Stat-403/ 650/ 890 (BPK)
Final Exam

Answer all questions in the exam booklets. All parts have equal worth. Be succinct - if you are writing a thesis, you are doing far too much!

Part I: Sex ratios under different rearing temperatures

The sex ratio of some insects depends on the temperature at which they are raised.

An experiment will be conducted where infested pods from canola plants are placed in a total of 12 cages. A number of parasitoids[1] are then introduced into each cage. The individual cages are kept at three different temperatures.

The parasitoids lay eggs. These eggs hatch and turn into larvae which eventually turn into adults. The final adults are sexed. We are interested in the effect of the temperature of the cage on the sex ratio of the adult parsitoids.

Some output is present at the end of the exam.

1. Draw a picture of the experimental layout and describe the treatment structure, the experimental units, the observational units, and the randomization structures.

   Solution:
   Treatment structure: There is a single factor (temperature) with 3 levels (10, 15, 20).
   Experimental unit structure: The experimental unit is the CAGE but the observational unit is the adult parasitoid. There is pseudo-replication occurring.
   Randomization structure: Temperature is completely randomized to

[1] Any of various insects whose larvae are parasites that eventually kill their hosts. The adult parasitoid deposits an egg on or inside the body of its host, typically the larva of another arthropod. When the egg hatches, the parasitoid larva feeds on the host's tissues, gradually killing it.

cages. We assume that the parasitoids are randomly assigned to cage and that the host larvae were randomly assigned to cages (self randomization).

2. A student starts with an analysis of data from cages 1–3. What is the null and alternate hypothesis and your conclusion.

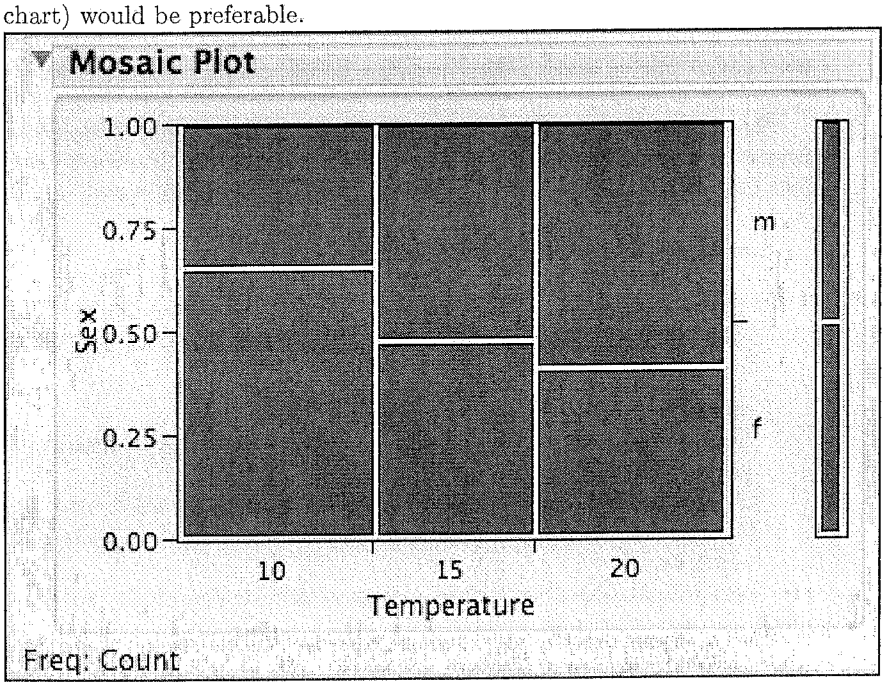
   Solution: The null hypothesis is that the POPULATION proportion of males (and females) is the same across the different temperatures. The alternate hypothesis is that the proportion of males (and females) differs across the temperatures.

   NOTE: The hypothesis is NOT that the sex ratio is 50:50. It may naturally occur that the sex ratio is different than 50:50 even in the absence of a temperature effect.

   The chi-square test statistic is 2.9. The p-value is .24. There is no evidence that the sex ratio changed across the three temperatures/cages. As noted below, because the cages and temperature are confounded, it is impossible to disentangle their effects.

3. Draw a suitable graphical display based on the data from cages 1–3. Certain types of displays that start with the letter p will earn you "negative marks".

   Solution: DON'T USE A PIE CHART! A segmented bar chart (mosaic chart) would be preferable.



