

CSDS 455: Homework 24

Shaochen (Henry) ZHONG, sxz517

Due and submitted on 11/16/2020
Fall 2020, Dr. Connamacher

I have consulted Yige Sun for this assignment.

Problem 1

Using the idea of "ribbons", modify CLIQUE example to determine the existence of 6-cliques instead of 4-cliques, and modify the TRIANGLE example to count the number of 5-cliques instead of triangles.

6-CLIQUE and 5-CLIQUE with utilization of ribbons can be expressed as:

$$6\text{-CLIQUE}(A_R, B_R) = \frac{1}{2^{15}} \sum_{R: \text{graph on } V(A_R \cup B_R)} \prod_{\{l_1, l_2\} \in E(R)} \chi_{\{l_1, l_2\}}$$

$$5\text{-CLIQUE}(A_R, B_R) = \frac{1}{2^{10}} \sum_{w_1 \in [n] \setminus V(A_R \cup B_R)} \cdot \sum_{R: \text{graph on } V(A_R \cup B_R \cup C_R)} \prod_{\{l_1, l_2\} \in E(R)} \chi_{\{l_1, l_2\}}$$

Problem 2

Is there a parity issue with this technique? Can we count the number of 6-cliques and determine the existence of 5-cliques? Show how you can adjust the algorithms for these problems or show why a straightforward modification will not work.

I don't quite get this question, if we can count the number of 6-cliques we can surely determine the existence of 5-cliques, as any 6-clique contains a 5-clique. But in the case of no 6-clique, there is still the possibility of having 5-clique. In such case we may remove a vertex x from one side of the ribbon and run the n -CLIQUE algorithm to count if there's a $(n-1)$ -CLIQUE.

Problem 3

Given your solutions above, define what the "ribbon" and "shape" would be for the solutions.

- Ribbon in 6-CLIQUE in **Problem 1** 2 is two sets of vertices of equal size (3).

- Shape in 5-CLIQUE in **Problem 1** is two sets of vertices of equal size (2) and another vertex from $[n]$.

Problem 4

Describe (general terms) the purpose of the "constraint graph" in section 3. What is it supposed to represent?

I haven't got this far yet but from a superficial scan it seems it a multi-graph representation of multiple graph matrices "gluting" together. Specifically it makes edge if there is an edge between two vertices from different partition of the ribbon.