MATH 307

Individual Homework 10

Instructions: Read textbook pages 51 to 54 before working on the homework problems. Show all steps to get full credits.

1. Problem 6 on page 56.
2. Let V be the vector space of all real coefficient polynomials over the interval $[0, 1]$, define an inner product $< f, g >= \int_0^1 f(x)g(x)dx$ for $f, g \in V$. Prove that $1, x, x^2$ are linearly independent in V but not orthogonal.
3. Given the vectors

   $$v_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},$$

   find the projection of v_1, v_2 along v_3 respectively, and then use them to find the projection of $2v_1 + v_2$ along v_3.
4. Let V be the vector space of all real coefficient polynomials over $[0, 1]$ with degree no more than 1. One can prove that $1, x$ over $[0, 1]$ form a basis of V. Let $p, q \in V$, define an inner product $< p, q >= \int_0^1 p(x)q(x)dx$. Use Gram-Schmidt to find an orthonormal basis for V.