

EECS 491 Assignment 3

Due Mon Mar 29 before midnight. 95 points total.

Submitting assignments to Canvas

- For jupyter notebooks, submit the .ipynb file and a pdf or html export of the notebook. Make sure the output represents the latest state of your notebook. If you use interactive plots, make sure the output for the static file is representative of the points you wish to make. If your are not using notebooks, writeup your assignment using latex and submit a pdf with your code. The writeup should include relevant code with description if it can fit on a page. Do not include binaries or large data files.
- Use the following format for filenames:
 - EECS491-A3-yourcaseid.ipynb
 - EECS491-A3-yourcaseid.pdf
- If you have more than these two files, put all your files in a directory named EECS491-A3-yourcaseid. Then zip the directory and submit it with the name EECS491-A3-yourcaseid.zip. Do not use other compression formats.

Exercise 1. MRFs and Images Denoising (40 points)

In this problem, you will implement the image de-noising example using a Markov Random Field (MRF). This material on MRFs is covered in the textbook (Barber) in chapter 4.2.5. The lecture and this problem is based on the presentation in Bishop in chapter 8.3, which is available online.

As discussed in class, energy function for this MRF is

E(x,y)=h∑i​xi​−β∑i​∑j∈ne(i)​xi​xj​−η∑i​xi​yi​

where the binary variables x_i represent the unknown, noise-free image pixels, which are binary, i.e. black or white, and $ne(i)$ indicates the neighbors of node i . The variables y_i represent the observed noisy pixels, i.e. the pixel could randomly change from black ($= -1$) to white ($= +1$) or vice-versa.

The corresponding joint probability distribution over the variables is

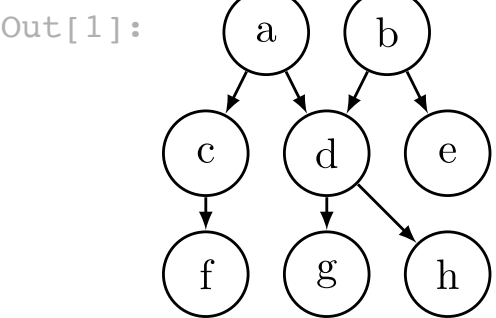
p(x,y)=1Zexp[−E(x,y)]

- 1.1 (5 pts) Derive the equation that specifies the change in the energy equation when one variable changes state.
- 1.2 (10 pts) Write a program to iteratively (or in random order) update the state variables to minimize the energy (maximize the probability). Explain the design of your code.
- 1.3 (5 pts) Show that your update algorithm minimizes the energy function $E(\mathbf{x}, \mathbf{y})$ and converges by plotting the energy vs the number of passes through the set of pixels.
- 1.4 (5 pts) Illustrate the model by showing the state of the denoised image at the points: at the start before updating, when it is about 50% converged in terms of energy minimization, and at the end when it converges. Choose images that aren't too high resolution so that the individual pixels are visible as squares. You may also do a live plot in a notebook to show it updating continuously, but make sure you have the static plots too in case the dynamic plot has portability issues.
- 1.5 (5 pts) Experiment with different settings of the energy equation parameters and explain your results in terms of their effect on the energy equation.
- 1.6 (10 pts) Generalize the energy equation so that the model better captures different structure. Explain your rationale behind this new model (i.e. terms in the equation). Illustrate it with denoising examples (other types of images) with are not well-handled by the previous model.

Exercise 2. Graphical Representation (15 points)

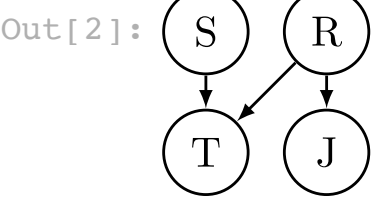
```
In [1]: using TikzPictures

# manual layout: straightforward, predictable
g = TikzPicture(L"" % latex/TikZ code
% define styles
\tikzstyle{every node} = [draw, circle, minimum size=7mm]
\tikzset{>=latex}
% add nodes
\foreach \n/\x/\y in {a/1.5/3, b/2.5/3, c/1/2, d/2/2, e/3/2, f/1/1, g/2/1, h/3/1}
\node (\n) at (\x,\y) {\n};
% draw links
\foreach \from/\to in {a/c, a/d, b/d, b/e, c/f, d/g, d/h}
\draw [->] (\from) -- (\to);
""", options="thick, scale=1.2, transform shape")
```