   This seems to be the most natural way to present the data, but a mosaic chart with a single bar for each sex that is divided by the proportion in the three cages would also be acceptable.

4. The student (who has NOT taken Stat403/ 650/ 890) submits the results from an analysis of cages 1–3 to the supervisory committee. Unfortunately, the supervisory committee includes a certain faculty member. The response from this faculty member is "Read Hurlbert (1984)". Explain.

   Solution: This simple analysis is an example of simple pseudo-replication. The experimental unit is the CAGE and there is only one cage per temperature level. The adult insects are pseudo-replicates. Because there is only one replicate at each temperature level, you cannot distinguish between the temperature effects and the cage effects.

5. The student (who still hasn't taken Stat403/ 650/ 890) then analyzes all of the data from all 12 cages together. The response from the certain faculty member is again "Read Hurlbert (1984)". Explain.

   Solution: This is now a case of sacrificial pseudo-replication. By pooling over cages, you have lost information about the cage-to-cage variation. The cage-to-cage variation is needed because the experimental unit (the cage) variation is the "noise" in the signal-to-noise F-statistic.

6. Suggest a way to properly analyze all of the data from this experiment. Be sure to give enough details that this student can run the analysis, i.e. you need to convince a certain faculty member that you know what you are doing!

   Solution: An approximate analysis could be run by finding the proportion of males in each cage. This now gives 12 values, 4 of which are available for each of the 3 temperatures. This now looks like a single factor CRD ANOVA.

   This is not entirely satisfactory as it ignores the different sample sizes in each cage, but will work well in practice. Refer to my notes on dealing with sacrificial pseudo-replication at `http://www.stat.sfu.ca/~cschwarz/CourseNotes` for more details where it is shown how to use a Generalized Linear Mixed Model to analyze this data.

Part II: Fatty acids in pig fat

Dugan, M.E.R. et al. (2004)[2] state:

> "Pork has been associated with an unhealthy image due to it's fat content and relative proportions of polyunsaturated and saturated

[2] Dugan, M.E.R., Aalhus, J.L., Robertson, W.M., Gibson, L.L., Rolland, D.C., and Larsen, I.L. Feeding flax seed to pigs: effects on pork fatty acid composition and palatability. Advances in Pork Production. Volume 15 (Abstract).

fatty acids. Flax seed oil contains approximately 72% polyunsaturates and is the richest oilseed source of linolenic acid (58%). Feeding flax, which contains 40 to 45% oil, to finishing pigs will increase the percentage beneficial omega-3 (n-3) fatty acids in carcass fat and produce pork with value added potential."

This question is based on an experiment conducted by the Agriculture and Agri-Food Canada experimental station in Alberta. The numbers have been modified to preserve confidentiality.

An experiment was conducted to investigate the impact of different amounts of flax (5%, 10%, and 15%) in the diets of pigs for different amounts of time (4, 8, or 12 weeks) upon the fatty acid composition of backfat. The amount of one of the fatty acids (Fatty Acid 1) was measured in the backfat just after slaughter. Eight pigs were treated with each combination of amount of flax and feeding time. Two pigs of each treatment combination were housed in separate barns (four barns in total), but each pig is in a separate stall within the barn.

Some output is available at the end of exam.

1. Draw a picture of the experimental layout (curly tails on the pigs leads to bonus marks) and describe the treatment, experimental unit, and randomization structures.

   Solution: The barns are the blocks. Within each block, two replicates of each treatment combination (flax percentage x length of time) are assigned randomly to a pig in its own stall. Because there are two replicates of each treatment in a barn, this is a (generalized) randomized complete block design.

   This design is NOT a split-plot-in-time because individual pigs are NOT repeatedly measured over time.

   Treatment Structure: There are two factors (flax percentage, 3 levels), and length of time feeding (3 levels). The structure is factorial as all treatment combinations appear in the experiment.

   Experimental Unit Structure: Because individual stalls are used for each pig, the experimental and observation unit is the pig. This is NOT a split-plot in time because each pig is only measured once; different pigs are used for the different length of time on the diet.

   Randomization Structure: The pigs are randomized to barns. Within barns, the pigs are randomized to individual stalls. Within barns, the diets are randomly assigned to pigs in their stalls. At each week, the stalls from each diet are randomly selected to be slaughtered and the fatty acid measured.

