

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

WINTER FINAL EXAMINATION
Duration: 3 Hours

WINTER SEMESTER, 2015-2016
FULL MARKS: 200

Math-4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **8 (four)** questions. Answer any **6 (three)** of them.
Figures in the right margin indicate marks.

- Define power function, polynomial function and transcendental function with example. 10.33
- Graphs the functions: i. $y = \frac{1}{x}$, ii. $y = \sqrt{-x}$. Write the symmetries, if any. Specify the interval over which the function is increasing or decreasing. 10
- What is composite function? If $y = \frac{x}{x-2}$, find a function $y=g(x)$ so that $(f \circ g)(x) = x$. 13
- Explain the shifting and scaling formulas for a function with each an example. 10
- What do you mean by exponential function? State whether the following functions are exponential or not, if not, give your reason: 8.33
- i. $f(x) = x^{10}$ ii. $f(x) = 10^x$ iii. $f(x) = \pi^n$ iv. $f(x) = \pi^x$
- Suppose you are interested to invest 5000 dollars in an account that earns 9% interest compounded annually, how long will it take the account to reach 12500 dollars? First derive the formula and then solve the above problem considering the logarithm at base 10. 15
- Give the basic definition of limit of a function. Prove the limit statements for the followings: 13.33
- i. $\lim_{x \rightarrow 3} 3x - 7 = 2$
- ii. $\lim_{x \rightarrow 0} \sqrt{4-x} = 2$ 10
- Let $f(x) = \begin{cases} 3-x, & x < 2 \\ \frac{x}{2} + 1, & x > 2 \end{cases}$
- Does $\lim_{x \rightarrow 2} f(x)$ exist or not? If not, explain why. Also find $\lim_{x \rightarrow 4^+} f(x)$ and $\lim_{x \rightarrow 4^-} f(x)$. 10
- A function is defined as follows:
- $f(x) = \begin{cases} 1+x, & -4 \leq x < -1 \\ 4, & -1 \leq x \leq 0 \\ 1+x^2, & 0 < x \leq 4 \end{cases}$
- Examine the continuity and discontinuity of the function at $x=0$ and -1 . 10.33
- What is differentiation? Explain the derivative of a function. Prove that every finitely derivable function is always continuous. 13
- Given $f(x) = |x-5|$. Does $f'(5)$ exist or not? 10
- Using definition, find $F'(0)$, $F'(1)$, if $F(x) = \sqrt{2x+1}$. 18
- Find $\frac{dy}{dx}$ for the following functions:
- i. $y = u^4, u = \sqrt{x} + x - \frac{1}{x} + 10$
- ii. $x^4 + \sin y = x^3 y^2$
- iii. $y^2 = x^2 + \sin xy$

- b) Find the lines that are tangent and normal to the given circle $x^2 + y^2 = 25$ at the point $(3, -4)$ and sketch the graph all together.
6. a) If $xy + y^2 = 1$, find $\frac{d^2y}{dx^2}$ at the point $(0, -1)$.
- b) Water is flowing at the rate of $6 \text{ m}^3/\text{min}$ from a reservoir shaped like a hemispherical bowl of radius 13 meter. If the volume of hemispherical bowl with radius R and height y is given by $V = \frac{\pi}{3} y^2 (3R - y)$. At what rate is the water level is changing when the water is 8 meter deep?
- c) Define linearization and find the linearization of $f(x) = \cos x$ at $x = \pi/2$.
7. a) Define local extreme values and write down its necessary conditions. Find the extreme values (local and absolute) of the function $y = x - 4\sqrt{x}$.
- b) Write the statement of the Mean-Value theorem. Verify it for the function $f(x) = x + \frac{1}{x}$ on the interval $[1/2, 2]$.
- c) Identify the point of inflection and test the concavity of the curve $y = \frac{1}{4}x^4 - 2x^2 + 4$.
8. a) Define monotonic function and write the first derivative test for local maximum or minimum. Find the critical points of $y = x^3 + 3x^2 + 10$ and identify the intervals on which the function is increasing or decreasing. At what points, if any, does the function assume local maximum or minimum values?
- b) A rectangle has its base on x -axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?