9 \end{aligned}$$

- Choice 1: $x^9 = 2$ false
 Choice 2: $x^2 = 2$ false
 Choice 3: $x^9 = 9$ false
 Choice 4: $x^2 = 7$ false

Choice 5: $x^2 = 9$ true

Question 15 Experience: 15 Order: c1 Level: c1 Question-ID: 163

Express $3 \log_3(a + b) + \log_3 4$ as a single logarithm

Solution 15

$$3 \log_3(a + b) + \log_3 4$$

Choice 1: $\log_3(a + b)^{3 \log_3(4)}$ false

Choice 2: $\log_3(12)$ false

Choice 3: $\log_3((a + b)^3 \times 4)$ false

Choice 4: $\log_3 4(a + b)^3$ false

Choice 5: $\log_3 4(a + b)^3$ true

Question 16 Experience: 15 Order: c2 Level: c2 Question-ID: 164

Express $\log_4(a^2 - b^2) - 2 \log_4(a + b)$ as a single logarithm

Solution 16

$$\log_4(a^2 - b^2) - 2 \log_4(a + b)$$

Choice 1: $\log_3 \frac{(a + b)^2}{a^2 - b^2}$ false

Choice 2: $\log_3((a^2 - b^2) - (a + b)^2)$ false

Choice 3: $\log_3 \left(\frac{(a^2 - b^2)(a - b)}{(a^2 + b^2)b^2} \right)$ false

Choice 4: $\log_3 \frac{a^2 + b^2}{a - b}$ false

Choice 5: $\log_3 \frac{a - b}{a + b}$ true

Question 17 Experience: 15 Order: c2 Level: c2 Question-ID: 165

Express $\log_x(4a - 6b) + \log_x \frac{1}{2}$ as a single logarithm

Solution 17

$$\log_x(4a - 6b) + \log_x \frac{1}{2}$$

$$= \log_x \frac{1}{2}(4a - 6b)$$

$$= \log_x(2a - 3b)$$

Choice 1: $\log_{(4a-6b)} \frac{1}{2}x$ false

Choice 2: $\log_x(4a - 6b)$ false

Choice 3: $\frac{1}{2} \log_{(4a-6b)} x$ false

Choice 4: $\frac{1}{2} \log_x(2a - 3b)$ false

Choice 5: $\log_x(2a - 3b)$ true

Question 18 Experience: 15 Order: d1 Level: d1 Question-ID: 166

Express $\log_4(6a) - \log_4(2a)$ as a single logarithm

Solution 18

$$\log_4(6a) - \log_4(2a)$$

$$= \log_4(6a \div 2a)$$

$$= \log_4 3$$

- Choice 1: $\log_a 2$ false
 Choice 2: $\log_a 3$ false
 Choice 3: $\log_4 3a$ false
 Choice 4: $\log_4 12a^2$ false
 Choice 5: $\log_4 3$ true

Question 19 Experience: 15 Order: d1 Level: d1 Question-ID: 167

Express $\log_{10}(15) - \log_{10}(3)$ as a single logarithm

Solution 19

$$\begin{aligned} & \log_{10}(15) - \log_{10}(3) \\ &= \log_{10}(15 \div 3) \\ &= \log_{10} 5 \end{aligned}$$

- Choice 1: $\log_{10} 3$ false
 Choice 2: $\log_5 45$ false
 Choice 3: $\log_{10} 45$ false
 Choice 4: $\log_5 10$ false
 Choice 5: $\log_{10} 5$ true

Question 20 Experience: 15 Order: d2 Level: d2 Question-ID: 168

Express $3 \log_y(5) + \log_y(4)$ as a single logarithm

Solution 20

$$\begin{aligned} & 3 \log_y(5) + \log_y(4) \\ &= \log_y 5^3 + \log_y 4 \\ &= \log_y(5^3 \times 4) \\ &= \log_y 500 \end{aligned}$$

- Choice 1: $\log_y 100$ false
 Choice 2: $\log_y 8000$ false
 Choice 3: $4 \log_y 125$ false
 Choice 4: $4 \log_y 50$ false
 Choice 5: $\log_y 500$ true

Question 21 Experience: 15 Order: d2 Level: d2 Question-ID: 169

Express $3 \log_a(4) - 4 \log_a(2)$ as a single logarithm

Solution 21

$$\begin{aligned} & \log_a(4^3) - \log_a(2^4) \\ &= \log_a(64) - \log_a(16) \\ &= \log_y(64 \div 16) \\ &= \log_y 4 \end{aligned}$$

- Choice 1: $\log_4 a^2$ false
 Choice 2: $\log_4 16$ false
 Choice 3: $\log_4 64$ false

Choice 4: $\log_y 16$ false

Choice 5: $\log_y 4$ true

Question 22 Experience: 10 Order: a1 Level: a1 Question-ID: 153

Express $\log_3 7 = a + b^2$ in power form

Solution 22

$$\log_3 7 = a + b^2$$

$$3^{a+b^2} = 7$$

Choice 1: $7^{a+b^2} = 3$ false

Choice 2: $3^7 = 7a + b^2$ false

Choice 3: $(a + b^2)^3 = 7$ false

Choice 4: $3^7 = a + b^2$ false

Choice 5: $3^{a+b^2} = 7$ true

Question 23 Experience: 30 Order: d3 Level: d3 Question-ID: 171

Express $4 \log_9 5 - 2 \log_3(15)$ as a single logarithm

Solution 23

$$4 \log_9 5 - 2 \log_3(9)$$

$$= 4 \left(\frac{\log_3 5}{\log_3 9} \right) - 2 \log_3(15)$$

$$= \left(\frac{4 \log_3 5}{2} \right) - \log_3(15^2)$$

$$= 2 \log_3 5 - \log_3(15^2)$$

$$= \log_3(5^2 \div 15^2)$$

$$= \log_3 \frac{25}{225}$$

$$= \log_3 \frac{1}{9}$$

Choice 1: $\log_9 225$ false

Choice 2: $\log_9 25$ false

Choice 3: $\log_9 \frac{1}{3}$ false

Choice 4: $\log_3 9$ false

Choice 5: $\log_3 \frac{1}{9}$ true

Question 24 Experience: 15 Order: c1 Level: c1 Question-ID: 160

Express $\log_a 4 + \log_a 5$ as a single logarithm

Solution 24

$$\log_a 4 + \log_a 5$$

$$= \log_a (4 \times 5)$$

$$= \log_a 20$$

Choice 1: $\log_4 5a$ false

Choice 2: $a \log_4 5$ false

- Choice 3: $4 \log_a 5$ false
 Choice 4: $5 \log_a 4$ false
 Choice 5: $\log_a 20$ true

Question 25 Experience: 20 Order: e2 Level: e2 Question-ID: 188

Express $\log_a \frac{a^2 b^5}{c^3}$ as a linear combination

Solution 25

$$\log_a \frac{a^2 b^5}{c^3}$$

$$= \log_a a^2 + \log_a b^5 - \log_a c^3$$

- Choice 1: $3 + 5 \log_a b - 2 \log_a c$ false
 Choice 2: $2 + 5 \log_a b - 3 \log_a c$ false
 Choice 3: $5 + 3 \log_a b - 2 \log_a c$ false
 Choice 4: $2 + 3 \log_a b - 5 \log_a c$ false
 Choice 5: $2 + 5 \log_a b - 3 \log_a c$ true