2. Use the standard "shorthand syntax" and specify the appropriate model, i.e. what is the response variable and what "effect" terms would be used and why.

   Solution: The model for the GRCB is

   $FattyAcid1 = Barn(R)\ \ Diet\ \ Time\ \ Diet{*}Time\ \ Barn{*}Diet{*}Time(R)$

   if individual data values are used.

   A model

   $$FattyAcid1 = Barn(R)\ \ Diet\ \ Time\ \ Diet * Time$$

   is also ok, but now you are making a strong assumption that treatments and block effects are additive which you can actually examine with the above model.

   You can also analyze the average of the 2 pigs for each treatment, in which case the model is

   $$AvgFattyAcid1 = Barn(R)\ \ Diet\ \ Time\ \ Diet * Time$$

   Specifying the barn as a random factor is optional. Because the blocks are complete (i.e. each block has all treatment combinations), comparisons of treatments and estimation of treatment effects can all take place WITHIN blocks and so the block effects will "cancel" out in the analysis.

   The *Diet*, *Time* and *Diet***Time* terms are a consequence of the treatment structure.

   Because there is only one size of experimental unit (the pig), it does NOT have to be explicitly entered into the model.

   NOTE: Because there are replicate pigs under each treatment combination in each barn, it is actually possible to assess the assumption of additivity between blocks and treatments – this is the purpose of the Barn*Diet*Time(R) term in the first model. [It is not necessary to specify this term for this course.]

3. Output is given at the end of the exam and presents some of the statistical tests conducted. State the hypothesis of interest for the last line of the *Fixed Effect Tests*, and your conclusion.

   Solution: The null hypothesis is that there is no interaction between the *Diet* and *Time* factors in their effect upon the MEAN fatty acid level in the population of pigs.

   The p-value is .28. This is large and there is NO evidence of an interaction effect.

4. State the hypothesis of interest for the first line in the *Fixed Effect Tests* and your conclusion.

Solution: The MEAN fatty acid level is the same for all diets (when averaged over the time feeding) in the population.

The p-value is .029. There is evidence against the hypothesis of equal MEAN fatty acid levels across all the diets. [At this point, we don't know which diets may have different means.]

NOTE: It is important to specify that the MEANS are being compared in the hypothesis.

5. Using the results, construct a "joined lines" diagram (a.k.a. connected letters report) for the (main) effect of *Diet* showing the results of the Tukey multiple comparison procedure. What do you conclude from this? Explain any "odd" results.

   Solution:

Level		Least Sq Mean
5	A	1.4862500
10	A B	1.4162500
15	B	1.2975000

   Levels not connected by same letter are significantly different.

   We cannot distinguish between the mean fatty acid level of the first two diets, nor the last two diets, but we can distinguish between the mean fatty acid levels of the highest and lowest flax levels.

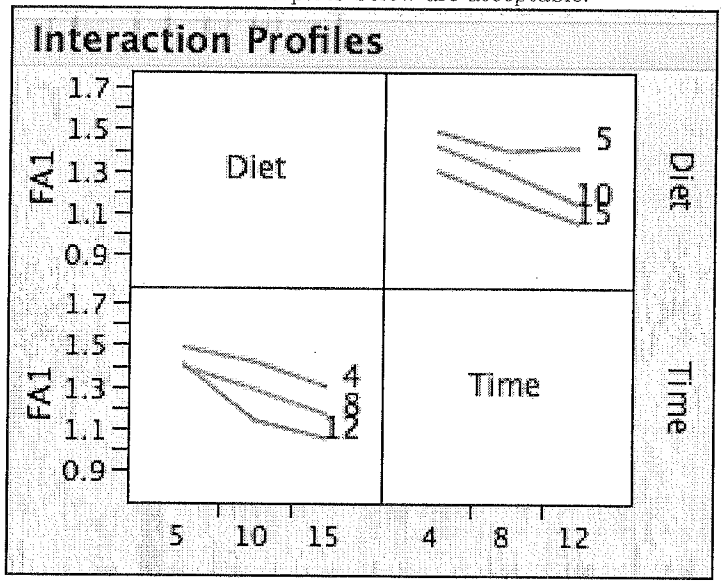
   There are two odd features of this result.

