MATH 307: Individual Homework 9

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Due and submitted on 03/08/2021
Spring 2021, Dr. Guo

Problem 1

See HW instruction.

Known that $< u, v >= \sum_{i=1}^{3} u_i \overline{v_i}$ for $u, v \in \mathbb{C}^3$. We have the induced norm to be:

$$\begin{aligned} \|u\| &= \sqrt{\sum_{i=1}^{3} u_i \overline{u_i}} \\ &= \sqrt{\sum_{i=1}^{3} |u_i|^2} = \|u\|_2 \end{aligned}$$

And we have the norm of $u = \begin{bmatrix} 1+i \\ 2i \\ 3+i \end{bmatrix}$ being:

$$\begin{aligned} \|u\|_2 &= (\sum_{i=1}^{3} |u_i|^2)^{\frac{1}{2}} = \sqrt{\sum_{i=1}^{3} |u_i|^2} \\ &= \sqrt{(1^2+1^2)+(2^2)+(3^2+1^2)} = \sqrt{2+4+10} \\ &= \sqrt{16} = 4 \end{aligned}$$

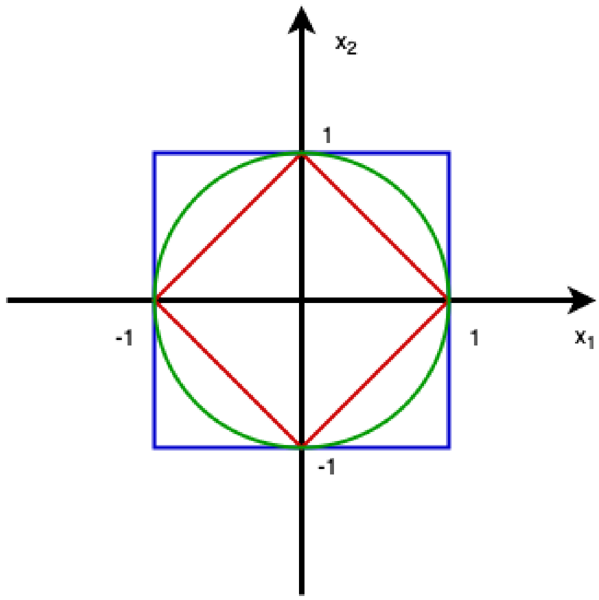
For the distance between given u, v, we have:

$$\begin{aligned} d(u,v) = \|u-v\| &= \left\| \begin{bmatrix} i \\ -3+3i \\ 3+2i \end{bmatrix} \right\| \\ &= \sqrt{1^2 + ((-3)^2+3^2) + (3^2+2^2)} \\ &= \sqrt{1+18+13} = \sqrt{32} = 4\sqrt{2} \end{aligned}$$

Problem 2

See HW instruction.

Since $x \in \mathbb{R}^2$ we have $x = [x_1, x_2]$ for $x_1, x_2 \in \mathbb{R}$.



Plots of 1-, 2-, and ∞-norm unit balls in $\mathbb{R}^2$

- **1-norm**: $|x_1| + |x_2| = 1$. Thus, we have all the points on the red diamond to be the unit ball of 1-norm.
- **2-norm**: $\sqrt{|x_1|^2 + |x_2|^2} = 1 = |x_1|^2 + |x_2|^2$. Thus, we have all the points on the green circle to be the unit ball of 2-norm.
- **∞-norm**: $\max(x_1, x_2) = 1$. Thus, we have all the points on the blue square to be the unit ball of ∞-norm.

For $x \in \mathbb{R}^2$ where $\|x\|_1 = \|x\|_2 = \|x\|_\infty = 1$, we must look at the intersections of the red, green, and blue plots – which are $\{(1, 0), (0, -1), (-1, 0), (0, 1)\}$.