MATH 307: Individual Homework 6

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Problem 1

Textbook page 41, problem 14.

No. It is not an inner product on $\mathbb{C}^4$ as we may have $z = w = i$ where $< \begin{bmatrix} i \\ i \\ i \\ i \end{bmatrix}, \begin{bmatrix} i \\ i \\ i \\ i \end{bmatrix} >= i^2 + 2i^2 + 3i^2 + 4i^2 = -10 \not\geq 0$, thus the *positivity* property doesn't hold.

Problem 2

Textbook page 41, problem 16.

It is an inner product as:

- *Symmetry*: $< x, y >= 10x_1y_1 + x_2y_2 + 3x_3y_3 = 10y_1x_1 + y_2x_2 + 3y_3x_3 =< y, x >$
- *Positivity*: $< v, v >= 10v_1^2 + v_2^2 + 3v_3^2 \neq 0$ if $v \neq 0$.
- *Sequlinearity*: $< av + w, z >= 10(au_1 + w_1)z_1 + (au_2 + w_2)z_2 + 3(au_3 + w_3)z_3 = a(10u_1z_1 + u_2z_2 + 3u_3z_3) + 10w_1z_1 + w_2z_2 + 3w_3z_3 = a < u, z > + < w, z >$

Now to calculate lenghts, we have:

$$\left\| \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\| = \sqrt{10(1)^2 + (-1)^2 + 3(2)^2} = \sqrt{23}$$
$$\left\| \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix} \right\| = \sqrt{10(0)^2 + (3)^2 + 3(-2)^2} = \sqrt{21}$$

And the cos angle between the two vectors is:

$$\cos\theta = \frac{< \begin{bmatrix}1\\-1\\2\end{bmatrix}, \begin{bmatrix}0\\3\\-2\end{bmatrix} >}{\left\| \begin{bmatrix}1\\-1\\2\end{bmatrix} \right\| \left\| \begin{bmatrix}0\\3\\-2\end{bmatrix} \right\|}$$
$$= \frac{-15}{\sqrt{23}\cdot\sqrt{21}}$$
$$= \frac{-15}{\sqrt{483}}$$

They are not orthogonal as $< \begin{bmatrix}1\\-1\\2\end{bmatrix}, \begin{bmatrix}0\\3\\-2\end{bmatrix} > \neq 0$.

Problem 3

Textbook page 41, problem 18.

Because it doesn't hold the *positivity* property for $< \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix} >= 1\cdot 1 - 2\cdot 2 = -3 \ngeq 0$.

Problem 4

See HW instruction.

Let $A = \begin{bmatrix} a_{1R} + a_{1I}i \\ a_{2R} + a_{2I}i \\ \cdots \\ a_{mR} + a_{mI}i \end{bmatrix}$ and same for B in terms of notation. For clarity of demostration, assume the a, b in below are in fact a_{jk}, b_{jk}; now to show the purposed mapping is an inner product:

- *Conjugate Symmetry*: $< A, B >= \sum_j \sum_k a_R b_R - a_R b_I i + a_I b_R i + a_I b_I$
$= \overline{\sum_j \sum_k b_R a_R - b_R a_I i + b_I a_R i + b_I a_I} = \sum_j \sum_k b_R a_R + b_R a_I i - b_I a_R i + b_I a_I = \overline{< B, A >}$
- *Positivity*: $< A, A >= \sum_j \sum_k a_R^2 - a_R a_I i + a_I a_R i + aI^2 = a_R^2 + aI^2 \geq 0$ as long as $A \neq 0$.
- *Sequlinearity*: $< \lambda u + w, z >= \lambda(u_R + w_R)z_R - \lambda(u_R + w_R)z_I i + \lambda(u_I + w_I)z_R i + \lambda(u_I + w_I)z_i$
$= \lambda(u_R z_R - u_R z_I i + u_I z_R i + u_I z_I) + \lambda(w_R z_R - w_R z_I i + w_I z_R i + w_I z_I)$
$= \lambda < u, z > + \lambda < w, z >$

As all necessary conditions have been achicved, the purposed mapping is a defined inner product.