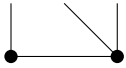


Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

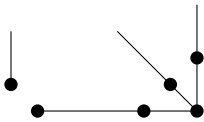
Problem 1: Prove that if $q(G - S) \leq |S|$ for all $S \subseteq V(G)$, then G has a 1-factor. (This is half of *Tutte's Theorem*.)

Problem 2: Given graph G and integer $k \leq \delta(G)$. Create a new graph G' where we take G and replace each vertex v of G with a $K_{d(v), d(v)-k}$ (a complete bipartite graph with $d(v)$ vertices in one bipartition and $d(v) - k$ vertices in the other). We will call the $d(v)$ side $A(v)$ and the $d(v) - k$ side $B(v)$. For each edge uv of G , G' will have an edge connecting one vertex of $A(u)$ with one vertex of $A(v)$. Every vertex of $A(v)$ will have exactly one such edge.

For example, if G has the edge:



and $k = 2$ then H has:



Prove that G contains a k -factor if and only if H contains a 1-factor.