CSDS 455: Applied Graph Theory
Homework 12
Due Monday, October 5 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

The next subject we will cover is graph coloring for general graphs. For your reading this weekend, take a look at Brook's Theorem and look up the *greedy algorithm* for coloring a graph.

Problem 1: Show that every graph G has a vertex ordering for which the greedy algorithm uses only $\chi(G)$ colors.

Problem 2: For every $n > 1$, find a bipartite graph on $2n$ vertices, ordered in such a way that the greedy algorithm uses n rather than 2 colors.

Problem 3: A k chromatic graph is called *critically k-chromatic*, or just *critical*, if $\chi(G - v) < k$ for every $v \in V(G)$. Show that every k-chromatic graph has a critical k-chromatic induced subgraph and that any such subgraph has minimum degree at least $k - 1$.

Problem 4: Let $\Delta(G)$ be the maximum degree of G. Prove that for any $\Delta \geq 4$, there exists a graph G with $\chi(G) \geq \Delta(G) - 1$ but G does not contain a $\Delta(G) - 1$ clique.