MATH 307: Individual Homework 4

Shaochen (Henry) ZHONG, sxz517@case.edu

Due and submitted on 02/17/2021
Spring 2021, Dr. Guo

Problem 1

Textbook page 40, problem 1.

For W to be a vector space, we must have $(u+v) \in W$ for $u, v \in W$. However for $u = v = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$, we have $(u+v) = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$; which is $\notin W$ as $|x_j| \nless 3$ for $i \leq j \leq 4$. Thus, we may conclude that W is not a vector space.

Problem 2

Textbook page 40, problem 5.

For $W \in \mathbb{R}^3 : x + 20y - 12z - 1 = 0$ for x, y, z as elements of W to be a vector space, we must have a zero vector 0 where $0 + u = u$ for $u \in W$. In this case the zero vector is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ but it is $\notin W$ as $0 + 20(0) - 12(0) - 1 \neq 0$. Thus, we may conclude that W is not a vector space.

Problem 3

See HW instruction.

For matrices $M, N \in A$, we know that $tr(M) = tr(N) = 0$. Since the trace of a matrix is only about its diagonal elements, lets assume we have the $tr(M) = M_{11} + M_{22} + ... + M_{nn} = 0$ and $tr(N) = N_{11} + N_{22} + ... + N_{nn} = 0$ (where the subscript is the index of element). Also assume we have scalar $\lambda \in F$. Since we already know that $F^{n \times n}$ is a vector space over F, we have:

1. $M + N \in A$ as $tr(M + N) = (M_{11} + N_{11}) + (M_{22} + N_{22}) + ... + (M_{nn} + N_{nn}) = 0$.
2. $\lambda M \in A$ as $tr(\lambda M) = \lambda(M_{11} + M_{22} + ... + M_{nn}) = 0$.
3. Zero vector $0 \in A$ as $tr(0) = 0 + 0 + ... + 0 = 0$.

Thus, we may conclude that A is a subspace of $F^{n \times n}$ over F.