Topics in Ergodic Theory (M24)

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Ergodic theory studies dynamical systems that are endowed with an invariant measure. There are many examples of such systems that originate from other branches of mathematics. This led to a fruitful interplay between ergodic theory and other fields, especially number theory.

I will explain some basic elements of ergodic theory, such as recurrence, ergodic theorems, mixing properties and entropy. I will also talk about some applications of the theory, such as Furstenberg's proof of Szemerédi's theorem, and Weyl's equidistribution theorem for polynomials.

I aim to cover the following topics:

- Furstenberg's correspondence principle,

- Poincaré recurrence, ergodicity,

- ergodic theorems,

- unique ergodicity,

- Weyl's equidistribution theorem for polynomials,

- mixing and weak mixing,

- the multiple recurrence theorem for weak mixing systems,

- entropy and its relation to mixing,

- Rudolph's theorem on $\times 2$, $\times 3$ invariant measures.

Pre-requisites

Measure theory, basic functional analysis, conditional expectation, Fourier transform.

Literature

Notes will be available on the lecturer's webpage.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.