MATH 307: Group Homework 5

Group 8
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Problem 1

See HW instruction.

The vector-in-question can be transform as:

$$= x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 2 \\ 3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 3 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Problem 2

See HW instruction.

Assume the original vector $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ with a length of r has a degree of θ, we first reflect it about y-axis by swapping the x and y value and make $A_1 \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_0 \\ -y_0 \end{bmatrix}$, this means $A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

Say the $\begin{bmatrix} x_0 \\ -y_0 \end{bmatrix}$, we call it $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$, got a degree of θ, we then rotate it ϕ (in this case $\phi = 90°$) degrees more couterclock wisely to have $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$:

$$
\begin{aligned}
x_2 &= r\cos(\theta+\phi) = r(\cos\theta\cos\phi - \sin\theta\sin\phi) \\
&= r\cos\theta\cos\phi - r\sin\theta\sin\phi \\
&= x_1\cos\phi - y_1\sin\phi \\
y_2 &= r\sin(\theta+\phi) = r(\sin\theta\cos\phi + \cos\theta\sin\phi) \\
&= r\sin\theta\cos\phi + r\cos\theta\sin\phi \\
&= y_1\cos\phi + x_1\sin\phi
\end{aligned}
$$

As we need $A\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$, we must have:

$$
\begin{aligned}
x_2 &= x_1\cos\phi - y_1\sin\phi \\
y_2 &= y_1\cos\phi + x_1\sin\phi \\
\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} &= \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \\
&\text{Since } \phi = 90^\circ \\
\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot (\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}) \\
&= (\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}) \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \\
\implies A &= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}
\end{aligned}
$$

Thus, $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

Problem 3

See HW instruction.

A is not a square matrix but a **retangular** one as it has unequal number of rows and columns. So it is also not a diagonal, upper-/lower-triangular as these require being a square matrix as prerequisite.

B is a **square** matrix (and therefore not retangular) as it has equal number of rows and columns. It is not diagonal, as we has non-zero entries outside of the main diagonal. It is a **lower-triangular** matrix as all entries above the main diagonal are zero.

C is a **square** matrix (and therefore not retangular) as it has equal number of rows and columns. It is also **diagonal** as it has zero entries except the main diagonal; it is also both an **upper-** and a **lower-triangular** matrix as all entries below/above its main diagonal are zero.