Question 26 Experience: 20 Order: e2 Level: e2 Question-ID: 189

Express $\log_c \frac{a^4}{b^2 c^6}$ as a linear combination

Solution 26

$$\log_c \frac{a^4}{b^2 c^6}$$

$$= \log_c a^4 - \log_c b^2 - \log_c c^6$$

$$= 4 \log_c a - 2 \log_c b - 6$$

- Choice 1: $6 \log_c a - 4 \log_c b - 2$ false
 Choice 2: $2 \log_c a - 6 \log_c b - 4$ false
 Choice 3: $2 \log_c a - 4 \log_c b - 6$ false
 Choice 4: $4 \log_c a - 6 \log_c b - 2$ false
 Choice 5: $4 \log_c a - 2 \log_c b - 6$ true

Question 27 Experience: 10 Order: a1 Level: a1 Question-ID: 180

Express $\log_a b - 4 = 7$ in power form

Solution 27

$$\log_a b - 4 = 7$$

$$a^7 = b - 4$$

- Choice 1: $a^{b-4} = 7$ false
 Choice 2: $(b-4)^7 = a$ false
 Choice 3: $7^a = b - 4$ false
 Choice 4: $(b-4)^a = 7$ false
 Choice 5: $a^7 = b - 4$ true

Question 28 Experience: 25 Order: g1 Level: g1 Question-ID: 198

Solve $3^x = 27$ for x

Solution 28

$$3^x = 27$$

$$x = \log_3 27$$

$$x = \frac{\log 27}{\log 3}$$

$$x = 3$$

Answer part 1: Label $x =$ Solution 3

Answer part 1 hint: x is an integer value

Question 29 Experience: 30 Order: d3 Level: d3 Question-ID: 170

Express $2 \log_{16} 8 - 4 \log_4(2)$ as a single logarithm

Solution 29

$$\begin{aligned} & 2 \log_{16} 8 - 4 \log_4(2) \\ &= 2 \left(\frac{\log_4 8}{\log_4 16} \right) - 4 \log_4(2) \\ &= \left(\frac{2 \log_4 8}{2} \right) - \log_4(16) \\ &= \log_4(8 \div 16) \\ &= \log_4 \frac{1}{2} \end{aligned}$$

Choice 1: $\log_8 4$ false

Choice 2: $\log_8 2$ false

Choice 3: $\log_8 \frac{1}{2}$ false

Choice 4: $\log_4 \frac{1}{2}$ false

Choice 5: $\log_4 \frac{1}{2}$ true

Question 30 Experience: 30 Order: d4 Level: d4 Question-ID: 181

Express $3 \log_4 5 + 4 \log_{16}(3)$ as a single logarithm.

Solution 30

$$3 \log_4 5 + 4 \log_{16}(3)$$

Choice 1: $\log_4 \frac{225}{16}$ false

Choice 2: $\log_4 5 + \frac{4}{2} \log_4 3$ false

Choice 3: $\log_{16} \frac{225}{16}$ false

Choice 4: $\log_4 5^3 + \frac{4}{2} \log_4 3$ false

Choice 5: $\log_4 5^3 + \log_4 3^2$ true

$$= \log_4 1125$$

Question 31 Experience: 30 Order: d4 Level: d4 Question-ID: 184

Express $8 \log_4 3 - 2 \log_2(4)$ as a single logarithm

Solution 31

$$\begin{aligned} & 8 \log_4 3 - 2 \log_2 4 \\ &= 8 \left(\frac{\log_2 3}{\log_2 4} \right) - 2 \log_2 4 \\ &= 4 \log_2 3 - 2 \log_2 4 \\ &= \log_2 3^4 - \log_2 4^2 \\ &= \log_2 \frac{81}{16} \end{aligned}$$

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- Choice 1: $\log_4 65$ false
 Choice 2: $\log_4 1296$ false
 Choice 3: $\log_2 \frac{27}{4}$ false
 Choice 4: $\log_2 65$ false
 Choice 5: $\log_2 \frac{81}{16}$ true

Question 32 Experience: 20 Order: e1 Level: e1 Question-ID: 185

Express $\log_x a^2 b^3 c^4$ as a linear combination

Solution 32

$$\log_x a^2 b^3 c^4$$

$$= \log_x a^2 + \log_x b^3 + \log_x c^4$$

$$= 2 \log_x a + 3 \log_x b + 4 \log_x c$$

- Choice 1: $4 \log_x a + 2 \log_x b + 3 \log_x c$ false
 Choice 2: $4 \log_x a + 3 \log_x b + 2 \log_x c$ false
 Choice 3: $2 \log_x a + 4 \log_x b + 3 \log_x c$ false
 Choice 4: $3 \log_x a + 2 \log_x b + 4 \log_x c$ false
 Choice 5: $2 \log_x a + 3 \log_x b + 4 \log_x c$ true

Question 33 Experience: 20 Order: e1 Level: e1 Question-ID: 186

Express $\log_a x^5 y^4 z^6$ as a linear combination

Solution 33

$$\log_a x^5 y^4 z^6$$

$$= \log_a x^5 + \log_a y^4 + \log_a z^6$$

$$= 5 \log_a x + 4 \log_a y + 6 \log_a z$$

- Choice 1: $5 \log_a x + 6 \log_a y + 4 \log_a z$ false
 Choice 2: $6 \log_a x + 4 \log_a y + 5 \log_a z$ false
 Choice 3: $4 \log_a x + 5 \log_a y + 6 \log_a z$ false
 Choice 4: $6 \log_a x + 5 \log_a y + 4 \log_a z$ false
 Choice 5: $5 \log_a x + 4 \log_a y + 6 \log_a z$ true

Question 34 Experience: 20 Order: e1 Level: e1 Question-ID: 187

Express $\log_b a^7 b^2 c^5$ as a linear combination

Solution 34

$$\log_b a^7 b^2 c^5$$

$$= \log_b a^7 + \log_b b^2 + \log_b c^5$$

$$= 7 \log_b a + 2 + 5 \log_b c$$

- Choice 1: $2 \log_b a + 5 + 7 \log_b c$ false
 Choice 2: $2 \log_b a + 7 + 5 \log_b c$ false
 Choice 3: $7 \log_b a + 5 + 2 \log_b c$ false
 Choice 4: $5 \log_b a + 2 + 7 \log_b c$ false
 Choice 5: $7 \log_b a + 2 + 5 \log_b c$ true

