CSDS 455: Homework 17

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Hi Kyle, I got one midterm comming so didn't invest as much of time on this as I used to. Sorry if the proofs is kind of sketchy.

Problem 1

I have consulted https://www.ti.inf.ethz.ch/ew/lehre/GA10/lec-nfz-new-nopause.pdf *for this problem.*

I think it has something to do with Tutte's flow conjectures where every bridgeless graph has a 5-NZF. We already know that the sum of flow along any edge-cut of the graph is 0. So every bridgeless graph must admit a k-NZF. Then we push the k with Tutte's flow conjectures till $k = 5$ as demonstrated in the referenced source.

The other way to think about it might be the fact that if G is 4-NZF, then the flow of an edge e of G is at most 3 and never 0. Assume e is an edge of verticies uv in G. Since $G - e$ is bridgeless, this suggests G is at least 3-connected. So maybe by distributing the $f(e) = 3$ to each edge, we promote some edge e' to at most $f(e') = 4$ and therefore become 5-NZF. But I haven't fully rationalized how to do this flow re-distribution yet.

Problem 2

I have consulted http://www.people.vcu.edu/~dcranston/slides/nowhere-zero-talk.pdf *and Yuhui Zhang for this problem.*

Let $G = G_1 \cup G_2$, say we have flow f_1, f_2 on G_1 and G_2 respectively. We extend f_1 to $\hat{f}_1$ by assigning weight 0 to edges $\in E(G) - E(G_1)$; and likewise, extend f_2 to $\hat{f}_2$ by by assigning weight 0 to edges $\in E(G) - E(G_2)$.

Let $f = \hat{f}_1 + k_1 \hat{f}_2$ which is a non-zero flow as f_1 and f_2 are non-zero. We know that $|f(e) \leq (k_1 - 1) + k_1(k_2 - 1) = k_1 k_2 - 1$. The statement is therefore proven.