Combinatorics (M16)

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The flavour of the course is similar to that of the Part II Graph Theory course, although we shall not rely on many of the results from that course.

We shall study collections of subsets of a finite set, with special emphasis on size, intersection and containment. There are many very natural and fundamental questions to ask about families of subsets; although many of these remain unsolved, several have been answered using a great variety of elegant techniques.

We shall cover a number of 'classical' extremal theorems, such as those of Erdős-Ko-Rado and Kruskal-Katona, together with more recent results concerning such topics as 'concentration of measure' and hereditary properties of hypergraphs. There will be several indications of open problems.

We hope to cover the following material.

Set Systems

Definitions. Antichains; Sperner's lemma and related results. Shadows. Compression operators and the Kruskal-Katona theorem. Intersecting families; the Erdős-Ko-Rado theorem.

Isoperimetric Inequalities

The vertex-isoperimetric inequality in the cube (Harper's theorem). Inequalities in the grid. The classical isoperimetric inequality on the sphere. The 'concentration of measure' phenomenon. Applications, including Katona's t-intersecting theorem.

Projections

The trace of a set system; the Sauer-Shelah lemma. Bounds on projections: the Bollobás-Thomason box theorem. Hereditary properties. Intersecting families of graphs.

Desirable Previous Knowledge

The only prerequisites are the very basic concepts of graph theory.

Introductory Reading

1. Bollobás, B., Combinatorics, C.U.P. 1986.