STAT 430: FINAL

Name:
Student ID:

Instructions:

- There are 11 pages in this exam booklet, including cover page.
- There are 9 questions in total.
- You must answer the first 8 questions. The last question is a bonus question.
- You are allowed to have a two-sided cheat sheet and a non-programmable calculator.

Good luck!

1. A completely randomized experiment was performed to compare four treatments A, B, C and D. The summary of the data is as follows:

Treatment	Number of observations	Sample mean	Sample variance
A	2	32	25
B	3	29	30
C	4	43	31
D	6	47	35

(a) Construct an ANOVA table.

(b) What can you conclude from the ANOVA table?

(c) Suppose that the experimenter found a mistake later on. It turns out that the treatment A and D he used are actually the same treatment called Treatment P while treatment B and C are from Treatment Q! In other words, the summary of data above becomes

Treatment	Number of observations	Sample mean	Sample variance
P	2	32	25
Q	3	29	30
Q	4	43	31
P	6	47	35

Reconstruct the ANOVA table. Is there any evidence of treatment effect?

2. Five statistics students decided to have a few poker games for the weekend.

   (a) How would you arrange the students in each of the following situations such that each student plays the same number of games and every two students play the same number of games together?

   Situation 1: Each poker game can have 3 players but they only have time to play 10 games.

   Situation 2: Each poker game can have 4 players but they only have time for 5 games.

   (b) Which arrangement above is a balanced incomplete block design? *Use the incidence matrix or the necessary conditions.*

3. A $p \times p$ hypersquare is a design in which three or more orthogonal $p \times p$ Latin squares are superimposed. Consider a $p \times p$ hypersquare that is obtained by superimposing three mutual orthogonal $p \times p$ Latin squares.

   (a) What can the design obtained by superimposing three mutual orthogonal $p \times p$ Latin squares be used for?

   (b) Write down the model for this design.

   (c) Develop an ANOVA table for such a $p \times p$ hypersquare, i.e. write down the formula for the sum of squares, degree of freedom, mean square and F_0.

4. Consider the following blocking schemes for blocking a full factorial design, 2^7, into 8 blocks.

   S1 : $B_1 = 123$, $B_2 = 234$, $B_3 = 345$

   S2 : $B_1 = 1234$, $B_2 = 2345$, $B_3 = 145$

   S3 : $B_1 = 1234$, $B_2 = 12345$, $B_3 = 245$

   (a) Rank the blocking scheme based on order.

   (b) Can you find a blocking scheme better than the above under the minimum aberration?

5. Suppose that a forgetful experimenter, Oksana, can only conduct 16 experiments that consist of 6 factors, A, B, C, D, E and F. She consulted a statistician, Ryan, for a suitable design but she has forgotten all the details he gave her except one aliasing pattern

$$I = ABCDE = BF = ACEF$$

~~$I = BCE = BCF = EF$~~

(a) Find the generator, defining relation and the resolution of the design.

(b) Is this design suitable for the requirement set

$$RS = \{A, B, C, D, E, F, AC, CD\}?$$

(c) Can you propose a better design suitable for the requirement set?

6. Suppose that the original experiment is based on a 2^{7-3}_{III} with generators

$$5 = 12, \quad 6 = 23, \quad 7 = 123 .$$

(a) Derek, the experimenter, is willing to perform another ~~eight~~ 16 experiments on top of the original experiments in order to estimate the effects **5**, **6**, **12**, and **23**. Suggest a suitable design for him.

(b) How do you construct an augmented design for the original experiment to de-alias all the main effects? What is the resolution of the augmented design?

7. In a single-wafer plasma etching process, the experimenters are interested to investigate 8 factors. Each of the factors have two levels. They considered a fractional factorial split-plot design, $2^{(4-1)+(4-2)}$.

   (a) How is a fractional factorial split-plot design different from a fractional factorial design?

   (b) i. Write down a suitable set of generators for the design to be used in this experiment.

   ii. What is the defining relation for your chosen design.

8. (a) Assume that there are p variables of interest. Write down the first-order model used in Response Surface Methodology.

   (b) The initial estimate of an optimum operating conditions for a system is usually far away from the actual optimum. The method of steepest ascent is used to move sequentially from the initial condition to the actual optimum. Describe the method of steepest ascent.

9. *(Bonus Question)* What is your favourite statistics course?

   A. STAT 430
   B. STAT 430
   C. STAT 430
   D. STAT 430
   E. None of the above.

IV Percentage Points of the F Distribution (*Continued*)

$F_{0.05,\nu_1,\nu_2}$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Degrees of Freedom for the Denominator (ν_2)

Degrees of Freedom for the Numerator (ν_1)