

MATH 307: Group Homework 4

Group 8

Shaochen (Henry) ZHONG, Zhitao (Robert) CHEN, John MAYS, Huaijin XIN
{sxz517, zxc325, jkm100, hxx200}@case.edu

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Problem 1

Textbook page 42, problem 22.

Since we know that $\|x\| = \sqrt{\langle x, x \rangle}$; and due to conjugate symmetry we know that $\langle x, y \rangle = \overline{\langle y, x \rangle}$, implying $\langle x, y \rangle + \langle y, x \rangle = 2\operatorname{Re} \langle x, y \rangle$. Thus, for $x, y \in V$, we may have:

$$\begin{aligned}\|x + y\|^2 &= \langle x + y, x + y \rangle \\ &= \langle x, x \rangle + \langle x, y \rangle + \langle y, x \rangle + \langle y, y \rangle \\ &= \|x\|^2 + 2\operatorname{Re} \langle x, y \rangle + \|y\|^2 \\ \|x - y\|^2 &= \langle x - y, x - y \rangle \\ &= \langle x, x \rangle + (-\langle x, y \rangle) + (-\langle y, x \rangle) + \langle y, y \rangle \\ &= \|x\|^2 - 2\operatorname{Re} \langle x, y \rangle + \|y\|^2 \\ \implies \|x + y\|^2 + \|x - y\|^2 &= 2\|x\|^2 + 2\|y\|^2\end{aligned}$$

The equality-in-question is therefore shown.

Problem 2

See HW instruction.

Known from the above problem, we have the following (note we don't have to specify $\operatorname{Re} \langle x, y \rangle$ here as the problem specified V is a vector space with a real valued inner product, so $\operatorname{Re} \langle x, y \rangle = \langle x, y \rangle$):

$$\begin{aligned}\|x + y\|^2 + \|x - y\|^2 &= \|x\|^2 + 2\langle x, y \rangle + \|y\|^2 - (\|x\|^2 - 2\langle x, y \rangle + \|y\|^2) \\ &= 4\langle x, y \rangle \\ \implies \langle x, y \rangle &= \frac{1}{4}(\|x + y\|^2 + \|x - y\|^2)\end{aligned}$$

The equality-in-question is therefore shown.

Problem 3

See HW instruction.

Known from the above two problem, we have:

$$\begin{aligned}\|u + v\|^2 &= \langle u + v, u + v \rangle \\ &= \|u\|^2 + 2\langle u, v \rangle + \|v\|^2 = 7^2 \\ \|u - v\|^2 &= \langle u - v, u - v \rangle \\ &= \|u\|^2 - 2\langle u, v \rangle + \|v\|^2 = 3^2 \\ &\Rightarrow 4\langle u, v \rangle = 49 - 9 = 40 \\ &\Rightarrow \langle u, v \rangle = 10 \\ &\Rightarrow 2\|u\|^2 + 2\|v\|^2 = 49 + 9 = 58 \\ &\Rightarrow \|u\|^2 + \|v\|^2 = 29\end{aligned}$$

Thus, we have $\langle u, v \rangle = 10$ and $\|u\|^2 + \|v\|^2 = 29$.