Statistics 270 Final Exam

You are allowed a calculator and one 8.5×11" sheet of notes, both sides. Write your answers in pen or pencil in an examination booklet. Please take a moment to read the instructions on the exam booklet, particularly the caution regarding academic integrity.

1. (3 marks) FEV_1, a measure of lung function, is recorded for six adolescents suffering from asthma: 2.30, 2.15, 3.50, 2.60, 2.75, 2.83.

   (a) (1 mark) Find the mean of these data.

   (b) (1 mark) Find the median of these data.

   (c) (1 mark) Sketch a histogram of these data with three bins of witdth 1/2, starting at 2.0.

2. (5 marks) Consider events A, B and C. Suppose $P(A) = P(B) = P(C) = 1/3$, $P(AB) = P(BC) = 1/8$ and $P(AC) = 0$.

   (a) (1 mark) Draw a Venn diagram to depict the three sets within a sample space $\mathcal{S}$.

   (b) (1 mark) What is $P(ABC)$?

   (c) (1 mark) What is $P(A \cup B \cup C)$?

   (d) (1 mark) What is $P(A|B)$?

   (e) (1 mark) Are A and B independent? Justify your answer.

3. (3 marks) A box contains 4 red balls, 3 white balls and 2 black balls of identical size. The experiment is to draw 3 balls at random, without replacement.

   (a) (1 mark) What is the probability of drawing all red balls? Simplify your answer.

   (b) (1 mark) What is the probability of drawing 2 red balls and 1 white ball? Simplify your answer.

   (c) (1 mark) What is the probability that the three balls are all different colour? Simplify your answer.

4. (5 marks) A box contains 3 white balls and 2 black balls of identical size. The box sits on a table that also has 2 white balls and 3 black balls on it. The experiment is comprised of draw/replace cycles:

   draw select a ball at random from the box

   replace put the ball that was drawn back in the box *and* put another ball of the same color from the table into the box

   The experiment ends when there are no more white balls on the table, or no more black balls on the table.

   (a) (1 mark) What is the probability that the experiment ends on the second draw/replace cycle?

   (b) (1 mark) What is the probability of drawing a white ball on the second draw?

   (c) (1 mark) What is the probability that the experiment ends on the third draw/replace cycle?

(d) (2 marks) What is the probability that the experiment ends with no white balls on the table?

5. (6 marks) The Evergreen Cultural Centre theatre seats 257. For an upcoming Christmas concert, adult tickets are \$20 each, and tickets for children and seniors are \$15 each. Based on previous years, 55% of concert-goers are adults. This year's concert is sold out.

   (a) (1 mark) What is the distribution of the number of adults, X, who attend the concert?

   (b) (1 mark) What is the expected total ticket revenue for the concert?

   (c) (1 mark) What is the variance of the total ticket revenue for the concert?

   (d) (3 marks) Approximately, what is the probability that total revenue is more than \$4600? Verify any conditions necessary to make your approximation.

6. (4 marks) Let $X \sim Poisson(\lambda)$. What is $E(e^{tX})$ for a constant t? ($E(e^{tX})$ is known as the moment generating function of X.)

7. (4 marks) Consider a random variable X with pdf $f(x) = |x|$ for $-1 < x < 1$.

   - (1 mark) Find the cdf of X.
   - (1 mark) Find the cdf of $Y = X^2$.
   - (1 mark) Find the pdf of $Y = X^2$.
   - (1 mark) Name the distribution of Y.

8. (5 marks) Let $U_1 \sim Uniform[0, 1/3]$, $U_2 = U_1 + 1/3$ and $U_3 = U_1 + 2/3$. Let X_1 be the number of U_i's in the interval $[0, 1/4)$, X_2 be the number of U_i's in the interval $[1/4, 3/4)$, and X_3 be the number of U_i's in the interval $[3/4, 1]$.

   (a) (3 marks) What is the joint pmf of the random vector (X_1, X_2, X_3)? (Hint: Break the interval [0,1] into twelfths, and consider possibilities for U_1.)