- 2.1 (5 pts) For the Bayesian network show above, draw the corresponding Markov Random Field (MRF), and write out the joint probability using potential functions. You do not need to specify the functions themselves, only which arguments they take. What are the potential functions in terms of the Bayes net?
- 2.2 (5 pts) Now specify the Bayes net as a factor graph. Again write the expression for the joint probability, but using factor functions.
- 2.3 (5 pts) Express the following Bayes net (from the sprinkler example) in two different factor graphs. For each network, write the factors as a function of the conditional probabilities and specify the joint probability.

```
In [2]: # automatic layout: usually more concise and compact but not always predictable
g = TikzPicture(L""
\graph [simple, grow down, branch right, edges=>=latex,
nodes={draw, circle, minimum size=7mm} ] {
{S,R} -> {T,J}, R -> T
};
""", options="thick, scale=1.2, transform shape",
preamble="\usetikzlibrary{graphs}")
```



Exercise 3. The Sum Product Algorithm (20 pts)

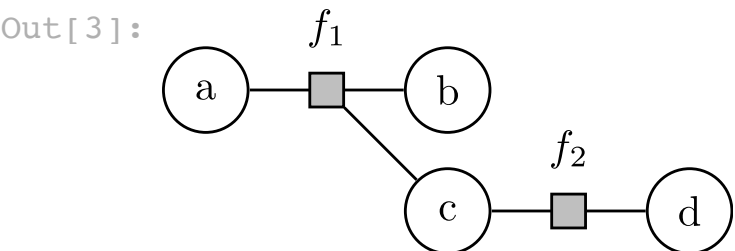
Consider the following factor graph.

```
In [3]: g = TikzPicture(L""
% define node styles
\tikzstyle{every node} = [draw, circle, minimum size=7mm]
\tikzstyle{every label} = [draw=none, inner sep=0pt]
\tikzstyle{factor} = [draw, rectangle, scale=0.4, fill=black!25]

% (Aside: Why can we add blank lines here, but not in the first graph??)
% variable nodes
\foreach \v/\x/\y in {a/1/2, b/3/2, c/3/1, d/5/1}
\node (\v) at (\x,\y) {\v};

% factor nodes
\foreach \n/\x/\y/\pos in {1/2/2/above, 2/4/1/above}
\node [factor, label=\pos:$f_{\n}$] (f\n) at (\x,\y) {};

% edges
\foreach \from/\to in {a/f1, f1/b, f1/c, c/f2, f2/d}
\draw (\from) -- (\to);
""", options="thick, scale=1.2, transform shape")
```



- 3.1 (5 pts) Apply the sum-product algorithm to compute the all messages when none of the variables are known. In your answers, you do not need to substitute in the values of other messages, i.e. your answers should be in terms of local factors and other messages.
- 3.2 (5 pts) Compute the marginal probability $p(c)$, expressing it in terms of the messages you derived in the previous question.
- 3.3 (5 pts) Verify that the marginal is the correct expression substituting in the message definitions.

