B. Engg. / HD CSE 2nd Semester (64)

23 October 2016

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION — SUMMER SEMESTER, 2015-2016

DURATION: 3 Hours — FULL MARKS: 150

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any **6 (six)** of them.

Figures in the right margin indicate marks.

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1. a) A band of 17 pirates stole a bag of gold coins. When they tried to divide the fortune into equal portions, 3 coins remained. In the ensuing brawl over who should get the extra coins, one pirate was killed. The wealth was redistributed, but this time an equal division left 10 coins. Again, another argument developed in which another pirate was killed. But now the total fortune was evenly distributed among the survivors. What is the least number of coins that could have been stolen? 10

   b) Show that if m is an integer greater than 1 and $a*c \equiv b*c(mod\ m)$, then $a \equiv b(mod\ m/gcd(c, m))$. 8

   c) Find 5^{2003} mod 13 and 23^{1002} mod 41. 7

2. a) Ten friends have several interests in common, as shown in table 1. 15

Table 1: Data Table for Question 2(a).

Basketball	Classic rock	Film	Painting	Political discussion	Wine tasting
Andrew Leah Megan Tanya	David Steve Whitney	Andrew Eric Megan Sarah	David Joe Sarah Whitney	Joe Leah Steve Whitney	Eric Megan Tanya

They decided to get together regularly to enjoy their interests. There is an activity center they can rent by the hour that has many rooms (and a gym), but they must rent the whole center, so they would like to schedule as many interest groups at the same time as possible. Of course, they cannot schedule two groups at the same time if there is someone who wants to participate in both groups. Suppose that each interest group wants to meet for two hours each week.

i. Draw the conflict graph for this situation.
ii. Color the vertices of the conflict graph according to the rules for graph coloring.
iii. How should the interest groups be scheduled? How many hours a week must the activity center be reserved in total?

   b) Define Dual graph and chromatic number. Find the chromatic number of the following graphs: 4+6

i. K_n
ii. W_n
iii. C_n
iv. Q_n

3. a) Define Proposition and Propositional Variable.
   b) If a sequence in the form of $s_n = a(2^n) + b*n + c$ is $\{s_1, s_2, \ldots\} = \{6, 11, 18, 29, 48, 83, 151, \ldots\}$ then what are the values of a, b, and c?
   c) Give a good big-O estimate for each of the followings:
      i. $(n \log n + 1)^2 + (\log n + 1)(n^2 + 1)$
      ii. $n^{2^n} + n^{n^2}$

4. a) Prove that the relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$
   b) Determine whether the following relations are equivalence relation:
      i. Let $a = (x1, y1)$, $b = (x2, y2)$ are ordered pairs of integers; $(a, b) \in R$ if $x1y2 = x2y1$
      ii. Let x, y be integers; $(x, y) \in R$ if $|x - y| = 2$,
      iii. Let R be the relation on the set of all URLs (or Web addresses) such that xRy if and only if the Web page at x is the same as the Web page at y.
   c) Define n-ary relation and transitive closure. What is the transitive closure for $\emptyset$?

5. a) Prove that the product of any three consecutive integers is divisible by 6.
   b) Consider two sets A and B, where the elements of A are the id of 2nd semester students and elements of set B contains age of all students. If function from set A to set B $f(a)=b$ indicates that age of student a is b.
   For each of the case below whether these functions are either one-to-one, onto or bijection. Justify your answer.
      i. More than one student having the same age.
      ii. Each student having unique age.
      iii. Set B contains age of the students of whole university, not only for 2nd year.
   Will this relation still be called a function if set A contains name of a student instead of id? Explain your answer.
   c) Using truth table find out whether this statement is tautology or not:
   $(p \wedge q) \rightarrow (p \rightarrow q)$.

6. Given a 7×7 matrix X whose i,j th entry is 1 if i+1 divides j+1 or j+1 divides i+1, $i \neq j$; whose i,j th entry is 2 if $i=j$; and whose i,j th entry is 0 otherwise.
   a) If X is an adjacency matrix representing graph G, then draw that graph.
   b) If X represents a relation, is it an equivalence relation?
   c) Many puzzles ask you to draw a picture in a continuous motion without lifting a pencil so that no part of the picture is retraced and to finish the drawing at the vertex where you started. To solve such puzzles we use Euler circuits and paths. In Figure 1 there is a graph. Your task is to determine whether the graph in Figure 1 have Euler Circuit? If yes, then show the steps to find it.