   First, the apparent violation of transitivity where "equality" of the means for the first two levels and last two levels does not translate in to equality of the means between the first and last levels, This apparent contradiction is explained by the "paint-chip analogy". Statistical equality is not the same as mathematical equality. While we cannot distinguish between the means at the two lowest levels of flax, the difference in means between the lowest and highest level of flax is large enough to be distinguished.

   Second, the original hypothesis of this experiment was that as you increase the amount of flax in the diet, the mean fatty acid level should INCREASE – it appears to decrease. The effects are "real" as there was evidence of a difference in the mean fatty acid levels among diets – the cause of this contradictory response is a mystery to the scientists involved.

6. Using the results, construct a profile plot. Carefully explain what features of the plot illustrate the main effects of each factor and their interaction.

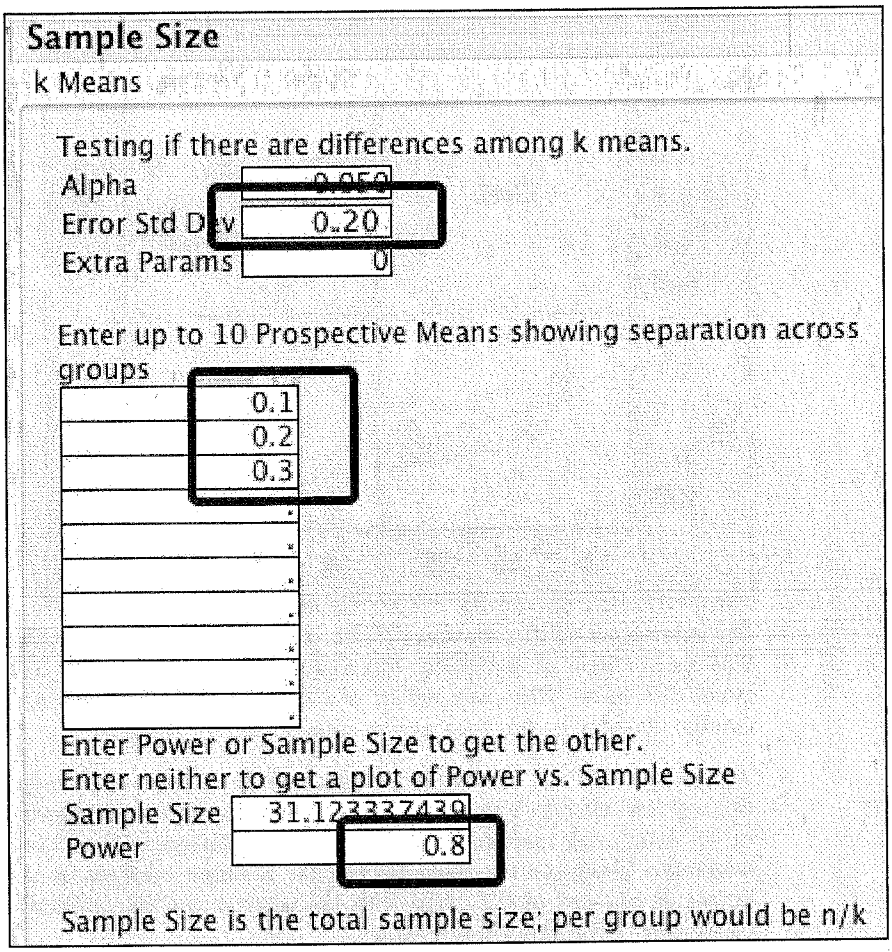
Solution: Either of the plots below are acceptable.



The interaction effect correspond to the non-parallelism of the profile lines. The main effects of one factor correspond to the average differences between the lines. The main effect of the other factor correspond to the average increase in the lines as you move from left to right.

7. The results for the *Time* factor were somewhat surprising to the scientists as they had expected to see an effect. Suppose that a difference of 0.2 in the fatty acid response variable leads to differences that need to be detected. Complete the dialogue box for a power analysis as shown in output at the end of the exam with the appropriate values (don't forget to transfer this to your exam book).

   Solution:



The estimate of the standard deviation is taken as the approximate average of the standard deviation values in the output or around 0.20. You could also have used the maximum value of the standard deviations.

It is not necessary to specify anything for the number of extra parameters. Normally, this would be the number of blocks-1, but this is usually ignored at the planning stage. Any 3 values could be entered for the means that differ by .2 from the highest to the lowest, e.g. .2, .3 and .4.

