CSDS 455: Applied Graph Theory
Homework 3
Due Wednesday, September 2 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

Problem 1: Let G be a bipartite graph, let A and B be the partition sets of $V(G)$, and suppose we have the following fact: for every $S \subseteq A$, $|S| \leq |N(S)|$. ($N(S)$ is the set of vertices of B adjacent to a vertex of S.) Let M be a matching of G and let $a \in A$ be an unmatched vertex. Prove that there exists an augmenting path in G with respect to M starting from a.

Problem 2: Let G be a bipartite graph, and let A and B be partition sets of $V(G)$. Given $S \subseteq A$, define the *deficiency of* S to be $|S| - |N(S)|$. (The deficiency of $\emptyset$ is 0.) Let $Def(A)$ be the maximum deficiency of over all sets $S \subseteq A$. Prove that the size of the maximum matching of G is equal to $|A| - Def(A)$.

Problem 3: Let $q(H)$ be the number of *components* of (not necessarily connected) graph H that contain an odd number of vertices. Prove that a tree T has a perfect matching if and only if $q(T - v) = 1$ for all $v \in V(T)$.