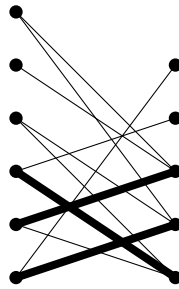


CSDS 455: Applied Graph Theory
Homework 2
Due Monday, August 31 before the start of class

Homework rules: You are welcome to work with others to solve these problems. If you do get help from someone else (or from some other resource), please indicate that on your homework.

For this assignment, you will need to read about *bipartite matchings*. Look up the *Hopcroft-Karp Algorithm* for finding a bipartite matching.

Consider the following graph and matching (bold edges) on the graph.



Problem 1: For the above bipartite graph and given *maximal matching* (the bold edges), prove that it is possible for the Hopcroft-Karp Algorithm to produce such a matching at the end of one of the algorithm's iterations.

Problem 2: Trace the remaining steps the Hopcroft-Karp algorithm will take, starting with the above matching, to produce a *maximum matching* for the graph. Be certain to identify the *augmenting paths* found by the algorithm.

Problem 3: Let M and N be different matchings in a bipartite graph G with $|M| > |N|$. Prove that there exist matchings M' and N' in G such that $|M'| = |M| - 1$ and $|N'| = |N| + 1$ and $M \cup N = M' \cup N'$ and $M \cap N = M' \cap N'$.