Question 35 Experience: 20 Order: e2 Level: e2 Question-ID: 190

Express $\log_x \frac{x^3 y^8}{z^2}$ as a linear combination

Solution 35

$$\log_x \frac{x^3 y^8}{z^2}$$

$$= \log_x x^3 + \log_x y^8 - \log_x z^2$$

$$= 3 + 8 \log_x y - 2 \log_x z$$

Choice 1: $8 + 2 \log_x y - 3 \log_x z$ false

Choice 2: $2 + 8 \log_x y - 3 \log_x z$ false

Choice 3: $2 + 3 \log_x y - 8 \log_x z$ false

Choice 4: $3 + 2 \log_x y - 8 \log_x z$ false

Choice 5: $3 + 8 \log_x y - 2 \log_x z$ true

Question 36 Experience: 20 Order: f1 Level: f1 Question-ID: 191

Express $\log_4 20$ in terms of \ln

Solution 36

Choice 1: $\log_4 20 \frac{\ln 20}{\ln 4}$ false

Choice 2: $\log_e 20 \frac{\ln 80}{\ln 24}$ false

Choice 3: $\log_e 4 \ln 80$ false

Choice 4: $\ln 20 \frac{\ln 5}{\ln 20}$ false

Choice 5: $\ln 4 \frac{\ln 20}{\ln 4}$ false

Question 37 Experience: 20 Order: f1 Level: f1 Question-ID: 192

Express $\log_8 24$ in terms of \ln

Solution 37

Choice 1: $\log_8 24 \frac{\ln 24}{\ln 192}$ false

Choice 2: $\log_e 24 \frac{\ln 192}{\ln 192}$ false

Choice 3: $\log_e 8 \ln 32$ false

Choice 4: $\ln 24 \frac{\ln 3}{\ln 24}$ false

Choice 5: $\ln 8 \frac{\ln 24}{\ln 8}$ false

Question 38 Experience: 20 Order: f1 Level: f1 Question-ID: 193

Express $\log_6 18$ in terms of \ln

Solution 38

Choice 1: $\log_6 18 \frac{\ln 6}{\ln 2}$ false

Choice 2: $\log_e 18 \ln 108$ false

Choice 3: $\log_e 6 \ln 24$ false

Choice 4: $\ln 18 \frac{\ln 3}{\ln 18}$ false

Choice 5: $\ln 6 \frac{\ln 18}{\ln 6}$ true

Question 39 Experience: 20 Order: f2 Level: f2 Question-ID: 194

Express $\log_2 8$ in terms of \ln

Solution 39

Choice 1: $\log_2 8 \frac{\ln 8}{\ln 16}$ false

Choice 2: $\log_e 8 \frac{\ln 10}{\ln 10}$ false

Choice 3: $\log_e 2 \ln 16$ false

Choice 4: $\ln 8 \frac{\ln 8}{\ln 2}$ true

Choice 4: $\frac{\ln 4}{\ln 8}$ false
 Choice 5: $\frac{\ln 8}{\ln 2}$ true

Question 40 Experience: 20 Order: f2 Level: f2 Question-ID: 195

Express $\log_6 15$ in terms of \ln

Solution 40

Choice 1: $\log_6 15 \frac{\ln 5}{\ln 2}$ false
 Choice 2: $\log_e 15 \frac{\ln 2}{\ln 9}$ false
 Choice 3: $\frac{\ln 21}{\log_e 6}$ false
 Choice 4: $\frac{\ln 15}{\ln 6}$ false
 Choice 5: $\frac{\ln 15}{\ln 6}$ true

Question 41 Experience: 25 Order: g1 Level: g1 Question-ID: 199

Solve $4^x = 64$ for x

Solution 41

$$4^x = 64$$

$$x = \log_4 64$$

$$x = \frac{\log 64}{\log 4}$$

$$x = 3$$

Answer part 1: Label $x =$ Solution 3

Answer part 1 hint: x is an integer value

Question 42 Experience: 25 Order: h1 Level: h1 Question-ID: 200

Solve $5^x = 35$ for x

Solution 42

$$5^x = 35$$

$$x = \log_5 35$$

$$x = \frac{\log 35}{\log 5}$$

$$x = 2.21$$

Answer part 1: Label $x =$ Solution 2.21

Answer part 1 hint: Write the answer to 2 d.p.

Question 43 Experience: 25 Order: h1 Level: h1 Question-ID: 201

Solve $4^x = 20$ for x

Solution 43

$$4^x = 20$$

$$x = \log_4 20$$

$$x = \frac{\log 20}{\log 4}$$

$$x = 2.16$$

Answer part 1: Label $x =$ Solution 2.16

Answer part 1 hint: Write the answer to 2 d.p.

Question 44 Experience: 25 Order: h1 Level: h1 Question-ID: 202

Solve $5^x = 75$ for x

Solution 44

$$5^x = 75$$

$$x = \log_5 75$$

$$x = \frac{\log 75}{\log 5}$$

$$x = 2.68$$

Answer part 1: Label $x =$ Solution 2.68

Answer part 1 hint: Write the answer to 2 d.p.

Question 45 Experience: 40 Order: i1 Level: i1 Question-ID: 203

Solve $3^{5x+3} = 5^{3x-1}$ for x

Solution 45

$$3^{5x+3} = 5^{3x-1}$$

$$\log(3^{5x+3}) \log(5^{3x-1})$$

$$(5x + 3) \log 3 = (3x - 1) \log 5$$

$$5x \log 3 + 3 \log 3 = 3x \log 5 - \log 5$$

$$5x \log 3 - 3x \log 5 = -\log 5 - 3 \log 3$$

$$x(5 \log 3 - 3 \log 5) = -(\log 5 + 3 \log 3)$$

$$x \log \frac{3^5}{5^3} = -\log(5 \times 3^3)$$

$$0.289x = -2.13$$

$$x = -7.38$$

Answer part 1: Label $x =$ Solution -7.38

Answer part 1 hint: Write the answer to 2 d.p.

Question 46 Experience: 40 Order: i1 Level: i1 Question-ID: 204

Solve $5^{2x-5} = 3^{6x+7}$ for x

Solution 46

$$5^{2x-5} = 3^{6x+7}$$

$$\log(5^{2x-5}) = \log(3^{6x+7})$$

$$(2x - 5) \log 5 = (6x + 7) \log 3$$

$$2x \log 5 - 5 \log 5 = 6x \log 3 + 7 \log 3$$

$$2x \log 5 - 6x \log 3 = 5 \log 5 + 7 \log 3$$

$$x(\log 5^2 - \log 3^6) = \log 5^5 + \log 3^7$$

$$x \left(\log \frac{5^2}{3^6} \right) = \log(5^5 \times 3^7)$$

$$-1.46x = 6.83$$

$$x = -4.67$$

Answer part 1: Label $x =$ Solution -4.67

Answer part 1 hint: Write the answer to 2 d.p.

Question 47 Experience: 40 Order: i2 Level: i2 Question-ID: 205

Solve $2^{4x-3} = 5^{7x+4}$ for x

Solution 47

$$2^{4x-3} = 5^{7x+4}$$

$$\log(2^{4x-3}) = \log(5^{7x+4})$$

$$(4x - 3) \log 2 = (7x + 4) \log 5$$

$$4x \log 2 - 3 \log 2 = 7x \log 5 + 4 \log 5$$

$$4x \log 2 - 7x \log 5 = 3 \log 2 + 4 \log 5$$

$$x(\log 2^4 - \log 5^7) = \log 2^3 + \log 5^4$$

$$x \left(\log \frac{2^4}{5^7} \right) = \log(2^3 \times 5^4)$$

$$-3.69x = 3.70$$

$$x = -1.00$$

Answer part 1: Label $x =$ Solution -1.00

Answer part 1 hint: Write the answer to 2 d.p.

Question 48 Experience: 40 Order: i2 Level: i2 Question-ID: 206

Solve $7^{6x+2} = 4^{4x-5}$ for x

Solution 48

$$7^{6x+2} = 4^{4x-5}$$

$$\log(7^{6x+2}) = \log(4^{4x-5})$$

$$(6x + 2) \log 7 = (4x - 5) \log 4$$

$$6x \log 7 + 2 \log 7 = 4x \log 4 - 5 \log 4$$

$$6x \log 7 - 4x \log 4 = -2 \log 7 - 5 \log 4$$

$$x(6 \log 7 - 4 \log 4) = -(2 \log 7 + 5 \log 4)$$

$$x(\log 7^6 - \log 4^4) = -(\log 7^2 + \log 4^5)$$

$$x \left(\log \frac{7^6}{4^4} \right) = -\log(7^2 \times 4^5)$$

$$2.66x = -4.70$$

$$x = -1.77$$

Answer part 1: Label $x =$ Solution -1.77

Answer part 1 hint: Write the answer to 2 d.p.

