EECS 340: Assignment 7

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EECS 340, Dr. Koyutürk

Problem 1

(a)

Set the each team as a node (vertex) in graph G and make every node fully connected (representing the fact that all teams have played against each other). If one team A beats another team B, we will have a directed edge of $A \to B$ betwen the nodes representing A and B.

Algorithm 1 DFS(G)

```
procedure DFS(G)
    for each u ∈ G.V do:
        u.color = WHITE
        u.z = 0
    for each u ∈ G.V do:
        if u.color == WHITE then
            DFS-GAME-VISIT(G, u)
```

Algorithm 2 DFS-Game-Visit(G, u)

```
procedure DFS-GAME-VISIT(G, u)
    z = NULL
    u.color = GRAY
    for each v ∈ Adj(u) do:
        if v.color == WHITE then
            z = v.rank
            z = min(z, DFS-GAME-VISIT(G, v))
    u.color = BLACK
    if z == NULL then
        u.z = 0
        z = u.rank
    else
        u.z = z
    return z
```

(b)

Justification for runtime The algorithm is $O(m+n)$ as DFS will first traverse every node, which means it will at least be $O(n)$. In addition, since the graph G is implemented using adjacency list, for each node we will have to traverse through all its adjacent edges – where such number can be at most m (total number of edges in G) for a single node. Thus, the total time complexity is $O(m+n)$.

Justification for correctness The algorithm basically performs a DFS travesal on the graph G and set the node `u` with smallest `rank` value as `z` to all of node `u`'s ancestor nodes. Due to the nature of DFS, node `u`'s ancestor nodes can legally have `u.rank` as their domination factor, as being `u`'s ancestors imply the fact these teams have won against team `u`. Also in DFS-Game-Visit we will not set a node's `.z` value unless such node is marked as `BLACK` (which means all of its decedents have been explored). Combine these two observations, each node has explored all of its decendents nodes, and the node's decendent node with the smallest `rank` will be assigned as all its ancestor nodes' `.z` value. The algorithm is a perfect mimic of the game logic of domination factor and guaranteed to be correct.

Problem 2

Note the tuple below each node represents their `(start-time, finish-time)`.

(a)

Proof with cases:

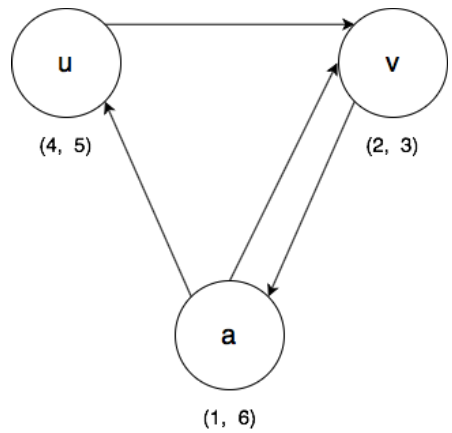
There are two possible cases where we can have a $v.f > u.f$:

- Case 1: $v.d < u.d < u.f < v.f$
  In this case u is a decent of v, which means there must be a path of $v \to u$; and known that we have $uv \in E$ (from the question instruction), this implies there is a direct connection of $u \to v$. Thus, now we have a path of $v \to u$ and a path of $u \to v$, uv is guaranteed to be on a circle.
- Case 2: $u.d < u.f < v.d < v.f$
  In this case we have u completely discovered before v. This suggest there shoule be no relationship between u and v, as v should not be a decendent nor ancestor of u. However, it is known from the question that we have $uv \in E$, which implies v must be a decendent of u. Thus, this case will not be valid under the question restriction and therefore disregarded.

As all legal case(s) shows uv is guaranteed to be on a circle, we may prove the statement to be true.

(b)

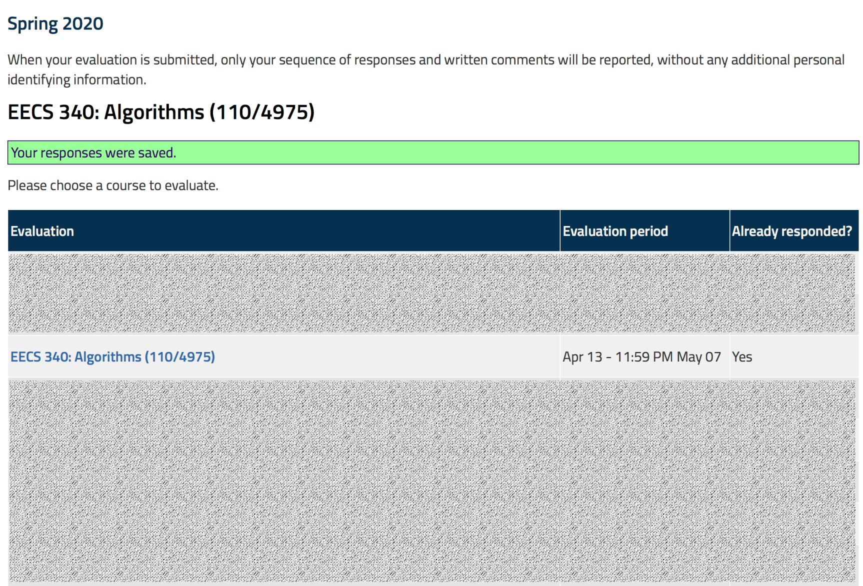
Disprove with conterexample:



It is shown that there is a path from v to u as $v \to a \to u$, however uv is still a cross edge as $v.d < v.f < u.d < u.f$ $(2 < 3 < 4 < 5)$. Thus, this conterexample disproves the statement.

Problem 4

Shaochen (Henry) ZHONG's proof of course evalution



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EECS 340: Algorithms (110/4975)

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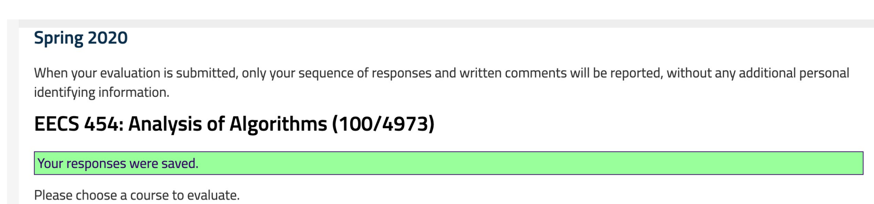
Yuhui ZHANG's proof of course evalution



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