Name: Student Number:

STAT 270:

Final Examination

Instructions: This is a closed book test but you are allowed four sheets of US letter sized paper with as much on them as you want. You may use a calculator but not a computer or a phone. Your work will be marked for clarity of explanation. Explain what assumptions you are making and comment if those assumptions seem unreasonable. I want the answers written on the paper. I have provided a normal table and a t-table on a separate sheet of paper. The exam is out of **52**.

1. Suppose X is a continuous random variable with density

$$f(x) = \begin{cases} 2x & 0 < x < c \\ 0 & \text{otherwise} \end{cases}.$$

(a) What is c? [2 marks]

(b) What is the variance of X? [2 marks]

2. Bag A contains 3 balls with the numbers 1, 2, and 3 on them. Bag B contains 2 balls numbered 1 and 2. I pull one ball from Bag A and one from Bag B.

   (a) What is the sample space? [1 mark]

   (b) What is the event that the sum of the two numbers drawn is even? [1 mark]

   (c) Let A be the event that the sum of the two numbers drawn is 4 and B be the event that the sum of the two numbers tossed is even. Find $P(A|B)$ and $P(B|A)$. [2 marks]

3. Full-term (40 weeks of gestational age) female babies of European heritage have approximately normally distributed birth lengths with a mean of 50.2 cm and an SD of 1.7 cm.

   (a) Suppose X is the birth length of a randomly selected baby in this population. What is $P(|X - 52| < 2)$? [2 marks]

   (b) Now suppose $\bar{X}$ is the average length of 6 babies randomly selected from this population. What is $P(|\bar{X} - 52| < 2)$? [2 marks]

4. A random sample of 400 students at SFU is drawn and an independent random sample of 600 UBC students is also drawn. Students are surveyed and asked the question: "How many hours of paid work did you do last week?" For this question you may assume that there was no non-response. At UBC the responses average 7.2 hours with an SD of 6 hours. At SFU the average was 8.5 hours with an SD of 8 hours.

   (a) Give a 90% confidence interval for the difference in average hours worked for all students at the two institutions. [4 marks]

   (b) Test the hypothesis that there is no difference in average hours worked last week between the two institutions. Use $\alpha = 1\%$. [4 marks]

5. A simple random sample of 400 houses in BC is drawn. A 90% confidence interval for the average finished floor areas in BC houses is 1255 square feet to 1295 square feet.

   (a) What is the sample mean for this survey? [1 mark]

   (b) What is the estimated standard error of the mean? [1 mark]

   (c) What is the sample standard deviation? [1 mark]

   (d) Give a 99% confidence interval for the same average. [1 mark]

6. A sample of 25 patients are tested on a drug intended to lower cholesterol levels. Their blood cholesterol levels are measured before and after they are treated with the drug. The average decrease observed is 1.0 mmol per dL with a standard deviation of 1.5 mmol per dL. At the 5% level is there evidence that the drug decreases average blood cholesterol levels? [4 marks]

7. I toss a pair of dice (the usual 6 sided kind). *Doubles* means the two dice show the same number.

   (a) What is the chance of throwing doubles on a single toss? [1 marks]

   (b) What is the chance of throwing doubles 3 times in a row? [2 marks]

   (c) If I toss the pair of dice 4 times what is the chance I throw doubles an even number of times. [2 marks]

8. I have two boxes with 100 balls in each of them. Box A has 80 green and 20 red balls. Box B has 40 green and 60 red. I pick a box at random and choose 2 balls from that box, without replacement. Given that both balls are green what is chance that I pick from Box A? [4 marks]

9. Suppose X and Y have joint density

$$f(x,y) = x + y \quad 0 \leq x \leq 1, 0 \leq y \leq 1.$$

(a) What is the marginal density of X? [1 mark]

(b) What is $\mathrm{E}(XY^2)$ [1 mark]

10. Suppose a random variable X has an exponential distribution with $\lambda = 1$. Let $Y = X^2$.

(a) Derive a formula for the cdf of Y. [2 marks]