The above is valid for the first experimental design, but need to be modified slightly for the second design. This is beyond the scope of this course.

Only 3 values are specified despite there being 9 treatment combinations. If there is no interaction between the effect of diet and time, then the time effect is the SAME as all the levels of diet and so it is NOT important to consider the diet levels in the power analysis.

8. The results from a sample size determination are shown in the output at the end of the exam. What sample size is needed for a 80% power to detect the important difference above. Express the sample size in terms of the number of animals needed for each COMBINATION of flax and time factor levels. Are you surprised?

   Solution: Reading across the graph from a power of .80, we find that a TOTAL sample size of 30-35 pigs is needed. This must be split between the 9 treatment combinations, or 4 pigs per combination of flax and time on the diet.

   We already have 8 pigs per treatment combination but failed to detect an effect. However, the observed difference is only about .1 which must have less power and hence was NOT detected.

9. The experimental design was somewhat constrained because the fatty acid composition could only be measured after slaughtering the animals. But suppose a non-lethal method was available to measure fatty acid composition. Outline an alternative experimental plan that does not require a separate animal for each combination of flax and time feeding. What are the potential advantages and disadvantages of this design?

   Solution: An alternate design could have EACH pig measured at each time point. This would be a variant of a split-plot in time.

   By measuring each pig at each time point, each pig serves as its own control. Hence, the unexplained variation should be reduced, and the power of the tests increased, and the *se* of the estimate reduced.

   One potential disadvantage is that measurements taken closer together in time may be more correlated than measurements taken further apart in time. Hence a simple split-plot-in-time analysis may need to modified to account for this changing correlation over time.

   You could also run a split-plot with the time as the main-plot factor, and the pigs doing all three diet combinations. This is a much less preferred option as you now must allow for wash-out periods between diet changes so that the fatty acid from one diet does not carry forward to the next diet.

   An RCB with each pig undergoing all 9 treatment combinations (diets and length of time on the diet) is also theoretically possible, but again has many logistical difficulties.

Output for Part I.

PART of the raw data for the insect question

	Cage	Temperature	Sex	Count
1	1	10	m	8
2	1	10	f	15
3	2	15	m	10
4	2	15	f	9
5	3	20	m	13
6	3	20	f	9
7	4	10	m	8
8	4	10	f	14
9	5	15	m	13
10	5	15	f	13
11	6	20	m	8
12	6	20	f	6

Analysis for cages 1–3

Contingency Table - Cages 1-3 Only

Sex (columns) by Temperature (rows)

Temperature	Count / Total % / Col % / Row %	f	m	
10	Count	15	8	23
	Total %	23.44	12.50	35.94
	Col %	45.45	25.81	
	Row %	65.22	34.78	
15	Count	9	10	19
	Total %	14.06	15.63	29.69
	Col %	27.27	32.26	
	Row %	47.37	52.63	
20	Count	9	13	22
	Total %	14.06	20.31	34.38
	Col %	27.27	41.94	
	Row %	40.91	59.09	
	Count	33	31	64
	Total %	51.56	48.44	

Tests

N	DF	-LogLike	RSquare (U)
64	2	1.4430432	0.0326

Test	ChiSquare	Prob>ChiSq
Likelihood Ratio	2.886	0.2362
Pearson	2.851	0.2404

Analysis for ALL cages

Contingency Table – ALL cages

Temperature	Count / Total % / Col % / Row %	Sex: f	Sex: m	
	10	53 21.37 43.09 60.23	35 14.11 28.00 39.77	88 35.48
	15	38 15.32 30.89 46.91	43 17.34 34.40 53.09	81 32.66
	20	32 12.90 26.02 40.51	47 18.95 37.60 59.49	79 31.85
		123 49.60	125 50.40	248

Tests

N	DF	-LogLike	RSquare (U)
248	2	3.4330534	0.0200

Test	ChiSquare	Prob>ChiSq
Likelihood Ratio	6.866	0.0323*
Pearson	6.823	0.0330*

Output for Part II.

Some preliminary information.

	Diet	Time	Std Dev(Fatty Acid 1)
1	5	4	0.2143386839
2	5	8	0.2965395661
3	5	12	0.1674813422
4	10	4	0.223287131
5	10	8	0.1328734629
6	10	12	0.1623433311
7	15	4	0.2049346586
8	15	8	0.2412368921
9	15	12	0.1486546813

Part of the effect tests.

