CALCULUS II ASSIGNMENT 2

DUE FEBRUARY 7, 2019

1. Compute the following trigonometric integrals:

(i) $\displaystyle\int \sin(\theta)^2 \cos(\theta)^3 \, d\theta$,

(ii) $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}} \, dx$,

(iii) $\displaystyle\int_0^{\pi/2} \sin(t)^2 \cos(t)^2 \, dt$,

(iv) $\displaystyle\int \tan(y)^2 \, dy$,

(v) $\displaystyle\int \tan(z)^3 \sec(z) \, dz$,

(vi) $\displaystyle\int \sin(8u) \cos(5u) \, du$.

2. In this Problem, we are going to compute the following relations: for positive integers n and m,

$$\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx = \begin{cases} 0 & \text{if } m \neq n, \\ \pi & \text{if } m = n, \end{cases}$$

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx = \begin{cases} 0 & \text{if } m \neq n, \\ \pi & \text{if } m = n, \end{cases}$$

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx = 0.$$

To do this, use the prosthaphaeresis formulae,

$$\sin(A)\sin(B) = \frac{1}{2}\big(\cos(A-B) - \cos(A+B)\big),$$

$$\cos(A)\cos(B) = \frac{1}{2}\big(\cos(A-B) + \cos(A+B)\big),$$

$$\sin(A)\cos(B) = \frac{1}{2}\big(\sin(A-B) + \sin(A-B)\big),$$

to express the integrands as sums of individual trigonometric functions. Then show that the integrands you get end up being even or odd functions, depending on whether you have $n \neq m$ or $n = m$. If it is helpful to you, feel free to choose specific positive integers n and m representing the cases above in doing this computation.

These relationships are effectively the starting point to Fourier analysis; these give you ways to tease out waves of a particular frequency in some given periodic signal!

3. Use the trigonometric substitution $x = 2\sin(\theta)$ to evaluate

$$\int_0^1 x^2 \sqrt{9 - x^2}\, dx.$$

Be careful about the bounds of integration once you do your substitution: what must θ be when $x = 0$ or $x = 1$?

4. Sometimes you are going to have to do some manipulations before being able to perform a trigonometric substitution. Here is an example:

(i) Write the polynomial $3 - 2x - x^2$ in the form $a - (x + b)^2$, for some numbers a and b, by completing the square.

(ii) Do the substitution $u = x + b$ followed by a trigonometric substitution to evaluate the integral

$$\int \sqrt{3-2x-x^2}\,dx.$$

5. Given a circle of radius a, its circumference is $2\pi a$ and its area is πa^2.

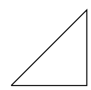
(i) Compute the integral $\int_0^a 2\pi r\,dr$.

(ii) Thinking about polar coordinates, try to explain how the computation in (i) is a way of computing the area of a circle of radius r.

As an analogy, it might be helpful to think about how the integral

$$\int_0^1 x\,dx$$

computes the area of the right triangle



where the vertices are at $(0,0)$, $(0,1)$, and $(1,1)$.