



## The Oxford Handbook of Linguistic Fieldwork

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### CHAPTER

## 14 Fieldwork in Ethnomathematics

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### Abstract

This article focuses on the subject of fieldwork research addressing mathematical concepts developed in elaborated traditional knowledge. Its goal is to give advice to fieldworkers from this particular point of view, and to draw their attention to methodological issues with respect to the completeness of data collection during fieldwork and the veracity of the interpretations and analyses subsequent researchers are able to undertake without visiting the field. The same holds for more recent books on a similar subject. One must distinguish a mathematical concept and its application. From an ethnomathematical point of view it is useful to make a few observations on the best way to record annotated new media while visiting the field, whether video or computer experiment, in order to make possible afterwards the exploration of their mathematical content. The study devoted to the question of completeness of data collection during fieldwork, a crucial point in ethnomathematics for checking the consistency of mathematical knowledge embedded in the data. The study tackles the question of vernacular lexicons used for numbers and measurement, and it will be seen that it only partly meet the general goals of an ethnomathematical approach. Furthermore, the article discusses the use of measurement terms. Finally, it addresses the question of mathematical operations on approximate quantities carried out in a society where there are no number words above five.

**Keywords:** [ethnomathematics](#), [mathematical content](#), [data collection](#), [vernacular lexicons](#), [ethnomathematical approach](#)

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## 14.1 Introduction<sup>1</sup>

This chapter will focus on the subject of fieldwork research addressing mathematical concepts developed in elaborated traditional knowledge. Its goal is to give advice to fieldworkers from this particular point of view, and to draw their attention to methodological issues with respect to the completeness of data collection during fieldwork and the veracity of the interpretations and analyses subsequent researchers are able to undertake without visiting the field. There is a wide range of activities which might evince mathematical structures such as games (e.g. cat's cradle), kinship structures, poetry, riddles, art and design, music. All belong to the intricate landscape that is ethnomathematics. Their full description involves mathematical concepts taken from various fields such as number theory, geometry, graph theory, algebra, or combinatorics on words. For instance, Stevens (1981) provides a rich collection of two-dimensional patterns from various parts of the world classified according to the crystallographic notions of symmetry. Such a book does not deal with fieldwork, and nothing is said about how to collect such patterns and how native artists conceive them. The same holds for more recent books on a similar subject (see e.g. Horne 2000). Even in works more related to the 'ethno' dimension of ethnomathematics, such as the website *Ethnomathematics in Australia* (Rudder n.d.), the methodology for fieldwork is not dealt with, since these studies are partly motivated by an educational concern about how to teach Western mathematics to Indigenous students. It appears that most of the work in ethnomathematics falls into one of these two categories. Either they are based on existing fieldwork data collection studied by subsequent researchers who have not visited the field, or they are made by fieldworkers working in the particular context of educational activity. As there exist only few studies in ethnomathematics that take account of fieldwork problems, this chapter is restricted to concrete examples, most of them encountered during our fieldwork conducted on the mathematics of divination in Madagascar. It is worth mentioning that many ethnomathematical topics are not covered. Thus the presentation of the Malagasy research takes up a large proportion of this chapter. This is not the place for primary presentation of such a practice and the reader is referred to other papers published elsewhere (Chemillier et al. 2007; Chemillier 2007; 2009). In this chapter I provide some basic definitions to explain the way mathematical concepts are extracted from fieldwork situations.

What is involved in doing mathematics? When playing the cat's cradle game, are the participants doing mathematics or merely engaging in an intricate cultural activity? When we layer western mathematical analyses on the playing of the game, is this 'doing mathematics'? None of these cases corresponds to what we will call 'doing mathematics'. In this chapter, I will refer to the definition of ethnomathematics given by Ascher and Ascher (1986: 125) as 'the study of mathematical ideas of nonliterate peoples'. When someone plays the cat's cradle game, he may only repeat known figures without developing mathematical ideas about them. In the same way, the use of western mathematical concepts to analyse traditional activities does not prove that these activities require mathematical ideas from people doing them (just as the use of symmetry groups in the study of crystal structures does not prove that minerals can have mathematical ideas!). The kind of ideas that we are interested in when doing ethnomathematical fieldwork can be characterized by the following features inspired by Pascal's definition of the 'mathematical mind' (Pascal 2008[1660]: 23). First of all, they are based on principles which are explicit and that one can see fully (e.g. the rules of the Malagasy divination system). Secondly, these principles are removed from ordinary use, and in some cases, there is a shift from reality to abstract speculation by introducing artefacts such as pebbles, notched sticks, knotted cords, seeds (as in the case of Malagasy divination), lines traced on the sand, strings plucked to play music. Note that these artefacts are of no mathematical importance as individual objects, because what is relevant here is their relationship at an abstract level. The third feature of mathematical ideas derived from these principles is that they proceed in a detailed, linear, systematic fashion following a deductive mode of reasoning. As we will see, these three features have direct consequences for fieldwork researchers, in

particular with ↵ respect to the completeness of their data recording (as Pascal noticed, principles underlying mathematical ideas must be ‘plain’ and ‘palpable’).

One must distinguish a mathematical concept and its application. Words for number and for measurement of time, weight, and distance can be involved in mathematical ideas, but they can also be applied to activities not mathematical *per se*. Selling goods at the market place using a weighing machine is business, but it is not mathematics. Words for number and measure are not necessarily organized in a systematic fashion because they are partly determined by practical constraints. For example, traditional measuring methods for short distances took advantage of the practical use of the human body. This gave birth to widespread units such as the foot and the inch (representing the width of a thumb as it is explicit in French, where *pouce* is the translation for both ‘inch’ and ‘thumb’), but these various units were not organized in a logical way. Number words can also have been altered by historical transformations. In French, integers 11–16 are named *onze*, *douze*, *treize*, *quatorze*, *quinze*, *seize*, sharing the same suffix *-ze*, whereas 17 is *dix-sept*, without this suffix. This difference is relevant when studying the evolution of the French language from a historical perspective, but it is not relevant from the point of view of mathematicians. In this chapter, we will describe basic mathematical constructions used for number words and measurement units, but we will also point out their practical and historical contingency, which is not relevant in the context of mathematical ideas.

The question of fieldwork in ethnomathematics raises another important issue concerning the relation between language and thought. A famous metaphor by Saussure quoted by Benveniste illustrates their intrinsic link: ‘Language can also be compared with a sheet of paper: thought is the front and the sound the back; one cannot cut the front without cutting the back at the same time; likewise in language, one can neither divide sound from thought nor thought from sound’ (Benveniste 1971: 45). Nevertheless, ethnomathematical fieldwork brings evidence of the fact that mathematical ideas are sometimes expressed without words and that gestures appear to be a fundamental feature in their development. While one may ask if this is not the case in the development of any ideas, the answer is that the possibility of expressing ideas by means of gestures seems to fit the particular features of what Pascal called the ‘mathematical mind’ (*esprit de géométrie*) as opposed to the ‘intuitive mind’ (*esprit de finesse*) (Pascal 2008[1660]: 23). For example, as we will see later in this chapter, it is possible to give a definition of the ‘evenness’ or the ‘oddness’ of an integer without saying a single word, by simply moving seeds on a mat. It is much more difficult to do so for holistic notions such as ‘beauty’ or ‘love’. In many cultures all over the world, especially in Africa, a precise system of gestures accompanies the use of number words. It seems obvious in such cases that the recording of language by linguists could efficiently make use of new technological approaches involving more than the simple recording of example words or sentences. This point is dealt with elsewhere in this volume ↵ (see above, Seyfeddinipur (Chapter 6) on gesture and Margetts and Margetts (Chapter 1) on recording), but as we will see, it is worth highlighting it with respect to ethnomathematics.

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From an ethnomathematical point of view it is useful to make a few observations on the best way to record annotated new media while visiting the field, whether video or computer experiment, in order to make possible afterwards the exploration of their mathematical content. Section two below will be devoted to the question of completeness of data collection during fieldwork, a crucial point in ethnomathematics for checking the consistency of mathematical knowledge embedded in the data. Sections three and four will tackle the question of vernacular lexicons used for numbers and measurement, and we will see that it only partly meet the general goals of an ethnomathematical approach. In section four I discuss the use of measurement terms. The fifth section addresses the question of mathematical operations on approximate quantities (addition, subtraction, comparison) carried out in a society where there are no number words above five. This section focuses on the basic numerical abilities of Mundurucu people from Amazonia, and illustrates the use of computer-based fieldwork experiments. The last three sections refer to our research on

Malagasy divination, which addresses a more complex traditional knowledge involving elaborated mathematical ideas. First of all, we show how gestures can be used by Indigenous experts as a means of explanation. This case study describes how a diviner explains the distinction between an even and an odd number of seeds. Secondly, we present an experimental task of mental calculation which shows that having a videorecording aids analysis because the transcription of an expert's comments while doing this task reveals afterwards the successive steps of his calculation. In the third example from Madagascar the diviner moves the seeds with his hand in order to execute a complex transformation of a tableau, illustrating a computational rather than explanatory use of gesture. In all these Malagasy examples, I will discuss the vocabulary used by diviners to show that language only makes up a small proportion of the knowledge underlying this complex traditional practice.