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Diet	2	2	58.99	3.7629	0.0290*
Time	2	2	58.99	1.0296	0.3635
Diet*Time	4	4	59.01	1.3044	0.2789

Part of the lsmeans portion of the output

Least Squares Means Table

Level	Least Sq Mean	Std Error
5	1.4862500	0.05984599
10	1.4162500	0.05984599
15	1.2975000	0.05984599

LSMeans Differences Tukey HSD

α= 0.050

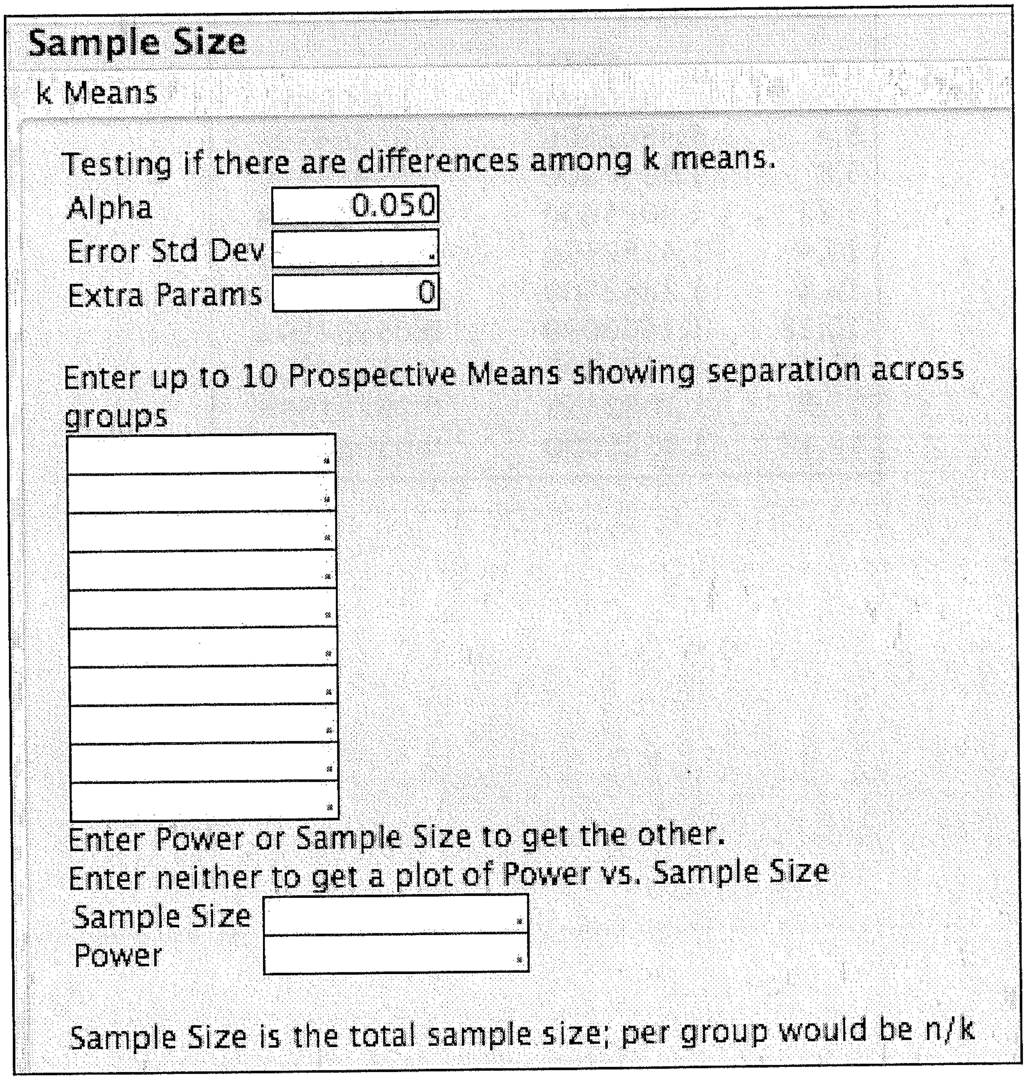
LSMean[i]	Mean[i]-Mean[j] Std Err Dif Lower CL Dif Upper CL Dif	LSMean[j] 5	10	15
	5	0 0 0 0	0.07 0.06956 -0.0973 0.23725	0.18875 0.06956 0.0215 0.356
	10	-0.07 0.06956 -0.2373 0.09725	0 0 0 0	0.11875 0.06956 -0.0485 0.286
	15	-0.1888 0.06956 -0.356 -0.0215	-0.1187 0.06956 -0.286 0.0485	0 0 0 0

