ÇÖZÜMLÜ PROBLEMLER

A) $\frac{0}{0}$ belirsizliği

1. $f(x)=\dfrac{x^2+2x}{x+2}$ fonksiyonu $x\to-2$ yerinde $\frac{0}{0}$ belirsizliğine sahiptir.

$$\lim_{x\to-2}\frac{x^2+2x}{x+2} \overset{\frac{0}{0}}{=} \lim_{x\to-2}\frac{x(x+2)}{(x+2)} = \lim_{x\to-2}x = -2 \text{ bulunur.}$$

2. $$\lim_{x\to0}\left[\left(\frac{1}{2+x}-\frac{1}{2}\right):x\right] \overset{\frac{0}{0}}{=} \lim_{x\to0}\left[\frac{-x}{2(2+x)}\cdot\frac{1}{x}\right] = -\lim_{x\to0}\frac{1}{2(2+x)} = -\frac{1}{4}$$

3. $$\lim_{x\to a}\frac{x^2-\sqrt{a^3x}}{\sqrt{ax}-a} \overset{\frac{0}{0}}{=} \lim_{x\to a}\frac{\sqrt{x}(\sqrt{x^3}-\sqrt{a^3})}{\sqrt{a}(\sqrt{x}-\sqrt{a})}$$

$$=\lim_{x\to a}\left(\frac{\sqrt{x}}{\sqrt{a}}\,\frac{(\sqrt{x}-\sqrt{a})(\sqrt{x^2}+\sqrt{ax}+\sqrt{a^2})}{(\sqrt{x}-\sqrt{a})}\right)$$

$$=\lim_{x\to a}\frac{\sqrt{x}}{\sqrt{a}}\cdot\lim_{x\to a}\frac{x+\sqrt{ax}+a}{1}=3a$$

4. $\lim\limits_{x\to0}\dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}=\dfrac{0}{0}$ belirsizliği. Burada $1+x=y^6$ diyelim. ($1+x$ in kesirli üslerinin paydalarının en küçük ortak katı 6 olduğundan).

$x\to0$ için $y\to1$ dir. O halde problemimiz $\lim\limits_{y\to1}\dfrac{y^3-1}{y^2-1}$ hesaplamaya dönüşür:

$$\lim_{y\to1}\frac{y^3-1}{y^2-1} \overset{\frac{0}{0}}{=} \lim_{y\to1}\frac{(y-1)(y^2+y+1)}{(y-1)(y+1)}=\lim_{y\to1}\frac{y^2+y+1}{y+1}=\frac{3}{2} \text{ bulunur.}$$

5. $$\lim_{x\to1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2} \overset{\frac{0}{0}}{=} \lim_{y\to1}\frac{y^2-2y+1}{(y^3-1)^2} \quad (x=y^3 \text{ dönüşümüyle})$$

$$=\lim_{y\to1}\frac{(y-1)^2}{(y-1)^2(y^2+y+1)^2} = \lim_{y\to1}\frac{1}{(y^2+y+1)^2}=\frac{1}{9}$$

6. $$\lim_{x\to1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1} \overset{\frac{0}{0}}{=} \lim_{x\to1}\frac{y^4-1}{y^3-1} \quad (x=y^{12} \text{ dönüşümüyle})$$

$$\overset{\frac{0}{0}}{=}\lim_{y\to1}\frac{(y-1)(y+1)(y^2+1)}{(y-1)(y^2+y+1)} = \frac{4}{3}$$

7. $$\lim_{x\to3}\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} \quad \frac{0}{0}$$

$$\overset{\frac{0}{0}}{=}\lim_{x\to3}\frac{x^2-2x+6-(x^2+2x-6)}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$=\frac{1}{6}\lim_{x\to3}\frac{-4x+12}{(x-3)(x-1)} = \frac{-4}{6}\lim_{x\to3}\frac{(x-3)}{(x-3)(x-1)}$$

$$=-\frac{2}{3}\lim_{x\to3}\frac{1}{x-1} = -\frac{2}{3}\cdot\frac{1}{2}=-\frac{1}{3}$$

8. $\lim\limits_{x\to0}\dfrac{\sin x}{x} = 1$ olduğu bilindiğine göre aşağıdaki limitleri hesaplayınız;

a) $\lim\limits_{x\to 0}\dfrac{\operatorname{tg}x}{x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\cdot\dfrac{1}{\cos x}\right) = 1$

b) $\lim\limits_{x\to 0}\dfrac{\sin 5x}{\sin 2x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 0}\dfrac{\dfrac{\sin 5x}{2\cdot 5x}}{\dfrac{\sin 2x}{5\cdot 2x}} = \dfrac{\frac{1}{2}}{\frac{1}{5}} = \dfrac{5}{2}$

c) $\lim\limits_{x\to 1}\dfrac{1-x^2}{\sin \pi x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 1}\dfrac{(1+x)(1-x)}{2\sin\frac{\pi}{2}x\cos\frac{\pi}{2}x}$