## 14.2 Completeness and Consistency of Data Collection During Fieldwork

p. 321 Some activities in traditional societies seem to rely on a complex set of formal procedures and precise rules that are not easy to describe when doing fieldwork. The development of the field of ethnomathematics has been motivated by the fact that sometimes these sets of rules or procedures appear to have properties of consistency and abstraction which make them close to what we call mathematical ideas, and actually they can be formalized in a mathematical framework. It is crucial to point out that the consistency of such sets of rules cannot be analysed properly if the description of the data lacks some particular element. Ethnologists have to work through every detail of the presentation of the procedures they are studying.

This question goes far beyond the attention to detail that characterizes the work of careful fieldworkers. There is a specific difficulty in the case of ethnomathematics due to the logical dependency linking the completeness of ethnographic data to the consistency of the mathematical knowledge underlying them. Let us take an example to illustrate this point. Traditions of sand drawing in different parts of the world have revealed an interest of native people in a mathematical concept named 'Eulerian path'. It means tracing a figure continuously and covering each line once and only once without lifting the finger from the surface. In order to check that a given path is Eulerian, one has to check that each line is covered 'once and only once' and thus to know exactly how each of them has been traced. As soon as this information is missing for one line, the data are incomplete and the whole mathematical structure becomes unreachable. Fortunately in the case of sand drawing, as we will recall below, we have extant records of these wonderful examples of cultural artefacts. It is not always the case, and one possible reason might be that many anthropologists in the past were limited in their understanding of mathematics. Today, fieldworkers having to record this kind of activity should be advised that the consistency of the described facts must be trusted even when the procedures seem complicated, not to say obscure, because these procedures are probably not at all inconsistent. This applies very well, for instance, to the complex tableaux of seeds used in Malagasy divination that we will study later in this chapter. They are so elaborated that Vérin and Rajaonarimanana (1991: 62) described ethnologists recording the workings of divination as being 'anxious'. As we will see, there exists behind the complexity of divination procedures a strongly consistent mode of thinking.

In his fieldworker's handbook for ethnologists, Mauss noted in 1947:

The most widespread game, reported worldwide, is the string game or cat's cradle. It is one of the most difficult games to describe. The fieldworker should learn how to make every figure so as to be able to reproduce the movements afterwards. It is preferable to use words and sketches to describe the string game since film blurs the figures. To make the sketch indicate the position of the string at each moment and also the direction in which it will be moved so as to pass from one position to the next. The written description will call on a precise vocabulary. (Mauss 2007[1947]: 72)

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The string figures of cat's cradle provide a good example of a traditional activity that leads to interesting mathematical problems. In fact how can we characterize the string figures that can be derived from a given one by a combination of simple  $\hookleftarrow$  operations? Some mathematicians have attempted to do so by using elaborated tools called 'knot polynomials' (Stewart 1997; Yamada et al. 1997). From a linguistic point of view, it is possible that in societies where this game is played one can find specific terms used by native people to designate the operations involved in cat's cradle and their combinations into subroutines. For instance, among the Inuit of Pelly Bay in Canada there exist words used for naming different steps of the realization of a figure. The final position is called *ayarauseq*, and various initial positions are called *pauriicoq* or *paurealik*. Even more complex combinations of gestures are named. The word *anitidlugo* has the meaning of a particular subroutine that consists of passing one loop through the other (Vandendriessche 2007: 47).

For such topics involving mathematical ideas in traditional activities that go beyond everyday expertise, it is important to focus on the consistency of the data, as noted earlier, because if they are altered in some way or even incomplete, the formalization of their mathematical content becomes impossible. First of all, fieldworkers have to record the data with the use of appropriate devices such as diagrammatic records. Fortunately, the history of ethnology provides examples of fieldwork conducted by accurate researchers who recorded their data in such a way that their mathematical study has been made possible even a long time afterwards. It is the case for the sand drawings recorded by Bernard Deacon in Vanuatu, as Ascher (1991: 64) notes:

In the 1920s, A. Bernard Deacon studied among the Malekula. With an eye and insight that were especially rare, he collected material that he believed demonstrated mathematical ability and evidence of abstract thought. One of the things he saw as mathematical [...] was 'the amazingly intricate and ingenious' geometrical-figure drawings. He was meticulous in recording about ninety figures, including their exact tracing path.

Let us have a look at the drawing recorded by Deacon reproduced in Fig. 14.1. One can easily recognize the form of a turtle represented on this picture, but there is additional information included in the figure which is very important from our mathematical point of view. All the lines involved in the tracing path have been numbered by Deacon from 1 to 103. Thanks to this crucial information one can have access not only to the form of the drawing but also to the gesture of the native people who produced it. This information allows us to study a particular kind of consistency of the tracing path, which is expressed by the concept of 'Eulerian path'. It appears that most of the sand drawings from Vanuatu are traced in this way. The interest of native people in 'Eulerian path' seems to be shared by different traditions of sand drawing all over the world, not only in Vanuatu but also, for instance, among the Tshokwe of Angola, and it has been studied by various ethnomathematicians (Ascher 1991; Gerdes 1995; 2006).

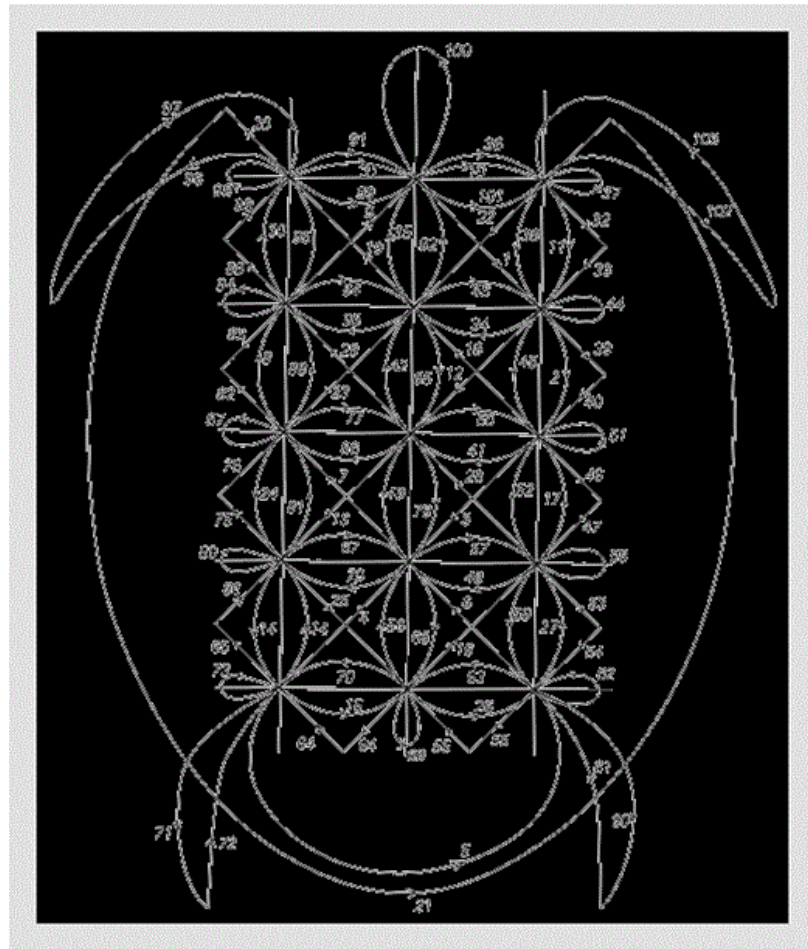
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In Vanuatu an indigenous word is directly related to the tracing path. A figure is called *suon* when the drawing ends at the point from which it began (Ascher  $\hookleftarrow$  1991: 45). The word *suon* may have a different meaning in another context, and we will encounter several examples of this type in this chapter. It must be stressed in such cases that the ethnomathematical context gives to the word a new meaning that can be fully understood only when one has a clear understanding of the underlying mathematical procedure. As pointed out by Ascher and Ascher (1986: 1  $\hookleftarrow$  26): 'If a word is adopted from an already existing word, it soon takes on a meaning appropriate to its new context. For example, when an English speaker says "a foot" in the context of measurement, no English hearer thinks he is thinking of a body part.'

