

1. (5 points) Three treatments are to be compared for a disease of fruit trees called fire blight: control, removal of affected branches and spraying with antibiotic. Each treatment is tried on 20 trees selected at random and the results are classified into one of three groups: trees that die in the year the disease is diagnosed, trees that die in the 2nd to 4th year after the disease is noticed, and trees that survive for more than 4 years. The data are:

	Treatment		
	Control	Pruning	Spring
1 Year	12	7	4
2 to 4 years	5	7	9
More than 4 years	3	6	7

Carry out a  $\chi^2$  test at significance level 0.05 to see if there is a relationship between treatment and survival time of the trees. As part of your answer you should complete the following tables:

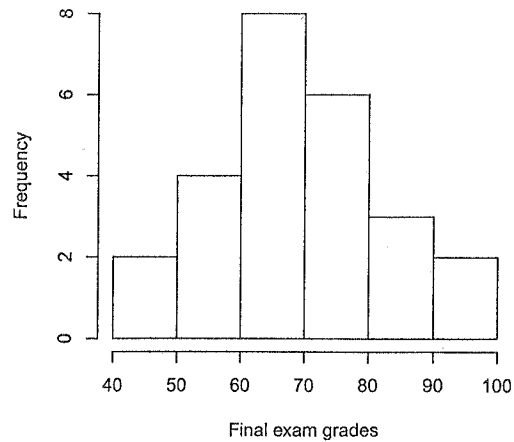
Expected Cell Counts

	Treatment		
	Control	Pruning	Spring
1 Year	7.667	7.667	7.667
2 to 4 years			
More than 4 years			

Components of  $\chi^2$ 

	Treatment		
	Control	Pruning	Spring
1 Year	2.449		1.754
2 to 4 years	0.571	0.000	
More than 4 years	1.021		

2. Looking at the final exam grades of a simple random sample of 25 students from a very large class, the average grade is 68.4 with a standard deviation 12.4. The histogram for the final exam grades is shown below.



- (a) (4 points) Construct a 95% confidence interval for the average final exam grade of the whole class. Be clear about any assumption you must make to do the problem.

- (b) (3 points) We want to use midterm grade to predict final exam grade. But we do not know the mean and standard deviation of the midterm grades of these 25 students. But from previous experience, we can assume the correlation between the midterm grades and finale exam grades is 0.6. If a student's midterm grade is about 2 standard deviations above the average, what would you predict is his final exam grade?

3. A study investigated the effect of two soporic drugs (increase in hours of sleep compared to control) on 10 patients. The following table reports the increase of sleep time (in hours) using the two drugs.

Patients	1	2	3	4	5	6	7	8	9	10	Mean	SD
Drug 1	0.7	-1.6	-0.2	-1.2	-0.1	3.4	3.7	0.8	0	2	0.75	1.79
Drug 2	1.9	0.8	1.1	0.1	-0.1	4.4	5.5	1.6	4.6	3.4	2.33	2.00
Drug 1 - Drug 2	-1.2	-2.4	-1.3	-1.3	0	-1	-1.8	-0.8	-4.6	-1.4	-1.58	1.23

Note that the sample means and sample standard deviations for these two samples were calculated. In addition, the mean and standard deviation for the differences in the increase of sleep time between drug 1 and drug 2 were also calculated. We can assume that the distributions of the increase of sleep time for the two drugs as well as the distribution of the differences are approximately Normal.

- (a) (5 points) Does the data suggest that these two drugs results in different average increase of sleep time at the significance level 0.05?

(b) (2 points) Will it be correct to use a two-sample  $t$  test on this data? Explain.

(c) (2 points) Can I interpret the  $P$ -value in (a) as the probability that the null hypothesis is true? Why?

4. In the past decades there have been intensive antismoking campaigns sponsored by both federal and private agencies. In one study of national smoking trends, two random samples of U.S. adults were selected in different years: The first sample, taken in 1995, involved 4276 adults, of which 1642 were smokers. The second sample, taken in 2010, involved 3908 adults, of which 1415 were smokers.
- (a) (4 points) Construct a 98% confidence interval for the difference in the proportions of all U.S. adults that smoked between 1995 and 2010.
- (b) (6 points) Does this data provide good evidence that the proportion of U.S. adults that smoke decline during the 15 year period, at the significance level 0.05.

5. (4 points) In an experiment, 7 rooms were carpeted and 7 were left uncarpeted. The rooms are similar in size and function. After a suitable period of time, the concentration of bacteria in the air was measured (in units of bacteria per cubic foot) in all of these rooms. We can assume that both the distributions of the bacteria concentration in carpeted and uncarpeted rooms are approximately Normally distributed. The data and summaries are provided:

	Bacteria concentration							Mean	SD
Carpeted rooms:	201	193	191	177	188	186	149	184	16.9
Uncarpeted rooms:	172	180	191	201	152	168	175	177	15.9

Construct a 96% confidence interval for the difference in the average bacteria concentration in air between carpeted rooms and uncarpeted rooms.