(b) Find the density of Y using part (a). [1 mark]

11. I shuffle a deck of 52 cards and deal out the top 3 cards. What is the chance that I get 2 spades and a diamond? [3 marks]

12. Suppose X, Y and Z are 3 random variables. The mean of X is 2. The mean of Y is 3. The mean of Z is 4. The standard deviations are 1, 2, and 3 (in the same order). The correlation between any two is -1/4. Find the mean and variance of $T = X + Y + 2Z$. [3 marks]

13. When you throw a thumbtack it can either land point up or tipped over. I throw a thumbtack 200 times. It lands point up 118 times. Is there significant evidence that the probability of landing point up is not 1/2? [4 marks]

Final Exam Grade Sheet

Name: Student Number:

1a		2	1b		2			
2a		1	2b		1	2c		2
3a		2	3b		2			
4a		4	4b		4	5a		1
5b		1	5c		1	5d		1
6		4						
7a		1	7b		2	7c		2
8		4	9a		1	9b		1
10a		2	10b		1			
11		3	12		3	13		4

Total		**52**

Normal Cumulative Distribution Function

z	$F(z)$	z	$F(z)$	z	$F(z)$	z	$F(z)$	z	$F(z)$
0.00	0.5000	0.41	0.6591	0.82	0.7939	1.23	0.8907	1.68	0.9535
0.01	0.5040	0.42	0.6628	0.83	0.7967	1.24	0.8925	1.72	0.9573
0.02	0.5080	0.43	0.6664	0.84	0.7995	1.25	0.8944	1.76	0.9608
0.03	0.5120	0.44	0.6700	0.85	0.8023	1.26	0.8962	1.80	0.9641
0.04	0.5160	0.45	0.6736	0.86	0.8051	1.27	0.8980	1.84	0.9671
0.05	0.5199	0.46	0.6772	0.87	0.8078	1.28	0.8997	1.88	0.9699
0.06	0.5239	0.47	0.6808	0.88	0.8106	**1.282**	**0.900**	1.92	0.9720
0.07	0.5279	0.48	0.6844	0.89	0.8133	1.29	0.9015	**1.960**	**0.9750**
0.08	0.5319	0.49	0.6879	0.90	0.8159	1.30	0.9032	2.00	0.9772
0.09	0.5359	0.50	0.6915	0.91	0.8186	1.31	0.9049	2.04	0.9793
0.10	0.5398	0.51	0.6950	0.92	0.8212	1.32	0.9066	2.08	0.9812
0.11	0.5438	0.52	0.6985	0.93	0.8238	1.33	0.9082	2.12	0.9830
0.12	0.5478	0.53	0.7019	0.94	0.8264	1.34	0.9099	2.16	0.9846
0.13	0.5517	0.54	0.7054	0.95	0.8289	1.35	0.9115	2.20	0.9861
0.14	0.5557	0.55	0.7088	0.96	0.8315	1.36	0.9131	2.24	0.9875
0.15	0.5596	0.56	0.7123	0.97	0.8340	1.37	0.9147	2.28	0.9887
0.16	0.5636	0.57	0.7157	0.98	0.8365	1.38	0.9162	2.32	0.9898
0.17	0.5675	0.58	0.7190	0.99	0.8389	1.39	0.9177	**2.326**	**0.990**
0.18	0.5714	0.59	0.7224	1.00	0.8413	1.40	0.9192	2.36	0.9909
0.19	0.5753	0.60	0.7257	1.01	0.8438	1.41	0.9207	2.40	0.9918
0.20	0.5793	0.61	0.7291	1.02	0.8461	1.42	0.9222	2.44	0.9927
0.21	0.5832	0.62	0.7324	1.03	0.8485	1.43	0.9236	2.48	0.9934
0.22	0.5871	0.63	0.7357	1.04	0.8508	1.44	0.9251	2.52	0.9941
0.23	0.5910	0.64	0.7389	1.05	0.8531	1.45	0.9265	2.56	0.9948
0.24	0.5948	0.65	0.7422	1.06	0.8554	1.46	0.9279	**2.576**	**0.995**
0.25	0.5987	0.66	0.7454	1.07	0.8577	1.47	0.9292	2.60	0.9953
0.26	0.6026	0.67	0.7486	1.08	0.8599	1.48	0.9306	2.64	0.9959
0.27	0.6064	0.68	0.7517	1.09	0.8621	1.49	0.9319	2.68	0.9963
0.28	0.6103	0.69	0.7549	1.10	0.8643	1.50	0.9332	2.72	0.9967
0.29	0.6141	0.70	0.7580	1.11	0.8665	1.51	0.9345	2.76	0.9971
0.30	0.6179	0.71	0.7611	1.12	0.8686	1.52	0.9357	2.80	0.9974
0.31	0.6217	0.72	0.7642	1.13	0.8708	1.53	0.9370	2.84	0.9977
0.32	0.6255	0.73	0.7673	1.14	0.8729	1.54	0.9382	2.88	0.9980
0.33	0.6293	0.74	0.7704	1.15	0.8749	1.55	0.9394	2.92	0.9982
0.34	0.6331	0.75	0.7734	1.16	0.8770	1.56	0.9406	2.96	0.9985
0.35	0.6368	0.76	0.7764	1.17	0.8790	1.57	0.9418	3.00	0.9987
0.36	0.6406	0.77	0.7794	1.18	0.8810	1.58	0.9429	3.04	0.9988
0.37	0.6443	0.78	0.7823	1.19	0.8830	1.59	0.9441	3.08	0.9990
0.38	0.6480	0.79	0.7852	1.20	0.8849	1.60	0.9452	3.12	0.9991
0.39	0.6517	0.80	0.7881	1.21	0.8869	1.64	0.9495	3.16	0.9992
0.40	0.6554	0.81	0.7910	1.22	0.8888	**1.645**	**0.950**	∞	1.0000