Question 49 Experience: 40 Order: i1 Level: i1 Question-ID: 234

Solve $5^{2-3x} = 4^{3+2x}$ for x

Solution 49

$$5^{2-3x} = 4^{3+2x}$$

$$\log(5^{2-3x}) = \log(4^{3+2x})$$

$$(2 - 3x) \log 5 = (3 + 2x) \log 4$$

$$2 \log 5 - 3x \log 5 = 3 \log 4 + 2x \log 4$$

$$-3x \log 5 - 2x \log 4 = 3 \log 4 - 2 \log 5$$

$$3x \log 5 + 2x \log 4 = -3 \log 4 + 2 \log 5$$

$$x(\log 5^3 + \log 4^2) = 2 \log 5 - 3 \log 4$$

$$x \log(5^3 \times 4^2) = \log\left(\frac{5^2}{4^3}\right)$$

$$3.30x = -0.408$$

$$x = -0.12$$

Answer part 1: Label $x =$ Solution -0.12

Answer part 1 hint: Write the answer to 2 d.p.

Question 50 Experience: 40 Order: i2 Level: i2 Question-ID: 235

Solve $3^{2x+1} = 7^{2x-5}$ for x

Solution 50

$$3^{2x+1} = 7^{2x-5}$$

$$\log(3^{2x+1}) = \log(7^{2x-5})$$

$$(2x + 1) \log 3 = (2x - 5) \log 7$$

$$2x \log 3 + \log 3 = 2x \log 7 - 5 \log 7$$

$$2x \log 3 - 2x \log 7 = -5 \log 7 - \log 3$$

$$x(2 \log 3 - 2 \log 7) = -(5 \log 7 + \log 3)$$

$$x(\log 3^2 - \log 7^2) = -(\log 7^5 + \log 3)$$

$$x \left(\log \frac{3^2}{7^2} \right) = -\log(7^5 \times 3)$$

$$-0.736x = -4.70$$

$$x = 6.39$$

Answer part 1: Label $x =$ Solution 6.39

Answer part 1 hint: Write the answer to 2 d.p.

Lesson 2 Applications of logarithms

Question 1 Experience: 50 Order: j1 Level: j1 Question-ID: 207

Solve $(5^x)^2 - 8(5^x) + 15 = 0$ for x

Solution 1

$$(5^x)^2 - 8(5^x) + 15 = 0 \quad y = 5^x$$

$$y^2 - 8y + 15 = 0 \quad P = 15 \quad S = -8$$

$$(y - 5)(y - 3) = 0 \quad (-5, -3)$$

$$y = 5 \quad y = 3$$

$$5^x = 5 \quad 5^x = 3$$

$$x = 1 \quad x = 0.68$$

Answer part 1: Label $x =$ Solution 0.68,1

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 2 Experience: 50 Order: j1 Level: j1 Question-ID: 208

Solve $(2^x)^2 - 9(2^x) + 14 = 0$ for x

Solution 2

$$(2^x)^2 - 9(2^x) + 14 = 0 \quad y = 2^x$$

$$y^2 - 9y + 14 = 0 \quad P = 14 \quad S = -9$$

$$(y - 7)(y - 2) = 0 \quad (-7, -2)$$

$$y = 7 \quad y = 2$$

$$2^x = 7 \quad 2^x = 2$$

$$x = 2.81 \quad x = 1$$

Answer part 1: Label $x =$ Solution 1,2.81

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 3 Experience: 50 Order: j2 Level: j2 Question-ID: 209

Solve $2^{2x+1} - 10(2^x) + 12 = 0$ for x

Solution 3

$$2^{2x+1} - 10(2^x) + 12 = 0$$

$$2(2^x)^2 - 10(2^x) + 12 = 0 \quad y = 2^x$$

$$2y^2 - 10y + 12 = 0 \quad P = 24 \quad S = -10$$

$$(y - 2)(y - 3) = 0 \quad (-4, -6) \quad (-2, -3)$$

$$y = 2 \quad y = 3$$

$$2^x = 2 \quad 2^x = 3$$

$$x = 1 \quad x = 1.58$$

Answer part 1: Label $x =$ Solution 1,1.58

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 4 Experience: 50 Order: j2 Level: j2 Question-ID: 210

Solve $3^{2x+1} - 17(3^x) + 20 = 0$ for x

Solution 4

$$3^{2x+1} - 17(3^x) + 20 = 0$$

$$3(3^x)^2 - 17(3^x) + 20 = 0 \quad y = 3^x$$

$$3y^2 - 17y + 20 = 0 \quad P = 60 \quad S = -17$$

$$(y - 4)\left(y - \frac{5}{3}\right) = 0 \quad (-12, -5) \quad \left(-4, -\frac{5}{3}\right)$$

$$y = 4 \quad y = \frac{5}{3}$$

$$3^x = 4 \quad 3^x = \frac{5}{3}$$

$$x = 1.26 \quad x = 0.46$$

Answer part 1: Label $x =$ Solution 0.46,1.26

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 5 Experience: 50 Order: k1 Level: k1 Question-ID: 212

Solve $3 \log_4 x - 4 \log_{16} x = 2$ for x

Solution 5

$$\begin{aligned}
3 \log_4 x - 4 \log_{16} x &= 2 \\
3 \log_4 x - 4 \left(\frac{\log_4 x}{\log_4 16} \right) &= 2 \\
3 \log_4 x - 4 \left(\frac{\log_4 x}{2} \right) &= 2 \\
3 \log_4 x - 2 \log_4 x &= 2 \\
\log_4 x^3 - \log_4 x^2 &= 2 \\
\log_4 \frac{x^3}{x^2} &= 2 \\
\log_4 x &= 2 \\
x &= 4^2 \\
x &= 16
\end{aligned}$$

Answer part 1: Label $x =$ Solution 16

Answer part 1 hint:

Question 6 Experience: 50 Order: k2 Level: k2 Question-ID: 213

Solve $5 \log_2 x - 9 \log_8 x = 10$ for x

Solution 6

$$\begin{aligned}
5 \log_2 x - 9 \log_8 x &= 10 \\
5 \log_2 x - 9 \left(\frac{\log_2 x}{\log_2 8} \right) &= 10 \\
5 \log_2 x - 9 \left(\frac{\log_2 x}{3} \right) &= 10 \\
\log_2 x^5 - \log_2 x^3 &= 10 \\
\log_2 \frac{x^5}{x^3} &= 10 \\
x^2 &= 2^{10} \\
x &= 32
\end{aligned}$$

Answer part 1: Label $x =$ Solution 32

Answer part 1 hint:

