Mixing Times of Markov Chains (L16)

Dr Jonathan Hermon

Elementary theory tells us that, after a sufficiently large time, the distribution of an irreducible, aperiodic, finite Markov chain is close to the invariant distribution. But how long should one wait in practice? For instance, how many times should a deck of cards be shuffled before its distribution is approximately uniform? This type of questions is at the heart of the theory of mixing times of Markov chains. It is a surprisingly rich question, with ramifications in analysis, geometry, combinatorics, representation theory, etc.

We shall focus on the basic theory and expose some of the main techniques which have been used to tackle this question. Our main goal will be to discuss the *cutoff phenomenon*, which says that a Markov chain reaches its stationary distribution in an abrupt fashion, after a well-defined number of steps called the *mixing time*. Surprisingly this phenomenon seems to be widespread.

A rough plan of the course is as follows:

Coupling method: convergence in total variation distance.

Spectral methods: eigenvalue decomposition and relaxation time.

Geometric methods: canonical paths, Cheeger's inequality, expanders.

Analytic methods: comparison theorem of Diaconis and Saloff-Coste.

Other notions of stationarity: strong stationary times, cover times, Lovász–Winkler theory.

Pre-requisites

This course assumes almost no background, except for prior exposure to Markov chains at an elementary level.

Literature

1. D. Levin, Y. Peres and E. Wilmer. *Markov Chains and Mixing Times.* American Mathematical Society, 2008.

2. N. Berestycki. *Mixing Times of Markov Chains: Techniques and Examples.* Available on the webpage of the author.

3. R. Montenegro and P. Tetali. *Mathematical aspects of mixing times in Markov chains.* Foundations and Trends in Theoretical Computer Science: Vol. 1: No. 3, pp 237–354, 2006.

Additional support

Examples sheets will be provided and associated examples classes will be given.