Schramm-Loewner Evolutions (L16)

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Schramm-Loewner Evolution (SLE) is a family of random curves in the plane, indexed by a parameter $\kappa \geq 0$. These non-crossing curves are the fundamental tool used to describe the scaling limits of a host of natural probabilistic processes in two dimensions, such as critical percolation interfaces and random spanning trees. Their introduction by Oded Schramm in 1999 was a milestone of modern probability theory.

The course will focus on the definition and basic properties of SLE. The key ideas are conformal invariance and a certain spatial Markov property, which make it possible to use Itô calculus for the analysis. In particular we will show that, almost surely, for $\kappa \leq 4$ the curves are simple, for $4 \leq \kappa < 8$ they have double points but are non-crossing, and for $\kappa \geq 8$ they are space-filling. We will then explore the properties of the curves for a number of special values of κ (locality, restriction properties) which will allow us to relate the curves to other conformally invariant structures.

The fundamentals of conformal mapping will be needed, though most of this will be developed as required. A basic familiarity with Brownian motion and Itô calculus will be assumed but recalled.

Literature

1. Nathanaël Berestycki and James Norris. Lecture notes on SLE. `http://www.statslab.cam.ac.uk/~james/Lectures`
2. Wendelin Werner. *Random planar curves and Schramm-Loewner evolutions*, arXiv:math.PR/0303354, 2003.
3. Gregory F. Lawler. *Conformally Invariant Processes in the Plane*, AMS, 2005.

Additional support

Two examples sheets will be provided and examples classes given. There will be a revision class in Easter Term.