L8L

B.Sc. Engg. CSE 2nd Semester 01 November 2018

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

FINAL SEMESTER EXAMINATION SUMMER SEMESTER, 2017-2018

DURATION: 3 Hours FULL MARKS: 150

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (Eight)** questions. Answer any **6 (Six)** of them.

Figures in the right margin indicate marks.

1\. a) Suppose that a and b are integers where, $a \equiv 11 \ (mod\ 19)$ and $b \equiv 3 \ (mod\ 19)$, find the integer c with $0 \leq c < 19$ such that, 3×3

i. $c \equiv a^3 + b^3 \ (mod\ 19)$

ii. $c \equiv 2a + 3b \ (mod\ 19)$

iii. $c \equiv a^3 - 4b^3 \ (mod\ 19)$

b) Find the Octal and Hexadecimal expansion of $(345678)_{10}$. 2×4

c) Compute the summation and multiplication of the following numbers, 8

i. $(11001)_2$ and $(11011)_2$

ii. $(10001)_2$ and $(10101)_2$

2\. a) Mr. Luke Skywalker is a leader of the resistance against the empire. He is stranded on a distant planet and now needs help from the resistance for rescuing him. Luke wants to send the message "SABOTAGE EMPIRE" to his friend R2D2 far away in planet Nebula so that the Empire does not understand the message he is sending. Luke has with him his robot friend C-3PO who is able to encrypt and send the message. Your task is to help C-3PO encrypt the message using a hybrid encryption involving Caesar's Cipher first and then Transposition Cipher with the following details in mind, 10

i. $\sigma = \{1, 2, 3, 4\}$

ii. $\sigma(1) = 3, \sigma(2) = 1, \sigma(3) = 4, \sigma(4) = 2$

b) Prove that, $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n+1)^2$ is valid using mathematical induction. 9

c) The most commonly used procedure for generating *pseudorandom* numbers is the *linear congruential method*. We choose four integers: the modulus m, multiplier a, increment c, and seed x_0, with $2 \leq a < m, 0 \leq c < m$, and $0 \leq x_0 < m$. We generate a sequence of pseudorandom numbers $\{x_n\}$, with $0 \leq x_n < m$ for all n, by successively using the recursively defined function 6

$$x_{n+1} = (ax_n + c) \ mod\ m.$$

Based on the following information and considering $m = 9$, $a = 7$, $c = 4$, $x_0 = 3$, find the sequence of pseudorandom numbers generated by the *linear congruential method*.

3\. a) Give a recursive algorithm for finding the reversal of a bit string. Using that algorithm find the reverse of the following bit strings. Show step by step execution of your algorithm. $7 + 4 + 4$

i. 101101

ii. 101010

b) If x is a real number then prove that, $\lfloor 2x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{4} \right\rfloor$ 10

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4. Consider the following adjacency matrix of a town map

$$\begin{array}{c|ccccccc} \ddots & a & b & c & d & e & f & g \\ \hline a & 0 & 1 & 0 & 0 & 2 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 0 & 2 & 0 \\ c & 0 & 1 & 0 & 1 & 2 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ e & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ f & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ g & 0 & 0 & 0 & 2 & 0 & 0 & 0 \end{array}$$

From the above information answer the following:

a) Draw the graph that is represented by the adjacency matrix. 3

b) What is an Euler path? Does the above graph have an Euler path or circuit? Explain your answer logically. 2+5

c) If this graph has an Euler path then find an Euler path for travelling from town a to town e. 3

d) With the help of Dirac's and Ore's Theorem find whether the graph has a Hamilton circuit or a Hamilton path or both. 6

e) From the above graph prove that *"An undirected graph has an even number of vertices of odd degree."* 6

5. a) Consider the following adjacency matrix for a directed graph

$$\begin{array}{c|ccccc} \ddots & a & b & c & d & e \\ \hline a & 1 & 1 & 1 & 0 & 1 \\ b & 0 & 0 & 0 & 1 & 0 \\ c & 0 & 1 & 1 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 1 & 0 & 0 & 1 & 1 \end{array}$$

From the above information do the following:

i. Draw the graph represented by the matrix. 2

ii. Mathematically show that $\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$. 3

iii. Does this graph have a Hamilton path and a circuit? Justify your answer. If there is any Hamilton path and/or circuit then write down the Hamilton path and/or circuit. 5+2

b) Determine whether the pair of graphs in Figure 1 are isomorphic. 10

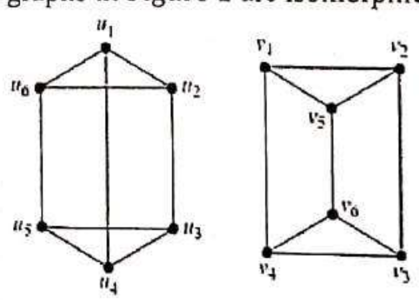

Figure 1: Graph for 5Question .(b)

c) Draw the following graphs Q_3, $K_{3,3}$, W_8, and determine their chromatic number χ. 3

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a) From Figure 2 generate the sequence of nodes in the following methods of tree traversal: 3×3

i. Pre-order traversal
ii. In-order traversal
iii. Post-order traversal

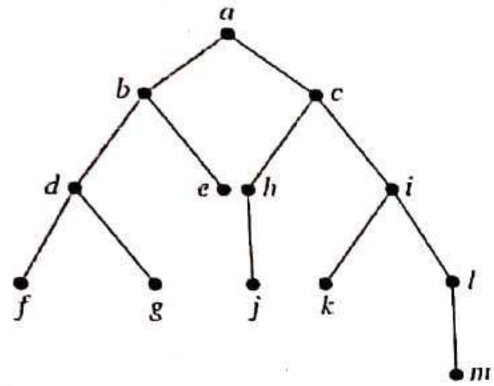

Figure 2: Graph for Question 6.(c)

b) From the expression $((x + y) \uparrow 2) + ((x - 4)/3)$ do the following: $3 + 9$

i. Draw the tree that represents this expression.
ii. Generate the prefix, infix and postfix notation for this expression.

c) Considering Figure 2, find out the following: $2 + 2$

i. Descendants of nodes b and i.
ii. Ancestors of nodes m and j

7. a) Using rules of inference show that the premises "*A student in this class has not read the book,*" and "*Everyone in this class passed the first exam*" imply the conclusion "*Someone who passed the first exam has not read the book.*" 7

b) Draw the combinatorial circuit for the following expressions: 2×3

i. $(p \vee (q \wedge \neg r)) \wedge (\neg q \vee \neg r)$
ii. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

c) Consider that f and g are the functions from the set of integers to the set of integers defined by $f(x) = \frac{2x+3}{x+2}$ and $g(x) = \frac{3x+2}{x-3}$.
Answer the following based on this information:

i. What is the composition of f and g? 2
ii. What is the composition of g and f? 2
iii. Determine whether $f(x)$ and $g(x)$ are one-one functions. 8

8. a) A father tells his two children, a boy and a girl, to play in their backyard without getting dirty. However, while playing, both children get mud on their foreheads. When the children stop playing, the father says "At least one of you has a muddy forehead," and then asks the children to answer "Yes" or "No" to the question: "Do you know whether you have a muddy forehead?" The father asks this question twice. What will the children answer each time this question is asked, assuming that a child can see whether his or her sibling has a muddy forehead, but cannot see his or her own forehead? Assume that both children are honest and that the children answer each question simultaneously. 7

b) Devise an algorithm for computing the quotient and remainder in a division operation. 10

c) Using the algorithm in 8.(b) find the quotient and remainder if divisor is 5 and dividend is -21. You have to show step by step operation of the algorithm. 8