

# MATH 307: Group Homework 5

Group 8

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## Problem 1

See HW instruction.

The vector-in-question can be transform as:

$$\begin{aligned} &= x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 2 \\ 3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 3 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \end{aligned}$$

## Problem 2

See HW instruction.

Assume the original vector  $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$  with a length of  $r$  has a degree of  $\theta$  and we intend to rotate it  $\phi$  (in this case  $\phi = 90^\circ$ ) more degrees to have  $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ , we have:

$$\begin{aligned} x_1 &= r \cos(\theta + \phi) = r(\cos \theta \cos \phi - \sin \theta \sin \phi) \\ &= r \cos \theta \cos \phi - r \sin \theta \sin \phi \\ &= x_0 \cos \phi - y_0 \sin \phi \\ y_1 &= r \sin(\theta + \phi) = r(\sin \theta \cos \phi + \cos \theta \sin \phi) \\ &= r \sin \theta \cos \phi + r \cos \theta \sin \phi \\ &= y_0 \cos \phi + x_0 \sin \phi \end{aligned}$$

As we need  $A \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ , we must have:

$$x_1 = x_0 \cos \phi - y_0 \sin \phi$$

$$y_1 = y_0 \cos \phi + x_0 \sin \phi$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

Since  $\phi = 90^\circ$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

Thus,  $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ .

### Problem 3

*See HW instruction.*

$A$  is not a square matrix but a **retangular** one as it has unequal number of rows and columns. So it is also not a diagonal, upper-/lower-triangular as these require being a square matrix as prerequisite.

$B$  is a **square** matrix (and therefore not retangular) as it has equal number of rows and columns. It is not diagonal, as we has non-zero entries outside of the main diagonal. It is a **lower-triangular** matrix as all entries above the main diagonal are zero.

$C$  is a **square** matrix (and therefore not retangular) as it has equal number of rows and columns. It is also **diagonal** as it has zero entries except the main diagonal; it is also both an **upper-** and a **lower-triangular** matrix as all entries below/above its main diagonal are zero.