CSDS 455: Applied Graph Theory
Homework 16
Due Monday, October 19 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

Our subject next week is flows. We will spend much of the time looking at k-flows, but for this weekend, I want to read about (or recall) network flows where we have a source and a sink on a graph.

Problem 1: Prove that you can reduce the *maximum bipartite matching* problem to the network flow problem. (Given a bipartite graph, turn it into a flow problem such that the value of the flow equals the maximum matching of the bipartite graph.)

Problem 2: Prove that you can reduce Menger's Theorem to the network flow problem. (Given a graph, turn it into a flow problem so that the flow size gives the number of vertex disjoint paths of the graph.)

Problem 3: Suppose we have a network (directed graph) D with edge weights (capacities) and with source node s and sink node t. Let $S \subset V(D)$ and $T \subset V(D)$ be subsets of nodes such that $s \in S, T$ but $t \notin S, T$. $(S, \overline{S})$ is a *cut* of the network, and $\mathrm{cap}(S, \overline{S})$ is the sum of weights of edges with their tail in S and head in $\overline{S}$. Prove that $\mathrm{cap}(S \cup T, \overline{S \cup T}) + \mathrm{cap}(S \cap T, \overline{S \cap T}) \leq \mathrm{cap}(S, \overline{S}) + \mathrm{cap}(T, \overline{T})$.