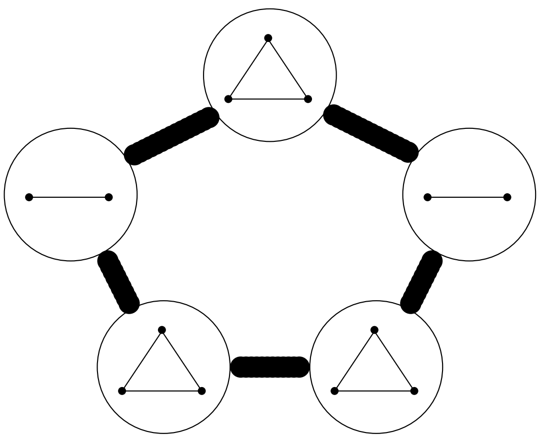
CSDS 455: Applied Graph Theory
Homework 14
Due Monday, October 12 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

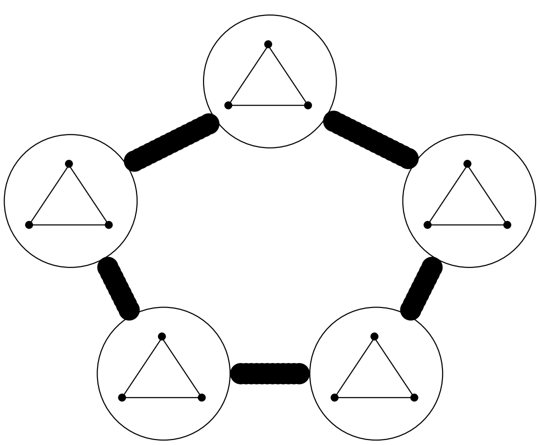
Hadwiger's Conjecture (1943 - still open): Every k-chromatic graph contains a K_k-minor (we can form a K_k through edge contractions or deletions of G).

Hajós's Conjecture (1961 - proved false in 1979): Every k-chromatic graph contains a subdivision of a K_k (we take a K_k and add extra vertices along the edges).

Problem 1: Prove the following graph has chromatic number 7 but does not contain a subdivision of a 7-clique.



Problem 2: Prove that the following graph has chromatic number 8 but does not contain a subdivision of an 8-clique.



Problem 3: In *list coloring*, each vertex is given a list of colors and must choose one color of the list. As before, we require each vertex to be assigned a different color. A graph is k-list colorable (or k-chooseable) if for any way we can assign a list of k colors to each vertex, there exists a legal coloring. $\mathrm{ch}(G)$ is the smallest value k such that G is k-list colorable.

Give an example of a graph that is 2-colorable but not 2-list colorable.

Problem 4: Prove that if G is k-list colorable then G is k-colorable.