EECS 340/454: Algorithms Spring 2020

Assignment 2: Asymptotic Notation

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Problem 1

For the following statements, consider the functions $f(n)$, $g(n)$ and constant c such that $f(n) \geq 0$, $g(n) \geq 0$, and $c > 0$. Indicate whether the statements are true or false. If true prove the statement by providing a formal argument based on the definition of asymptotic notation, otherwise, provide a counter-example to prove that they are false.

(a) $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.

(b_1) $f(n) + c = O(f(n))$.

(b_2) If $f(n) \geq 1$, then $f(n) + c = O(f(n))$.

(c_1) If $f(n) = O(g(n))$, $\log(f(n)) \geq 0$ and $\log(g(n)) \geq 0$, then $\log(f(n)) = O(\log(g(n)))$.

(c_2) If $f(n) = O(g(n))$, $\log(f(n)) \geq 0$ and $\log(g(n)) \geq 1$, then $\log(f(n)) = O(\log(g(n)))$.

(d_1) $f(2n) = \Theta(f(n))$.

(d_2) If $f(n) = O(n^c)$, then $f(2n) = O(n^c)$.

(d_3) If $f(n) = \Theta(n^c)$, then $f(2n) = \Theta(f(n))$.

Problem 2

Assume ϵ, a and b are constants such that $0 < \epsilon < 1 < a < b$. Sort the following functions in asymptotically increasing order and indicate when two or more functions are asymptotically equivalent (see definition below). Briefly justify your answers (no formal proof is needed).

n^ϵ	ϵ^n	a^n	b^n
$a^{\log_a n}$	$\log(n^a)$	$\log(n^b)$	n/a
ϵn	$(n+a)^b$	n^{a+b}	$(n+b)^a$
$n^{-\epsilon}$	n^{-a}	n^a	$\log(n^\epsilon)$
$\log_{1/\epsilon} n$	$(\log n)^a$	$\log(bn)$	$a^{\epsilon n}$

Definition. Two functions $f(n)$ and $g(n)$ are **asymptotically equivalent** if and only if $f(n) = \Theta(g(n))$.

Example. Suppose we are sorting the following functions: n, $\log(n)$, $2n$, 2^n. Then, the asymptotical ordering is: $\log(n) \ll n \equiv 2n \ll 2^n$.