   (b) (1 mark) What is the marginal distribution of X_2?

   (c) (1 mark) What is the expected value of X_2?

9. (3 marks) A mill produces logs whose weights (measured in kg) are normally distributed. A random sample of 25 logs yields an average weight of 8439 kg, with sample SD 120 kg. Calculate a 99% confidence interval for the mean weight of logs.

10. (4 marks) A new radar device is being considered for a missile defence system. The system is checked by experimenting with aircraft in which a kill or no kill is simulated. There is interest in whether the kill percentage exceeds 80%. In 300 independent trials, 250 kills are observed.

    (a) (1 mark) State the statistical model.

    (b) (1 mark) State appropriate hypotheses.

    (c) (1 mark) Calculate a p-value.

    (d) (1 mark) State your conclusion in a complete sentence.

Appendix B: Statistical Tables

left tail prob	0.800	0.900	0.950	0.975	0.990	0.995	0.999
right tail prob	0.200	0.100	0.050	0.025	0.010	0.005	0.001
ν							
1	1.376	3.078	6.314	12.706	31.821	63.657	318.309
2	1.061	1.886	2.920	4.303	6.965	9.925	22.327
3	0.978	1.638	2.353	3.182	4.541	5.841	10.215
4	0.941	1.533	2.132	2.776	3.747	4.604	7.173
5	0.920	1.476	2.015	2.571	3.365	4.032	5.893
6	0.906	1.440	1.943	2.447	3.143	3.707	5.208
7	0.896	1.415	1.895	2.365	2.998	3.499	4.785
8	0.889	1.397	1.860	2.306	2.896	3.355	4.501
9	0.883	1.383	1.833	2.262	2.821	3.250	4.297
10	0.879	1.372	1.812	2.228	2.764	3.169	4.144
12	0.873	1.356	1.782	2.179	2.681	3.055	3.930
14	0.868	1.345	1.761	2.145	2.624	2.977	3.787
16	0.865	1.337	1.746	2.120	2.583	2.921	3.686
18	0.862	1.330	1.734	2.101	2.552	2.878	3.610
20	0.860	1.325	1.725	2.086	2.528	2.845	3.552
22	0.858	1.321	1.717	2.074	2.508	2.819	3.505
24	0.857	1.318	1.711	2.064	2.492	2.797	3.467
26	0.856	1.315	1.706	2.056	2.479	2.779	3.435
28	0.855	1.313	1.701	2.048	2.467	2.763	3.408
30	0.854	1.310	1.697	2.042	2.457	2.750	3.385
35	0.852	1.306	1.690	2.030	2.438	2.724	3.340
40	0.851	1.303	1.684	2.021	2.423	2.704	3.307
50	0.849	1.299	1.676	2.009	2.403	2.678	3.261
60	0.848	1.296	1.671	2.000	2.390	2.660	3.232
80	0.846	1.292	1.664	1.990	2.374	2.639	3.195
∞	0.842	1.282	1.645	1.960	2.326	2.576	3.090

Table B.1: Probability points corresponding to Student(ν).

