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B.Sc. Engg. CSE 2nd Semester **23 October 2018**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2017-2018**

DURATION: 3 Hours **FULL MARKS: 200**

Math 4241: Integral Calculus and Differential Equations

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any **6 (six)** of them.

Give figure(s) where necessary. Figures in the right margin indicate marks.

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1. a) Write the fundamental theorem of Calculus. Find dy/dx of $y = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}$ i. by using Fundamental theorem, ii. by evaluating the integral and then differentiating the result. 5+16.33

   b) Find the total area between the region and the x-axis formed by the curve $y = 3x^2 - 3$, $-2 \le x \le 2$ 12

2. a) Evaluate the followings: 3×5

   i. $\int_{-4}^{4} |x-2|\, dx$, ii. $\int_1^{e^{\pi/4}} \frac{4}{x(1+\ln^2 x)}\, dx$, iii. $\int_{-1}^{-1/2} x^{-2} \sin^2(1+\frac{1}{x})\, dx$

   b) Find the area of the regions enclosed by the lines and the curves as follows: 9+9.33

   i. $y = 7 - x^2$ and $y = x^2 + 4$

   ii. $x - y^2 = 0$ and $x + 2y^2 = 3$

3. a) The solid lies between the planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the x-axis are circular disks whose diameter run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$. Find the volume of the solid. 18.33

   b) Find the volume of the solid generated by revolving the regions bounded by the given curve and the lines $y = 2\sqrt{x}$, $y = 2$, $x = 0$ about x-axis. 15

4. a) Define length of curve. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$. 5.33+7

   b) Find the surface area generated by revolving the curve $x = \left(\frac{1}{3}\right) y^{3/2} - y^{1/2}$, $1 \le y \le 3$, about y- axis. 10

   c) Find the lateral surface area of the cone generated by revolving the line segment $y = x/2$, $0 \le x \le 4$, about y-axis. Check your answer with the following formula: 7+4

   Lateral surface area = (1/2) × base circumference × slant height.

5. a) Using Trapezoidal and Simpson's rules with n = 4 and 8, find the approximate value of $\int_0^3 \sqrt{x+1}\, dx$. Finally compare your results with true value and comments on it. 10+5.33

   b) Define proper and improper integrals with examples. Evaluate the following integrals and then state whether they are convergent or not : 4+2×7

   i. $\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}}\, dx$, ii. $\int_0^{\ln 2} x^{-2} e^{-1/x}\, dx$

6. a) Define ordinary and partial differential equations with examples. Form an ordinary differential equation corresponding to the family of curves $y = k(x-k)^2$, where k is an arbitrary constant. Finally, identify it. 4+10+3

b) Define is exact differential equation and write its necessary condition. Test whether the following differential equations are exact or not. 4.33+4×3

i. $(2y \sin x \cos x + y^2 \sin x)dx + (\sin^2 x + 2y \cos x)dy = 0$, ii. $(y^2 + 2xy)dx - x^2\,dy = 0$

iii. $(4x + 3y^2)dx + 2xy\,dy = 0$, iv. $\left(\frac{x}{y^2} + x\right)dx + \left(\frac{x^2}{y^3} + y\right)dy = 0$

7. a) Determine the constant A such that the given equation is an exact differential equation (DE) and then solve it. 5.33+12

$$\left(\frac{Ay}{x^3} + \frac{y}{x^2}\right)dx + \left(\frac{1}{x^2} - \frac{1}{x}\right)dy = 0$$

b) Solve the following DEs: 2×8

i. $4xy\,dx + (x^2+1)dy = 0$, ii. $(x^2 + 3y^2)dx - 2xydy = 0$

8. a) What is first order linear differential equation? Explain with examples, when Bernoulli's DE becomes a first order linear DE. 4.33+2

b) Solve the following initial value problems: 3×9

i. $x\frac{dy}{dx} - 2y = 2x^4$, $y(2)=8$, ii. $\frac{dy}{dx} - y = \sin 2x$, $y(0)=0$ iii. $\frac{dy}{dx} + y = xy^3$, $y(0)=1$