$= \lim\limits_{x\to 1}\dfrac{1+x}{2\sin\frac{\pi}{2}x}\cdot\lim\limits_{x\to 1}\dfrac{1-x}{\cos\frac{\pi}{2}x} = \lim\limits_{x\to 1}\dfrac{1-x}{\sin\frac{\pi}{2}(1-x)}$

$= \lim\limits_{x\to 1}\dfrac{\frac{\pi}{2}(1-x)}{\frac{\pi}{2}\sin\frac{\pi}{2}(1-x)} = \dfrac{2}{\pi}$

d) $\lim\limits_{x\to 1}\dfrac{\sin \pi x}{\sin 3\pi x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 1}\dfrac{\sin \pi x}{\sin \pi x\cos 2\pi x + \sin 2\pi x\cos \pi x}$

$= \lim\limits_{x\to 1}\dfrac{\sin \pi x}{\sin \pi x(\cos 2\pi x + 2\cos^2 \pi x)} = \dfrac{1}{3}$

e) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{x^2} = \dfrac{1}{2}\lim\limits_{x\to 0}\left(\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = \dfrac{1}{2}$

f) $\lim\limits_{x\to -2}\dfrac{\operatorname{tg}\pi x}{x+2} \overset{\frac{0}{0}}{=} \lim\limits_{x\to -2}\dfrac{\sin \pi x}{x+2}\cdot\dfrac{1}{\cos \pi x}$

$= \lim\limits_{x\to -2}\dfrac{\sin(2\pi+\pi x)}{x+2} = \lim\limits_{x\to -2}\dfrac{\pi\sin \pi(x+2)}{\pi(x+2)} = \pi$

g) $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{1-2\cos x}{\pi-3x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to\frac{\pi}{3}}\dfrac{2(\cos\frac{\pi}{3}-\cos x)}{3(\frac{\pi}{3}-x)}$

$= \dfrac{2}{3}\lim\limits_{x\to\frac{\pi}{3}}\dfrac{-2\sin\frac{1}{2}(\frac{\pi}{3}+x)\sin\frac{1}{2}(\frac{\pi}{3}-x)}{2\cdot\frac{1}{2}(\frac{\pi}{3}-x)} = -\dfrac{2}{3}\dfrac{\sqrt{3}}{2} = -\dfrac{\sqrt{3}}{3}$

h) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\operatorname{tg}x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to\frac{\pi}{4}}\dfrac{-\cos x(1-\operatorname{tg}x)}{(1-\operatorname{tg}x)} = -\dfrac{1}{\sqrt{2}}$

B) $\dfrac{\infty}{\infty}$ **belirsizliği**

$\lim\limits_{x\to\infty}\dfrac{P(x)}{Q(x)} = \lim\limits_{x\to\infty}\dfrac{a_n x^n a_1 x^{n-1}+\ldots+a_n}{b_0 x^p + b_1 x^{p-1}+\ldots+b_p}$

$= \lim\limits_{x\to\infty}\dfrac{a_0 x^n(1+\frac{a_1}{a_0 x}+\ldots+\frac{a_n}{a_0 x^n})}{b_0 x^p(1+\frac{b_1}{b_0 x}+\ldots+\frac{b_p}{b_0 x^p})} = \lim\limits_{x\to\infty}\dfrac{a_0 x^n}{b_0 x^p}$

1. $\lim\limits_{x\to\infty}\dfrac{x^2-3x+1}{3x^2+7x-5} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\infty}\dfrac{x^2(1-\frac{3}{x}\frac{1}{x^2})}{3x^2(1+\frac{7}{3x}-\frac{5}{3x^2})} = \lim\limits_{x\to\infty}\dfrac{x^2}{3x^2} = \dfrac{1}{3}$

2. $\lim\limits_{x\to\infty}\dfrac{x^2}{10+x\sqrt{x}} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\infty}\dfrac{x^2}{x^{3/2}} = \infty$

3. $\lim\limits_{n\to\infty}\dfrac{2^{n+1}+3^{n+1}}{2^n+3^n} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{n\to\infty}\dfrac{3^{n-1}\left[(\frac{2}{3})^{n+1}+1\right]}{3^n\left[(\frac{2}{3})^n+1\right]} = 3$