Part of the lsmeans portion of the output

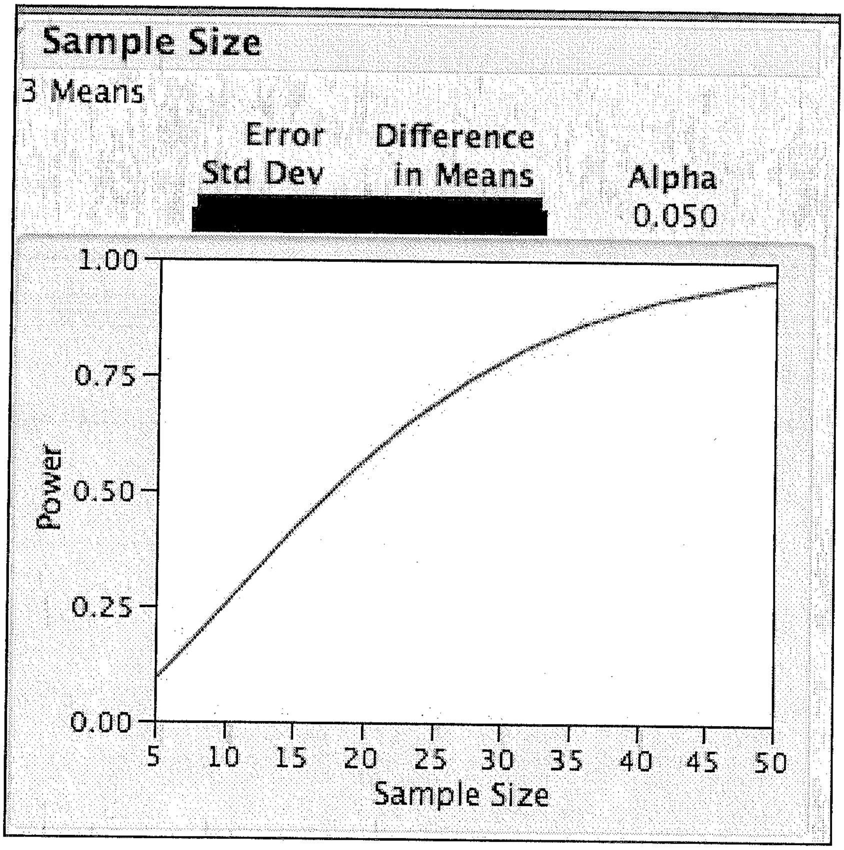
Least Squares Means Table

Level	Least Sq Mean	Std Error
5,4	1.4862500	0.05984599
5,8	1.3937500	0.05984599
5,12	1.4075000	0.05984599
10,4	1.4162500	0.05984599
10,8	1.2862500	0.05984599
10,12	1.1400000	0.05984599
15,4	1.2975000	0.05984599
15,8	1.1682356	0.06281556
15,12	1.0525000	0.05984599

Part of the power analysis dialogue box. DON'T FORGET TO TRANSFER YOUR ANSWERS TO THE EXAM BOOKLET!

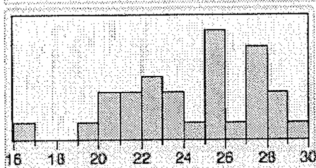


Part of the results from a power analysis.



Statistics about student performance

Final Exam - 2014 - Spring (Out of 30)



Quantiles

100.0%	maximum	30
99.5%		30
97.5%		30
90.0%		28
75.0%	quartile	27
50.0%	median	25
25.0%	quartile	21
10.0%		18.4
2.5%		12
0.5%		12
0.0%	minimum	12

Summary Statistics

Mean	23.540541
Std Dev	4.0592459
Std Err Mean	0.6673359
Upper 95% Mean	24.893961
Lower 95% Mean	22.187121
N	37

Correlations - 2014 - Term tests and final exam

	T01	T02	Final
T01	1.0000	0.6651	0.6832
T02	0.6651	1.0000	0.7749
Final	0.6832	0.7749	1.0000

There are 5 missing values. The correlations are estimated by REML method.

Scatterplot Matrix