

1242

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

WINTER SEMESTER, 2017-2018

DURATION: 1 Hour 30 Minutes

FULL MARKS: 100

Math 4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. a) Find the domain and range of the following functions 12
 i. $f(x) = \sqrt{x^2 - 1} |x + 30|$ ii. $g(x) = \sec x$ iii. $F(x) = \frac{x-3}{|x-3|}$ iv. $h(x) = \sin^{-1} x$
- b) Express the function $f(x) = x + |x + 1|$ in piecewise form and graph the function. 8
- c) Graph the function $f(x) = 3 \sin \frac{\pi}{2} x + 1$ using period and amplitude and write their domain and range. 7
- d) Find the inverse of the function $f(x) = \ln(x-3) + 1$ and graph the f and f^{-1} in the same plane. 6.33
2. a) Write down the precise definition of limit. Using formal definition of limit prove that 8
 $\lim_{x \rightarrow 2} \sqrt{9x-2} = 4$.
- b) Find the Limit of the following functions: 16
 i. $\lim_{x \rightarrow 2^+} \frac{1}{|2-x|}$ ii. $\lim_{x \rightarrow 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2}$
 iii. $\lim_{t \rightarrow +\infty} \frac{6-t^3}{7t^4+3}$ iv. $\lim_{x \rightarrow -\infty} \frac{1-e^x}{1+e^x}$
- c) Discuss the Limit and Continuity of the following function at, $x = 3$ 9.33

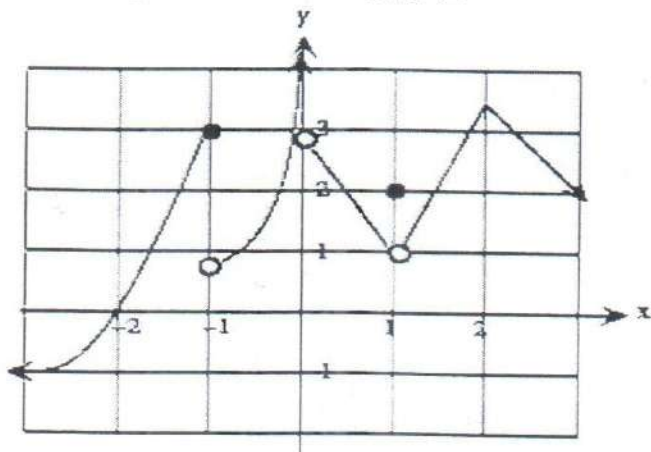
$$f(x) = \begin{cases} 4-x^2 & x < 3 \\ 6 & x = 3 \\ x-8 & x > 3 \end{cases}$$
 and draw the graph.

3. a) State Sandwich theorem and using this theorem show that

13.33

i. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ ii. $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$

- b) Whether the following function f graphed is continuous on $[-3, 3]$ or not. If not where does it fail to be continuous and why? What is the $\lim_{x \rightarrow 1} f(x)$? 8



- c) Use Limit to determine for all horizontal and oblique asymptotes from following equations: 12

i. $F(x) = \frac{x^3 - 2}{|x|^3 + 1}$

ii. $h(x) = \frac{x^2 - 4}{x - 1}$

4. a) Find the tangent line to the curve $f(x) = \sqrt{x-1} + 2$ at $x = 3$. 8

- b) At what points do the graph of the function $g(x) = x^2 - 7x + 12$ have horizontal tangent line and show in your graph. 10

- c) The power P , in Watts, supplied to a circuit by a battery is given by the formula $P = 6I - 0.5I^2$ where I is the current in amperes. 10

- i. What is the average rate of change of power when current is 1 to 4 amperes?
ii. What is the approximate instantaneous rate of change of power when current is 4 amperes?

- d) Show that $f(x) = |2x + 1|$ continuous everywhere. 5.33