6. As a homework exercise each student in a class selected two separate simple random samples from two different populations. Each students used the sample to construct a 90%  $Z$  confidence interval for the population mean for each population.
- (a) (8 points) There are 200 students in the class. If the students work independently, what is the chance that between 155 and 170 get the two confidence intervals both cover the true value of the population means.



- (b) (4 points) One of the 90%  $Z$  confidence intervals for the first population mean is (95.1, 104.9). What would be a 95%  $Z$  confidence interval for the first population mean based on the same sample data?

7. (2 points) At a small college, all entering freshmen must take a foreign language class, chosen from the languages Spanish, French, Swahili, Chinese, and Arabic. The probability distribution for the language studied by a randomly selected freshman is summarized in the following table:

Language	French	Swahili	Chinese	Arabic
Probability	0.12	0.09	0.19	0.12

What is the probability that a freshman is studying Spanish or French?

8. (4 points) Poliomyelitis, also known as infantile paralysis, is an infectious viral disease that enters through the mouth and is usually spread by contaminated drinking water or food. Severe polio tends to be rare in communities with poor hygiene. The reason is that the virus is abundant in such communities, so babies are likely to be exposed to the virus early, while still protected with antibodies from their mothers. Later (assuming that they survive other diseases associated with poor hygiene), these children develop their own antibodies to the virus. To test for the effect of a polio vaccine, a trial was conducted. In this trial, parents of second-graders at selected schools in selected areas of the country were asked to consent to treatment with vaccine. The consenting parents tended to have higher income than the non-consenting parents. The children whose parents consented were given vaccine. The vaccinated second-graders would form the treatment group. The first and third-graders at the schools would not be given the vaccination, and would form the control group. Would you use this design to test for the effect of the vaccine? Explain the reasons you use or not use.

9. There are  $n = 100$  observations from a Normal population with known population standard deviation  $\sigma = 5$  with unknown population mean  $\mu$ . We will test  $H_0 : \mu = 0$  and  $H_a : \mu < 0$  with fixed significance level  $\alpha = 0.05$ . Find the test power, the probability of rejecting  $H_0$  when  $\mu = -2$ , by the following these steps.

(a) (2 points) The  $z$  test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 0}{5/\sqrt{100}} = 2\bar{x}.$$

What values of  $z$  lead to rejecting  $H_0$  at the 5% significance level?

(b) (1 point) Based on your result in (a), what values of  $\bar{x}$  lead to rejecting  $H_0$ ?

(c) (1 point) What is the sampling distribution of  $\bar{x}$  when  $\mu = -2$ ?

(d) (3 points) The power is the probability that you reject  $H_0$  when  $\mu = -2$ . What is the probability?

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## 676 TABLES

Table entry for  $z$  is the area under the standard Normal curve to the left of  $z$ .

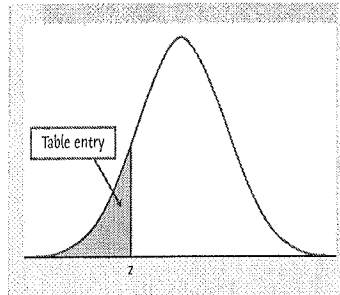


TABLE A Standard Normal cumulative proportions

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for  $z$  is the area under the standard Normal curve to the left of  $z$ .

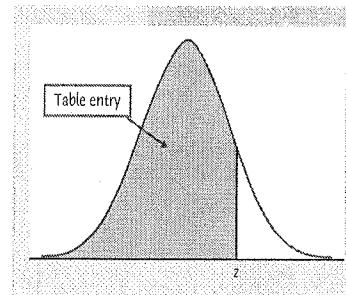


TABLE A Standard Normal cumulative proportions (continued)

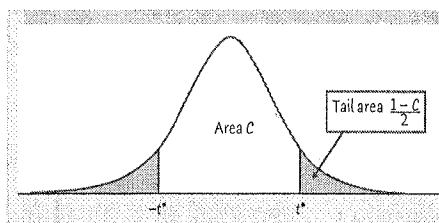
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Table entry for C is the critical value  $t^*$  required for confidence level C. To approximate one- and two-sided P-values, compare the value of the  $t$  statistic with the critical values of  $t^*$  that match the P-values given at the bottom of the table.

TABLE C  $t$  distribution critical values

DEGREES OF FREEDOM	CONFIDENCE LEVEL C											
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$z^*$	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
One-sided P	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
Two-sided P	.50	.40	.30	.20	.10	.05	.04	.02	.01	.005	.002	.001

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## 680 TABLES

Table entry for  $p$  is the critical value  $\chi^*$  with probability  $p$  lying to its right.

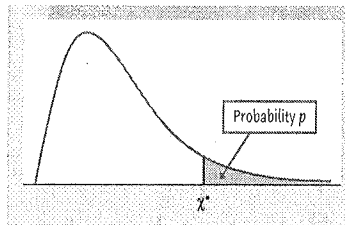


TABLE D Chi-square distribution critical values

df	$p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

