CSDS 440: Assignment 7

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Problem 31

$$\begin{aligned}
K(x, y) &= (x \cdot y + c)^3 \\
&= (xy)^3 + 3(xy)^2 c + 3(xy)c^2 + c^3 \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} (x_i y_i)(x_j y_j)(x_k y_k) + 3c \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i y_i)(x_j y_j) + 3c^2 \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i y_i)(x_j y_j) + c^3 \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} (x_i x_j x_k)(y_i y_j y_k) + \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i x_j \cdot \sqrt{3c})(y_i y_j \cdot \sqrt{3c}) \\
&\quad + \sum_{i=1}^{n} (x_i \cdot c\sqrt{3})(y_i \cdot c\sqrt{3}) + \sqrt{c^3}\sqrt{c^3} \\
&= \varphi(x)\varphi(y)
\end{aligned}$$

Where

$$\begin{aligned}
\varphi(a) = [&a_1 a_1 a_1, \ldots, a_i a_j a_k, \ldots, a_n a_n a_n, \\
&a_1 a_1 \sqrt{3c}, \ldots, \sqrt{2} a_i a_j \sqrt{3c}, \ldots, a_n a_n \sqrt{3c}, \\
&a_1 c\sqrt{3}, \ldots, a_i c\sqrt{3}, \ldots, a_n c\sqrt{3}, \\
&\sqrt{c^3}]
\end{aligned}$$

which is a non-linear mapping of input a. So we have showed $K(x, y) = \varphi(x)\varphi(y)$, we will continue to show that K is also symmetric and PSD in *Question 32*.