fixed number of elements (the value of the base), and then determining how many such sets can be obtained so that the total amount of units does not exceed the value represented. The representation is thus reduced to two pieces of information: a number of sets and a number of remaining units not grouped into sets. For instance, in the decimal numeration system, the word 'sixty-three' corresponds to the grouping 63 = 6×10 + 3, which means 'six sets of ten units and three remaining units' ('sixty' and 'three'). This system is called 'decimal' because its base is 10, but in a similar way one can find base-5 systems called 'quinary' or base-20 systems called 'vigesimal'. There exist around the world many systems with different values of the base—10, 5, 20 as previously mentioned, but also 4 or 8, for instance.

Numeration systems can also combine different bases. In such situations, the grouping is first processed with sets containing a number of elements equal to the greater base. The remaining units are then grouped into sets containing a number of elements equal to the next base in decreasing order, and so on. For instance, Zaslavsky noticed that 'counting based on five and twenty, called quinquavigesimal counting, is widespread throughout the world' (1973: 36). Such a system has a word for five; beyond this value, numbers are represented by adding 1, 2, 3, or 4 to combinations of 5, until the secondary base, 20, is reached. Thus twenty-eight is represented by 28 = 1×20 + 1×5 + 3.

Note that in counting systems where a unique base is defined, there are in fact many grouping sizes which are used, since units are grouped according to the value not only of the base, but also of its square, cube, and successive powers such as 10, 100, 1000 in the decimal system. For instance, the number word for 143 is defined by the decomposition 143 = 1×100 + 4×10 + 3.

From a linguistic point of view, the construction of number words according to these principles has been analysed by Salzmann (1950), who pointed out three main dimensions. The first is the frame, which consists of a class of elementary numerals (basic words existing in any given language for 1, 2, 3, 4, and so on). The second is the cycle defined by the periodic return of one or several basis terms in the sequence of their successive powers (such as 10, 100, 1000, and so on, in decimal systems) for grouping units to represent numbers. The third is formed by the rules applied to the other two components involving arithmetical operations in order to derive actual patterns of words in the given language, which can express any number used in that language.

p. 327 The written system of numbers used in western mathematics is called a 'place value system' because the place of a numeral together with its face value determine its meaning. For instance, in 128 the last digit has a value of 8 but the next one does not have a value of 2. Its value is 20 since its place value indicates that it must be multiplied by ten. Thus the digits 1, 2, and 8 in the written representation of 128 correspond to sets of different sizes 128 = 1×100 + 2×10 + 8 in a way that is similar to the grouping of units we have described previously in the oral representation of numbers. As Ascher noticed, there are some similarities between the concepts underlying such a place value system, 'and the cycles and arithmetic relationships that are seen in the number words of many cultures' (Ascher 1991: 23). The main difference is that the mathematical decimal system used for writing numbers in a symbolic form is strongly consistent because the principle of grouping units is applied in a very uniform way which simplifies and enables the progress of arithmetic and calculation, whereas oral systems generally admit exceptions to the rules, or additional rules based on different principles.

For example, in Africa the Yoruba system has an unusual feature because it relies upon subtraction to a very high degree in a way that is similar to some aspects of Roman numerals, as in IX, for instance, which means 10 − 1 = 9, or to the English reading of time when one says 'twenty (minutes) to three'. Zaslavsky, who studied the system of number words found among the Yoruba, wrote: 'One must be a mathematician to learn this complex system' (Zaslavsky 1973: 204). Some numbers are used as intermediate figures, which means that their successors are calculated as a quantity less than the next higher stage.