CSDS 440: Assignment 1

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Problem 2

We can view this problem as having two points x_1 and x_2 uniformaly distributed on a line with a length of $\sqrt{2}$, since this is the length of function $x + y = 1$ in interval $(0, 1)$ is $\sqrt{2}$). Let x be a random varianle $\in [0, \sqrt{2}]$, the PDF of this x would be:

$$f(x) = \begin{cases} \frac{1}{\sqrt{2}} & x \in [0, \sqrt{2}] \\ 0 & \text{otherwise} \end{cases}$$

For x_1 and x_2, since the placement of two points are independent, we have the joint PDF of x_1, x_2 to be:

$$f(x_1, x_2) = \begin{cases} \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} & x_1, x_2 \in [0, \sqrt{2}] \\ 0 & \text{otherwise} \end{cases}$$

Since the square distance is $D = (x_1 - x_2)^2$, its expected value is $E[(x_1 - x_2)^2]$, which is:

$$
\begin{aligned}
E[(x_1 - x_2)^2] &= \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} (x_1 - x_2)^2 \cdot f(x_1, x_2) \cdot dx_1 dx_2 \\
&= \frac{1}{2} \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} (x_1^2 - 2x_1 x_2 + x_2^2) \cdot dx_1 dx_2 \\
&= \frac{1}{2} \int_0^{\sqrt{2}} \frac{x_1^2}{3} - 2\frac{x_1^2}{2} x_2 + x_2 x_1 \Big|_0^{\sqrt{2}} \cdot dx_2 \\
&= \frac{1}{2} \cdot \frac{2}{3} \\
&= \frac{1}{3}
\end{aligned}
$$