CSDS 455: Applied Graph Theory
Homework 21
Due Wednesday, November 4 at the start of class

**Homework rules:** You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

**Problem 1:** Assume that you have a tree decomposition for graph $G$ and a second graph $H$ without a decomposition. Assume the decomposition is adjusted so that every bag has size exactly $k+1$ and every bag has 0 or 2 children.

Write a dynamic programming solution to: *Graph Isomorphism*: Given two graphs $G$ and $H$, determine if $G \cong H$.

Assume you have a tree decomposition for $G$ but not for $H$. As a hint, first spend $O(n^{k+1})$ time to find all separators of size $k+1$ in $H$. A dynamic programming solution uses a table of at most $2n^{k+1}(k+1)!$ booleans for each bag of the decomposition.

**Problem 2:** Assume that you have a tree decomposition for graph $G$. Assume the decomposition is adjusted so that every bag has size exactly $k+1$ and every bag has 0 or 2 children.

Write a dynamic programming solution to: *Hamiltonian Circuit*: Given a graph $G$, does there exist a cycle that visits each vertex of $G$ exactly once?

A dynamic programming solution uses a table of at most $\sum_{s=1}^{\lfloor \frac{k+1}{2} \rfloor} \binom{k+1}{2s} S(2s,2) 2^{k+1-2s}$ booleans for each bag of the decomposition. $S(a,b)$ is the Stirling number that counts the number of ways to partition $a$ labelled elements into $b$ non-empty, unlabelled subsets.