CSDS 455: Applied Graph Theory
Homework 20
Due Monday, November 2 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

For this assignment, lookup how *dynamic programming* algorithms work.

Recall the *maximum independent set* problem: Given a graph G find the largest independent set of vertices of G. An independent set is an induced subgraph that has no edges.

Problem 1: Give a dynamic programming algorithm that finds the maximum independent set on a tree T. The algorithm should start at the leaves of the T and work to the root. The algorithm should run in time $O(|T|)$.

Problem 2: Give a dynamic programming algorithm that finds the maximum independent set on a k-tree T_k. The algorithm should start at the "leaves" of T_k (the vertices whose neighborhood is a k-clique) and work to the "root" (a K_k clique that is part of the K_{k+1} clique in the base case in the recursive definition of T_k). The algorithm should run in time $O(k|T_k|)$.

Problem 3: Give a dynamic programming algorithm that finds the maximum independent set on a graph G with treewidth k. The algorithm should start at the leaves of the tree and work to the root. The algorithm should run in time $O(2^k|G|)$.