CSDS 455: Applied Graph Theory
Homework 1
Due Wednesday, August 26 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

Problem 1: Prove that a self-complementary simple graph with n vertices exists if and only if $n \equiv 1 \mod 4$ or $n \equiv 0 \mod 4$. (Hint, generalize problems 5 and 6 from the class.)

Problem 2: Let P and Q be paths of maximum length in a connected graph G. Prove that P and Q have a common vertex.

Problem 3: Prove that the following assertions are equivalent for a graph T:
(i) T is a tree
(ii) any two vertices of T are linked by a unique path in T
(iii) T is connected but $T - e$ is disconnected for every edge e of T
(iv) T does not contain a cycle but $T + xy$ contains a cycle for any two non-adjacent vertices x, y of T.

Problem 4: Use induction on the size or structure of T to prove that if T is a tree and G is any graph with $\delta(G) \geq |T| - 1$, then $T \subseteq G$. $|T|$ is the number of vertices of T, $\delta(G)$ is the smallest vertex degree in G, and $T \subseteq G$ means we can create an isomorphic copy of T by deleting vertices and edges of G.