

MATH 307

Group Homework 3

Instructions: Read textbook pages 29 to 31 before working on the homework problems. Show all steps to get full credits.

1. Let

$$u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Prove that $\mathbb{R}^2 = \text{span}(u, v, w)$.

2. Prove that $P^4 = \text{span}(-x^4, x^3, -x^2, x, -1)$.

3. Determine whether each of the following lists of vectors is linearly independent and provide justification.

(a)

$$\begin{pmatrix} 1 \\ 1+i \end{pmatrix}, \begin{pmatrix} 1-i \\ 2 \end{pmatrix} \text{ in } \mathbb{C}^2$$

(b)

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

(c)

$$\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$$

4. Provide a basis for the vector space of $\mathbb{C}^{2 \times 3}$ over \mathbb{C} and show it is indeed a basis.

5. Is

$$\begin{pmatrix} 1 \\ i \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

a basis of \mathbb{C}^2 ? Justify your answer.