Elliptic Partial Differential Equations (L24)

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This course is intended as an introduction to the theory of linear second order elliptic partial differential equations. Second order elliptic equations play a fundamental role in many areas of mathematics including geometric analysis and mathematical physics. A strong background in the linear theory provides a foundation for studying a number of non-linear problems including minimal submanifolds, harmonic maps, geometric flows and general relativity. We will discuss both classical and weak solutions to linear elliptic equations focusing on the question of existence and uniqueness of solutions to the Dirichlet problem and the question of regularity of solutions. This involves establishing maximum principles, Schauder estimates and other a priori estimates for the solutions. As time permits, we will discuss other topics including the De GiorgiNashMoser theory (which provides the Harnack inequality and establishes Holder continuity for weak solutions), applications of the linear theory to quasilinear elliptic equations, regularity for hypoelliptic operators and spectral analysis of Schrödinger operators.

Pre-requisites

Lebesgue integration, Lebesgue spaces, Sobolev spaces and basic functional and Fourier analysis.

Literature

1. David Gilbarg and Neil S. Trudinger, Elliptic Partial Differential Equations of Second Order. Springer-Verlag (1983).
2. Lawrence Evans, Partial Differential Equations. AMS (1998)
3. Qing Han and Fanghua Lin, Elliptic partial differential equations. Courant Lecture Notes, Vol. 1 (2011)
4. Lars Hörmander, The Analysis of Linear Partial Differential Operators, vols. I and II. Springer-Verlag (1983).

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.