Question 7 Experience: 50 Order: k2 Level: k2 Question-ID: 214

Solve $2 \log_2 x + 5 \log_4 x = 18$ for x

Solution 7

$$\begin{aligned}
2\log_2 x + 5\log_4 x &= 16 \\
2\log_2 x + 5\left(\frac{\log_2 x}{\log_2 4}\right) &= 18 \\
2\log_2 x + 5\left(\frac{\log_2 x}{2}\right) &= 18 \\
\log_2 x^2 + \log_2 x^{\frac{5}{2}} &= 18 \\
\log_2 x^{\frac{9}{2}} &= 18 \\
\frac{9}{2}\log_2 x &= 18 \\
\log_2 x &= 4 \\
x &= 2^4 \\
x &= 16
\end{aligned}$$

Answer part 1: Label $x =$ Solution 16

Answer part 1 hint:

Question 8 Experience: 35 Order: k3 Level: k3 Question-ID: 216

Solve $\log_2(3x + 2) - \log_2(x - 8) = 4$ for x

Solution 8

$$\begin{aligned}
\log_2(3x + 2) - \log_2(x - 8) &= 4 \\
\log_2 \frac{3x + 2}{x - 8} &= 4 \\
\frac{3x + 2}{x - 8} &= 16 \\
3x + 2 &= 16(x - 8) \\
3x + 2 &= 16x - 128
\end{aligned}$$

Answer part 1: Label $x =$ Solution 10
 Answer part 1 hint: $13x = 130$
 $x = 10$

Question 9 Experience: 50 Order: k3 Level: k3 Question-ID: 217

Solve $\log_4(6x + 2) - \log_4(x - 3) = 2$ for x

Solution 9

$$\begin{aligned}
\log_4(6x + 2) - \log_4(x - 3) &= 2 \\
\log_4 \frac{6x + 2}{x - 3} &= 2 \\
\frac{6x + 2}{x - 3} &= 16 \\
6x + 2 &= 16(x - 3) \\
6x + 2 &= 16x - 48 \\
10x &= 50 \\
x &= 5
\end{aligned}$$

Answer part 1: Label $x =$ Solution 5

Answer part 1 hint:

Question 10 Experience: 45 Order: 12 Level: 12 Question-ID: 220

Solve $\log_5 x = 8 - \frac{15}{\log_5 x}$ for all values of x

Solution 10

$$\log_5 x = 8 - \frac{15}{\log_5 x}$$

$$(\log_5 x)^2 = 8 \log_5 x - 15$$

$$(\log_5 x)^2 - 8 \log_5 x + 15 = 0 \quad P = 15 \quad S = -8$$

$$(\log_5 x - 5)(\log_5 x - 3) = 0 \quad (-5, -3)$$

$$\log_5 x = 5 \quad \log_5 x = 3$$

$$x = 5^5 \quad x = 5^3$$

$$x = 3125 \quad x = 125$$

Answer part 1: Label $x =$ Solution 125,3125

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 11 Experience: 45 Order: 14 Level: 14 Question-ID: 225

Solve $\log_2(x - 6) = \log_4(5x) + \log_4(2x + 3)$ for all values of x

Solution 11

$$\log_2(x - 6) = \log_4(5x) + \log_4(2x + 3)$$

$$\log_2(x - 6) = \log_4 5x(2x + 3)$$

$$\log_2(x - 6) = \frac{\log_2 5x(2x + 3)}{\log_2 4}$$

$$\log_2(x - 6) = \frac{\log_2 5x(2x + 3)}{2}$$

$$2 \log_2(x - 6) = \log_2 5x(2x + 3)$$

$$\log_2(x - 6)^2 = \log_2 5x(2x + 3)$$

$$(x - 6)^2 = 5x(2x + 3)$$

$$x^2 - 12x + 36 = 10x^2 + 15x$$

$$9x^2 + 27x - 36 = 0$$

$$x^2 + 3x - 4 = 0 \quad P = -4 \quad S = 3$$

$$(x + 4)(x - 1) = 0 \quad (4, -1)$$

$$x = -4 \quad x = 1$$

Answer part 1: Label $x =$ Solution -4,1

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 12 Experience: 45 Order: 13 Level: 13 Question-ID: 223

Solve $\log_2 x + \log_2(x + 1) = \log_2(x + 28) - \log_2 2$ for all values of x .

Solution 12

$$\log_2 x + \log_2(x + 1) = \log_2(x + 28) - \log_2 2$$

$$\log_2 x(x + 1) = \log_2 \frac{x + 28}{2}$$

$$x(x + 1) = \frac{x + 28}{2}$$

$$2x^2 + 2x = x + 28$$

$$2x^2 + x - 28 = 0 \quad P = -56 \quad S = 1$$

$$\left(x - \frac{7}{2}\right)(x - 4) = 0 \quad (-7, 8) \quad \left(-\frac{7}{2}, 4\right)$$

$$x = \frac{7}{2} \quad x = 4$$

$$x = 3.5$$

Answer part 1: Label $x =$ Solution 3.5,4

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 13 Experience: 45 Order: l3 Level: l3 Question-ID: 222

Solve $\log_4 3 - \log_4(x + 4) = \log_4 2 - 2\log_4 x$ for all values of x .

Solution 13

$$\log_4 3 - \log_4(x + 4) = \log_4 2 - 2\log_4 x$$

$$\log_4 \frac{3}{x + 4} = \log_4 \frac{2}{x^2}$$

$$\frac{3}{x + 4} = \frac{2}{x^2}$$

$$3x^2 = 2(x + 4)$$

$$3x^2 - 2x - 8 = 0 \quad P = -24 \quad S = -2$$

$$(x - 2)\left(x + \frac{4}{3}\right) = 0 \quad (-6, 4) \quad \left(-2, \frac{4}{3}\right)$$

$$x = 2 \quad x = -\frac{4}{3}$$

$$x = -1.33$$

Answer part 1: Label $x =$ Solution -1.33,2

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 14 Experience: 45 Order: l4 Level: l4 Question-ID: 224

Solve $\log_9(6x^2 - 12x + 1) = \log_3(x - 5)$ for all values of x

Solution 14

$$\log_9(6x^2 - 12x + 1) = \log_3(x - 5)$$

$$\frac{\log_3(6x^2 - 12x + 1)}{\log_3 9} = \log_3(x - 5)$$

$$\log_3 6x^2 - 12x + 1 = 2\log_3(x - 5)$$

$$\log_3 6x^2 - 12x + 1 = \log_3(x - 5)^2$$

$$6x^2 - 12x + 1 = (x - 5)^2$$

$$6x^2 - 12x + 1 = x^2 - 10x + 25$$

$$5x^2 - 2x - 24 = 0 \quad P = -120 \quad S = -2$$

$$(x + 2)\left(x - \frac{12}{5}\right) = 0 \quad (10, -12) \quad \left(2, -\frac{12}{5}\right)$$

$$x = -2 \quad x = -\frac{12}{5}$$

$$x = -2.4$$

Answer part 1: Label $x =$ Solution -2.4,-2

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 15 Experience: 45 Order: 11 Level: 11 Question-ID: 218

Solve $\log_4(x + 9) + \log_4(x + 3) = 2$ for all values of x

Solution 15

$$\log_4(x + 9) + \log_4(x + 3) = 2$$

$$\log_4(x + 9)(x + 3) = 2$$

$$(x + 9)(x + 3) = 4^2$$

$$x^2 + 12x + 27 = 16$$

$$x^2 + 12x + 11 = 0 \quad P = 11 \quad S = 12$$

$$(x + 11)(x + 1) = 0 \quad (11, 1)$$

$$x = -1$$

Answer part 1: Label $x =$ Solution -1

Answer part 1 hint:

