CSDS 455: Homework 27

Shaochen (Henry) ZHONG, sxz517

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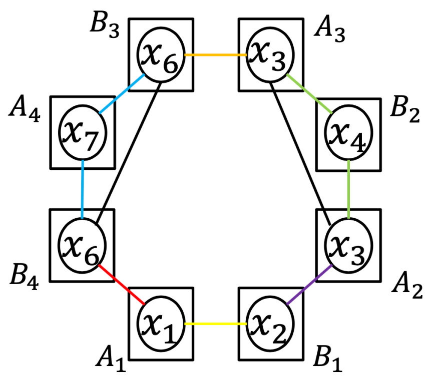
Problem 1

Read sections 5 and 6 of the paper, and in your own words explain why they are able to bound the norm of the matrix by looking at the size of a separator set.

Known the tracing power $tr((MM^T)^q) \geq ||M^{2q}||$ (due to some supposely well-know linear stuffs that I don't know). We are interesting in finding the non-zero expected value of this $tr((MM^T)^q)$. By doing the constraint graph, we discovered that if a constraint graph c has edge that not appeared in even time, it has a non-zero expected value. Like the orange and yellow edges in the following graph.



Thus, wheather or not a constraint graph c may have non-zero expected value for its tracing power will related to the distinct indicies it has. Let S being the minimum vertex cut between shapes U and V in H. For distinct indicies, we know that the upper bound will be $|V(H)|q$ as every index can at most appear q times. Each $p \in S$ will lower this bound by $q - 1$ (why?).

Then with Menger's Theorem we know that the maximum size of S is dependend on the smaller of U or V. WLOG we assume it is $|V|$ then we may update the bound to be $|V(H)|q - S(q - 1)$. As it bounds the tracing power, it therefore bounds the norm $||M^{2q}||$ due to the shown inequality.

I have refered A LOT (if not straightly taken) from the Lecture 12 slide of https://canvas.uchicago.edu/courses/17604.

ALL DONE THANK YOU !!!!!!!!!!!!!!!!!!!!