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**Figure 14.1.**



Sand drawing from Vanuatu recorded by Deacon (1934). His numbering of the tracing path shows that each line is covered once and only once in a continuous way, except the jump that occurs between lines 31 and 32 near the head of the turtle.

The fact that Deacon recorded the details of the tracing paths is of crucial importance for the study of the mathematical knowledge embedded in sand drawing. Note that in the particular case of the turtle, the analysis of the sequence of numbers reveals an amazing fact (Chemillier 2002; 2007: 56). Strictly speaking the tracing path is not Eulerian. If you look at it carefully, you will find a discontinuity between lines 31 and 32 (near the head of the turtle). It seems not to be a flaw in the record, since various videos available on the Internet showing people from Vanuatu drawing the turtle always follow the tracing path recorded by Deacon. What is amazing in this case, from a mathematical point of view, is that the turtle could have been traced in a different way, by following Euler's conditions. The fact that it did not remains an unsolved ethnomathematical question.

As soon as a simple detail is missing in the recording of the ethnographic data, then the consistency of the underlying knowledge can be destroyed. In another example taken from our fieldwork on Malagasy divination, Fig. 14.2 shows the gesture of randomly placing seeds on the ground in the process of divination. The diviner takes two fistfuls of seeds from his bag in a random way, and then lumps the seeds into piles on the ground. Then he reduces each pile by deleting the seeds until either one or two are left, and places the remaining seeds in a tableau. On the right part of the picture, the diviner's hand is reducing one pile, and a second pile has not yet been reduced. On the left, some elements have already been placed—a complete column with four entries equal to two, one, two, two, and the beginning of a second column with entries two and one. At the end of the process the final tableau has four rows and four columns, the entries of which can only be one seed or two seeds.

**Figure 14.2.**



Seeds are set up on the ground in Malagasy divination (© 2000 Victor Randrianary). Each randomly chosen pile of seeds is reduced by deleting the seeds two at a time so that at the end only one or two remain.

p. 325 A fundamental detail is missing in the previous description. When the diviner deletes the seeds of the piles, he does it two at a time with his forefinger and middle finger. It is not very easy to see this on the fixed photograph reproduced in Fig. 14.2, but on the original video from which it has been taken the process is more visible. This detail is of great importance for the consistency of the procedure because it explains why the remaining seeds in each pile can only be one or two seeds. In fact it is a well-known principle of Euclidian division learnt at school that the remainder obtained by dividing an integer is always less than its divisor. This means that the number of possible values for the remainder cannot exceed the value of the divisor. It is precisely the reason why the process of picking up seeds always ends up with one or two seeds, because the diviner picks them up two at a time. Thus in this case the divisor is equal to two and the remainder can only take two values. As we will see later, Malagasy diviners are clearly aware of this mathematical rule, which determines the parity of the number of seeds contained in each fistful that are pulled out of the bag.

### 14.3 Number Words, Their Mathematical Construction, and Their Historical Contingency

It is well known that many kinds of numeration systems exist all over the world, which may vary considerably from a few number words to elaborate constructions in which counting extends into the millions. Some societies only have words for 'one', 'two', and a third word signifying 'many'. For example, the Bushmen of South Africa or the communities of the savannah and the tropical forest of South America use very rudimentary numerical systems. We shall discuss later the question of how to describe mathematical abilities in such societies by presenting fieldwork experiments for accessing this kind of ability, a question related to the current debate about Everett's (2005) claim that Pirahã has no number. For the moment, it must be pointed out that numeration systems are developed mainly in relation to practical constraint such as economic need, and that it is obvious in societies in which necessities of life are produced within the community that people do not have to rely on a complex system of counting.

p. 326 A simple way to represent numbers with sounds is to repeat a simple-sounding event as many times as the value of the number. Actually, this principle is used in the peals of a bell to indicate what time it is. As the value increases, the perception of the corresponding number becomes harder. Thus in languages dealing with numbers not reduced to one, two, and a few positive integer values, the numeration system must involve a grouping of units according to a fixed reference value called a 'base'. Crump noticed that 'the representation of numbers by the single repetition of a simple sound is not to be found in any spoken language' (Crump 1990: 33), highlighting the fact that the construction of number words is not reduced to the simple juxtaposition of units. In most languages, it involves the grouping of units into sets containing a

fixed number of elements (the value of the base), and then determining how many such sets can be obtained so that the total amount of units does not exceed the value represented. The representation is thus reduced to two pieces of information: a number of sets and a number of remaining units not grouped into sets. For instance, in the decimal numeration system, the word ‘sixty-three’ corresponds to the grouping  $63 = 6 \times 10 + 3$ , which means ‘six sets of ten units and three remaining units’ (‘sixty’ and ‘three’). This system is called ‘decimal’ because its base is 10, but in a similar way one can find base-5 systems called ‘quinary’ or base-20 systems called ‘vigesimal’. There exist around the world many systems with different values of the base—10, 5, 20 as previously mentioned, but also 4 or 8, for instance.

Numeration systems can also combine different bases. In such situations, the grouping is first processed with sets containing a number of elements equal to the greater base. The remaining units are then grouped into sets containing a number of elements equal to the next base in decreasing order, and so on. For instance, Zaslavsky noticed that ‘counting based on five and twenty, called quinquavigesimal counting, is widespread throughout the world’ (1973: 36). Such a system has a word for five; beyond this value, numbers are represented by adding 1, 2, 3, or 4 to combinations of 5, until the secondary base, 20, is reached. Thus twenty-eight is represented by  $28 = 1 \times 20 + 1 \times 5 + 3$ .

Note that in counting systems where a unique base is defined, there are in fact many grouping sizes which are used, since units are grouped according to the value not only of the base, but also of its square, cube, and successive powers such as 10, 100, 1000 in the decimal system. For instance, the number word for 143 is defined by the decomposition  $143 = 1 \times 100 + 4 \times 10 + 3$ .

From a linguistic point of view, the construction of number words according to these principles has been analysed by Salzmann (1950), who pointed out three main dimensions. The first is the frame, which consists of a class of elementary numerals (basic words existing in any given language for 1, 2, 3, 4, and so on). The second is the cycle defined by the periodic return of one or several basis terms in the sequence of their successive powers (such as 10, 100, 1000, and so on, in decimal systems) for grouping units to represent numbers. The third is formed by the rules applied to the other two components involving arithmetical operations in order to derive actual patterns of words in the given language, which can express any number used in that language.

p. 327 The written system of numbers used in western mathematics is called a ‘place value system’ because the place of a numeral together with its face value determine its meaning. For instance, in 128 the last digit has a value of 8 but the next one does not have a value of 2. Its value is 20 since its place value indicates that it must be multiplied by ten. Thus the digits 1, 2, and 8 in the written representation of 128 correspond to sets of different sizes  $128 = 1 \times 100 + 2 \times 10 + 8$  in a way that is similar to the grouping of units we have described previously in the oral representation of numbers. As Ascher noticed, there are some similarities between the concepts underlying such a place value system, ‘and the cycles and arithmetic relationships that are seen in the number words of many cultures’ (Ascher 1991: 23). The main difference is that the mathematical decimal system used for writing numbers in a symbolic form is strongly consistent because the principle of grouping units is applied in a very uniform way which simplifies and enables the progress of arithmetic and calculation, whereas oral systems generally admit exceptions to the rules, or additional rules based on different principles.

For example, in Africa the Yoruba system has an unusual feature because it relies upon subtraction to a very high degree in a way that is similar to some aspects of Roman numerals, as in IX, for instance, which means  $10 - 1 = 9$ , or to the English reading of time when one says ‘twenty (minutes) to three’. Zaslavsky, who studied the system of number words found among the Yoruba, wrote: ‘One must be a mathematician to learn this complex system’ (Zaslavsky 1973: 204). Some numbers are used as intermediate figures, which means that their successors are calculated as a quantity less than the next higher stage.



