ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING

Mid Semester Examination | Winter Semester: 2019-2020
COURSE NO.: Math-4541 | TIME : 1½ Hours
COURSE TITLE: Multivariable Calculus and Complex Variables | FULL MARKS: 75

There are 4 (Four) questions. Answer any 3 (Three) of them. Programmable calculators are not allowed. Do not write anything on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning.

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1. a) (i) Let z_1, z_2, z_3 represent vertices of an equilateral triangle. Prove th

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$

(ii) An airplane travels 150 miles southeast, 100 miles due west, 225 miles 30^0 north of east, and then 200 miles northeast. Determine by the concept of polar form of a complex number (a) analytically and (b) graphically how far and in what direction it is from its starting point.

b) (i) Find an equation using the complex number system for (a) a circle of radius 4 with center at (2, 1), (b) an ellipse with major axis of length 10 and foci at (3, 0) and (3, 0). **13**

(ii) State De Moivre's Theorem and using this theorem prove that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

2. a) Find each of the indicated roots and locate them graphically **12**

$$(i)\ (-1+i)^{1/3} \quad (ii)\ \left(-2\sqrt{3}-2i\right)^{1/4}$$

b) Solve the equation: **13**

$$(i)\ z^2 + (2i-3)z + 5 - i = 0 \quad (ii)\ z^5 = 1$$

3. a) Consider the transformation $w = \ln z$. **12**

Show that

(i) circles with center at the origin in the z plane are mapped into lines parallel to the v axis in the w plane.

(ii) lines or rays emanating from the origin in the z plane are mapped into lines parallel to the u axis in the w plane.

(iii) the z plane is mapped into a strip of width 2π in the w plane. Illustrate the results graphically.

b) (i) Suppose the principal branch of $\sin^{-1} z$ to be that one for which $\sin^{-1} 0 = 0$. **13**

Prove that $\sin^{-1} z = \frac{1}{i}\ln\left(iz + \sqrt{1-z^2}\right)$

(ii) Prove that $f(z) = z^2$ is uniformly continuous in the region $|z| < 1$

4. a) (i) Write necessary and sufficient conditions of $f(z) = u(x,y) + v(x,y)i$ be analytic in a region R. **12**

(ii) Prove that $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic and Find v such that $f(z) = u + iv$ is analytic.

b) Locate and name all the singularities of $f(z) = \dfrac{z^8 + z^4 + 2}{(z-1)^3(3z+2)^2}$

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