


**Exercise 9 (for grade)** ~ Monday, December 5, 2022 ~ CPSC 535.01 Fall 2022

Write one submission for your entire group, and write all group members' names on that submission. Turn in your submission before the end of class. The  symbol marks where you should write answers.

Recall that our recommended problem-solving process is:

1. **Understand** the problem definition. What is the input? What is the output?
2. **Baseline** algorithm for comparison
3. **Goal** setting: improve on the baseline how?
4. **Design** a more sophisticated algorithm
5. **Inspiration** (if necessary) from patterns, bottleneck in the baseline algorithm, other algorithms
6. **Analyze** your solution; goal met? Trade-offs?

Follow this process for each of the following computational problems. For each problem, your submission should include:

- a. State are the input variables and what are the output variables
- b. Pseudocode for your baseline algorithm, that needs to include the data type and an explanation for any variable other than input and output variables
- a. The  $\Theta$ -notation time complexity of your baseline algorithm, with justification.

and if you manage to create an improved algorithm:

- c. Answer the question: how is your improved algorithm different from your baseline; what did you change to make it faster?
- d. Pseudocode for your improved algorithm, that needs to include the data type and an explanation for any variable other than input and output variables
- a. The  $\Theta$ -notation time complexity of your improved algorithm, with justification.

Today's problems are:

1. *(linear programming)*

A farmer has 110 acres of land that he plans to grow wheat or barley to be sold with the maximum profit. He wants to know how many acres he needs to plant the wheat and how many acres to be used for barley. To plant wheat on one acre, one spends \$100 per acre and uses 10 days of one man skill. To plant barley on one acre, one spends \$200 per acre and uses 30 days of one man skill. The farmer has \$10000 and 1,200 man-days available. The farmer can obtain \$5000 profit per acre of wheat and \$12,000 profit per acre of barley. Express the assignment problem as an integer linear programming problem.

2. *(linear programming)*

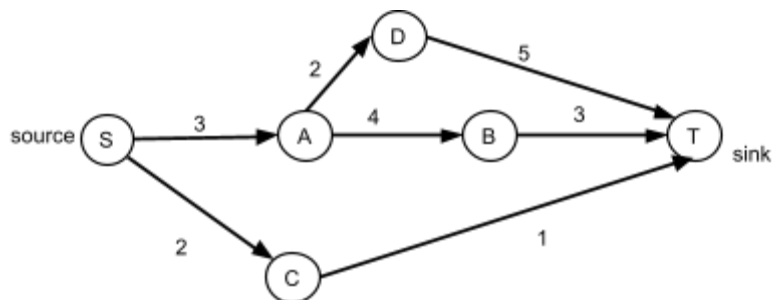
Consider the linear programming problem

$$\begin{array}{ll}\text{minimize} & 2x + 3y \\ \text{subject to} & x + y \geq 4 \\ & x + 3y = 6 \\ & x \geq 0\end{array}$$

- Convert the LP problem from general form to standard form
- Convert the standard form to slack form and state what are the nonbasic and what are the basic variables
- Solve the LP problem using simplex method

3. *(linear programming)*

Given the flow network below:



- Convert the maximum-flow problem as a linear programming problem.
- Formulate the linear programming problem into the standard form.
- Formulate the linear programming problem into the slack form.

## Names

Write the names of all group members below.

✖ Rosa Cho

## Exercise 1: Solve and provide answer

✖

If we represent our variables as:

$W_a$  = wheat (acres)

$B_a$  = barley (acres)

$W_m$  = wheat (man-days)

$B_m$  = barley (man-days)

$W_p$  = wheat (dollars)

$B_p$  = barley (dollars)

With  $w_a, w_p, b_a, b_p \geq 0$

Our objective of “how many total acres our crop yields” would look like:

$$W_a + b_a = 110 \text{ acres}$$

Constraints:

“With \$100/acre + 10 man-days for wheat and \$200/acre + 30 man-days for barley”

Wheat = \$100/acre ( $w_a$ ) with 10 man-days ( $w_m$ )

Barley = \$200/acre ( $b_a$ ) with 30 man-days ( $b_m$ )

“The farmer has \$10000 and 1,200 man-days available. ”

$$100w_p + 200b_p = \$10,000$$

$$10w_m + 30b_m = 1200 \text{ man-days}$$

“The farmer can obtain \$5000 profit per acre of wheat and \$12,000 profit per acre of barley. ”

$$W_a + w_m + W_p = \$5000$$

$$B_a + b_m + B_p = \$12000$$

Then our integer linear programming would be:

Minimize:  $100w_a + 10w_m + 10000w_p + 200b_m + 30b_p$

Subject to:  $w_a + b_a \leq 110$

$$100w_p + 200b_p = \$10,000$$

$$10w_m + 30b_m = 1200 \text{ man-day}$$

$$W_a, w_m, w_p, b_a, b_m, b_p \geq 0$$



## Exercise 2: Solve and provide answer



General form:

$$\begin{array}{ll} \text{minimize} & 2x + 3y \\ \text{subject to} & x + y \geq 4 \\ & x + 3y = 6 \\ & x \geq 0 \end{array}$$

Slack form:

$$\begin{array}{l} z = 2x + 3y \\ x + y + u \geq 4 \text{ where } u \geq 0 \\ x + 3y + v = 6 \text{ where } v \geq 0 \\ x \geq 0 \end{array}$$

Solve:

$$\begin{array}{cccccc} X[1] & + & y & [1] & + & u[1] & + & v[0] & = & [4] & \text{with } x, y, u, v \geq 0 \text{ and } z - 2x - 3y = 0 \\ [1] & & [3] & & [0] & & [1] & & [6] \end{array}$$

1. Where  $x=0, y=0, u=4, v=6$

	x	y	u	v	
u	1	1	1	0	4
v	1	3	0	1	6
	-2	-3	0	0	$z=0$

2. Divide 2nd row by 3

	x	y	u	v	
u	1	1	1	0	4
v	1	$3/3 = 1$	0	1	$6/3 = 2$
	-2	-3	0	0	$z=0$

3. Subtract 2nd row from 1st

	x	y	u	v	
u	0	0	-1	1	2
v	1	1	0	1	6

	-2	-3	0	0	$z=0$
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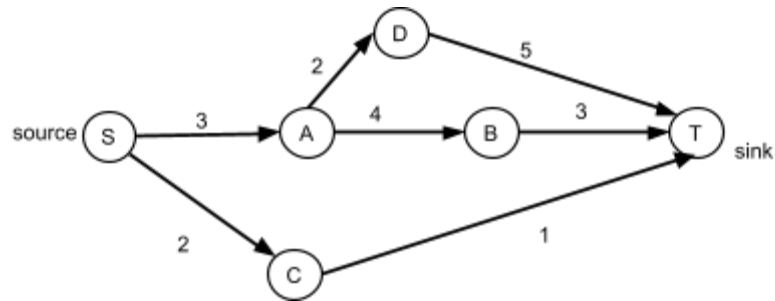
4.

	x	y	u	v	
u	0	0	0	1	3
v	0	0	1	0	2
	0	0	0	0	$z=0$

Solution:  $x_1 = 3, y_2 = 2$

### Exercise 3: Solve and provide answer

✖ Given the flow network below:



- Convert the maximum-flow problem as a linear programming problem.  
Create variable  $f$ :  
Maximize:  
Subject to:
- Formulate the linear programming problem into the standard form.
- Formulate the linear programming problem into the slack form.