CSDS 455: Homework 21

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Problem 1

I have consulted `https://www.sebastian-kuhnert.de/cs/paper/ktree.pdf` *and* `https://edoc.hu-berlin.de/bitstream/handle/18452/18099/kuhnert.pdf?sequence=1` *for this problem.*

We first obtain a tree decomposition $T(G)$ of G where every bag of $V(T(G))$ is a $(k+1)$-clique in G that is not contaned in exactly one $(k+1)$-clique, this will take $O(n^{(k+1)})$ time.

We know that a k-tree G is $(k+1)$-colorable, as it starts from a k-clique which takes k colors, and for every new node it is conncted to a k colors so it can be colored as the $k+1$ color. There will be $(k+1)!$ different coloring for G as for the first node you may have $k+1$ choices, then k choices, then $k-1$ choices...

Let $T(G, \pi)$ to be $T(G)$ with the $(k+1)$-coloring π, we may than compute a canoical labeling of G in linear time by doing a BFS from the center of G to all verticies, which will take $O((k+1)!n)$ since there are n nodes. We may then compare the canoical labeling of H with G and see if they are same – as two isomorphic graphs will have identical canoical labeling. Thus, we have a total tuntime of $O(n^{(k+1)}) \cdot O((k+1)!n) = O(2n^{k+1}(k+1)!)$

Problem 2

I have consulted Daniel Shao for this problem.

Note that in a tree decomposition $T(G)$ of G with maximum bag size of $k+1$, there can be at most $\frac{(k+1)}{2}$ unique edges in this bag that can be in a cycle of all verticies (as one vertex is connected between 2 edges). We loop over these edges as set $s = 1 \to \frac{(k+1)}{2}$, to see if they will be part of a Hamiltonian Curcuit.

Know that a Hamiltonian Curcuit will visit a vertex twice, for a bag of $k+1$ verticies, there will be $\binom{k+1}{2s}$ edges to be a part of a Hamiltonian Curcuit. We then partition these edges into two sets, where there can be $S(2s, 2)$ ways of doing the partition. So far we got the $\sum_1^{\frac{(k+1)}{2}} \cdot \binom{k+1}{2s} \cdot S(2s, 2)$ were formed. But I don't have enough time/brain cell left to figure out the 2^{k+1-2s} stands for, gotta nervously freshing social media about the election, sorry!