CALCULUS II ASSIGNMENT 6

DUE MARCH 7, 2019

This xkcd comic would have served well for the introduction of this course. In any case, time to continue on the series...

1. Justify why the following series converge and find their sums.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4+n^2}$,

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$,

(iii) $\displaystyle\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$,

(iv) $\displaystyle\sum_{n=1}^{\infty} \big(\sin(1/n) - \sin(1/(n+1))\big)$.

2. Determine whether or not the following series converge or diverge. If they converge, determine their sum.

(i) $\displaystyle\sum_{n=1}^{\infty} \cos(n)$,

(ii) $\displaystyle\sum_{k=1}^{\infty} \sin(100)^k$,

(iii) $\displaystyle\sum_{m=2}^{\infty} \frac{1}{m^3-m}$,

(iv) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^2-2n+5}$,

(v) $\displaystyle\sum_{i=1}^{\infty} \frac{3^{i+1}}{(-2)^i}$,

(vi) $\displaystyle\sum_{\ell=1}^{\infty} \frac{1}{1+(2/3)^{\ell}}$.

3. The Comparison Test we discussed in class says that given two series $\sum a_i$ and $\sum b_i$ with positive terms, then

(i) if $\sum b_i$ is convergent and $a_n \leq b_n$ for all sufficiently large indices n, then $\sum a_i$ is convergent; and
(ii) if $\sum b_i$ is divergent and $a_n \geq b_n$ for all sufficiently large indices n, then $\sum a_i$ is divergent.

Formulate analogues of statements (i) and (ii) for series $\sum a_i$ and $\sum b_i$ with negatives terms. Try to justify your statements using (i) and (ii) above.

4. Use the Comparison Test to determine whether or not the following series converge or diverge:

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$,

(ii) $\displaystyle\sum_{m=1}^{\infty} \frac{\log(m)}{m}$,

(iii) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^k}$,

(iv) $\displaystyle\sum_{\ell=1}^{\infty} \frac{e^{1/\ell}}{\ell}$,

(v) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$,

(vi) $\displaystyle\sum_{m=1}^{\infty} \frac{9^m}{3+10^m}$.

In some of the comparisons above, it might be useful to know that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges when $p > 1$ and diverges when $p \leq 1$; note that the $p = 1$ case is the Harmonic Series, which I mentioned in class. We will see why these statements are true, soon!