MATH 307: Group Homework 4

Group 8
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Problem 1

Textbook page 42, problem 22.

Since we know that $\|x\| = \sqrt{< x, x >}$; and due to conjugate symmetry we know that $< x, y >= \overline{y, x}$, implying $< x, y > + < y, x >= 2Re < x, y >$. Thus, for $x, y \in V$, we may have:

$$\begin{aligned} \|x+y\|^2 &=< x+y, x+y > \\ &=< x, x > + < x, y > + < y, x > + < y, y > \\ &= \|x\|^2 + 2Re < x, y > + \|y\|^2 \\ \|x-y\|^2 &=< x-y, x-y > \\ &=< x, x > +(- < x, y >) + (- < y, x >) + < y, y > \\ &= \|x\|^2 - 2Re < x, y > + \|y\|^2 \\ &\implies \|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2 \end{aligned}$$

The equality-in-question is therefore shown.

Problem 2

See HW instruction.

Known from the above problem, we have the following (note we don't have to specify $Re < x, y >$ here as the problem specified V is a vector space with a real valued inner product, so $Re < x, y >=< x, y >$):

$$\begin{aligned} \|x+y\|^2 - \|x-y\|^2 &= \|x\|^2 + 2 < x, y > + \|y\|^2 - (\|x\|^2 - 2 < x, y > + \|y\|^2) \\ &= 4 < x, y > \\ \implies < x, y > &= \frac{1}{4}(\|x+y\|^2 + \|x-y\|^2) \end{aligned}$$

The equality-in-question is therefore shown.

Problem 3

See HW instruction.

Known from the above two problem, we have:

$$\begin{aligned}
\|u+v\|^2 &=< u+v, u+v > \\
&= \|u\|^2 + 2Re < u,v > + \|v\|^2 = 7^2 \\
\|u-v\|^2 &=< u-v, u-v > \\
&= \|u\|^2 - 2Re < u,v > + \|v\|^2 = 3^2 \\
\Rightarrow < u+v, u+v > - < u-v, u-v > &= 4Re < u,v > = 49 - 9 = 40 \\
\implies Re < u,v > &= 10 \\
\Rightarrow < u+v, u+v > + < u-v, u-v > &= 2\|u\|^2 + 2\|v\|^2 = 49 + 9 = 58 \\
\implies \|u\|^2 + \|v\|^2 &= 29
\end{aligned}$$

Thus, we have $< u,v >= 10$ and $\|u\|^2 + \|v\|^2 = 29$.