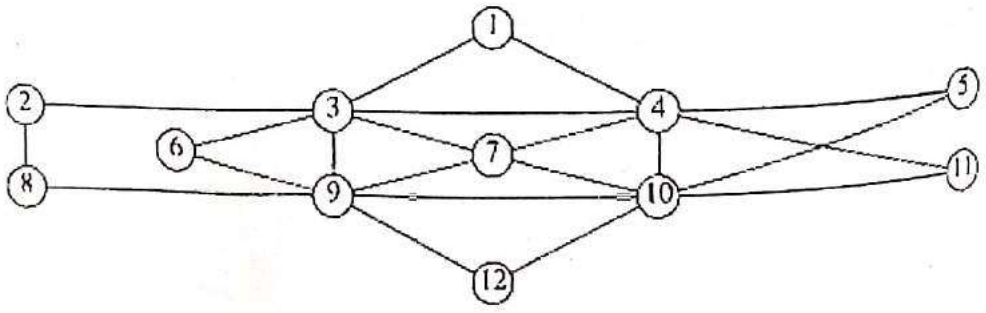

Figure 1: Graph for Question 6(c).

a) If a football game remains tied even after the extra time we go for tie breaker. This year, BFF has modified the rules a bit. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious. 12

How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

b) How many nonnegative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 < 40$ if we must satisfy $x_1 \geq 5$, $x_2 \geq 4$, $x_3 \geq 3$, $x_4 \geq 2$, $x_5 \geq 1$? 8

c) A key in the Vigenère cryptosystem is a string of English letters, where the case of the letters does not matter. How many different keys for this cryptosystem are there with three, four, five, or six letters? 5

a) Construct a Huffman code for the letters of the English alphabet where the frequencies of letters in typical English text are as shown in Table 2. 12

Table 2: English letter frequency

Letter	Frequency	Letter	Frequency
E	12.02	M	2.61
T	9.10	F	2.30
A	8.12	Y	2.11
O	7.68	W	2.09
I	7.31	G	2.03
N	6.95	P	1.82
S	6.28	B	1.49
R	6.02	V	1.11
H	5.92	K	0.69
D	4.32	X	0.17
L	3.98	Q	0.11
U	2.88	J	0.10
C	2.71	Z	0.07

b) Define ancestors. Show ***inorder traversals*** of the tree in Figure 2. 7

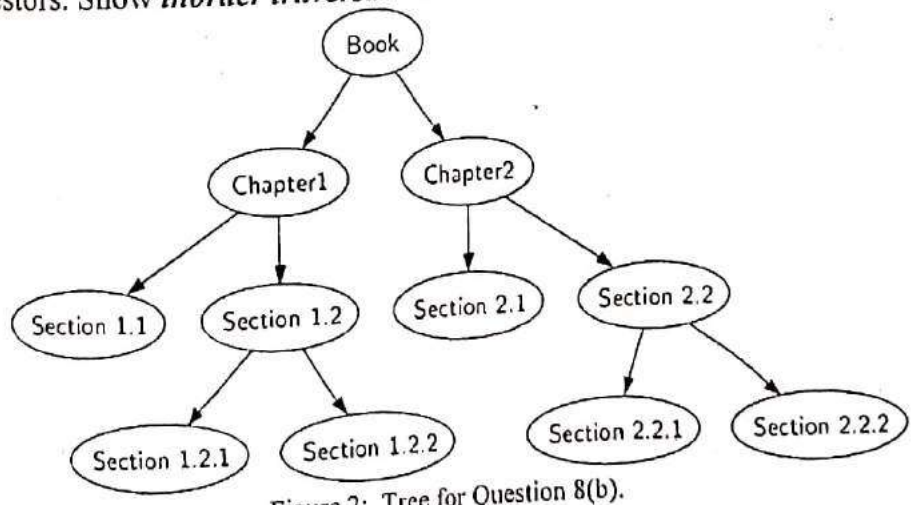

Figure 2: Tree for Question 8(b).

c) What is the value of each of the following expressions: 6

i. + − ↑ 3 21 ↑ 2 3 / 6 − 4 2 (Prefix)

ii. 3 2 * 2 ↑ 5 3 − 8 4 / * − (Postfix)