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B.Sc. Engg. SWE 3rd Semester **04 March 2019 (Afternoon)**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2018-2019**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. **Question no.4 is Mandatory to answer.**

Answer any **2 (two)** from the remaining.

Figures in the right margin indicate marks.

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1. a) The matrices shown below represent a system of linear equations Ax=b:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} b = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

i. Write the equations representing the system and draw Column Picture for the equations. 3

ii. Solve this system of linear equation using Gaussian Elimination. 6

iii. Perform 'EA=U' and 'A=LU' factorization on A. 5

iv. Compare the two factorizations mentioned above and determine which one gives better insight about the elimination process. 2

v. Finally show the factorizations 'PA=LU' and 'PA=LDU' for matrix A. 4

b) Prove that, Inverse of A^T can be found by taking the Transpose of A^{-1}. 5

2. a) Invert these matrices A by the Gauss-Jordan method starting with [A I]: 10

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \ and\ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

b) Consider the matrices E and F. 9

$$E = \begin{bmatrix} 1 & 0 & -2 \\ 2 & -1 & 5 \\ 4 & 1 & 0 \end{bmatrix} \qquad F = \begin{bmatrix} 4 & 3 & 0 \\ 1 & 2 & 7 \\ 0 & 0 & 7 \end{bmatrix}$$

Multiply the matrices in the following ways to determine EF:

i. Linear combination of the Columns

ii. Linear combination of the Rows

iii. Columns of E times Rows of F.

c) Do the vectors lying on the line 2x+y=7 form a subspace? Justify your answer. 3

d) S = {x, y ϵ R: x>0, y>0 or x<0, y<0} Does this set of vectors form a valid subspace? Justify your answer. 3

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3\. a) Define 'Column Space' of a matrix. What is the significance this space? 5

b) i. If A is any 5 by 5 invertible matrix, then its column space is __. why? 8

ii. If the 9 by 12 system Ax = b is solvable for every b, then C (A) = __. Why?

iii. The column space of 2A equals the column space of A. (True or False? Why?)

iv. A square matrix will always have free variables. (True or False? Why?)

c) Suppose four matrices A,B, C and D are defined as $[\overrightarrow{V_1}]$, $[\overrightarrow{V_3}\,\overrightarrow{V_1}]$, $[\overrightarrow{V_1}\,\overrightarrow{V_2}\,\overrightarrow{V_3}]$, $[\overrightarrow{V_1}\,\overrightarrow{V_2}\,\overrightarrow{V_3}\,\overrightarrow{V_4}]$, where, 4×3

$$\overrightarrow{V_1} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \overrightarrow{V_2} = \begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix}, \overrightarrow{V_3} = \begin{bmatrix} 2 \\ 9 \\ 0 \end{bmatrix}, \overrightarrow{V_4} = 2\overrightarrow{V_1}$$

Answer the followings:

i. What is the column space of each matrices?

ii. Modify the matrix D so that B & D have the same column space.

iii. How many vectors will be in the Null Space of A, B, C and D?

[Mandatory]

4\. a) Consider the matrices below:

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 1 & 3 & 2 & 0 \\ -1 & 3 & -2 & -6 \end{bmatrix} x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Answer the followings:

i. Find out the Null Space of A. 8

ii. What is the rank of the matrix? What are the pivot variables and free variables? What is the largest possible rank of a matrix with same dimension as A? 3

iii. What is the condition on 'b' for Ax=b to have solution? 4

iv. What will be the solution of the system Ax= $\begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$. 3

v. Find out the complete solution of Ax= $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ 4

vi. What is the shape of N(A) and C(A)? 1.5

vii. What will be the dimension of each vectors in N(A) and C(A)? 1.5