ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION   SUMMER SEMESTER, 2018-2019
DURATION: 3 Hours   FULL MARKS: 150

Math 4441: Probability and Statistics

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **8 (eight)** questions. Answer any **6 (six)** of them.
Figures in the right margin indicate marks. The Symbols have their usual meaning.

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1. a) Test two integrated circuits one after the other. On each test, the possible outcomes are **a,**   10
        (accept) and r (reject). Assume that all circuits are acceptable with probability 0.95 and
        that the outcomes of successive tests are independent. Count the number of acceptable
        circuits X and count the number of successful tests Y before you observe the first reject. (If
        both tests are successful, let $y = 2$) Draw a tree diagram for the experiment and find the joint
        PMF $P_{X,Y}(x, y)$.

    b) Suppose that the joint probability density equation of X_1 and X_2 is given by   15

    $$f(x_1, x_2) = Cx_1^2 x_2 \quad for \quad x_1^2 \leq x_2 \leq 4$$
    $$= 0 \ , \quad\quad \text{Otherwise}$$

    i.   Final the value of C
    ii. Calculate $P(X_1 \geq X_2)$

2. a) Find the marginal distribution of X and Y from the following joint distribution function and   15
        verify that marginal distributions are also probability distribution.

    $$f(x, y) = \frac{1}{8}(6 - x - y), \quad 0 < x < 2, \ 2 < y < 4$$
    $$= 0 \ , \ \text{Otherwise}$$

    Also complete $P(X + Y < 3)$ and $P\left(X < \frac{3}{2}, Y < \frac{5}{2}\right)$.

    b) The average score of a sixth-grander in a certain school district on a math aptitude exam is   10
        72 with a standard deviation of 64. A random sample of 120 students in one school was
        taken. The mean score of these students was 70. Does this indicate that the students of this
        school are significantly slower in their mathematical ability? (Given that $Z_{.05} = \pm 1.645$).

3. a) The joint pdf of the random variable X and Y is   10
        $$f(x, y) = 2(2x + 3y - 4xy), \quad 0 < x < 1, \ 0 < y < 1$$
        $$= 0 \ , \quad\quad \text{Otherwise}$$

    (i) Find E(X+Y), E(XY)
    (ii) Also verify whether E(X+Y),= E(X)+E(Y).

    b) Define Degrees of Freedom, Type-I and Type-II errors, Test Statistics., Null hypothesis.   5

    c) The mean weekly sale of a certain brand of chocolate bar in candy stores was 156. 35 bars   10
        per store. After an advertising campaign, the mean weekly sale in 26 stores for a typical
        week increased to 163.75 and showed a standard deviation of 15.9. Was the advertising
        successful? (Use a 5 % level of significance). Necessary chart-1 is attached.

4. a) It has been observed that the joint distribution of a number X, $X \geq 1$ and it's fractional part   15
        Y which is the result of a computer operation involving a random generator is

$$f(x, y) = \frac{4}{5}\frac{x+y}{x^3}, \ 0 \leq y < 1, \quad 1 < x < \infty.$$

   (i) Find $P\left(0 < Y < \frac{1}{2} \Big| X = 2\right)$

   (ii) For what value of m is it true that $\left(0 < Y < \frac{1}{2} \Big| X > 2\right) = \frac{15}{16}$?

   (iii) Are X and Y independent?

   b) Define correlation and covariance. Prove that ρ assume the values between -1 and 1   10
       inclusive.

5. a) Let the joint p.d.f of X and Y be   15

$$f(x, y) = \frac{x^2 + y^2}{4\pi} \exp\left[-\left(\frac{x^2 + y^2}{2}\right)\right], \quad -\infty < x, \ y < \infty$$

   Show that X and Y are not independent but are uncorrelated.

   b) Police plan to enforce speed limits by using radar traps at four different locations within the   10
       city limits. The radar traps at each of the locations L1, L2, L3, and L4 will be operated 40%,
       30%, 20%, and 30% of the time. If a person who is speeding on her way to work has
       probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what
       is the probability that she will receive a speeding ticket?

6. a) Let the conditional PDF of X given Y is   8

$$f_{X|Y}\left(x|y\right) = \frac{f_{X,Y}\left(x, y\right)}{f_Y\left(y\right)} = \begin{cases} \dfrac{2}{(2-y)}, & 0 \leq y \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

   Find the conditional expected values (i) $E(X|Y = y)$ (ii) $E(X|Y)$.

   b) The television picture tubes of manufacturer A have a mean lifetime of 6.5 years and a   9
       standard deviation of 0.9 year, while those of manufacturer B have a mean lifetime of 6.0
       years and a standard deviation of 0.8 year. What is the probability that a random sample of
       36 tubes from manufacturer A will have a mean lifetime that is at least 1 year more than the
       mean lifetime of a sample of 49 tubes from manufacturer B?

   c) Suppose , X is a discrete Random Variable having pdf   8

$$f\left(x\right) = \frac{e^{-m} m^x}{x!}, \ x = 0, 1, 2...$$

   Find the Moment Generating Function of X and its mean and variance.

7. a) Two proofreaders were given the same manuscript to read. If proofreader 1 found n_1 errors,   10
        and proofreader 2 found n_2 errors, with $n_{1,2}$ of these errors being found by both
        proofreaders, **estimate N**, the total number of errors that are in the manuscript.

   b) A west coast publishing company keeps accurate records of its monthly expenditure for
       advertising and its total monthly sales. For the first ten months of 2012, the records showed
       the following (note that units are in US dollars):

Advertising (in thousands): 43 44 36 38 47 40 41 54 37 46

Sales (in millions): 74 76 60 68 79 70 71 94 65 78

(i)   Find the least-squares prediction equation appropriate for the data.   10
(ii) If the company plans to spend \$90,000 for advertising next month, what is its   5
      predicated sale? (Assume that all other factors can be neglected).

8. a) The joint pdf of the random variable X and Y is   12

$$f(x,y) = \begin{cases} 6xy(4 - 2x - y), 0 \le x \le 1, 0 \le y \le 1 \\ 0, \ else\ where \end{cases}$$

Determine (i) (i) $\sigma_{X,Y}$ (ii) $\rho_{X,Y}$
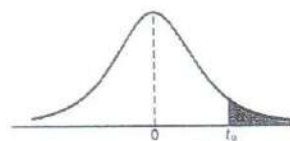
b) Calculate the two regression equations and the coefficients of correlation from the data   13
    given below:

    Age of husband   : 22   25   28   31   35   32   37   39   40   42
    Age of wife   : 19   18   22   29   31   23   30   33   34   35

Also estimate the most likely age of wife when husband's age is 36 and the age of husband
when wife's age is 24.

Table A.4 Critical Values of the *t*-Distribution

v	α						
	0.40	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Table A.4 (continued) Critical Values of the t-Distribution

v	0.02	0.015	0.01	α 0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290