MATH 307: Individual Homework 7

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Problem 1

See HW instruction.

X is vector of $[x_1, x_2, \ldots, x_n]$. And we know that $< u, v > \leq |u||v|$

$$\begin{aligned} \sum_i |x_i| &= < X, 1 > \\ \sqrt{n}\sqrt{\sum_i |x_i|^2} &= \sqrt{n}\,|X| = |X|\,|\sqrt{n}| \\ \text{Let } & n = 1 \text{ and according to the Cauchy-Schwartz inequality} \\ < X, 1 > &\leq |X|\,|1| \\ \sum_i |x_i| &\leq \sqrt{n}\,|X| = |X|\,|\sqrt{n}| \end{aligned}$$

for all $n \geq 1$.

Problem 2

See HW instruction.

For the ease of expression, for A of $\mathbb{C}^{m\times n}$ where on index (i,j) we have a_{ij}, A^k has $(a_{ij})^k$. Base on the Cauchy-Schwartz inequality, we have:

$$\begin{aligned} < A^2, B^3 > &\leq |A^2||B^3| \\ &\leq \sqrt{< A^2, A^2 >} \cdot \sqrt{< B^3, B^3 >} \\ \sum_i \sum_j |a_{ij}|^2 |b_{ij}|^3 &\leq \sqrt{\sum_i \sum_j |a_{ij}|^2 |a_{ij}|^2} \cdot \sqrt{\sum_i \sum_j |b_{ij}|^3 |b_{ij}|^3} \\ \sum_i \sum_j |a_{ij}|^2 |b_{ij}|^3 &\leq \sqrt{\sum_i \sum_j |a_{ij}|^4} \cdot \sqrt{\sum_i \sum_j |b_{ij}|^6} \end{aligned}$$