CSDS 455: Applied Graph Theory, Midterm Test
Due October 14, 2020

You have 2 hours to take this test. The test is closed book and closed notes. The question with the lowest grade will be dropped from your total.

At the end of the test, please write "I have neither given nor received aid on this examination, and I did not exceed the allowed time" and sign your name.

Problem 1:
Let G be a simple, undirected graph with an odd number of vertices. Let X be a non-empty set of all vertices with degree $n(G)-1$. Prove that if each component of $G-X$ is a complete graph and if the number of odd components of $G-X$ is less than or equal to $|X|$, then if we remove any vertex v of G, $G-v$ will have a 1-factor.

Problem 2:
Let G be an undirected, simple graph with edge lengths that are positive real numbers. We would like a spanning tree of G that minimizes the maximum distance between any two leaves of the tree. Prove that we can find such a spanning tree with an algorithm whose running time is a polynomial in terms of the number of vertices and edges of G.

Problem 3:
Let G be a connected, undirected graph. We will call a graph *k-resilient* if we can delete any k vertices of G and for any pair of vertices that remain, there exists a simple cycle containing that pair that also avoids any deleted vertex. What is the minimum connectivity (either vertex or edge) needed for G to be k-resilient? Prove your answer correct.

Problem 4:
Let G be a bipartite graph with partition sets X and Y. Suppose that for all $A \subseteq X$ we have $|N(A)| \geq |A|$. Prove that every edge of G is part of some matching of size $|X|$.

Problem 5:
Let G be a plane graph. Prove that if G is isomorphic to its dual G^* then G has $2n-2$ edges. Give an example of one such graph.