CSDS 440: Assignment 4

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Problem 17

Proof. To prove by contradiction. Assume we have a local minimum x in a convex function f but there is another global minimum x', where f(x') < f(x).

Since f is convex, by Jensen's inequality we must have:

$$f(\lambda x + (1 - \lambda)x') \le \lambda f(x) + (1 - \lambda)f(x')$$
$$< \lambda f(x) + (1 - \lambda)f(x)$$
$$< f(x)$$

Let $\lambda = 1$, we will have the below contradiction:

$$f(x) < f(x)$$

Thus, by contradiction, the local minimum of a convex function is always the global minimum.