4. $\lim\limits_{b\to\infty}\dfrac{n\sin n!}{n^2+1} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{n\to\infty}\dfrac{n\sin n!}{n^2(1+\frac{1}{n^2})} = \lim\limits_{n\to\infty}\dfrac{\sin n!}{n} = 0$

5. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\mathrm{tg}\,3x}{\mathrm{tg}\,5x} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin 3x}{\cos 3x}\cdot\dfrac{\cos 5x}{\sin 5x}$

$\overset{\frac{0}{0}}{=} \lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin 3x}{\sin 5x}\cdot\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos 5x}{\cos 3x} \overset{\frac{0}{0}}{=} -\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos 2x-\sin 3x\sin 2x}{\cos 3x}$

$= -\left[\lim\limits_{x\to\frac{\pi}{2}}\cos 2x+\lim\limits_{x\to\frac{\pi}{2}}\dfrac{2\sin x\cos x}{\cos 2x\cos 3x-\sin 2x\sin x}\right]$

$= -\left[-1+\lim\limits_{x\to\frac{\pi}{2}}\dfrac{2\sin x}{\cos 2x-2\sin^2 x}\right] = -\left[-1+\dfrac{2}{-1-2}\right] = 1+\dfrac{2}{3} = \dfrac{5}{3}$

6. $\lim\limits_{x\to\infty}\dfrac{500+2^x}{500-2^x} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to\infty}\dfrac{2^x(\frac{500}{2^x}+1)}{2^x(\frac{500}{2^x}-1)} = -1$

C) $0\cdot\infty$ belirsizliği

$A\cdot B=\dfrac{A}{\frac{1}{B}}=\dfrac{B}{\frac{1}{A}}$ özelliklerinden yararlanarak belirsizlik önce $\dfrac{0}{0}$ veya $\dfrac{\infty}{\infty}$ haline getirilir.

1. $\lim\limits_{x\to 1}(1-x)\,\mathrm{tg}\dfrac{\pi x}{2} \overset{0\cdot\infty}{=} \lim\limits_{x\to 1}\dfrac{1-x}{\mathrm{ctg}\frac{\pi x}{2}} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 1}\dfrac{1-x}{\mathrm{tg}(\frac{\pi}{2}-\frac{\pi}{2}x)}$

$= \lim\limits_{x\to 1}\dfrac{\frac{2}{\pi}\cdot\frac{\pi}{2}(1-x)}{\mathrm{tg}\frac{\pi}{2}(1-x)} = \dfrac{2}{\pi}$

2. $\lim\limits_{x\to\infty}\mathrm{ctg}\,2x\,\mathrm{ctg}(\frac{\pi}{2}-x) \overset{\infty\cdot 0}{=} \lim\limits_{x\to 0}\dfrac{\mathrm{ctg}(\frac{\pi}{2}-x)}{\mathrm{tg}\,2x}$

$= \lim\limits_{x\to 0}\dfrac{\mathrm{tg}\,x}{\frac{2\,\mathrm{tg}\,x}{1-\mathrm{tg}^2 x}} = \lim\limits_{x\to 0}\dfrac{1-\mathrm{tg}^2 x}{2}=\dfrac{1}{2}$

D) $\infty-\infty$ belirsizliği

Önce belirsizlik $\dfrac{0}{0}$ veya $\dfrac{\infty}{\infty}$ belirsizliklerinden birine dönüştürülür.

1. $\lim\limits_{x\to\infty}(\sqrt{x+a}-\sqrt{x}) \overset{\infty-\infty}{=} \lim\limits_{x\to\infty}\dfrac{x+a-x}{\sqrt{x+a}+\sqrt{x}}=0$

2. $\lim\limits_{x\to\infty}(x+\sqrt[3]{1-x^3})=\lim\limits_{x\to\infty}\dfrac{x^3+(1-x^3)}{x^2-x\sqrt[3]{1-x^3}+\sqrt[3]{(1-x^3)^2}}$

$\lim\limits_{x\to\infty}\dfrac{1}{x^2(1-\sqrt[3]{\frac{1}{x^3}-1}+\sqrt[3]{(\frac{1}{x^3}-1)^2})}=0$

3. $\lim\limits_{x\to-\infty}\left[\sqrt{x^2-4x+3}+x\right] \overset{\infty-\infty}{=} \lim\limits_{x\to-\infty}\dfrac{x^2-4x+3-x^2}{\sqrt{x^2-4x+3}-x} \overset{\frac{\infty}{\infty}}{=}$

$$\lim_{x\to-\infty}\frac{x(-4+\frac{3}{x})}{\sqrt{1-\frac{4}{x}+\frac{3}{x^2}}+1}=2$$

4. $\lim\limits_{x\to\infty}\left[\sqrt{x^2+px+q}-\sqrt{x^2+px+\frac{p^2}{4}}\right] = 0$ olduğundan

$\lim\limits_{x\to\infty}\sqrt{x^2+px+q} = \lim\limits_{x\to\infty}(x+\frac{p}{2})$ ve

$\lim\limits_{x\to-\infty}\sqrt{x^2+px+q} = -\lim\limits_{x\to-\infty}(x+\frac{p}{2})$ alınabilir.