Question 16 Experience: 45 Order: 11 Level: 11 Question-ID: 219

Solve $\log_3(x + 12) + \log_3(x + 4) = 2$ for all values of x

Solution 16

$$\log_3(x+12) + \log_3(x+4) = 2$$

$$\log_3(x+12)(x+4) = 2$$

$$(x+12)(x+4) = 3^2$$

$$x^2 + 16x + 48 = 9$$

$$x^2 + 16x + 39 = 0 \quad P = 39 \quad S = 16$$

$$(x+13)(x+3) = 0 \quad (13, 3)$$

$$x = -3$$

Answer part 1: Label $x =$ Solution -3

Answer part 1 hint:

Question 17 Experience: 45 Order: l1 Level: l1 Question-ID: 221

Solve $\log_{10}(x+10) + \log_{10}(x+4) = 2$ for all values of x

Solution 17

$$\log_{10}(x+10) + \log_{10}(x+4) = 2$$

$$\log_{10}(x+10)(x+4) = 2$$

$$(x+10)(x+4) = 10^2$$

$$x^2 + 14x + 140 = 100$$

$$x^2 + 14x + 40 = 0 \quad P = 40 \quad S = 14$$

$$(x+10)(x+4) = 0 \quad (10, 4)$$

$$x = -4$$

Answer part 1: Label $x =$ Solution -4

Answer part 1 hint:

Question 18 Experience: 50 Order: j1 Level: j1 Question-ID: 236

Solve $3^{2x} - 12(3^x) + 27 = 0$ for x

Solution 18

$$3^{2x} - 12(3^x) + 27 = 0$$

$$(3^x)^2 - 12(3^x) + 27 = 0 \quad y = 3^x$$

$$y^2 - 12y + 27 = 0 \quad P = 27 \quad S = -12$$

$$(y-9)(y-3) = 0 \quad (-9, -3)$$

$$y = 9 \quad y = 3$$

$$3^x = 9 \quad 3^x = 3$$

$$x = 2 \quad x = 1$$

Answer part 1: Label $x =$ Solution 1,2

Answer part 1 hint: Give all solutions correct to 2 d.p, exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 19 Experience: 50 Order: j2 Level: j2 Question-ID: 237

Solve $2(5^{2x+1}) - 17(5^x) + 3 = 0$ for x

Solution 19

$$2(5^{2x+1}) - 17(5^x) + 3 = 0$$

$$2 \cdot 5 \cdot (5^{2x}) - 17(5^x) + 3 = 0$$

$$10(5^x)^2 - 17(5^x) + 3 = 0 \quad y = 5^x$$

$$10y^2 - 17y + 3 = 0 \quad P = 30 \quad S = -17$$

$$\left(y - \frac{3}{2}\right) \left(y - \frac{1}{5}\right) = 0 \quad (-15, -2) \quad \left(-\frac{3}{2}, -\frac{1}{5}\right)$$

$$y = \frac{3}{2} \quad y = \frac{1}{5}$$

$$5^x = \frac{3}{2} \quad 5^x = \frac{1}{5}$$

$$x = 0.25 \quad x = -1$$

Answer part 1: Label $x =$ Solution 0.25,-1

Answer part 1 hint: Give all solutions correct to 2 d.p., exact integer solutions are also allowed, separate solutions with comma(s). (e.g: 2,-3 or 1.5,0.6)

Question 20 Experience: 50 Order: k1 Level: k1 Question-ID: 211

Solve $2 \log_3 x - 12 \log_9 x = 4$ for x

Solution 20

$$\log_3 x - 6 \log_9 x = 4$$

$$\log_3 x - 6 \left(\frac{\log_3 x}{\log_3 9} \right) = 4$$

$$\log_3 x - 6 \left(\frac{\log_3 x}{2} \right) = 4$$

$$\log_3 x - 3 \log_3 x = 4$$

$$\log_3 x - \log_3 x^3 = 4$$

$$\log_3 \frac{x}{x^3} = 4$$

$$\frac{1}{x^2} = 3^4$$

$$\frac{1}{x} = 3^2$$

$$x = \frac{1}{9}$$

$$x = 0.11$$

Answer part 1: Label $x =$ Solution 0.11

Answer part 1 hint: Write the answer to 2 d.p.

End of Chapter Questions

OCR A Level Maths

To be added

AQA A Level Maths

To be added

MEI A Level Maths

To be added

Unit 1 C 1

Chapter 1 asdfasfd

End of Chapter Questions

Edexcel A Level Further Maths

To be added

MEI A Level Further Maths

To be added

Unused Questions

Question 1 Experience: 50 Order: Level: Question-ID: 44

A sequence is defined by $2U_{n+4} = 3U_{n+3} - U_n$, $U_1 = 2, U_5 = 5, U_2 = 2U_4$, find the values of U_2, U_4 and U_6 .

Solution 1

$$2U_5 = 3U_4 - U_1$$

$$2(5) = 3U_4 - 2$$

$$3U_4 = 12$$

$$U_4 = 4$$

$$U_2 = 2U_4$$

$$U_2 = 2(4)$$

$$U_2 = 8$$

$$2U_6 = 3U_5 - U_2$$

$$2U_6 = 3(5) - 8$$

$$U_6 = \frac{7}{2}$$

Choice 1: $U_2 = 10$ $U_4 = 3$ $U_6 = 2$ false

- Choice 2: $U_2 = 8 \ U_4 = 3 \ U_6 = \frac{7}{2}$ false
- Choice 3: $U_2 = 6 \ U_4 = 4 \ U_6 = \frac{7}{2}$ false
- Choice 4: $U_2 = 6 \ U_4 = 3 \ U_6 = \frac{7}{2}$ false
- Choice 5: $U_2 = 8 \ U_4 = 4 \ U_6 = \frac{7}{2}$ true

Question 2 Experience: 50 Order: Level: Question-ID: 45

A sequence is defined by $3x_{n+4} = \frac{2x_{n+3}}{x_n}$, $x_1 = 3, x_4 = 9, \frac{x_6}{x_3} = 6$, find the value of x_5 and x_7 .

Solution 2

$$3x_5 = \frac{2x_4}{x_1}$$

$$3x_5 = \frac{2(9)}{3}$$

$$x_5 = 2$$

$$3x_7 = \frac{2x_6}{x_3}$$

$$3x_7 = 2(6)$$

$$x_7 = 4$$

- Choice 1: $x_5 = 2 \ x_7 = 8$ false
- Choice 2: $x_5 = 6 \ x_7 = 4$ false
- Choice 3: $x_5 = 6 \ x_7 = 8$ false
- Choice 4: $x_5 = 6 \ x_7 = 2$ false
- Choice 5: $x_5 = 2 \ x_7 = 4$ true

Question 3 Experience: 60 Order: Level: Question-ID: 61

A sequence is defined by the recurrence relation $Y_{n+1} = \frac{a^2}{Y_n} + b, Y_1 = 3, a, b \in \mathbb{N}$, given that $Y_2 = 7$ and $Y_3 = \frac{37}{7}$ find the value of a and b .