The combination of different formulae based on different arithmetic principles can sometimes alter seriously the consistency of the whole system. Lévi-Strauss observed:

As has been emphasized by certain authors, many systems defy all attempts at classification. They make up certain numbers by aggregation and change the formula according to whether the numbers are less than or equal to 10, between 10 and 20, or over 20. Some seemingly identical systems build up the numbers from 6 to 9 and those expressing tens, either by addition or subtraction. (1990[1968]: 336)

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French offers a good example of deviance from the base-10 system, which illustrates a change in the formula. For instance, the word for 87 in French is *quatre-vingt-sept* ('four-twenty' and 'seven'), clearly derived from a cyclic pattern with a base of 20. Furthermore, for any number between 80 and 99, the grouping is expressed as a multiple of 20. Thus there are two inconsistent systems that coexist in French: the base-10 decimal system and a base-20 system, probably originating from an older underlying Celtic system. As Crump pointed out: 'The deviant cases often represent lexical survivals' (1990: 36). In fact there is no mathematical reason why French uses a base-10 system for number 60 whereas it uses a base-20 system for number 80. It is not a matter of mathematics, it is a matter of history. There is still a generalization available: that the larger base is used for the larger numbers (and not vice versa). Nevertheless, there is no general and 'palpable' principle able to explain why 60 is not named *trois-vingt* ('three-twenty'); nor is the decimal naming *septante*, *octante*, and *nonante* used in France for 70, 80, 90 whereas it is in other French-speaking countries (such as Belgium).

## 14.4 Measurement of Time, Weight, and Distance in Relation to Practical Constraints

The measurement of time in traditional societies does not necessarily rely on number words. Frequently it is related to the principal activities of the day. Zaslavsky gives examples among the Ankole from Uganda where one talks about 'milking time' (6 a.m., *akasheshe*), 'resting time' (12 noon, *bari omubirago*), 'drawing water time' (1 p.m., *baaza ahamaziba*), 'drinking time' (2 p.m., *amazyo niganywa*) (Zaslavsky 1973: 260). When people in traditional societies reckon time they have to deal with the notion of succession, which implies the possibility of a systematic order based on uniform events. Such events can be of a social nature (natural life cycle between birth and death) or of a physical one (nights and days, seasons). As soon as they return, these events determine a periodic cycle, and its duration can be taken as a reference value for reckoning time. It must be stressed that this value needs not be of a numerical nature. Crump introduces a distinction between linguistic and arithmetical aspects of the measurement of time:

The linguistic system is concerned to name different, and possibly recurrent, points in time, whether these be days, years, or whatever, whereas the arithmetical system measures the lapse of time, again in different units, according to the end in view. The former is characteristic of 'traditional' time, where it is above all important to begin the harvest or observe a festival at the correct time. The latter is pre-eminently an institution of the modern world, in which the use of accurate clocks has enabled time to be equated with other numerical factors, such as distance or money. (Crump 1990: 83)

The events most suitable for the measurement of time are the position of the sun, the moon, the planets, and the stars. The periodic movement of these heavenly bodies in the cosmos is the basis for the reckoning of long time durations. Crump notes: 'This provides the starting point for a system of numeration based on the mathematical theory of congruences, which has been used for counting different units of time—from hours to units comprising several years—in many quite unrelated cultures' (1990: 84). It is not so easy to

combine in a consistent way the different periods of the movement of the planets and other heavenly bodies.

p. 329 For instance, as pointed out by Crump, observing the phases of the moon is simple but it is also 'misleading, for any calendar based on it inevitably gets out of step with the seasons' (p. 84). The duration from one full moon to the next appears to be about twenty-nine and a half days, and the solar year is not a multiple of it. Thus the months of the Gregorian calendar are not equal in length, and most of them except February are longer than a true lunar month. Ascher noticed that unlike these various non-congruent cycles, the week is different in kind, because it 'has no intrinsic relationship to any physical cycle; it is, instead, a completely arbitrary grouping of some number of days' (Ascher 2002: 40).

Following these observations, it is not surprising that in many cultures around the world one can find different ways of measuring the periods of time. In Africa, for instance, the Yoruba, Igbo, and Bini of southern Nigeria have a four-day week (Zaslavsky 1973: 64). Among the native people of America, '[t]he numeral type of calendar, in which the series of months or particular months were referred to by means of figures and not descriptive terms, used to be found along a continuous area of the Pacific coast, from the Aleutian Islands and adjacent lands to northern California; inland, the area included part of the River Columbia basin' (Lévi-Strauss 1990[1968]: 338). Some societies between southern Oregon and northern California have a calendar consisting of ten lunar months named after the fingers. In this particular part of America, the total year is the result of the addition of five winter months and five summer months (p. 337).

The fact that time measurement requires the combination of cycles with different periods explains why numerical representations of time generally rely on a multiple base counting system. In the western system of time measurement, 60 is the base for grouping seconds and minutes around the clock, but hours are grouped by 24 according to the duration of the day, and days are grouped by 7 according to the duration of the week. These different bases are combined so that we can perceive durations in an easier way. Let us take an example. Can you perceive the exact duration of 98,745 seconds? You will probably find it a bit difficult, but it should become easier if you represent it as the duration of 'one day three hours twenty-five minutes and forty-five seconds'.

The measurement of weight and distance in societies where there is a need for it makes use of various kinds of auxiliary instruments, such as the ruler for measuring length or the balance for measuring weight. As Crump pointed out, these tools are 'conceptual means by which two different entities can be compared in numerical terms'. For the class to which the measure is applicable, 'this implies that some abstract property is recognized as being common to all members of the class' (Crump 1990: 73). Furthermore, different abstract properties can be linked through the method used for measuring particular aspects of reality. For instance, the area under cultivation 'can be related to time, in terms of the labour input, just as the English acre was originally defined as the area which could be ploughed in a single day' (p. 74).

p. 330 Let us take an example illustrating a simple but ingenious tool for dividing a given length. In the Solomon Islands, there exist different types of musical wind instruments made of several bamboo tubes called 'panpipes'. Zemp has described the making of this kind of flute:

When cutting new panpipes, the instrument-maker measures octaves not only on two different-sized instruments, but also on pipes of the same instrument. He then either doubles the length of a pipe, or halves it, thus obtaining, respectively, the lower and higher octaves. The instrument-maker then blows simultaneously into the two pipes, to check the accuracy of the tuning by ear. (Zemp 1979: 13)

It is not difficult to double the length of a pipe because you just have to place it in two adjacent positions, but how to 'halve' it? In a film devoted to the making of panpipes, Zemp (1994) reveals the process: the instrument-maker takes a string to measure the length of the pipe, then he joins the two ends of this string and pulls the resulting loop. The length that he obtains is exactly half of the previous one.

Another fascinating activity that led people to measure distances on a much greater scale is the sea travel of navigators from the Central Caroline Islands of the north Pacific. Their ability is known to us thanks to the work of Gladwin (1970). He pointed out the amazing skill developed by each navigator

in judging the speed of his canoe under various conditions of wind, a skill sharpened by long experience, and his attention to the time which has passed as shown by the movement of sun and stars. Strictly speaking, it is not proper even to speak, as I did, of the number of miles the navigator has travelled. In our speech we find it natural to estimate (or measure) distance in arbitrary units. For a Puluwatan the estimate is relative. It is akin to a person walking across a familiar field in the dark. (Gladwin 1970: 184)

There exists a cognitive dimension in this practice that is not of a mathematical nature, since it relies on principles not fully explicit, thus not emanating from the ‘mathematical mind’.

## 14.5 Experimental Task for Accessing Non Verbal Numerical Knowledge

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It is important to point out that the use of language may involve the manipulation of numbers without explicit number words to designate them. In fact, as observed by Schmidl: ‘The absence of counting words by no means indicates a lack of counting concepts, since the number concept and the designation for the numbers need not always coincide’ (Schmidl 1915: 196, translated by Zaslavsky 1973: 14). Zaslavsky studied African counting systems and observed that standardized  $\downarrow$  gestures often accompany, or even replace, the number words, sometimes in relation to a taboo on counting living creatures (Zaslavsky 1973: 7). In the marketplace, for instance, the prices are indicated by moving the fingers. She recorded various systems of gestures by taking a series of photographs (published in Zaslavsky 1973: 243–5, 249–51). Among the Shambaa of northeast Tanzania, some numbers are named on the principle of two equal terms. The number six is named *mutandatu* as the result of adding two equal quantities  $6 = 3 + 3$ . To illustrate this, she gives a picture of a man showing the last three fingers of his two hands (p. 30). The same holds for eight, which is named *ne na ne* as the result of adding  $8 = 4 + 4$ . She remarks that during fieldwork, when one has to record properly the gesture involved in the representation of numbers, ‘only motion pictures can do justice to this gesture’ (p. 241). (See also Seyfeddinipur, Chapter 7 above.)

