CSDS 455: Homework 8

Shaochen (Henry) ZHONG, sxz517

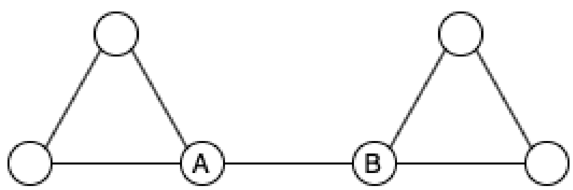
Due and submitted on 09/21/2020
Fall 2020, Dr. Connamacher

Problem 1

W.T.S $\lambda(G) \leq \delta(G)$**:**

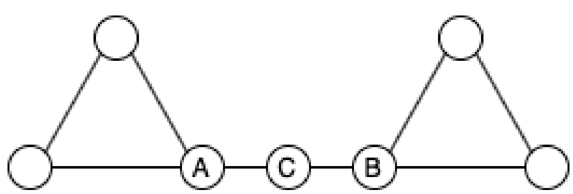
Let $v \in V(G)$ with $\delta(v) = \delta(G)$, this suggest by removing every edges connected to v we can clearly have disconnected part (the isolated v). This suggest $\lambda(G) = \delta(G)$.

However it is possible to have $\delta(V) < \delta(G)$, in the below example we have $\delta(G) = 2$, however by simply remove the AB edge we can have a disconnected graph. Thus, $\lambda(G) \leq \delta(G)$.



W.T.S $\kappa(G) \leq \lambda(G)$**:**

We know that we may have $\kappa(G) = \lambda(G)$ in a sense of $P2$ (two-vertex line), where to break it into a disconnected graph we will have $\kappa(G) = \lambda(G)$. However, it is possible to have $\kappa(G) < \lambda(G)$. In the below example we will need to remove edge AC and CB to make G disconnected, which suggest $\lambda(G) = 2$; but we can also make G disconnected by removing C, which yields a $\kappa(G) = 1$. Thus, we have $\kappa(G) \leq \lambda(G)$.



Combine with the above findings, we have $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

Problem 2

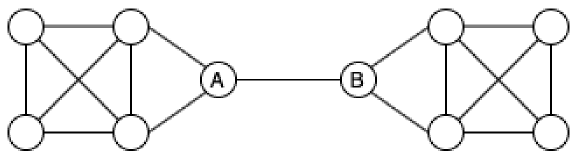
With a 1-dimentional cube, which is a line, the connectivity is trivially 1. With a 2-dimentional cube, which is a square, the connectivity is 2 (removing two non-adjacient vertices). For the base of induction, we assume a d-dimentional cube has a connectivity of d.

Since we can build a $d + 1$-dimentional cube with two d-dimentional cubes, and do a perfect matching between them. This suggest $d + 1$-dimentional cube has a connectivity of $d + 1$. As one

d-dimentional cube, needs to remove a minimum of d vertices to disconnect internally, and it will need to remove a matching vertex from the other d-dimentional cube to make this $d+1$-dimentional cube become disconnected.

As we have d and $d+1$ to be hold for the induction, we may conclude that a $d+1$-dimentional cube has a connectivity of $d+1$, as $\kappa(G) = d$.

Problem 3



With $\lambda(G) = 1$ we know the cut-edge must be a bridge. So at the two sides of the bridge (we denote them as H_1 and H_2) we should also have two 3-regular subgraphs. Known that the smallest (in terms of number of vertuces) 3-regular graph will have 4 vertices, call this K_{3min}, however we can't get a bridge with two of K_{3min} graph without breaking the 3-regular property. So we will insepct the next smallest K_3, which has 5 vertices, and we can build a bridge upton two of them as shown above.

Problem 4

We know that $\lambda(G) > 1$. Since if we remove e, we will have a cycle on the vertices of C_1 and C_2 and the new graph is still connected.

We also know that $\lambda(G) > 2$, as by removing e and another differnt e' in C_1 or C_2, we will still have a path on the vertices of C_1 and C_2 and the new graph is still connected.

With $\lambda(G) = 3$ we may find a way to disconnect G. By removing edge e we have a cycle on vertices of C_1 and C_2, then we remove another edge in C_1 (WLOG) to get a path on the vertices of C_1 and C_2, then we remove an edge from such a path. A path with a edge removed is disconncted, so G is 3-edge-connected.