z	$F(z)$	z	$F(z)$	z	$F(z)$	z	$F(z)$	z	$F(z)$
0.00	0.5000	0.41	0.6591	0.82	0.7939	1.23	0.8907	1.68	0.9535
0.01	0.5040	0.42	0.6628	0.83	0.7967	1.24	0.8925	1.72	0.9573
0.02	0.5080	0.43	0.6664	0.84	0.7995	1.25	0.8944	1.76	0.9608
0.03	0.5120	0.44	0.6700	0.85	0.8023	1.26	0.8962	1.80	0.9641
0.04	0.5160	0.45	0.6736	0.86	0.8051	1.27	0.8980	1.84	0.9671
0.05	0.5199	0.46	0.6772	0.87	0.8078	1.28	0.8997	1.88	0.9699
0.06	0.5239	0.47	0.6808	0.88	0.8106	**1.282**	**0.900**	1.92	0.9726
0.07	0.5279	0.48	0.6844	0.89	0.8133	1.29	0.9015	**1.960**	**0.975**
0.08	0.5319	0.49	0.6879	0.90	0.8159	1.30	0.9032	2.00	0.9772
0.09	0.5359	0.50	0.6915	0.91	0.8186	1.31	0.9049	2.04	0.9793
0.10	0.5398	0.51	0.6950	0.92	0.8212	1.32	0.9066	2.08	0.9812
0.11	0.5438	0.52	0.6985	0.93	0.8238	1.33	0.9082	2.12	0.9830
0.12	0.5478	0.53	0.7019	0.94	0.8264	1.34	0.9099	2.16	0.9846
0.13	0.5517	0.54	0.7054	0.95	0.8289	1.35	0.9115	2.20	0.9861
0.14	0.5557	0.55	0.7088	0.96	0.8315	1.36	0.9131	2.24	0.9875
0.15	0.5596	0.56	0.7123	0.97	0.8340	1.37	0.9147	2.28	0.9887
0.16	0.5636	0.57	0.7157	0.98	0.8365	1.38	0.9162	2.32	0.9898
0.17	0.5675	0.58	0.7190	0.99	0.8389	1.39	0.9177	**2.326**	**0.990**
0.18	0.5714	0.59	0.7224	1.00	0.8413	1.40	0.9192	2.36	0.9909
0.19	0.5753	0.60	0.7257	1.01	0.8438	1.41	0.9207	2.40	0.9918
0.20	0.5793	0.61	0.7291	1.02	0.8461	1.42	0.9222	2.44	0.9927
0.21	0.5832	0.62	0.7324	1.03	0.8485	1.43	0.9236	2.48	0.993
0.22	0.5871	0.63	0.7357	1.04	0.8508	1.44	0.9251	2.52	0.9941
0.23	0.5910	0.64	0.7389	1.05	0.8531	1.45	0.9265	2.56	0.9948
0.24	0.5948	0.65	0.7422	1.06	0.8554	1.46	0.9279	**2.576**	**0.995**
0.25	0.5987	0.66	0.7454	1.07	0.8577	1.47	0.9292	2.60	0.9953
0.26	0.6026	0.67	0.7486	1.08	0.8599	1.48	0.9306	2.64	0.9959
0.27	0.6064	0.68	0.7517	1.09	0.8621	1.49	0.9319	2.68	0.9963
0.28	0.6103	0.69	0.7549	1.10	0.8643	1.50	0.9332	2.72	0.9967
0.29	0.6141	0.70	0.7580	1.11	0.8665	1.51	0.9345	2.76	0.9971
0.30	0.6179	0.71	0.7611	1.12	0.8686	1.52	0.9357	2.80	0.9974
0.31	0.6217	0.72	0.7642	1.13	0.8708	1.53	0.9370	2.84	0.9977
0.32	0.6255	0.73	0.7673	1.14	0.8729	1.54	0.9382	2.88	0.9980
0.33	0.6293	0.74	0.7704	1.15	0.8749	1.55	0.9394	2.92	0.9982
0.34	0.6331	0.75	0.7734	1.16	0.8770	1.56	0.9406	2.96	0.9985
0.35	0.6368	0.76	0.7764	1.17	0.8790	1.57	0.9418	3.00	0.9987
0.36	0.6406	0.77	0.7794	1.18	0.8810	1.58	0.9429	3.04	0.9988
0.37	0.6443	0.78	0.7823	1.19	0.8830	1.59	0.9441	3.08	0.9990
0.38	0.6480	0.79	0.7852	1.20	0.8849	1.60	0.9452	3.12	0.9991
0.39	0.6517	0.80	0.7881	1.21	0.8869	1.64	0.9495	3.16	0.9992
0.40	0.6554	0.81	0.7910	1.22	0.8888	**1.645**	**0.950**	∞	1.0000

Table B.2: Table entries for the cumulative distribution function $F(z)$ corresponding to $Z \sim \text{normal}(0, 1)$. Boldfaced entries are commonly used values.