

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION
DURATION: 1 Hour 30 Minutes

SUMMER SEMESTER, 2015-2016

FULL MARKS:100

Math 4241: Integral Calculus and Differential Equation

Programmable calculators are not allowed. Do not write anything on the question paper.

Give the figure or figures where necessary.

There are 4 (four) questions. Answer any 3 (three) of them.

Figures in the right margin indicate marks.

1. a) Define integration, integrand, and integral. Explain with examples, why it is necessary to add the integral constant to the result in indefinite integral. 10
- b) Suppose that x months from now, the population $P(x)$ of Gazipur City will be changing at the rate of $(500+0.15x)^{3/2}$ people per month. If the current population is 995000 then what will be the populations after 5 years from now? 8.33
- c) Evaluate the following: 15
 - i. $\int x \sin^{-1} x \, dx$
 - ii. $\int e^{-3x} \cos 4x \, dx$
 - iii. $\int \sin^3 x \cos^3 x \, dx$
2. a) Evaluate the following using the method of trigonometric substitutions: 12
 - i. $\int \frac{1}{a \sin x + b \cos x} \, dx$
 - ii. $\int \frac{\sin x}{\sqrt{1+\sin x}} \, dx$
- b) Obtain the reduction formula for $\int \cos^n x \, dx$, and then evaluate $\int \cos^5 x \, dx$. 12
- c) Evaluate $\int \frac{2x^2-1}{(4x-1)(x^2+1)} \, dx$ by using partial fractions. 9.33
3. a) Find the area that is above the x -axis but below the curve $f(x) = 4 - x^2$ and the left side bounded by the lines $x=0$ and the right side by the line $x=1$ with $n=5$ equal subintervals using left hand, right hand and mid-point approximations of integration. 20
 - i. Make a sketch of the region
 - ii. Find the approximate area in each case
 - iii. Find the exact area
 - iv. Finally, find the percentage of error in each case and then write your comments on your results.
- b) Write the statement of fundamental theorem of definite integral. Evaluate $\int_0^4 f(x) \, dx$, 13.33

where $f(x) = \begin{cases} \sqrt{x}, & 0 \leq x < 1 \\ \frac{1}{x^2}, & x \geq 1 \end{cases}$
4. a) Evaluate the following: 15
 - i. $\int_1^{\sqrt{2}} \frac{dx}{x^2 \sqrt{4-x^2}}$
 - ii. $\int_0^1 \sin^{-1} x \, dx$
 - iii. $\int_1^3 \frac{1}{\sqrt{x}(x+1)} \, dx$
- b) State the Mean-Value theorem and verify it for the function $f(x) = \sin x$ in the interval $\pi \leq x \leq \pi$. 10
- c) Evaluate and sketch the region whose area is represented by the integral $\int_{-1}^5 |x-3| \, dx$ and then verify it using appropriate formula from geometry. 8.33