In many languages all over the world there exists evidence of a former use of gestures for expressing numbers in ancient counting systems. The name for ‘five’ often comes from the number of fingers on a person's hand. The corresponding word is often the same as ‘hand’, and in some languages where twenty is used as a base, the name for it is ‘man complete’, which means ‘ten fingers of both hands and ten toes of both feet’. Lévi-Strauss gives examples from Mexico and Central America where twenty is the ‘complete number’: ‘It was referred to by a word meaning “a body” in Yaqui, “a person” in Opata, “a man” in Maya-Quiché and also in Arawak, so that the practice extended also to the northern regions of South America’ (Lévi-Strauss 1990[1968]: 336). Note that conversely, the word for 5 can be used to designate the hand as in English, where a ‘high five’ means a hand gesture that occurs when two people simultaneously raise one hand and push the flat of their palm against the one of their partner. The same holds in Madagascar, where young people sometimes replace the word ‘hand’ by the word ‘five’ (*dimy*) in expressions such as ‘*Raiso ny dimy*’, which means ‘Let us beat one's hand’, that is to say ‘Agreed’.

Recent work on the subject of numerical abilities in various cultural contexts tends to prove that sophisticated competence can be present in the absence of a rich lexicon of number words. We will give examples that are interesting for our purpose because they illustrate the use of new technologies in fieldwork research, namely computerized experiments. It concerns numerical cognition in native speakers

of Mundurucu, a language of the Tupi family from the Para state of Brazil where one can find number words only for the numbers 1 through 5. The research team collected trials in classical arithmetical tasks on a computer, including a chronometric comparison test. With such technologies they were able to test whether competence for numerical operations such as addition or comparison are present in the absence of a well-developed language for numbers (Pica et al. 2004). Later the same team enriched their fieldwork by studying core knowledge of geometry (Dehaene et al. 2006).

p. 332 The first task consisted in presenting displays of 1 to 15 dots in randomized order, and asking the people in their native language to say how many dots were present. As can be expected due to the limitation of the numerical lexicon, there was little consistency in language use above five. For instance, a response to 13 dots was: 'all the fingers on the hands and then some more'. The word for 5, which can be translated as 'one hand' or 'a handful', was used for 5 but also 6, 7, 8, 9 dots. The authors conclude: 'With the exception of the words for 1 and 2, all numerals were used in relation to a range of approximate quantities rather than to a precise number' (Pica et al. 2004: 500). Furthermore, they remark: 'This response pattern is comparable to the use of round[ed] numbers in Western languages, for instance when we say "10 people" when there are actually 8 or 12.'

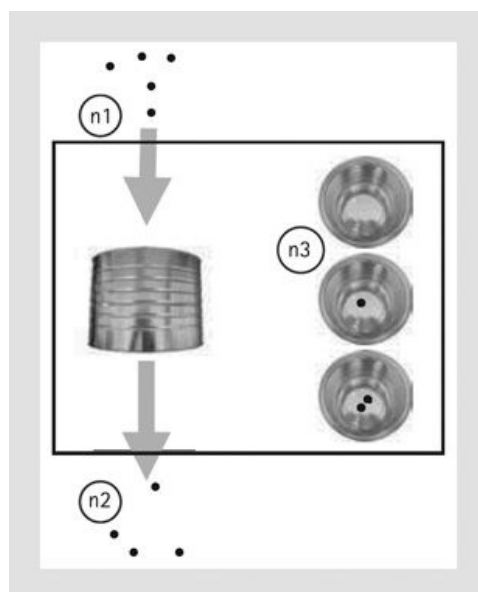
This by no means indicates that these peoples are unable to achieve arithmetic operations on approximate numbers. This was established by an approximate addition task, which is thought to be independent of language in western participants. Simple animations on the laptop screen illustrated a physical addition of two large sets of dots into a can. The participants had to approximate the result and compare it to a third set. Mundurucu participants had no difficulty in adding and comparing approximate numbers, with a precision identical to that of the French controls (i.e. a group of native speakers of French who were asked to do the same tests).

The question then arose of the ability of Mundurucu speakers to manipulate exact numbers. An exact subtraction task was proposed to the participants, who were asked to predict the outcome of a subtraction of a set of dots from a given set (see Fig. 14.3). The displayed animation showed a set of  $n_1$  dots entering the can and another set of  $n_2$  dots coming out of it. The question was: how many dots remain in the can? The initial number of dots  $n_1$ , could be up to 8 dots, but the result of the subtraction  $n_3 = n_1 - n_2$ , was always small enough to be named. Participants responded by pointing to the correct result among three alternatives  $n_3$ , (0, 1, or 2 objects left: see Fig. 14.3). It appeared that the Mundurucu were close to 100 per cent correct when the initial number was below 4, but their performance decreased sharply as the size of the initial number  $n_1$  increased.

p. 333 The authors conclude:  $\hookrightarrow$

Our results shed some light on the issue of the relation between language and arithmetic. They suggest that a basic distinction must be introduced between approximate and exact mental representations of number [...]. With approximate quantities, the Mundurucu do not behave qualitatively differently from the French controls. They can mentally represent very large numbers of up to 80 dots, far beyond their naming range, and do not confuse number with other variables such as size and density. They also spontaneously apply concepts of addition, subtraction and comparison to these approximate representations. (p. 502)

**Figure 14.3.**



Screen display of an exact subtraction task (Pica et al. 2004). The animated picture shows  $n_1$  dots entering the can, then  $n_2$  dots coming out of it. The subject is asked to choose on the right the correct number of dots  $n_3$  remaining in the can.

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## 14.6 Gestures as a Means of Explanation for Mathematical Concepts

As there exist numbers without number words, there is also mathematical knowledge without verbalization. We will give examples taken from our fieldwork on Malagasy divination illustrating such situations, and will describe techniques that can be used to access this kind of non-verbal knowledge. As we will see, the fact that this knowledge is not of a verbal nature does not mean that its lexicon is empty, but the meaning of the words of this lexicon cannot be accessed without an understanding of the related knowledge.

The figures used in Malagasy divination are based on one or two seed elements arranged by fours. As each of these four elements can only take two values (one or two seeds), their combination gives  $2 \times 2 \times 2 \times 2 = 16$  possible such figures. These are all displayed in Fig. 14.4 with their vernacular name, some of them derived from Arabic terms, since Malagasy divination has its origin in Arabic geomancy (e.g. *tareky* derives from *al-tariq* meaning 'the way, the pass'). Malagasy diviners group them according to a particular predefined classification. Eight figures are designated princes (*mpanjaka*) while the others are called slaves (*andevo*).

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This classification into princes and slaves is given Fig. 14.4. If you look carefully at them, you will find that each series shares a particular mathematical property. Before discussing this property, let us elaborate on the practical use of this classification.

**Figure 14.4.**

Princes ( <i>mpanjaka</i> )								Slaves ( <i>andevo</i> )							
<i>alokala</i>	<i>alohotsy</i>	<i>adalo</i>	<i>alasaady</i>	<i>adabara</i>	<i>tareky</i>	<i>asombola</i>	<i>alotsimay</i>	<i>aliksiy</i>	<i>alakarabo</i>	<i>alakaosy</i>	<i>reniliza</i>	<i>alibiavo</i>	<i>karija</i>	<i>alimizandà</i>	<i>alaimora</i>
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•

Malagasy divination figures based on one or two seed elements arranged by fours, with their vernacular name and their classification into princes and slaves.

In proceeding with the divination, particular figures are set up by randomly choosing piles of seeds, as we saw at the beginning of this chapter, and additional figures are computed by applying formal operations to be explained later. Thereafter, figures obtained in this way are read according to some basic rules related to their predefined classification. A principle for interpreting the randomly generated data given by the seeds is that princes are more powerful than slaves. In her seminal article of 1997 on *sikidy* divination, which is the starting point of our works on the subject, Ascher gives an example of a divination session related to illness that makes use of this relationship (Ascher 1997: 389). When the client appears to be a slave while the illness is a prince, the former is dominated by the latter so that the illness is considered to be serious.

The mathematical criterion that distinguishes princes from slaves among the figures is the evenness of their total number of seeds. It is easy to verify in Fig. 14.4 that princes can have four, six, or eight seeds, each of which is an even number, whereas slaves can have five or seven seeds, each of which is an odd number. During fieldwork it appeared to be difficult to establish such a relationship between the native classification into princes and slaves and the mathematical property of evenness. When asking questions such as: ‘What is particular concerning the number of seeds of the princes?’, the answer always was: ‘They can have four, six, or eight seeds’, with no mention of the particular property of evenness shared by these numbers. In fact, how do we express such an abstract concept as evenness? I have made many attempts to do so in the context of fieldwork, but did not succeed until the answer came up in an unexpected way, as we will see.

