ASSIGNMENT 7: GRAPH ALGORITHMS

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Problem 1

There are n basketball teams in the world. The ranking of these teams from the previous year is available. This year, some of these n teams played against each other and the winner of each game was determined. There were m games in total. The International Basketball Association wants to introduce a new performance criterion, called "domination factor", defined as follows: Team i is said to "dominate" team j if we can find a chain of games such that j was beaten by a team that was beaten by a team that was beaten by a team ... that was beaten by i (observe that, according to this definition, domination can be bi-directional, i.e., i and j can dominate each other). Then, for each team i, the domination factor z_i is defined as the rank of the best team (that is, the highest ranked team according to last year's rankings) that is dominated by team i.

   (a) Describe an $O(m + n)$ time algorithm to compute the domination factor for all the n teams. (*Hint:* Use Depth-First-Search)

   (b) Prove that your algorithm is correct.

Problem 2

Prove or disprove the following statements:

   (a) Let $G = (V, E)$ be a directed graph. For any $uv \in E$, if some run of Depth-First-Search (DFS) on G results in $v.f > u.f$, then uv must be on a cycle.

   (b) Consider any run of DFS on a directed graph $G = (V, E)$. For any edge $uv \in E$, if there is a path from v to u in G, then uv cannot be a cross edge.

Problem 3 - Removed

This problem was removed because Dijkstra's algorithm has not yet been taught.

Problem 4

Please upload evidence of completion of course evaluation (a screenshot of the confirmation page will suffice). Note: This is 20% of assignment grade.