CSDS 455: Applied Graph Theory
Homework 7
Due Wednesday, September 16 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

Problem 1: Prove that the graph obtained from K_n by deleting one edge has $(n-2)n^{n-3}$ spanning trees.

Problem 2: Let G be a connected graph in which each edge has a positive weight/length. Recall that Dijkstra's algorithm takes a vertex s of G and creates a *shortest path spanning tree* T from s. If T is a shortest path spanning tree from s, then $d_T(s,v) = d_G(s,v)$ for all $v \in V(G)$.

We can also create a shortest path spanning tree from a *point* on an edge. Consider edge uv of G and any point χ on uv. Let T_χ be a shortest path spanning tree from χ such that $d_{T_\chi}(\chi, v) = d_G(\chi, v)$ for all $v \in V(G)$.

There are an infinite number of points along any single edge, and we can create a shortest path spanning tree from each of these points. However, there are only a finite number of possible spanning trees of G (at most n^{n-2} from Monday's class); therefore, there will be many points χ and χ' for which the shortest path spanning trees T_χ and $T_{\chi'}$ are identical.

Furthermore, let's say two spanning trees T_1 and T_2 are *equivalent* if for all $x \in V$, $d_{T_1}(u,x) = d_{T_2}(u,x)$ and $d_{T_1}(v,x) = d_{T_2}(v,x)$. Let S be a set of spanning trees. Let's define a *class* of equivalent trees to be a maximum subset of S such that all trees in the subset are equivalent to each other.

Let S_{uv} be the set of all shortest path spanning trees that are rooted at a point along the edge uv. How many separate classes of equivalent spanning trees can there be in S_{uv}? Prove your answer.