Solution 3

$$Y_2 = \frac{a^2}{Y_1} + b$$

$$7 = \frac{a^2}{3} + b \quad (1)$$

$$Y_3 = \frac{a^2}{Y_2} + b$$

$$\frac{37}{7} = \frac{a^2}{7} + b \quad (2)$$

$$(1) - (2) \quad 7 - \frac{37}{7} = \frac{a^2}{3} + b - \left(\frac{a^2}{7} + b \right)$$

$$\frac{12}{7} = \frac{4a^2}{21}$$

$$a^2 = 9$$

$$a = 3$$

$$\text{Sub into(1)} \quad 7 = \frac{3^2}{3} + b$$

$$b = 4$$

Choice 1: $a = 3$ $b = 3$ false

Choice 2: $a = 2$ $b = 3$ false

Choice 3: $a = 2$ $b = 4$ false

Choice 4: $a = 4$ $b = 2$ false

Choice 5: $a = 3$ $b = 4$ true

Question 4 Experience: 60 Order: Level: Question-ID: 60

A sequence is defined by the recurrence relation $u_{n+1} = \sqrt{a} \left(u_n - \frac{1}{b} \right)$, $5u_1 = 4$, given that $u_2 = 7$ and $u_3 = 13$ find the value of a and b .

Solution 4

$$u_2 = \sqrt{a} \left(u_1 - \frac{1}{b} \right)$$

$$7 = \sqrt{a} \left(4 - \frac{1}{b} \right) \quad (1)$$

$$7 = 4\sqrt{a} - \frac{\sqrt{a}}{b} \quad (2)$$

$$u_3 = \sqrt{a} \left(u_2 - \frac{1}{b} \right)$$

$$13 = \sqrt{a} \left(7 - \frac{1}{b} \right)$$

$$13 = 7\sqrt{a} - \frac{\sqrt{a}}{b} \quad (3)$$

$$(3) - (2) \quad 13 - 7 = 7\sqrt{a} - \frac{\sqrt{a}}{b} - \left(4\sqrt{a} - \frac{\sqrt{a}}{b} \right)$$

$$6 = 3\sqrt{a}$$

$$2 = \sqrt{a}$$

$$a = 4$$

$$\text{Sub into (1)} \quad 7 = \sqrt{4} \left(4 - \frac{1}{b} \right)$$

$$\frac{7}{2} = 4 - \frac{1}{b}$$

$$-\frac{1}{2} = -\frac{1}{b}$$

$$b = 2$$

Choice 1: $a = 3$ $b = 2$ false

Choice 2: $a = 4$ $b = 3$ false

Choice 3: $a = 3$ $b = 3$ false

Choice 4: $a = 2$ $b = 3$ false

Choice 5: $a = 4$ $b = 2$ true

Question 5 Experience: 50 Order: Level: Question-ID: 59

A sequence is defined by the recurrence relation $Y_{n+1} = \frac{a^2}{Y_n} + b$, $Y_1 = 3$, $a, b > 0$, given that $Y_2 = 7$ and $Y_3 = \frac{37}{7}$ find the value of a and b .

Solution 5

$$Y_2 = \frac{a^2}{Y_1} + b$$

$$7 = \frac{a^2}{3} + b \quad (1)$$

$$Y_3 = \frac{a^2}{Y_2} + b$$

$$\frac{37}{7} = \frac{a^2}{7} + b \quad (2)$$

$$(1) - (2) \quad 7 - \frac{37}{7} = \frac{a^2}{3} + b - \left(\frac{a^2}{7} + b \right)$$

$$\frac{12}{7} = \frac{4a^2}{21}$$

$$a^2 = 9$$

$$a = 3$$

$$\text{Sub into(1)} \quad 7 = \frac{3^2}{3} + b$$

$$b = 4$$

Choice 1: $a = 3$ $b = 3$ false

Choice 2: $a = 2$ $b = 3$ false

Choice 3: $a = 2$ $b = 4$ false

Choice 4: $a = 4$ $b = 2$ false

Choice 5: $a = 3$ $b = 4$ true

Question 6 Experience: 100 Order: Level: Question-ID: 38

A sequence is defined by $X_n = \frac{a+1}{n} + b$, given the Sum of the first three terms is $\frac{2}{3}$ and the fifth term is $-\frac{3}{5}$, find the values of a and b .

Solution 6

$$\begin{aligned}
S_3 &= \left(\frac{a+1}{(1)} + b \right) + \left(\frac{a+1}{(2)} + b \right) + \left(\frac{a+1}{(3)} + b \right) \\
S_3 &= a + \frac{a}{2} + \frac{a}{3} + 3b + 1 + \frac{1}{2} + \frac{1}{3} \\
S_3 &= \frac{11}{6}a + 3b + \frac{11}{6} \quad S_3 = \frac{2}{3} \\
\frac{11}{6}a + 3b + \frac{11}{6} &= \frac{2}{3} \\
11a + 18b + 11 &= 4 \quad (1)
\end{aligned}$$

$$\begin{aligned}
X_5 &= \frac{a+1}{5} + b \quad X_5 = -\frac{3}{5} \\
\frac{a+1}{5} + b &= -\frac{3}{5} \\
a + 1 + 5b &= -3 \quad (2) \\
11a + 11 + 55b &= -33 \quad (3)
\end{aligned}$$

$$\begin{aligned}
(3) - (1) \quad 11a + 11 + 55b - (11a + 18b + 11) &= -33 - 4 \\
37b &= -37 \\
b &= -1
\end{aligned}$$

$$\begin{aligned}
\text{sub into } (2) \quad a + 1 + 5(-1) &= -3 \\
a - 4 &= -3 \\
a &= 1
\end{aligned}$$

Choice 1: $a = 1 \quad b = 2 \quad$ false
 Choice 2: $a = 3 \quad b = -1 \quad$ false
 Choice 3: $a = 3 \quad b = 2 \quad$ false
 Choice 4: $a = 2 \quad b = -1 \quad$ false
 Choice 5: $a = 1 \quad b = -1 \quad$ true

Question 7 Experience: 35 Order: z Level: z Question-ID: 215

Solve $\log_3(x+6) - \log_3\left(\frac{x}{2} - 4\right) = 2$ for x

Solution 7

$$\begin{aligned}
\log_3(x+6) - \log_3\left(\frac{x}{2} - 4\right) &= 2 \\
\log_3 \frac{x+6}{\left(\frac{x}{2} - 4\right)} &= 2 \\
\frac{2(x+6)}{x-8} &= 9 \\
2(x+6) &= 9(x-8) \\
2x+12 &= 9x-72 \\
7x &= 84 \\
x &= 12
\end{aligned}$$

Answer part 1: Label $x =$ Solution 12

Answer part 1 hint:

Question 8 Experience: 45 Order: l1 Level: l1 Question-ID: 241

Solve $\log_2(x - 3) + \log_2(3x - 7) = 4$ for all values of x

Solution 8

$$\log_2(x - 3) + \log_2(3x - 7) = 4$$

$$\log_2(x - 3)(3x - 7) = 4$$

$$(x - 3)(3x - 7) = 2^4$$

$$3x^2 - 16x + 21 = 16$$

$$3x^2 - 16x + 5 = 0 \quad P = 15 \quad S = -16$$

$$(x - 5)\left(x - \frac{1}{3}\right) = 0 \quad (-15, -1) \quad \left(-5, -\frac{1}{3}\right)$$

$$x = 5$$

Answer part 1: Label $x =$ Solution 5

Answer part 1 hint:

