ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2018-2019**

DURATION: 3 Hours **FULL MARKS: 150**

CSE 4403: Algorithms

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. **Question no *2,3 & 4* are mandatory to answer.**

Answer any **3 (three)** from the remaining.

Figures in the right margin indicate marks.

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1. a) Friday has many distinguishing features and virtues that Allah has bestowed upon this day and not others and so it is considered as the most important day of the whole week. There are a number of different Sunnah of Prophet Muhammad(saw) on this day. For example, on this day Muslims are encouraged to go to the mosque early on foot and sit near Imam. It's also said that whoever reads Surah Al-Kahf on Friday makes him illuminated with light between the two Fridays. Before going to the mosque, everyone should finish some activities like cutting nails, taking a bath and then go to the mosque wearing clean clothes. Applying perfume, doing miswaak are also important Sunnah before coming to the mosque. In addition, every Muslim should make a lot of Dua and read durood on Prophet Muhammad(saw) on this day. Based on the above-mentioned facts, find out the correct order of performing these Sunnahs and show the dependency of the activities (Sunnahs) using DAG in a linear fashion where all the directed edges go from left to right. 18

   b) Suppose a set of Strongly Connected Components (SCC) were found from a directed graph. Afterward, an edge was added into the graph and the procedure was performed again, but it resulted in no change in the set of SCC. What are the possible positions for the inclusion of that edge? 7

[Mandatory]

2. a) Showing proper argument, establish the fact whether Dijkstra's algorithm for solving the single-source shortest-paths problem on a weighted directed graph G = (*V, E*) follows Dynamic Programming or Greedy approach. 10

   b) Free medicines are being distributed in a camp where patients are sitting maintaining a line. The medical officer has examined their health status and given a rating based on the necessity of drugs. A patient with a higher rating means s/he is in the bad physical condition and requires more medicine. 15

   A nurse is distributing the medicines based on rating and she is instructed to give at least one strip of a tablet to each patient. If two patients are sitting next to each other, the one with the higher rating must get at least one more strip of medicines. Again, if two patients with equal rating sitting next to each other will get the same number of medicines. The nurse is given a goal to minimize the overall number of strips of medicine given to the patients.

   For example, if the ratings of the patients are (2,5,5) the distribution of medicines should be (1,2,2) that is 5 strips in total. Again, if the ratings are (4,6,6,10,10,1,4,2,1) the distribution should be (1,2,2,3,3,1,2,2,1) that is 17 strips in total.

   Analyze the scenario and characterize the structure of the optimal solution. Show whether this problem can be further divided into subproblems or not. Finally, propose an algorithm to find out the optimal solution in time-complexity $\theta(n)$, where n is the total number of patients.

[Mandatory]

3. a) Supposing that a data file contains a sequence of 3-bit characters such that all possible characters in the system are about equally common: the maximum character frequency is less than twice the minimum character frequency. 10
   Based on the facts mentioned above, justify the following claim, "Huffman coding, in this case, is efficient than using an ordinary 3-bit fixed-length code."

   b) Suppose an edge (u, v) exist in a directed graph. After applying Depth-first-search (DFS) on the graph; the discovery time and finishing time for every vertex are stored using the attributes d & f respectively. 15
   Match the two sides of the following table by showing appropriate explanations.

Edge (u, v) is:	
A back edge if and only if	$v.d \leq u.d < u.f \leq v.f$
A forward edge or tree edge if and only if	$v.d < v.f < u.d < u.f$
A cross edge if and only if	$u.d < v.d < v.f < u.f$

[Mandatory]

4. a) Long-time ago, there was a village named Lake-valley. The place consisted of $\boldsymbol{n}$ lakes represented using the set $S = \{L_1, L_2. \ldots .L_n\}$. Two types of fish lived in these lakes, but only one type in each lake. Both types of fish looked exactly the same, smelled exactly the same when cooked and had the exact same delicious taste – except one was poisonous and would turn any villager into flesh-craving zombies. This happened because the villagers angered a magician who cursed $\boldsymbol{k}$ lakes from those $\boldsymbol{n}$ lakes to have poisonous fish $(k \leq n)$. Unfortunately, the only known test to determine the condition of the lake is to serve some of the fish to a human and see whether they turn into zombies or not! Several brave volunteers offered to undergo such an experiment, but they are only willing to try their luck once. Each volunteer is willing to eat a single serving of fish that mixes together fishes from $S'(\subseteq S)$ lakes, which reveals at least one lake in S' has poisonous fish. Your goal is to use the fewest possible volunteers in order to determine, for each lake L_i, whether the fish in it is poisonous or not. 15
   Save the world by designing an algorithm to determine which $\boldsymbol{k}$ of the $\boldsymbol{n}$ lakes have poisonous fish using $O(klog_2(n))$ experiments.

   b) Show that Merge sort requires $\theta(nlog(n))$ auxiliary space to sort n numbers which can further be optimized to $\theta(n)$. 10

5. a) Design an algorithm that will find the longest-path from a source vertex to every other vertex in a directed weighted graph that contains no cycles. The parameters of the function will be the graph(G), source(s) and set of vertices(v). For every vertex, the algorithm should show the longest distance from the source vertex along with the vertices comprising that path. Additionally, show the unreachable nodes from the source vertex. 10

   b) Explain the difficulty faced in single-source shortest-path problem if Negative Weights are allowed for the edges. How does the Bellman-Ford algorithm deal with this issue? 8

   c) Suppose the vertices of a directed weighted graph are represented using a min-priority queue(Q) and the Bellman-Ford algorithm is applied to find the shortest path from the source vertex to every other vertex. Explain the impact this will bring on the overall complexity of the algorithm. 7

6. a) Find an optimal parenthesizing of a matrix-chain product whose sequence of dimensions is 15
   $(4, 10, 3, 12, 20, 7)$

   b) Let $X = (x_1, x_2, \ldots x_m)$ and $Y = (y_1, y_2, \ldots y_n)$ be sequences, and let $Z = (z_1, z_2, \ldots z_k)$ be any Longest common subsequence(LCS) of X and Y. 10
   Characterize the LCS and propose a recursive definition utilizing the optimal substructure of the problem.

7. a) Use the master method to give tight asymptotic bounds for the following recursions: 9

   i. $T(n) = 3T\left(\frac{n}{4}\right) + nlog(n)$

   ii. $T(n) = T\left(\frac{n}{2}\right) + 2^n$

   iii. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

   b) Assume that, instead of always selecting the first activity to finish, we select the last activity to start that is compatible with all previously selected activities. Describe how this approach is a greedy algorithm, and prove that it yields an optimal solution. 8

   c) Given an adjacency-list representation of a directed graph, how to compute the out-degrees and in-degrees of every vertex? What would be the case if the graph was represented using adjacency-matrix? What will be the time taken in both cases? 8

8. a) Build a Max-Heap from the following set of numbers {3,6,27,17,1,5,12,7,4,0,10,8,13,9} 10

   b) Show that, if a node in a Binary Search Tree has two children, then its successor has no left child and its predecessor has no right child. 8

   c) Prove the correctness of the Insertion-sort algorithm. 7