CSDS 440: Assignment 7

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Due and submitted on 10/23/2020

Fall 2020, Dr. Ray

Problem 30

For K_1 to be a valid kernel, we must have $K_1(x, y) = \varphi_1(x)\varphi_1(y)$ where ϕ is a non-linear mapping of its input. Due to the same principle, we also have $K_2(x, y) = \varphi_2(x)\varphi_2(y)$. Which gives us:

$$
\begin{aligned}
K(x, y) &= aK_1(x, y)K_2(x, y) \\
&= a(\varphi_1(x)\varphi_1(y) \cdot \varphi_2(x)\varphi_2(y) \\
&= a[(\sum_{i=1}^{n} \varphi_{1i}(x)\varphi_{1i}(y)) \cdot (\sum_{j=1}^{n} \varphi_{2j}(x)\varphi_{2j}(y))] \\
&= a\sum_{i=1}^{n}\sum_{j=1}^{n} \varphi_{1i}(x)\varphi_{1i}(y)\varphi_{2j}(x)\varphi_{2j}(y) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} (\sqrt{a}\, \varphi_{1i}(x)\varphi_{2j}(x)) \cdot (\sqrt{a}\, \varphi_{1i}(y)\varphi_{2j}(y)) \\
&= \varphi(x)\varphi(y) \begin{cases} \varphi(x) = \sqrt{a}\, \varphi_{1i}(x)\varphi_{2j}(x) \\ \varphi(y) = \sqrt{a}\, \varphi_{1i}(y)\varphi_{2j}(y) \end{cases}
\end{aligned}
$$

As the above φ is created upon multiplying a constant $\sqrt{a}$ with non-linear mapping φ_1 and φ_2, it is also a non-linear mapping; and K is therefore a valid kernel.