One day the diviner we were working with put all the princes on the ground and said, ‘I will show you something.’ The elements of the figures consist of one or two seeds and the diviner grouped the seeds by two in each prince containing isolated seeds. At the end all the princes were reduced to pairs, and he concluded that the result is *tsy ota*. Then he put all the slaves on the ground and grouped the seeds by two in the same way, but as the number of seeds was odd he removed one remaining seed in each of them. He said that the result was *ota*. The English translation for *ota* is ‘sin’ and *tsy ota* means ‘no sin’. The diviner’s comment suggests that the grouping procedure succeeded in the case of princes (no sin) because all the isolated seeds have been grouped by two, whereas it failed in the case of slaves.

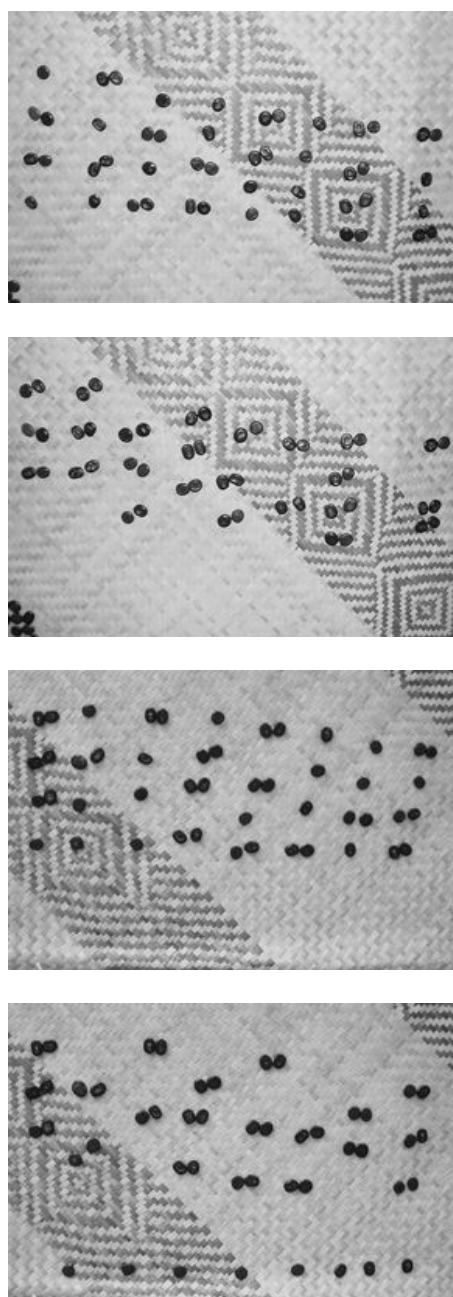
The diviner’s procedure may be considered as a mathematical definition of evenness. In this context the expressions *ota* and *tsy ota* can be translated into the English words ‘odd’ and ‘even’ respectively. It is interesting to note that there is a negative connotation associated with the numerical notion of oddness. The same holds in English, where ‘odd’ can mean bizarre and may also be used to indicate something not paired properly (‘an odd glove’). A similar remark can be made in French where the word *impair* (‘odd’) can be used as a noun with the meaning of a blunder.

It is obvious that in such cases the recording of the mathematical meaning of the terms involves more than the simple recording of example words or sentences. It relies to some degree upon the detailed recording of the corresponding gestures. In this example, the ethnographer is expected to have learnt about the use of the pair of fingers for grouping seeds and noted it by tracing a precise diagram in their notebook, or taking a



p. 336 picture of the seeds as shown in Fig. 14.5. Furthermore, as we have pointed out, it was not easy to ask for such an explanation. Actually, when the diviner gave us this explanation for the first time, we were not able to fully understand what he did with the seeds. In this case the video recording was clearly helpful, as we were able to play his gestures back again at night and ask him for the same explanation the next day. This is a simple example showing to some degree that having a video recording of the relevant activity aided analysis. In the next section, the example of a test situation better illustrates the use of video recording an experiment involving computer animation.

**Figure 14.5.**



Evenness (top) and oddness (bottom) of the number of seeds in each figure (© 2001 Annick Armani). Seeds can be grouped by pairs in (a) and (b), whereas in (c) and (d) one isolated seed remains in each figure.

## 14.7 Experimental Task Inducing the Verbalization of an Action

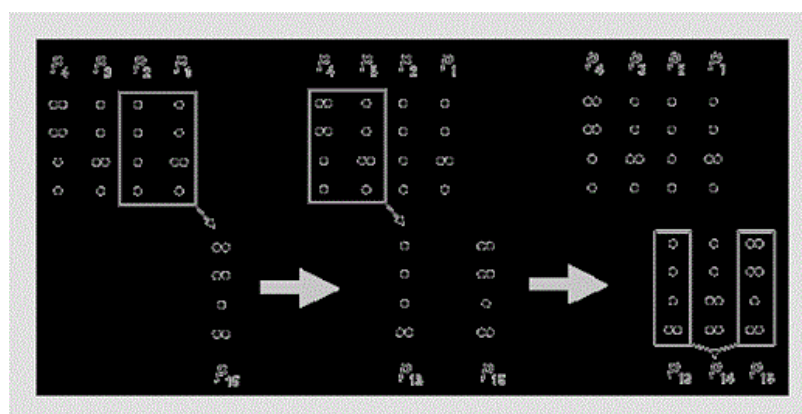
There are some simple techniques to induce the verbalization of an action. When people are placed in unusual conditions to carry out a task, they have a tendency to talk to themselves while achieving the successive steps of their task. A computer can be the appropriate tool for creating these particular conditions, and here I give an example related to the construction of tableaux of seeds used in divination.

As we have seen, the divination procedure begins with the placing of a tableau of seeds on the ground. The first part of the tableau, called the *mother-sikidy*, consists of four rows and four columns, the elements of which are single or double dots chosen randomly by taking piles of seeds and reducing their content two at a time, as we have described earlier. Then a second part is computed consisting of eight additional columns of four elements each called the ‘daughters’, placed below the previous ones. They are obtained as the addition of figures according to the rule that adding their one- or two-seed elements gives two when they are identical and one when they are different.

Successive generations of daughters are thus computed, the first ones deriving from the rows and columns of the *mother-sikidy* and the following ones deduced from the preceding. We will denote the eight daughters  $P_9$  to  $P_{16}$  according to the position where they are placed in the lower part of the tableau. Fig. 14.6 shows the first three steps of the process. Daughter  $P_{15}$  (named *safary*) is computed as the addition of *mother-sikidy* columns  $P_1$  and  $P_2$ . Then daughter  $P_{13}$  (named *asorità*) is computed as the addition of *mother-sikidy* columns  $P_3$  and  $P_4$ . The third step is the computation of second-generation daughter  $P_{14}$  (named *saily*), which derives from the two previous ones and is placed between them. In this example, the second-generation daughter  $P_{14}$  *saily* contains the figure one, one, two, two resulting from the two neighbouring first-generation daughters  $P_{13}$  *asorità* with figure one, one, one, two, and  $P_{15}$  *safary* with figure two, two, one, two.

p. 337 The whole process involves much more than the computation of first- and second-generation daughters  $P_{15}$ ,  $P_{13}$ ,  $P_{14}$  derived from the *mother-sikidy* columns. On the left part of the tableau, three more daughters  $P_{11}$ ,  $P_9$ ,  $P_{10}$  are derived from the *mother-sikidy* rows, which are read from right to left. Daughter  $P_{11}$  is derived from the first two rows, followed by  $P_9$  derived from the last two rows, and  $P_{10}$  between them results from the addition of both. Then a new-generation daughter  $P_{12}$  is placed at the middle (third generation) by adding  $P_{10}$  and  $P_{14}$ . At last, a fourth-generation one  $P_{16}$  is placed at the rightmost position by adding  $P_{12}$  and  $P_1$ .

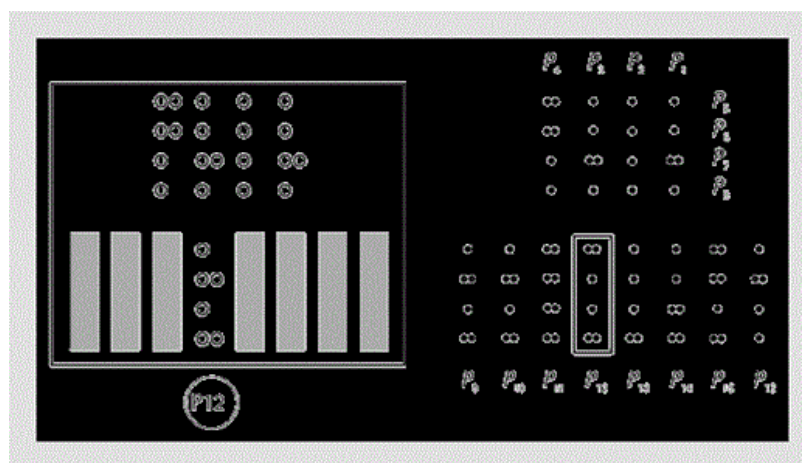
**Figure 14.6.**



First three steps of the computation of daughters denoted  $P_9$  to  $P_{16}$  according to their position. Daughters  $P_{15}$  (*safary*) and  $P_{13}$  (*asorità*) are first computed as the addition of *mother-sikidy* columns, then  $P_{14}$  (*saily*) is computed as the addition of its two neighbours.

