EECS 455: Applied Graph Theory
Homework 11
Due Wednesday, September 30 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

Problem 1: Let G be a simple, planar graph. Give each vertex v a "charge" of $d(v) - 4$ where $d(v)$ is the degree. Give each face f a "charge" of $|f| - 4$ where $|f|$ is the number of vertices on the face. Prove that the total charge of the graph is negative.

Problem 2: Let G be a simple, planar graph with $\delta(G) \geq 3$. Give each vertex v a "charge" of $d(v) - 4$, and give each face f a "charge" of $|f| - 4$. Prove that if every vertex of degree 3 is adjacent to three faces of size 6 or larger, then we can "discharge" the degree 3 vertices by transfering the negative charge of the degree 3 vertices its adjacent faces such that every vertex of G and every face of G of size 6 or larger has non-negative charge and such that the total charge of G does not change.

Problem 3: Prove that for any simple, planar graph G with $\delta(G) \geq 3$, there exists a vertex v on face f such that $d(v) + |f| \leq 8$. (Assume there is no such vertex or face and give a discharging rule that results in every vertex and face of G having non-negative charge.)