EECS 340: Assignment 6

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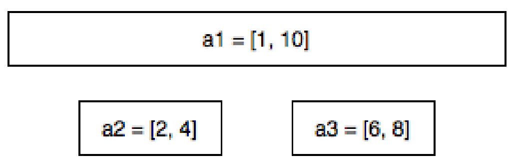
EECS 340, Dr. Koyutürk

Problem 1

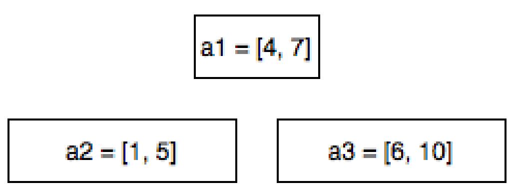
Hi Grader! Please note the boxes in following diagrams are merely provided for visualization purposes, they are not propertionaly graphed. Thanks.

(a)



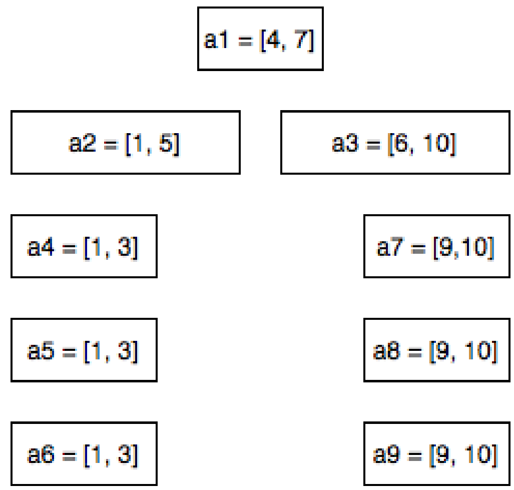
- Greedy Choice: `(a1)`
- Optimal Choice: `(a2, a3)`

(b)



- Greedy Choice: `(a1)`
- Optimal Choice: `(a2, a3)`

(c)



- Greedy Choice: `(a1)`
- Optimal Choice: Any combination between one of `{a2, a4, a5, a6}` and one of `{a3, a7, a8, a9}`.

Problem 2

Problem 3

(a)

Assume we have a set of coins with value of $\{\$1, \$5, \$8\}$ and we are looking to exchange $n = 20$.

The greedy chioce will lead to a solution $\{2 \times \$8, 4 \times \$1\}$ for a totle of 6 coins, however the optimal solution should be $\{4 \times \$5\}$ for total of 4 coins. $4 < 6$ and thus disprove the greedy choice property.

(b)

Assuming we have an optimal solution of $S =< x_o, x_1, x_2, ..., x_k >$, where each x_i respectively represent the amount of c_i coin in the avalivable coin denominations $\{c_1, c_2, ..., c_k\}$ for $1 \leq i \leq k$.

We may observe that there must be $x_i < \frac{c_{i+1}}{c_i}$ for all $1 \leq i \leq k$; as otherwise we may have solution $S' =< x'_o, x'_1, x'_2, ..., x'_k >$ with $x'_i = x_i - \frac{c_{i+1}}{c_i}$ and $x'_{i+1} = x_{i+1} + 1$[1]. Due to the fact that $\frac{c_{i+1}}{c_i}$ is at least 2, S' will be a more optimal solution than S' and therefore voids the optimal assumption of S. Thus, the property of $x_i < \frac{c_{i+1}}{c_i}$ must be held in the optimal solution.

Knowing this property, we may safely reach the optimal solution S by keep greedily picking the coins with largest c value possible ($c_{\max} \leq n$). Since if we don't pick – or don't pick the largest amount possible of – c_i coin(s) at one point, we will have to pick more c_j coins after (for $1 \leq j < i \leq n$), and that will result in a less optimal solution.

[1] This increment might have to be repeatedly done until the condition of $x_i < \frac{c_{i+1}}{c_i}$ is reached.