Student's Name: Course Name:Stat 380

SFU id number:

You are permitted one letter sized page of notes.

Calculators and electronic devices are not allowed

The exam is 3 hours long

Let X and Y be independent normal random variables each having parameters μ and σ^2. Show that X+Y is independent of X-Y.(6pts)

Show that if X_1, X_2,...X_n are iid bernoulli trials with parameter ß, then $Z=\sum_i X_i$ is a binomial rv with parameters n and ß. (3pts)

If X is exponentially distributed with mean 1/ß, find the distribution of X given that X>1. (4pts)

A Manuscript is sent to a typing firm consisting of typists A, B and C. If it is typed by A, then the number of errors made is a Poisson random variable with mean 2.6, if typed by B then the number of errors is a Poisson random variable with mean 3 and if typed by C then the number of errors is a Poisson random variable with mean 3.4. Let X denote the number of errors in the typed manuscript. Assume that typist A does the work with probability 1/2 and typists B and C do the work with probability 1/4 each.

Find E(X) (3pts)

Find Var(X) (3pts)

(6pts) There are 4 probability transition matrices for Markov chains. All states should be labelled starting at 0,1,...

Find the classes of the Markov chains.

Label each class, state or Markov chain (which ever is most appropriate) with as many of these labels as possible: Transient (T), Recurrent (R), Irreducible (I), Ergodic (E).

Find the period.

$$P_1 = \left\| \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 & 0 \end{matrix} \right\|$$

$$P_2 = \left\| \begin{matrix} 1/2 & 0 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \end{matrix} \right\|$$

$$P_3 = \left\| \begin{matrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{matrix} \right\|$$

On any given day Gary is either Cheerful (C), so-so (S) or glum (G). If he is cheerful today then he will be C, S or G tomorrow with respective probabilities 0.5,0.4 and 0.1. If he is feeling so-so today, then he will be C, S or G tomorrow with probabilities 0.3, 0.4, 0.3. If he is glum today, then he will be C, S or G tomorrow with probabilities 0.2, 0.3, 0.5.

a) If he is S on day 1, what is the probability that he will be S again on day 3? (6pts)

b) We don't know what mood Gary is in today. Find the expected number of days until he has been glum for three consecutive days. Assume that the limiting probabilities are πc=.1, πs=.3, πg=.6, (6pts)

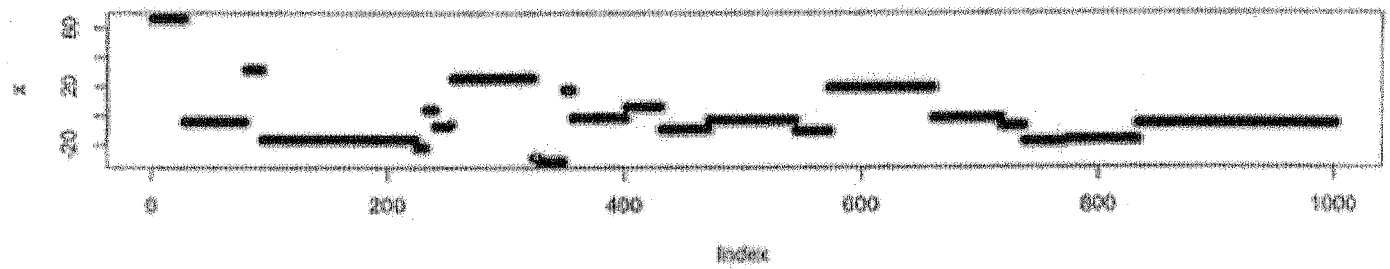
For a branching process

a) calculate π_0 when $P_0=1/4$ and $P_2=3/4$. (4pts)

b) What is the probability that the process will die out if $P_0=1/4$, $P_1=1/2$ and $P_2=1/4$? (4pts)

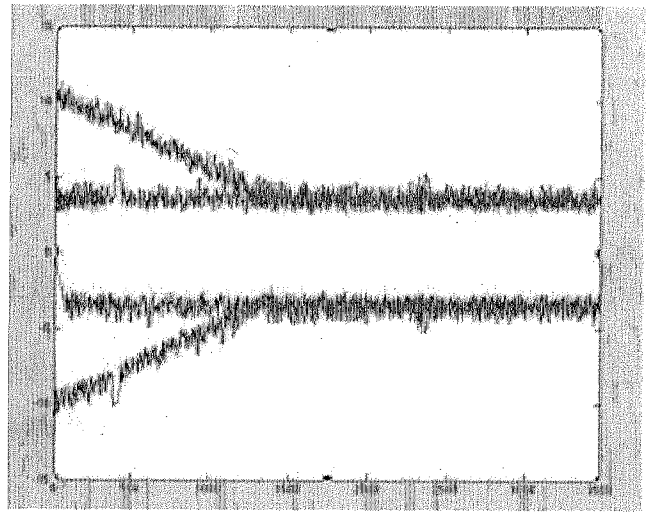
A taxi driver provides service in 2 zones of a city. Fares picked up in zone A will have destinations in zone A with probability 0.4 or in zone b with probability 0.6. Fares picked up in zone B will have destinations in zone A with probability 0.7 or in zone B with probability 0.3. The driver' receives a tip from fares picked up in zone A with probability 1 and the driver receives a tip from fares picked up from zone B with probability .4. The driver's first fare is picked up in zone A. What is the probability that the drivers 3rd fare results in a tip? (5pts)

You run a Metropolis Hastings Markov chain attempting to produce a sample from X_t. After 1000 sampled values you plot the progress and see the following.



