B.Sc. Engg. (CSE) 2nd Semester

30 August 2019 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION | **SUMMER SEMESTER, 2018-2019**

DURATION: 1 Hour 30 Minutes | **FULL MARKS: 75**

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them including **Question - 3**.

Figures in the right margin indicate marks.

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1. a) Consider the following conditional statements: 4+8

"If the file system is not locked, then new messages will be queued."
"If new messages are not queued, then they will be sent to the buffer."
"If the file system is not locked, then new messages will be sent to the buffer."
"It is not the case that if there are new messages, they will be sent to the buffer."

Based on these conditionals, answer the following questions.

i. Express the above statements using propositional logic.
ii. Rewrite each of these conditional statements without using the conditional.

b) When three professors are seated in a restaurant, 4

The ***hostess*** asks them, *"Does everyone want coffee?"*
The ***first*** professor says, *"I do not know."*
The ***second*** professor then says, *"I do not know."*
Finally, the ***third*** professor says, *"No, not everyone wants coffee."*

The hostess comes back and gives coffee to the professors who want it. Which professors did the hostess serve coffee to and how did she figure out who wanted coffee?

c) Suppose the domain consists of all tools and, 3×3

C(x): *"x is in the correct place."*
E(x): *"x is in excellent condition."*
T(x): *"x is a tool".*

Translate each of the following statements into logical expressions using predicates, quantifiers and logical connectives:

i. Something is not in the correct place.
ii. All tools are in the correct place and are in excellent condition.
iii. Nothing is in the correct place and is in excellent condition.

2. a) Find out if the following expressions are logically equivalent without using truth table: 2×2

i. $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$
ii. $(p \rightarrow q) \rightarrow (q \wedge \neg r)$ and $(p \vee (q \wedge \neg r)) \wedge (\neg q \vee \neg r)$

b) Consider the following two statements: 4×2

a. Every student in the class, has got at least two A+ grades.
b. There is a student in this class who has less than 50% attendance.

Express the above statements in terms of predicates and quantifiers. Based on their type of quantification, prove the validation of the following two laws:

i. $\neg\exists x Q(x) \equiv \forall x \neg Q(x)$
ii. $\neg\forall x P(x) \equiv \exists x \neg P(x)$

c) Consider the following two statements: 4×2

i. Some teachers in CSE department have published more than one journal.

ii. Every teacher in CSE department has published at least two articles or one conference paper.

Express the statements using predicates and quantifiers when ,

1. The domain consists of the teachers of CSE department
2. The domain consists of all the teachers of the university.

d) Give a proof by contradiction of the theorem, *"If 3n+2 is odd, then n is odd."* 5

3. **[Mandatory]**

a) Consider these statements, 12

"All hummingbirds are richly colored."
"No large birds live on honey."
"Birds that do not live on honey are dull in color."
"Hummingbirds are small."

Where,

P(x): *"x is a hummingbird."*
Q(x): *"x is large."*
R(x): *"x lives on honey."*
S(x): *"x is richly colored."*

Answer the following questions:

i. Express the statements logically.

ii. Using rules of inference logically prove that the last statement is a valid conclusion when the first three are premises.

b) Prove that if $n = ab$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$. 5

c) Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Explain your reasoning. 8

i. Either Kevin or Heather, or both, are chatting.

ii. Either Randy or Vijay, but not both, are chatting.

iii. If Abby is chatting, so is Randy.

iv. If Heather is chatting, then so are Abby and Kevin.

4. a) Determine if the following compound propositions are satisfiable: 3×2

i. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

ii. $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$

b) The English alphabet consists of 26 letters. There are 5 vowels and the rest are consonants. Among these 26 letters, 13 of them are odd and 13 are even. For instance, $(a,c,e,...)$ are odd letters and $(b,d,f,...)$ are even letters. Consider the set of lowercase English alphabets as the universal set U, where elements are arranged in alphabetical order from a to z. Consider five sets A,B,C,D and E where $A \subset U$, $B \subset U, C \subset U, D \subset U$ and $E \subset U$. The sets A,B,C,D and E are as follows:

$A = \{x_i | x_i \in U \; and \; \boldsymbol{x_i} \; is \; a \; vowel\}$
$B = \{x_i | x_i \in U \; and \; \boldsymbol{x_i} \; is \; an \; odd \; perfect \; square\}$
$C = \{x_i | x_i \in U \; and \; \boldsymbol{i} \; is \; a \; multiple \; of \; 3\}$
$D = \{x_i | x_i \in U \; and \; \boldsymbol{i} \; is \; a \; perfect \; square\}$
$E = \{x_i | x_i \in U \; and \; \boldsymbol{i} \; is \; a \; multiple \; of \; 7\}$

Based on the above scenario, answer the following questions:

i. What are the elements of sets *A, B, C, D, E*? 5

ii. Find $P(E)$. Show that, $|P(E)| = 2^{|E|}$. 3

iii. Find out the bit-string representation of the sets A,B,C, D and E. 5

iv. Calculate the following and find out the elements of the resultant set, 6
$(A \cap B), (C \cap D), (B \backslash A), (D \backslash B)$.