MATH 307: Individual Homework 1

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Section 2.4

Problem 1

(a)

It is a monoid as it has the properties of *closure* for each $a, b \in A$ we have $a + b \in A$, *associativity* for $a, b, c \in A$ we have $a + (b + c) = (a + b) + c$, and *identity* for $e = 0$.

(b)

It is not a group as for any $a, b \in A$ and either of $a, b \neq 0$, we can't have $a + b = b + a = e = 0$.

(c)

As we have checked above that $\{+, \mathbb{N}, 0\}$ is not a group, it is also not an Abelian group.

Problem 2

$\{+, \mathbb{Z}, 0\}$ is an Abelian group as it has the properties of *closure* for each $a, b \in A$ we have $a+b \in A$, *associativity* for $a, b, c \in A$ we have $a + (b + c) = (a + b) + c$, and *identity* for $e = 0$.

It also has *inverse* for all $x \in A$ as $-x$, where $-x$ is also $\in A$ and $x + (-x) = 0 = e$. And it certainly shows *commutativity* property with $a + b = b + a$ for all $a, b \in A$.

However, $\{\times, \mathbb{Z}, 1\}$ is only a monoid as it has the properties of *closure* for each $a, b \in A$ we have $a \times b \in A$, *associativity* for $a, b, c \in A$ we have $a \times (b \times c) = (a \times b) \times c$, and *identity* for $e = 1$. But we can't say it is a group as we might have something $a = 2$ where the *inverse* is $\frac{1}{2}$ which is $\notin \mathbb{Z}$.