What does this plot tell you? If there is a problem how do you fix it? (4pts)

You run another Markov chain from a few different starting points and plot the Markov chains on top of each other. You see the following:



What does this plot tell you? (3pts)

If you increase the number of samples from each of the 4 Markov Chains, how would this affect the Gelman-Rubin R statistic?(3pts)

If you increase the number of Markov chains from 4 to 10 what would be the effect on the Gelamn-Rubin R statistic? (3pts)

Another Markov Chain gives you the following result. You wish to compute the mean and 95% interval for X.

The Raftery-Lewis diagnostic information is given:

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Selection: 3

RAFTERY AND LEWIS CONVERGENCE DIAGNOSTIC
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Iterations used = 1:12961
Thinning interval = 1
Sample size per chain = 12961

$chain1

Quantile (q) = 0.025
Accuracy (r) = +/- 0.005
Probability (s) = 0.95

 Burn-in  Total Lower bound  Dependence
 (M)      (N)   (Nmin)       factor (I)
 114      12374 3746         3.3
```

a) If we ignore the burn-in and use all of the samples to compute the mean what could happen?(2pts)

b) How could we increase the dependence factor?(2pts)

c) Draw what the Autocorrelation function might look like for the above Markov Chain. Also draw how it might look if we increase the dependence factor. (4pts)

Along with the CODA output from the previous page, the software also gives us the following:

Iterations = 1:12961

Thinning Interval = 1

Number of chains = 1

Sample size per chain 12961

quantiles for each variable:

2.5%	25%	50%	75%	97.5%
0.0021	0.0024	.0032	.0054	.0075

Do we have enough samples to reasonably estimate the 95% interval? If not explain why not. If we can, then what is the interval estimate?(4pts)

In an experiment where N_v women were given a vaccine and $N_{no\ v}$ were not. Researchers were interested in knowing if the vaccine could prevent the rate of cervical cancer in women. For this problem we let:

b = the probability of getting cervical cancer,

(b|vaccine) = the probability of getting cancer with receiving the vaccine,

(b|no vaccine) = the probability of getting cancer without receiving the vaccine.

Y= the number of women who got cancer, and received the vaccine

X= the number of women who got cancer, and did not received the vaccine

We decide that $f(b \mid vaccine) = 2 - 2b$ when $0 \leq b \leq 1$.

And we wish to obtain $f(b \mid Y = y, N_v, vaccine)$.

a) In words what is $f(b \mid Y = y, N_v, vaccine)$? (2pts)

b) We are also interested in knowing about (b| vaccine) so we decide to begin with $f(b \mid vaccine) = f(b \mid X = x,\ no\ \ vaccine\ )$. What does this mean? (3pts)

c) Why is this a good idea? (3pts)

Consider a Brownian motion process $B(t), t \geq 0$. Compute the conditional distribution of $B(s)$ given that $B(t_1)=A$, $B(t_2)=A+C$, where $0<t_1<s<t_2$. (4pts)

Consider a Brownian motion process $B(t), t \geq 0$. What is the distribution of $B(s)+B(t)$, where $s \leq t$? (4pts)

If X and Y are discrete random variables, show that $E(XY^2) = E[\, X\, E(Y^2|X)\,]$ (4pts)

Customers arrive at an ice cream shop according to a Poisson process with rate ß and are served their ice cream with an exponentially distributed service time with rate Ω. There are two servers that can sell ice cream and having two servers with one customer does not improve the service time distribution. New customers will not wait in line for ice cream if there are already 3 customers in the store including the 2 being served. If ß=5 per hour and Ω=1/6 hours, what proportion of the time will the shop be empty? (5pts)

Every morning an individual leaves his house and goes for a run. He is equally likely to leave either from his front or back door. Upon leaving the house he chooses a pair of shoes (or goes running barefoot if there are no shoes at the door from which he departed). On his return he is equally likely to enter, and leave his shoes, either by the front or back door. If he owns a total of 4 pairs of shoes what proportion of the time does he run barefoot? (8pts)

Along with your answer provide the probability transition matrix and justify it's entries

A personal chef cooks meals for Gary every day that are either soup (s), toast (t) or rice (r). Let p_i be the probability that Gary enjoys meal i. and suppose that $p_s=.3$, $p_t=.6$ and probability $p_r=.9$. If Gary enjoys the meal, then the next meal is equally likely to be any of the three types. If Gary does not enjoy the meal then the chef will always make soup. What proportion of the meals are type i, where i=s,t,r? (5pts)

Prove the memoryless property of the exponential distribution (5pts)

Show that the acceptance probability for a Gibbs sampler equals 1. (5pts)