CSDS 455: Homework 11

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I have discussed with Yuhui Zhang and Yige Sun for this assignment. Also sorry if this one is a bit "sloppy" – have to watch the debate!

Problem 1

We have the total charge of a graph being $C = C_v + C_f$, know that $\sum_{f \in F} |f| = \sum_{v \in V} d(v) = 2|E|$ and by Euler's formular $|V| - |E| + |F| = 2$we have:

$$\begin{aligned} C &= \sum_{v \in V} (d(v) - 4) + \sum_{f \in F} (|f| - 4) \\ &= 2|E| - 4|V| + 2|E| - 4|F| \\ &= 4|E| - 4|V| - 4|F| \\ &= 4(|F| - 2) - 4|F| \\ &= -8 < 0 \end{aligned}$$

Thus, the total charge is negative.

Problem 2

A v with $d(v) = 3$ will have a charge of -1, and by requirement it will be one three faces with size 6 or larger. This suggests if we need to "discharge" this v, each face will get $-\frac{1}{3}$ negative charge upon it from this v.

For a face f, assume all of its vertices are 3-degree and have a -1 charge, then the total charge of this face will be $\frac{|f|}{3} + |f| - 4$. By observation we may tell as long as $|f| \geq 6$, such equation is ≥ 0. Combined with the fact that all the negative charges of on-face vertices are now "discharged", the "every vertex of G and every face of G of size 6 or larger has non-negative charge" requirement is now statsfied. Also because we simply added the negative charge of a 3-degree vertex onto a 6-size-or-greater face it is on, the total charge of G will be unaffected.

Problem 3

Proof. To proof by contradiction. Assume that every vertex in G on face f has $d(v) + |f| > 8$, which suggests for every vertex v on face f, there is $d(v) - 4 + |f| - 4 > 0$. For $\delta(G) \geq 3$, this suggests for a 3-degree vertex v, its neighbored face f must be $|f| \geq 6$ to statsfies the pervious inequality.

Now we may utilized the discharging rule introduced in *Problem 2*. Which will make every vertex and faces of G having a non-negative charge (as every vertex is neighbored with faces with $|f| \geq 6$), thus lead to a G with a total non-negative charge. This is a contradiction to *Problem 1* conclustion and the statement is therefore proved by contradiction. $\square$