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Solution to 1st QUIZ SUMMER SEMESTER, 2019-2020

CSE 4403: Algorithms

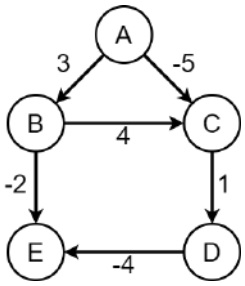
1. Consider that you have a directed graph $G = (V, E)$, where $|V| = X$. For a source vertex $s \in V$, there exists a path from s to all $v \in V$. If we run DFS from s, what is the number of tree edges in the graph?
   $X - 1$

2. An N-Progressive Graph is a directed graph with N vertices, $v_1, v_2, \ldots, v_n$. For all $i < j$, there's an edge from vertex v_i to vertex v_j. Let's say we run DFS on the graph considering v_1 as the source node. The adjacency list itself, and the linked list in each index of the adjacency list is sorted in lexicographic order.
   Let's say we have a X-progressive graph. After running DFS, what will be the number of forward edges?
   $\frac{(X-1)\times(X-2)}{2}$

3. Consider that we are running Dijkstra's Algorithm to compute the shortest path in a directed weighted graph, $G = (V, E)$. If all the outgoing edges of a vertex $v \in V$ is processed, we store that vertex in a set, P. If none of the outgoing edges of a vertex $v \in V$ is processed, we store that vertex in another set, U.
   Now consider the following 4 statements. If the statement is true, put a T in the blank. If the statement is false, put an F in the blank.

   - For every node $u \in U$, $d[v] = \infty$. F
   - For every node $u \in P$ and every node $v \in U$, $d[u] \leq d[v]$. T
   - For every node $u \in P$ and every node $v \in U$, $\delta(s, u) \leq \delta(s, v)$. T
   - If $u, v \in U$ and $d[u] < d[v]$, then $\delta(s, u) < \delta(s, v)$. F

4. Consider the following graph:

   In the worst-case, Bellman-Ford Algorithm runs 4 iterations to find the shortest path in this graph. However, if we relax the outgoing edges of the vertices in topologically sorted order, wit will take only one iteration to find out the shortest paths for all the vertices. That means, after that, no matter how many iterations we run, the distance from the source vertex will not change. We then say that Bellman-Ford will converge after one iteration.
   Here, you need to change the direction of one of the edges to make sure that no matter which ordering of edge relaxation we pick, Bellman-Ford will take more than one iterations to converge. You cannot change the weight of any of the edges. Which edge will you pick?
   (B, E)

5. You are given a connected weighted directed graph, $G = (V, E)$. All the edges in the graph have a negative weight. Which of the following is the most efficient algorithm to determine whether the graph contains a negative-weight cycle?
   DFS

6. Let $G = (V, E)$ be a complete binary tree with N nodes. You can find out the shortest path from the root to a vertex $v \in V$ in $O(\log_2 N)$ time. False

7. In a weighted undirected tree, $G = (V, E)$, we can run DFS from a source vertex $s \in V$ to find out the shortest path from s to all other vertices. True

8. Let, $G = (V, E)$ be an unweighted connected undirected graph. Here, $|E| = |V| - 1$. We can find the shortest path from source $s \in V$ to all the vertices in $O(|V|)$ time. True