During fieldwork we discovered that diviners are able to compute the whole series of eight daughters in a strictly mental way, that is to say by saying the name of the corresponding figures without laying out the seeds. In order to study this mathematical skill we designed a computerized experiment similar to the ones previously described among the Mundurucu of Amazonia. A few series of screen displays were prepared showing *sikidy* tableaux where all the daughters were hidden but one. The participant had to check if the visible daughter was correct according to the mother-*sikidy* displayed above it and to press a specific key to answer yes or no (Chemillier 2007; Chemillier et al. 2007). To achieve this unusual task, the diviner began to talk to himself by mentioning all the intermediate operations he was doing in his mind. Fig. 14.7 shows a screen display taken from this experiment with seven hidden figures among the eight daughters, and the corresponding full tableau (further examples are given in Chemillier 2009). The question was whether or not figure one, two, one, two was correct as the third-generation daughter  $P_{12}$ . The answer is no, since the correct figure is two, one, one, two as shown Fig. 14.8. In proceeding with the task the diviner first pronounced the following words: 'Alibiavo safary, alasady saily, karija asorità...'. According to the name of the figures (see Fig. 14.5) and the position in the tableau (see Fig. 14.6), this could be translated into: 'figure two, two, one, two at position  $P_{15}$ , figure one, one, two, two at position  $P_{14}$ , figure one, one, one, two at position  $P_{13}$ ...'.  
 p. 338

**Figure 14.7.**



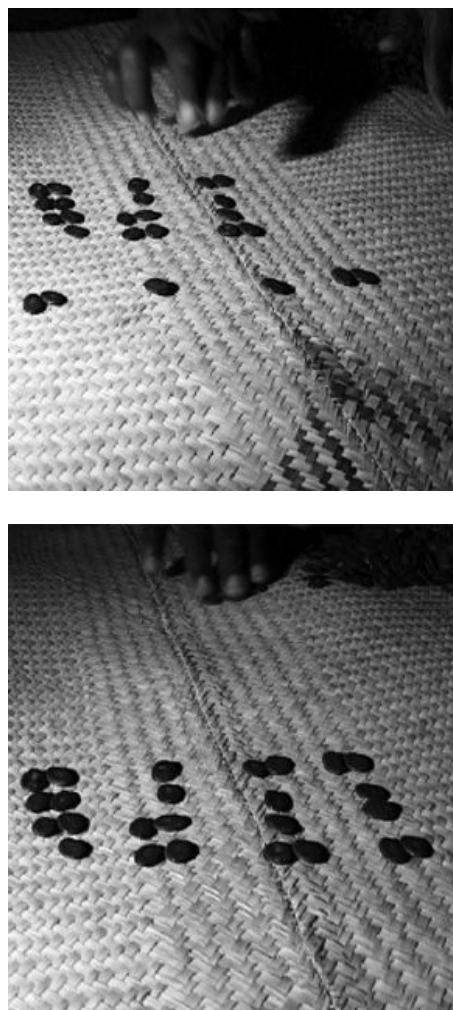
Screen display of an experimental task about the mental computation of daughters (on the left). The displayed daughter  $P_{12}$  (one, two, one, two) is wrong, since the third-generation daughter should be (two, one, one, two) as shown on the right.

The diviner's comment proves that in order to check the proposed daughter, he had to compute in his mind some daughters of the older generation as intermediate results. The amazing fact is that he mentioned a second-generation daughter (*saily*,  $P_{14}$ ) before the first-generation one from which it derives (*asorità*,  $P_{13}$ ). This is evidence of the fact that the second-generation daughter is computed by the diviner not as the addition of two first-generation daughters but directly from the mother-*sikidy*. Diviners are probably aware of a particular mathematical property. Since  $P_{15}$  and  $P_{13}$  are obtained by adding respectively the two rightmost and the two leftmost columns of the mother-*sikidy*, then  $P_{14}$  is obtained as the sum of the four mother-*sikidy* columns. Thus its four elements are determined by the parity of the mother-*sikidy* rows, which in this case are odd, odd, even, even (or slave, slave, prince, prince in the language of diviners), so that  $P_{14}$  contains figure one, one, two, two. The point that must be stressed is that when the diviners were asked about the way they compute the daughter columns, they always answered by referring to the formal definition of the daughters indicated in Fig. 14.6, that is to say  $P_{14}$  obtained by the addition of  $P_{13}$  and  $P_{15}$ , not to the actual computation they achieved in their  
 p. 339

mind. From a linguistic point of view, it appears that the simple recording of verbal explanations given by the native expert fails to access this kind of knowledge. We have designed various experimental tasks of this kind when at base between two fieldwork trips with the help of two cognitive psychologists involved in the project, Denis Jacquet and Marc Zabalía. Such experiments can easily be used and adapted in the field, since they are based on simple software environments (the one we used in this case was SuperLab<sup>2</sup>). As shown in this example, researchers involved in linguistic fieldwork dealing with ethnomathematical activities should be aware of the fact that new technologies are sometimes a necessary tool for understanding what native people think beyond what they say.

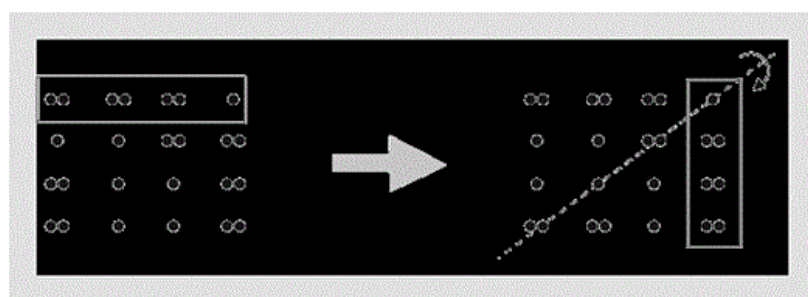
Figure 14.8.(a)





Videorecording of the successive steps of a transformation of the mother-*sikidy* by exchanging rows and columns (ꠘ Victor Randrianary).

**Figure 14.8.(b)**



As shown in the diagram above, the transformation is a reflection by the second diagonal starting from the top right.

## 14.8 Gestures Achieving a Complex Mathematical Transformation

In the quotation from Mauss at the beginning of this chapter, he advised the fieldworker not to use film in recording cat's cradle figures since 'film blurs the figures'. Despite Mauss's recommendation, we will give as the final case study of this chapter an example related to the tableaux of seeds where it appears that film (as video) can be a useful tool for recording the successive steps of a complex formal procedure.



p. 341 Fig. 14.8 shows a series of pictures taken from a video shot during fieldwork. If you look carefully, you can see a particular type of transformation applied to the seeds of the mother-*sikidy*. The upper row read from right to left (one, two, two, two) is displaced and oriented in the vertical direction. Then the second row (two, two, one, one) is transformed in the same way and placed as a new column beside the previous one. The same operation is applied to the other rows so that at the end of the process a new mother-*sikidy* is obtained in which rows and columns have been exchanged. The diagram displayed following the series of pictures in Fig. 14.8 summarizes the whole process. Notice that the first two rows are equal to the corresponding columns, since the mother-*sikidy* is partly symmetrical.

This operation creates a new matrix by inverting rows and columns of the initial one. Thus it is similar to the matrix transposition used in linear algebra, except that in mathematics the reflection is done by the main diagonal, which starts from the top left, whereas in *sikidy* divination it is done by the second diagonal, which starts from the top right. Obviously, the properties of a transpose in matrix algebra are not drawn in the context of divination since they are mainly related to the matrix product, which does not seem to be relevant in this context as far as we know. Recall that the transpose is used for defining an ‘orthogonal matrix’, which is a square matrix whose product with their transpose is equal to the identity matrix. Despite the fact that the function of matrix transposition is different in the context of divination, as we will explain, it is worth mentioning that there exists a close similarity between this transformation and the formal operation used by mathematicians.

