CSDS 440: Assignment 7

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Problem 32

First it is clear that $K(x, y) = K(y, x)$ as $(xy + c)^3 = (yx + c)^3$, so K is symmetric.

Now to show that K is also PSD, we need that for $K : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, K is valid for all $\{x^{(1)}, \ldots, x^{(m)}\}$. With such information on x we may form the equality this way:

$$
\begin{aligned}
v^T K v &= \sum_{i=1}^{m}\sum_{j=1}^{m} v_i v_j K(x^{(i)}x^{(j)}) \\
&= \sum_{i=1}^{m}\sum_{j=1}^{m} v_i v_j (x^{(i)}x^{(j)} + c)^3 \\
&= \sum_{i=1}^{m}\sum_{j=1}^{m} v_i v_j (y^{(i)}y^{(j)})^3 \quad \text{absorb } +c \text{ to } x \text{ and make } y \\
&= \sum_{i=1}^{m}\sum_{j=1}^{m} v_i v_j (\sum_{a=1}^{n} y_a^{(i)} y_a^{(j)}) \cdot (\sum_{b=1}^{n} y_b^{(i)} y_b^{(j)}) \cdot (\sum_{c=1}^{n} y_c^{(i)} y_c^{(j)}) \\
&= \sum_{i=1}^{m}\sum_{j=1}^{m} v_i v_j \sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n} (y_a^{(i)} y_b^{(i)} y_c^{(i)})(y_a^{(j)} y_b^{(j)} y_c^{(j)}) \\
&= \sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n} \sum_{i=1}^{m} v_i (y_a^{(i)} y_b^{(i)} y_c^{(i)}) \quad \sum_{j=1}^{m} v_j (y_a^{(j)} y_b^{(j)} y_c^{(j)}) \\
&= \sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n} \big(\sum_{i=1}^{m} v_i (y_a^{(i)} y_b^{(i)} y_c^{(i)})\big)^2
\end{aligned}
$$

Since whole equation is inside a square, K is PSD. And combined with the above finding of K being symmetric, K is a valid kernel.