Table B1: Student t critical points

left tail prob	0.800	0.900	0.950	0.975	0.990	0.995	0.999
right tail prob	0.200	0.100	0.050	0.025	0.010	0.005	0.001
ν							
1	1.376	3.078	6.314	12.706	31.821	63.657	318.309
2	1.061	1.886	2.920	4.303	6.965	9.925	22.327
3	0.978	1.638	2.353	3.182	4.541	5.841	10.215
4	0.941	1.533	2.132	2.776	3.747	4.604	7.173
5	0.920	1.476	2.015	2.571	3.365	4.032	5.893
6	0.906	1.440	1.943	2.447	3.143	3.707	5.208
7	0.896	1.415	1.895	2.365	2.998	3.499	4.785
8	0.889	1.397	1.860	2.306	2.896	3.355	4.501
9	0.883	1.383	1.833	2.262	2.821	3.250	4.297
10	0.879	1.372	1.812	2.228	2.764	3.169	4.144
12	0.873	1.356	1.782	2.179	2.681	3.055	3.930
14	0.868	1.345	1.761	2.145	2.624	2.977	3.787
16	0.865	1.337	1.746	2.120	2.583	2.921	3.686
18	0.862	1.330	1.734	2.101	2.552	2.878	3.610
20	0.860	1.325	1.725	2.086	2.528	2.845	3.552
22	0.858	1.321	1.717	2.074	2.508	2.819	3.505
24	0.857	1.318	1.711	2.064	2.492	2.797	3.467
26	0.856	1.315	1.706	2.056	2.479	2.779	3.435
28	0.855	1.313	1.701	2.048	2.467	2.763	3.408
30	0.854	1.310	1.697	2.042	2.457	2.750	3.385
35	0.852	1.306	1.690	2.030	2.438	2.724	3.340
40	0.851	1.303	1.684	2.021	2.423	2.704	3.307
50	0.849	1.299	1.676	2.009	2.403	2.678	3.261
60	0.848	1.296	1.671	2.000	2.390	2.660	3.232
80	0.846	1.292	1.664	1.990	2.374	2.639	3.195
∞	0.842	1.282	1.645	1.960	2.326	2.576	3.090