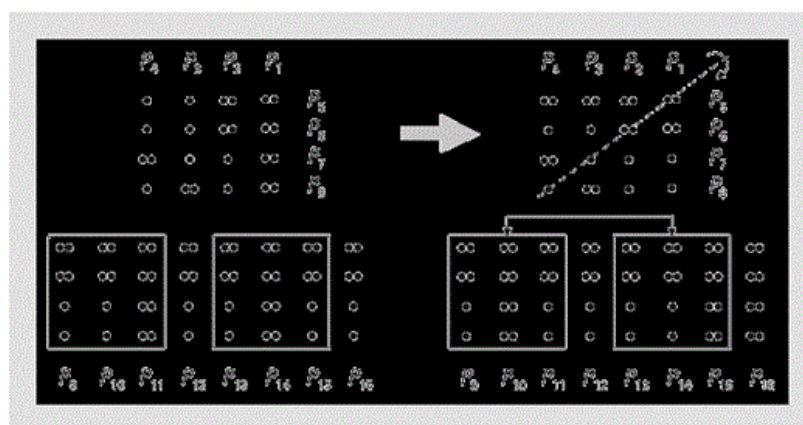
Malagasy diviners use a specific word *avaliky* to name this formal transformation in the south of the country. The corresponding verb is *mivaliky*, which means ‘to invert’. In official Malagasy it corresponds to the verb *mivadika* because in the dialect of the south words are often derived by replacing the letter ‘d’ by the letter ‘l’ (in fact the word for divination itself in the South is *sikily*). For instance, a sentence like ‘*Mivadika ny akanjoko*’ (‘inverted-the-shirt’) can be translated into ‘I put my shirt on the wrong way round’.

What is the function of matrix transposition in *sikidy* divination? The diviner's interest for this formal procedure is related to particular types of tableaux that are of great importance in their practice. One of them is called *fohatse*, and refers to tableaux where the same figure is repeated many times among both its mother-*sikidy* and daughters. Noël Gueunier, a linguist specializing in southwestern Malagasy dialects, noticed that *fohatse* is a variant of the word *vokatse*, which is used in the south with the meaning of ‘coming out of the earth’ (Chemillier et al. 2007: 28). Lanto Raonizanany, a member of our fieldwork team, observed that the corresponding word in official Malagasy is *vokatra* (with the usual suffix replacement leading to ‘tra’ instead of ‘tse’) and that its meaning is ‘harvest’ so that one can say, for instance, *Vokatra ny katsaka* (‘harvest-the-maize’) meaning ‘the harvest will give a lot of maize’. A possible explanation could be that the repetition of a ♄ figure in a tableau called *fohatse* is compared to an abundant harvest of maize ear (or whatever cereal it might be).

The relation between the repetition of a figure in a tableau and the exchange of rows and columns in its mother-*sikidy* relies on the following property. If a figure is repeated at least  $n$  times, then the same figure is repeated at least  $n - 1$  times in the new tableau obtained by transposing the mother-*sikidy*. Indeed, the matrix transposition preserves most of the daughters of the initial *sikidy* tableau. In Fig. 14.9 the daughters are the same in both tableaux except that some of them have been permuted. The three daughters on the right  $P_{13}, P_{14}, P_{15}$  have been exchanged with the three daughters on the left  $P_9, P_{10}, P_{11}$ ; the daughter  $P_{12}$  in the middle remains unchanged. The only daughter that can be changed in this process is  $P_{16}$ . The tableau *fohatse* on the left is called *adabarà sivy*, which means ‘nine occurrences of figure *adabarà* (two, two, one, one)’. One can easily verify that this figure is repeated nine times (namely at position  $P_2, P_5, P_6, P_9, P_{10}, P_{12}, P_{13}, P_{15}, P_{16}$ ), and that the tableau obtained on the right by transposing the mother-*sikidy* is also *fohatse* with eight occurrences of the same figure (namely at position  $P_1, P_2, P_3, P_9, P_{11}, P_{12}, P_{13}, P_{14}$ ).

Generally speaking, the *fohatse* property applies to tableaux where the repeated figure occurs at least seven times. It follows that tableaux with at least eight repetitions of the same figure always give a new *fohatse* when their mother-*sikidy* is transposed. Notice that in Fig. 14.9, the second column  $P_2$  is equal to the second row  $P_6$ . More generally, as soon as a figure is repeated many times among the positions of a tableau, it is obvious that the rows of the mother-*sikidy* tend to be equal to the corresponding columns, so that the mother-*sikidy* becomes partly symmetrical. In most cases, when the number of repetitions of a figure in a tableau takes the greatest possible value, the mother-*sikidy* becomes fully symmetrical, that is to say equal to its transpose. Diviners are aware of these mathematical results, and the function of matrix transposition is clearly to preserve formal properties such as the *fohatse* property, and others of the same kind that are considered as strongly powerful at the symbolic level.

Figure 14.9.



The matrix transposition applied to the mother-*sikidy* preserves the daughters by permuting some of them. The *fohatse* tableau on the left with nine repetitions of figure two, two, one, one remains *fohatse* on the right with eight repetitions.

## 14.9 Conclusion

To sum up the ideas that we have developed in this chapter, it appears that linguistic fieldwork dealing with ethnomathematics has to cope with two kinds of relation between language and mathematical ideas. On the one hand, words for number and measurement are partly based on mathematical constructions more or less related to the theory of congruence, as we have seen in numeration systems with different bases, or measurement systems with various units. These specific lexicons may reflect mathematical ideas from people using them, but their consistency is limited in some way by the conjunction of historical transformations and practical constraints which are not of a mathematical nature. On the other hand, the development of mathematical ideas does not necessarily require a rich lexicon to express them. It also appears that gesture can play an important role in the creation of mathematical ideas, be they geometrical, algebraic, or simply logical. The relatively small size of the set of words involved in this process may be related to the fact that, as opposed to the scientific tradition of literate societies, mathematical activities of non-literate peoples need not be transmitted in a systematic, normalized, and exhaustive way. We have observed during fieldwork on *sikidy* divination that the exploration of their mathematical system by Malagasy diviners was carried out in a quite solitary way. Moreover, part of the knowledge that they develop during their activity is kept secret, as it contributes to their prestige.

How, then, to record ethnomathematical knowledge when doing linguistic fieldwork? The fact that gesture appears to be an essential dimension in the expression of mathematical ideas does not mean that no words are used in this context. On the contrary, we have described in this chapter many examples of already

existing words taking on a new meaning when they were adapted to a mathematical context. A first lesson that can be drawn from our observations is that the vocabulary used in these activities must be recorded in close relation to the gestures that express the corresponding ideas with the help of accurate systems of notation. Activities of this type in traditional societies rely on precise procedures <sup>1</sup> and formal rules, which need to be recorded with extreme care if one wants to preserve their consistency and allow subsequent study from a mathematical point of view. It requires from fieldworkers an ability to make use of appropriate diagrammatic records in order to faithfully capture the whole thing (Deacon's notation of the tracing path of sand drawings is a reference example from this point of view).

The second lesson that can be drawn is that video can be a useful tool for recording the successive steps of formal transformations provided that the centring of the image is done properly and that the recording is made of the whole process from beginning to end. Obviously, the ethnographer may note the details of the procedure without a video recording, but the use of media is clearly helpful. Furthermore, as I have shown in this chapter, it happens that important gestures can escape notice, and in this case the ability to play back a video recording is essential to the ethnographer. We have encountered this situation when a diviner moved the seeds on the mat to explain the difference between an even and an odd number of seeds. The same holds for the gesture we have described about the matrix transposition of a mother-*sikidy*. One can even say, in this case, that the advantage of video recording applies to the second situation to a much higher degree, since in the first case, the diviner's purpose was clearly to point out something with his fingers so that the attention of the ethnographer was drawn to his gesture, whereas in the second, his purpose was to make the gesture for himself without any intention of communicating.

Finally, a third lesson can be drawn from this study with the aim of promoting computer experiments during fieldwork (see also Majid, Chapter 2 above). As we have seen in two different situations (basic numerical abilities of Mundurucu peoples and complex mental calculation of Malagasy diviners), specific tests using a computer can provide additional information on some aspects of the knowledge of native peoples that are not expressed as a verbal comment on their activity. Moreover, in the case of Malagasy diviners, we have seen that the test situation provides an efficient means for inducing the verbalization of an action. In such situations, the role of video recording is essential in assisting analysis, since the capture of an expert's comments needs to be synchronized with the corresponding screen display, so that the ethnographer can transcribe and analyse them afterwards in order to fully understand the underlying cognitive process.

## Notes

- 1 The author would like to acknowledge the reviewers for their rich comments which helped to improve this chapter.
- 2 <http://www.superlab.com/>