$\lim\limits_{x\to-\infty}\left[\sqrt{x^2-4x+3}+x\right]$ limiti bu yazışla

$\lim\limits_{x\to-\infty}\left[-(x-2)+x\right]=2$

5. $\lim\limits_{x\to\infty}\left[3x+4-\sqrt{9x^2-6x+8}\right] \overset{\infty-\infty}{=} \lim\limits_{x\to\infty}\left[3x+4-(3x-1)\right] = 5$

6. $\lim\limits_{x\to-\infty}\left[3x-4+\sqrt{9x^2-6x+8}\right] = \lim\limits_{x\to-\infty}\left[3x-4-(3x-1)\right] = -3$

E) 1^∞ belirsizliği

e sayısının hesabı.

$e=\lim\limits_{n\to\infty}(1+\frac{1}{n})^n$ (n tam) şeklinde tanımlanır. Bu limitin *e* ile gösterilişi Leonard Euler (1707 – 1738) e aittir.

Binom formülünden

$$(1+\frac{1}{n})^n = 1+\frac{n}{1}\cdot\frac{1}{n}+\frac{n(n-1)}{1\cdot2}\cdot\frac{1}{n^2}+\frac{n(n-1)(n-2)}{1\cdot2\cdot3}\cdot\frac{1}{n^3}+\ldots+$$

$$+\frac{n(n-1)\cdot\ldots\cdot(n-p+1)}{1\cdot2\cdot3\cdot\ldots\cdot p}\frac{1}{n^p}+\ldots$$

$$= 1+1+\frac{1-\frac{1}{n}}{1\cdot2}+\frac{(1-\frac{1}{n})(1-\frac{2}{n})}{1\cdot2\cdot3}+\ldots+$$

$\lim\limits_{n\to\infty}(1+\frac{1}{n})^n = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{p!}+\ldots$ elde edilir.

Bu limitin *e* ile gösterilen değeri

$$e=\lim_{n\to\infty}(1+\frac{1}{n})^n \cong 1+\frac{1}{1}+\frac{1}{2}=2{,}5$$

$$e=\lim_{n\to\infty}(1+\frac{1}{n})^n<1+\frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^p}+\ldots=1+\frac{1}{1-\frac{1}{2}}=3$$

eşitsizlikleri sebebiyle, $2{,}5<e=\lim\limits_{n\to\infty}(1+\frac{1}{n})^n<3$ ve 8 terim olarak

$e=\lim\limits_{n\to\infty}(1+\frac{1}{n})^n\cong2{,}7182818$ bulunur.

$\lim\limits_{n\to\infty}(1+\frac{1}{n})^n=e$ olduğuna göre aşağıdaki limitleri hesaplayınız.

1. $\lim\limits_{x\to-\infty}(1+\frac{1}{x})^x=?$

$x=-y$ dönüşümüyle

$\lim\limits_{y\to\infty}(1+\frac{1}{-y})^{-y} = \lim\limits_{y\to\infty}(1-\frac{1}{y})^{-y}$

$=\dfrac{1}{\lim\limits_{y\to\infty}(\frac{y-1}{y})^y} = \lim\limits_{y\to\infty}(\frac{y}{y-1})^y = \lim\limits_{y\to\infty}(1+\frac{1}{y-1})^y=e$ dir.

2. $\lim\limits_{x\to\infty}(1+\dfrac{p}{x})^x = \lim\limits_{x\to\infty}\left[(1+\dfrac{1}{\frac{x}{p}})^{\frac{x}{p}}\right]^p = e^p$ dir.

3. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = 1^\infty$

$y=\dfrac{1}{x}$ dönüşümüyle

$\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = \lim\limits_{y\to\infty}(1+\dfrac{1}{y})^y = e$ elde edilir.

