CSDS 455: Applied Graph Theory
Homework 8
Due Monday, September 21 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

Next week, we will be studying k-connected graphs. You should look through the section of your text on k-connectivity and/or cuts. You should be familiar with the terms *connectivity*, *edge connectivity*, *cut vertex*, *bridge*, and *block*.

Problem 1: Let $\kappa(G)$ be the connectivity of G, let $\lambda(G)$ be the edge connectivity of G, and let $\delta(G)$ be the minimum degree of G. Prove that $\kappa(G) \leq \lambda(G) \leq \delta(G)$ (assuming G has at least two vertices).

Problem 2: The d-dimensional cube is a graph with 2^d vertices. Each vertex has a unique label of $\{0,1\}^d$ (each label is a unique sequence of d 0's and 1's), and two vertices are adjacent if and only if their labels differ in exactly one position. Determine the connectivity of the d-dimensional cube.

Problem 3: Find the smallest 3-regular (simple) graph with connectivity 1, and prove your answer.

Problem 4: Let G be a connected graph in which for every edge e there are cycles C_1 and C_2 both containing e and for which e is their only common edge. Prove that G is 3-edge-connected.