CSDS 440: Assignment 7

Shaochen (Henry) ZHONG, sxz517

Mingyang TIE, mxt497

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Problem 29

We try to show that the new $K = aK_1 + bK_2$ is a valid kernel as it compliants to the Mercer's conditions.

$$\begin{aligned} K(x,y) &= aK_1(x,y) + bK_2(x,y) \\ &= aK_1(y,x) + bK_2(y,x) \quad \text{as } K_1 \text{ and } K_2 \text{ are valid kernels.} \\ &= K(y,x) \end{aligned}$$

So K is symmetry. Now suppose $\forall v \neq 0$, we have:

$$\begin{aligned} v^T \cdot Kv &= v^T(aK_1 + bK_2)v \\ &= a\underbrace{(v^T K_1 v)}_{\geq 0} + b\underbrace{v(v^T K_2 v)}_{\geq 0} \\ &\geq 0 \end{aligned}$$

So K is also PSD. We may say K is a valid kernel as both of the Mercer's conditions are met.