

CSDS 455: Applied Graph Theory

Homework 9

Due Wednesday, September 23 at the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

Problem 1: Let a and b be distinct vertices of G . Prove that the minimum number of edges separating a from b in G is equal to the maximum number of edge-disjoint $a - b$ paths in G . (Hint: look at the *line graph* of G .)

Problem 2: Let G be a k -connected graph for $k \geq 2$. Prove that any k vertices lie on a common cycle of G .

Problem 3: Let G/xy be the simple graph created by *contracting* edge xy to create a new vertex v_{xy} such that for each $u \in G$ distinct from x and y , uv_{xy} is an edge of G/xy if and only if ux or uy is an edge of G . (G/xy is the same as $G \cdot xy$ where we remove all loops and multiedges.)

Let G be a 3-connected graph. Prove that G/xy is 3-connected if and only if $G - \{x, y\}$ is 2-connected.