General Instructions: Write or type your answers neatly and remember to show all relevant work. All questions are worth 10 points. Each answer should be a separate pdf, and you can turn in the pdfs on canvas in the appropriate assignment. Some questions may be very challenging; significant partial credit is available for reasonable attempts at solutions. Since each question is worth the same number of points, do not waste too much time on any one. Ask me or the TAs for help if stuck.

Some of the questions require you to write short programs to simulate things. You can use any language/software to do this, and you do not need to turn in your code.

Upload your answers to Canvas as a pdf file by 11:59pm on the due date specified after the question. You will receive a 10% bonus for a solution turned in a week or more in advance of the due date. You can use one late day each week (up to Saturday 11:59pm) with a penalty of 20%. Submissions after Saturday 11:59pm for any week will not be graded.

Each group must do their own work. Only one submission is needed from each group. Do not use any source other than the lecture notes, textbook(s) and readings on the class website to answer these questions. Only those who contributed equally to a submission should have their names and Case IDs on the submission. Those not listed as contributing will not receive points.

28. For a constrained programming problem $min_w\ f(w)$ s.t. $g_i(w) \leq 0$, $h_j(w)=0$, the generalized Lagrangian is defined by $L(w,\alpha,\beta)=f(w)+\sum \alpha_i g_i(w)+ \sum \beta_j h_j(w)$, $\alpha_i \geq 0$. A primal linear program is a constrained program of the form: $min_x\ c'x$ s.t. $Ax\geq b$, $x\geq 0$. Using the generalized Lagrangian, show that the *dual form* of the primal LP is $max_u\ b'u$ s.t. $A'u \leq c$, $u \geq 0$.

29. Suppose K_1 and K_2 are two valid kernels. Show that for positive a and b, aK_1+bK_2 is also a valid kernel.

30. Suppose K_1 and K_2 are two valid kernels. Show that for positive a and b, aK_1K_2 is also a valid kernel, where the product is the Hadamard product: if $K=K_1K_2$ then $K(x,y)=K_1(x,y)K_2(x,y)$.

31. Define $K(x,y)=(x\cdot y+c)^3$, where c is a positive constant. Prove that K is a valid kernel by finding φ so that $K= \varphi(x)\cdot\varphi(y)$.

32. Prove that K above is a valid kernel by showing that K is symmetric positive semidefinite.