MATH 307: Group Homework 1

Group 8
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Problem 1

$f : \mathbb{N} \to \mathbb{Z}$, $f(n) = z$ where $z^2 = n$

It is not a function as $\exists n_0 \in \mathbb{N}$, assume $z_0 = \sqrt{n_0}$, we may have $z = \{z_0, -z_0\}$ both satisifying the requirement of $z^2 = n$. This suggests a one-to-many relationship and therefore not a function.

$f : \mathbb{Q} \to \mathbb{Z}$, $f(m/n) = 3m + 2n$

To prove by contradiction. Assume $\frac{m}{n} = z$, we have $z = \frac{km}{kn}$ for $k \in \mathbb{Q}$; thus for the same input z, we will have multiple output values as $3km + 2kn$. This suggests a one-to-many relationship and therefore not a function.

$g : \mathbb{R} \to \mathbb{R}$, $g(x) = x^3$

It is a function as for every input $x \in \mathbb{R}$, there are only one and only one output value x^3. This function is *bijective* as it shows perfect matching between its domain and codomain – that every element in its domain is matched to an unique element in its codomain, and vice versa.

$g : \mathbb{R} \to \mathbb{R}$, $g(x) = x^2$

It is a function as for every input $x \in \mathbb{R}$, there are only one and only one output value x^2. This function is not *injective* as for $\{x_0, x_1\} \in \mathbb{R}$, let $x_0 = -x_1$, we have $f(x_0) = f(x_1) = (x_0)^2$; this suggests a many-to-one relationship and therefore not an injective function. This function is also not *surjective* as we have $\mathbb{R}$ being the codomain of the function, but any $\mathbb{Z}^-$ among the codomain can't be matched to any domain in $\mathbb{R}$. So it is just a general function.

Problem 2

We have $f \circ g(x) = f(g(x)) = \sin(x^2)$ and $g \circ f(x) = g(f(x)) = (\sin x)^2$. It is clear they are both defined, but not equal.

Problem 3

To permute the elements of sets, we have $A = \{0, 1\}$ and $B = \{0, \pm 1, \pm 2, -3, -4, -5, ...\}$. Thus:

- $A \cup B = \{0, \pm 1, \pm 2, -3, -4, -5, ...\} = B$
- $A \cap B = \{0, 1\} = A$
- $A \backslash B = \{x \mid x \in A, x \notin B\} = \emptyset$

As every element of A is an element of B, we have $A \subset B$.