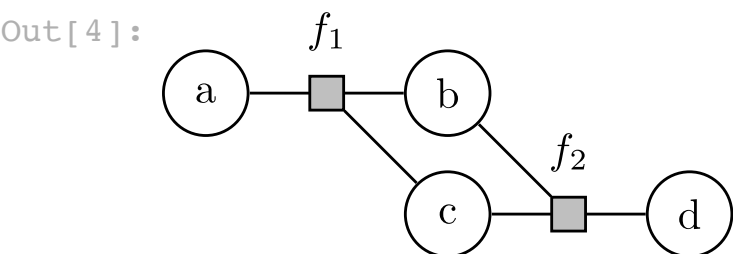
Now consider adding a loop to the graph.

```
In [4]: g = TikzPicture(L""
% define node styles
\tikzstyle{every node} = [draw, circle, minimum size=7mm]
\tikzstyle{every label} = [draw=none, inner sep=0pt]
\tikzstyle{factor} = [draw, rectangle, scale=0.4, fill=black!25]

% (Aside: Why can we add spaces here, but not above??)
% variable nodes
\foreach \v/\x/\y in {a/1/2, b/3/2, c/3/1, d/5/1}
\node (\v) at (\x,\y) {\v};

% factor nodes
\foreach \n/\x/\y/\pos in {1/2/2/above, 2/4/1/above}
\node [factor, label=\pos:$f_{\n}$] (f\n) at (\x,\y) {};

% edges
\foreach \from/\to in {a/f1, f1/b, b/f2, f1/c, c/f2, f2/d}
\draw (\from) -- (\to);
""", options="thick, scale=1.2, transform shape")
```



- 3.4 (5 pts) Explore the consequences of applying the sum-product algorithm to this graph. Can the algorithm still be applied?

Exploration (20 points)

Like in previous assignments, in this exercise you are meant to do creative exploration. You don't need to write a book chapter, but the intention is for you to go beyond what's been covered above or explore new topic altogether.

One suggestion: Select a probabilistic programming package and work through and explain in your own words an example of interest to you from the documentation or tutorial. Many of the simple tutorials will cover models similar to what we have discussed in class. Be sure your explanation is from your own perspective and understanding; do not simply copy the tutorial. You should add your own unique variations and modifications to better present the ideas or other things you found interesting. The purpose of this exercise is simply to make you aware of some of the modern tools that are available for probabilistic modeling. There are several packages available to choose from and they are all under active development as this is an active field of research. In python, popular packages are [PyMC3](#) and [TensorFlow Probability](#). In Julia, popular packages are [Gen](#) and [Turing](#). There are many other choices, so feel free to choose something else if you find it interesting.

Alternatively, you can select a topic from the readings (Barber chapters 4, 5, and 27; Murphy chapters 19, 20, and 23; or Bishop Chapter 8) and write your own exercise. It should aim to teach or explore a concept you don't understand or found interesting. Like before, you don't do an excessive amount of work. This isn't a project. Aim for something worth about 20 points, i.e. about half as much work has exercise 1 but more than exercise 2. See the rubric below for grading criteria.

Exploration Grading Rubric

Exploration problems will be graded according the elements in the table below. The scores in the column headers indicate the number of points possible for each rubric element (given in the rows). A score of zero for an element is possible if it is missing entirely.

	Substandard (+1)	Basic (+2)	Good (+3)	Excellent (+5)
Pedagogical Value	No clear statement of idea or concept being explored or explained; lack of motivating questions.	Simple problem with adequate motivation; still could be a useful addition to an assignment.	Good choice of problem with effective illustrations of concept(s). Demonstrates a deeper level of understanding.	Problem also illustrates or clarifies common conceptual difficulties or misconceptions.
Novelty of Ideas	Copies existing problem or makes only a trivial modification; lack of citation(s) for source of inspiration.	Concepts are similar to those covered in the assignment but with some modifications of an existing exercise.	Ideas have clear pedagogical motivation; creates different type of problem or exercise to explore related or foundational concepts more deeply.	Applies a technique or explores concept not covered in the assignment or not discussed at length in lecture.
Clarity of Explanation	Little or confusing explanation; figures lack labels or useful captions; no explanation of motivations.	Explanations are present, but unclear, unfocused, wordy or contain too much technical detail.	Clear and concise explanations of key ideas and motivations.	Also clear and concise, but includes illustrative figures; could be read and understood by students from a variety of backgrounds.
Depth of Exploration	Content is obvious or closely imitates assignment problems.	Uses existing problem for different data.	Applies a variation of a technique to solve a problem with an interesting motivation; explores a concept in a series of related problems.	Applies several concepts or techniques; has clear focus of inquiry that is approached from multiple directions.

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In [ ]:
```