Question 9 Experience: 50 Order: k1 Level: k1 Question-ID: 238

Solve $4\log_2 x + 3\log_8 x = 10$ for x

Solution 9

$$4\log_2 x + 3\log_8 x = 10$$

$$4\log_2 x + 3\left(\frac{\log_2 x}{\log_2 8}\right) = 10$$

$$4\log_2 x + 3\left(\frac{\log_2 x}{3}\right) = 10$$

$$4\log_2 x + \log_2 x = 10$$

$$\log_2 x^4 + \log_2 x = 10$$

$$\log_2 x^4 \times x = 10$$

$$x^5 = 2^{10}$$

$$x = 2^2$$

$$x = 4$$

Answer part 1: Label $x =$ Solution 4

Answer part 1 hint:

Question 10 Experience: 50 Order: k2 Level: k2 Question-ID: 239

Solve $\log_3 x - 8\log_9 x = 6$ for x

Solution 10

$$\log_3 x - 8 \log_9 x = 6$$

$$\log_3 x - 8 \left(\frac{\log_3 x}{\log_3 9} \right) = 6$$

$$\log_3 x - 8 \left(\frac{\log_3 x}{2} \right) = 6$$

$$\log_3 x - 4 \log_3 x = 6$$

$$\log_3 x - \log_3 x^4 = 6$$

$$\log_3 \frac{x}{x^4} = 6$$

$$\frac{1}{x^3} = 3^6$$

$$\frac{1}{x} = 3^2$$

$$x = \frac{1}{9}$$

$$x = 0.11$$

Answer part 1: Label $x =$ Solution 0.11

Answer part 1 hint: Write the answer to 2 d.p.

Question 11 Experience: 50 Order: z Level: z Question-ID: 229

Find values for x and y from the following, $x > 0, y > 0$:

$$3 \log_y x^2 = 4$$

$$y^2 = x^2 + 12x$$

Solution 11

$$3 \log_y x^2 = 4 \quad (1)$$

$$y^2 = x^2 + 12x \quad (2)$$

$$\text{Simplifying (1)} \quad \log_y x^2 = \frac{4}{3}$$

$$x^2 = y^{\frac{4}{3}}$$

$$x^6 = y^4$$

$$x^3 = y^2 \quad (3)$$

$$\text{Sub (3) into (2)} \quad x^3 = x^2 + 12x$$

$$x^3 - x^2 - 12x = 0$$

$$x(x^2 - x - 12) = 0 \quad P = -12 \quad S = -1$$

$$x(x - 4)(x + 3) = 0 \quad (-4, 3)$$

$$x = 4$$

$$\text{Sub } x = 4 \text{ into (3)} \quad 4^3 = y^2$$

$$y^2 = 64$$

$$y = 8$$

Answer part 1: Label $x =$ Solution 4

Answer part 1 hint:

Answer part 2: Label $y =$ Solution 8

Answer part 2 hint:

Question 12 Experience: 50 Order: z Level: z Question-ID: 228

Find values for x and y from the following, $x > 0$:

$$3 \log_x y - \log_x 125 = 2$$

$$125y = x^2$$

Solution 12

$$3 \log_x y - \log_x 125 = 2 \quad (1)$$

$$125y = x^2 \quad (2)$$

Simplifying (1) $3 \log_x y - 3 \log_x 5 = 2$

$$\log_x y - \log_x 5 = \frac{2}{3}$$

$$\log_x \frac{y}{5} = \frac{2}{3}$$

$$\frac{y}{5} = x^{\frac{2}{3}}$$

$$y = 5x^{\frac{2}{3}} \quad (3)$$

Simplifying (2) $y = \frac{x^2}{125} \quad (4)$

$$(3) = (4) \quad 5x^{\frac{2}{3}} = \frac{x^2}{125}$$

$$x^{\frac{4}{3}} = 625$$

$$x = 625^{\frac{3}{4}}$$

$$x = 125$$

Sub $x = 125$ into (2) $125y = (125)^2$

$$y = 125$$

Answer part 1: Label $x =$ Solution 125

Answer part 1 hint:

Answer part 2: Label $y =$ Solution 125

Answer part 2 hint:

Question 13 Experience: 50 Order: z Level: z Question-ID: 227

Find values for x and y from the following:

$$\log_4 4x + \log_5 5y = 4$$

$$\log_2 2x + 6 \log_{25} y = 6$$

Solution 13

$$\log_4 4x + \log_5 5y = 4 \quad (1)$$

$$\log_2 2x + 6 \log_{25} y = 6 \quad (2)$$

Simplifying (1) $\log_4 4 + \log_4 x + \log_5 5 + \log_5 y = 4$

$$1 + \frac{\log_2 x}{\log_2 4} + 1 + \log_5 y = 4$$

$$\frac{\log_2 x}{2} + \log_5 y = 2$$

$$\log_2 x + 2 \log_5 y = 4 \quad (3)$$

Simplifying (2) $\log_2 2 + \log_2 x + \frac{6 \log_5 y}{\log_5 25} = 6$

$$1 + \log_2 x + \frac{6 \log_5 y}{2} = 6$$

$$\log_2 x + 3 \log_5 y = 5 \quad (4)$$

(4) - (3) $\log_2 x + 3 \log_5 y - (\log_2 x + 2 \log_5 y) = 5 - 4$

$$\log_5 y = 1$$

$$y = 5$$

Sub $y = 5$ into (3) $\log_2 x + 2 \log_5 5 = 4$

$$\log_2 x + 2 = 4$$

$$\log_2 x = 2$$

$$x = 2^2$$

$$x = 4$$

Answer part 1: Label $x =$ Solution 4

Answer part 1 hint:

Answer part 2: Label $y =$ Solution 5

Answer part 2 hint:

Question 14 Experience: 50 Order: z Level: z Question-ID: 226

Find values for x and y from the following:

$$5 \log_3 x - 2 \log_5 y = 8$$

$$3 \log_{27} x = 4 \log_{25} y$$

Solution 14

$$5 \log_3 x - 2 \log_5 y = 8 \quad (1)$$

$$3 \log_{27} x = 4 \log_{25} y \quad (2)$$

Simplifying (2)

$$\frac{3 \log_3 x}{\log_3 27} = \frac{4 \log_5 y}{\log_5 25}$$

$$\frac{3 \log_3 x}{3} = \frac{4 \log_5 y}{2}$$

$$\log_3 x = 2 \log_5 y \quad (3)$$

Sub (3) into (1) $5(2 \log_5 y) - 2 \log_5 y = 8$

$$10 \log_5 y - 2 \log_5 y = 8$$

$$8 \log_5 y = 8$$

$$\log_5 y = 1$$

$$y = 5$$

Sub $y = 5$ into (1) $5 \log_3 x - 2 \log_5 5 = 8$

$$5 \log_3 x - 2 = 8$$

$$\log_3 x = 2$$

$$x = 3^2$$

$$x = 9$$

Answer part 1: Label $x =$ Solution 9

Answer part 1 hint:

Answer part 2: Label $y =$ Solution 5

Answer part 2 hint:

Question 15 Experience: 35 Order: k3 Level: k3 Question-ID: 240

Solve $\log_3(4x - 5) - \log_3(x - 5) = 2$ for x

Solution 15

$$\log_3(4x - 5) - \log_3(x - 5) = 2$$

$$\log_3 \frac{4x - 5}{x - 5} = 2$$

$$\frac{4x - 5}{x - 5} = 9$$

$$4x - 5 = 9(x - 5)$$

$$4x - 5 = 9x - 45$$

$$5x = 40$$

$$x = 8$$

Answer part 1: Label $x =$ Solution 8

Answer part 1 hint: