CSDS 455: Homework 15

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I have consulted https://scholarworks.wmich.edu/cgi/viewcontent.cgi?article=3802&context=honors_theses *for this assignment.*

Problem 1

It is because for the regularity to be hold, we will need the relationship of $|d(X,Y)| - d(A,B)| < \epsilon$. For $e(X,Y)$ representing the number of edges between X and Y, the density of pair (X,Y) is determind by $\frac{e(X,Y)}{|X||Y|}$ and will therefore be a value between 0 to 1. This is because $e(X,Y)$ can be at most $|X||Y|$, and at least 0 as neither X or Y can be an empty set.

For the relationship $|d(X,Y)| - d(A,B)| < \epsilon$ to be hold, tthe X, Y sets we pick can't be too small. An extreme case will be $|X| = |Y| = 1$, where each set has one and only one vertex, then $d(X,Y)$ will be either 0 or 1 depending on if there's an edge between the two vertices. In the case of $d(X,Y) = 0$, we will inevitably have $|d(X,Y)| - d(A,B)| = d(A,B)$, which might not be $< \epsilon$. Therefore, a minimum vertex cardinality of X, Y is required to give the subsets similar density to their parent sets.

Problem 2

Due to the nature of complement graph, we will have $d_{\bar{G}}(X,Y) = 1 - d_G(X,Y)$ (W.L.O.G). This is because as the edges not connected in G are now connected in $\bar{G}$, for the density formula $d(X,Y) = \frac{e(X,Y)}{|X||Y|}$[1] the denominator will be the same, but for the numerator the edges between $\bar{X}$ and $\bar{Y}$ will be $|X||Y| - e(X,Y)$.

Subsituting this discover into the ϵ-regularity inequality and knowing that $|d_G(X,Y)| - d_G(A,B)| < \epsilon$, we have:

$$\begin{aligned} |d_{\bar{G}}(X,Y)| - d_{\bar{G}}(A,B)| &= |1 - d_G(X,Y) - (1 - d_G(A,B))| \\ &= |d_G(A,B) - d_G(X,Y)| \\ &< \epsilon \end{aligned}$$

Thus for an ϵ-regular G, we have an ϵ-regular $\bar{G}$.

[1] X, Y here are just two example sets.