MATH 307: Individual Homework 3

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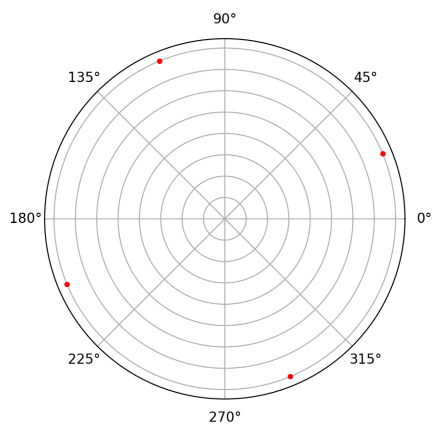
Problem 1

Compute all 4th roots of i. Express them in polar form and plot them in the complex plane.

Known that $i = e^{i\theta} = e^{i\frac{\pi}{2}}$, we have its 4th roots being $e^{i(\frac{\theta}{k} + \frac{2j\pi}{k})}$ for $k = 4$ and $j = 0, 1, 2, 3$. So the 4th roots are $e^{i(\frac{\pi}{8} + \frac{j\pi}{2})}$ for $j = 0, 1, 2, 3$ and $r^4 = 1 \implies r = \pm 1$. Here's the plotting of these roots where each root is marked in red:



Problem 2

Find the multiplicative inverse of the complex number $z = 2(\cos \pi/3 + i \sin \pi/3)$ and write it in the form $a + ib$.

For multiplicative inverse, we must have $z \cdot \frac{1}{z} = 1$.

$$z = 2(\cos \pi/3 + i \sin \pi/3) = 2 \cdot e^{\frac{\pi}{3}i}$$
$$z^{-1} = \frac{1}{2} \cdot e^{\frac{-\pi}{3}i}$$
$$\Rightarrow \begin{cases} a = \frac{1}{2} \cdot \cos \frac{-\pi}{3} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \\ b = \frac{1}{2} \cdot \sin \frac{-\pi}{3} = \frac{1}{2} \cdot \frac{-\sqrt{3}}{2} = \frac{-\sqrt{3}}{4} \end{cases}$$
$$\implies z^{-1} = \frac{1}{4} - \frac{\sqrt{3}}{4} i$$

Problem 3

Let $z = 1 - i$, find z^{10} using polar representation and write the answer in the form of $a + ib$.

To convert to polar form $z = 1 - i = \sqrt{2}e^{\tan^{-1}(\frac{-1}{1})} = \sqrt{2}e^{-\frac{\pi}{4}i}$. Now for z^{10}:

$$
\begin{aligned}
z^{10} &= \sqrt{2}^{10} \cdot e^{-\frac{10\pi}{4}i} \\
&= 32 \cdot e^{-\frac{5\pi}{2}i} \\
&\Rightarrow \begin{cases} a = 32 \cdot \cos -\frac{5\pi}{2} = 32 \cdot 0 = 0 \\ b = 32 \cdot \sin -\frac{5\pi}{2} = 32 \cdot -1 = -32 \end{cases} \\
\Longrightarrow z^{10} &= 0 - 32i = -32i
\end{aligned}
$$