5. $\lim\limits_{x\to\infty}(\dfrac{x-1}{x+1})^x \overset{1^\infty}{=} \lim\limits_{x\to\infty}(1-\dfrac{2}{x+1})^x$

$\lim\limits_{x\to\infty}\left[(1-\dfrac{2}{x+1})^{x+1}/(1-\dfrac{2}{x+1})^{-1}\right] = e^{-2} = \dfrac{1}{e^2}$

veya diğer bir yazışla

$\lim\limits_{x\to\infty}(\dfrac{x-1}{x+1})^x = \lim\limits_{x\to\infty}\dfrac{(1-\frac{1}{x})^x}{(1+\frac{1}{x})^x} \dfrac{\lim\limits_{x\to\infty}(1-\frac{1}{x})^x}{\lim\limits_{x\to\infty}(1+\frac{1}{x})^x}$ ü

$\dfrac{e^{-1}}{e} = e^{-2} = \dfrac{1}{e^2}$ bulunur.

6. $\lim\limits_{x\to\infty}(\dfrac{x}{x+1})^x = \lim\limits_{x\to\infty}\dfrac{1}{(1+\frac{1}{x})^x} = \dfrac{1}{\lim\limits_{x\to\infty}(1+\frac{1}{x})^x} = \dfrac{1}{e}$

7. $\lim\limits_{x\to\infty}(\dfrac{x-1}{x+3})^{x+2} = \dfrac{\lim\limits_{x\to\infty}(1-\frac{1}{x})^{x+2}}{\lim\limits_{x\to\infty}(1+\frac{3}{x})^{x+2}} = \dfrac{e^{-1}}{e^3} = e^{-4}$

8. $\lim\limits_{x\to\infty}(1+\sin x)^{\frac{1}{x}} = \lim\limits_{x\to\infty}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\frac{\sin x}{x}} = e$

9. $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x^2}} = \lim\limits_{x\to 0}[1+(\cos x-1)]^{\frac{1}{x^2}}$

$\lim\limits_{x\to\infty}(1-2\sin^2\dfrac{x}{2})^{\frac{1}{x^2}} = \lim\limits_{x\to 0}\left[(1-2\sin^2\dfrac{x}{2})^{\frac{1}{-2\sin^2\frac{x}{2}}}\right]^{\frac{-2\sin^2\frac{x}{2}}{x^2}}$

$= e^{\lim\limits_{x\to 0}(\frac{-2\sin^2\frac{x}{2}}{x^2})} = e^{-\frac{1}{2}\lim\limits_{x\to 0}(\frac{\sin\frac{x}{2}}{\frac{x}{2}})^2} = e^{-\frac{1}{2}} = \dfrac{1}{\sqrt{e}}$

F) Karışık Çözümlü Problemler

1. $\lim\limits_{x\to\infty}[\ln(2x+1)-\ln(x+2)] = \lim\limits_{x\to\infty}\ln\dfrac{2x+1}{x+2}$

$\ln(\lim\limits_{x\to\infty}\dfrac{2x+1}{x+2}) = \ln 2$

2. $\lim\limits_{x\to 0}\dfrac{\log(1+10x)}{x} = \lim\limits_{x\to 0}\log(1+10x)^{\frac{1}{x}}$

$=\log\left[\lim\limits_{x\to 0}(1+10x)^{\frac{1}{10x}}\right] = \log e^{10} = 10\log e$

3. $\lim\limits_{x\to 0}\dfrac{1}{x}\ln\sqrt{\dfrac{1+x}{1-x}} = \dfrac{1}{2}\lim\limits_{x\to 0}\ln(\dfrac{1+x}{1-x})^{\frac{1}{x}}$

$\dfrac{1}{2}\ln\lim\limits_{x\to 0}\left[(1+\dfrac{2x}{1-x})^{\frac{1-x}{2x}}\right]^{\frac{2}{1-x}} = \dfrac{1}{2}\ln e^2 = 1$

4. $\lim_{x\to\infty} x\left[\ln(x+1)-\ln x\right] = \lim_{x\to\infty} x\ln\frac{x+1}{x}$

$\lim_{x\to\infty}\ln(\frac{x+1}{x})^{x} = \ln\lim_{x\to\infty}(1+\frac{1}{x})^{x} = \ln e = 1$

5. $\lim_{x\to0}\frac{\ln\cos x}{x^2} = \ln\lim_{x\to0}(\cos x)^{\frac{1}{x^2}} = \ln e^{-1/2} = -\frac{1}{2}$

6. $\lim_{x\to0}\frac{e^x-1}{x} = \lim_{h\to0}\frac{h}{\ln(1+h)}$

$e^x-1 = h$ ile

$\lim_{h\to0}\frac{1}{\frac{1}{h}\ln(1+h)} = \frac{1}{\lim_{h\to0}\ln(1+h)^{1/h}} = 1$