CSDS 455: Homework 19

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I have consulted Yige Sun for the following problems.

Problem 1

To prove by induction. It is trivial to show that a 2-node tree is a 1-tree, due to the fact that a single vertex is a 2-complete graph and by definition a 1-tree; then by connecting a new vertex to one of the first vertex, the first vertex itself is a 1-clique.

Now assume this is true for a tree T with n verticies. For the $n+1$ vertex v, we must have $d(v) = 1$ as otherwise the graph will no longer be a tree. Say v is connected to u, this u itself is a 1-clique, and $T + v$ is therefore a 1-tree.

As we have showed that a tree with 2, n, or $n+1$ verticies is always a 1-tree, the statement is therefore proven.

Problem 2

To prove by induction. For a 2-tree we start with 3-complete graph, we denote the three verticies as a, b, c, which includes K_3 the smallest cycle.

Assume we may develop a 2-tree T that includes a cycle with n length. To further develop a 2-tree that contains a cycle with $n+1$ length, we may simply identify an edge xy in T where such edge is part of the n-cycle we found, then we again add a new vertex v to connect to both x and y. The graph is still a 2-tree due to the fact that xy is a 2-clique; but now, by tracing every edge of the n-cycle excepts edge xy but add with edges vx and vy, we have a $n+1$ cycle.

As we have showed that a cycle of length 3, n, and $n+1$ can be found in some 2-tree, the statement is therefore proven.

Problem 3

If a graph G is a subgraph of a k-tree T, then it is trivial to say that the minors of G obtained by edge or vertex deletions are still subgraph of the same T. The interesting part is the edge contraction. We also notice that the proposed question is same as asking a minor of a k-tree is still a k-tree; since a subgraph of k-tree is essentialy a minor of k-tree.

Say we have a k-tree T and we'd like to add a vertex to it, the newly added vertex v will form a $k+1$-cliques with its neighbors, as $N(v)$ should be a k-clique and v is connected to every of them.

Also considered the fact that a “minimum” k-tree is a complete graph with $k+1$ verticies, namely a $k+1$-clique, we may safely conclude that any edge among the a k-tree is inside a $k+1$-clique.

Thus, doing edge contraction on any edge will cause one or some of the $k+1$-cliques to become k-cliques. This is still a legal k-tree as any vertex addition to a k-tree is done by connecting a new vertex to k verticies on a k-clique.