ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2018-2019**

DURATION: 1 Hours 30 Minutes **FULL MARKS:100**

Math 4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 4 **(four)** questions. Answer any 3 **(three)** of them.

Figures in the right margin indicate marks.

Draw the figure or figures where necessary.

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1. a) Find the change in the coordinates of a point when the direction of axes is turned through an angle θ without changing the origin. Transform the equation $2x^2+4xy+4y^2-2x-4y-2=0$ to rectangular axes through the point (2,-1) and inclined at angle 45^0. 15

   b) Find the value of K, so that the conic $12x^2+36xy+ky^2+6x+6y+3=0$ represent a pair of straight lines. 8.33

   c) Write the different conditions need to be fulfilled so that the general equation of second degree represents a conic. Discuss the nature of the conic $x^2+6xy+9y^2+4x+12y-5=0$. 10

2. a) Define direction ratios and cosines of a line. Suppose OP be any line, where O be the origin and P (2, 1,-1) be any point, find the direction angles of the line. 10

   b) Find the Projection of a line AB on a line CD by using two different methods, where the points A, B, C, and D are respectively (1, 1,-1), (2, 1, 0), (-1, 0, 1) and (1, 2, 3). 10

   c) If the edges of a rectangular parallelepiped are L, H and W then show that the angles between the four diagonals are given by $cos^{-1}\left(\frac{\pm L^2 \pm H^2 \pm W^2}{L^2+H^2+W^2}\right)$. 13.33

3. a) Classify the following function as even, odd or neither. 8

   i. $f(x)=\frac{-3x^2}{x^3+1}$ ii. $g(x)=\cos x+1$ iii. $F(x)=\frac{x-3}{|x-3|}$ iv) $h(x)=x^4+1$

   b) If $f(x)=\begin{cases}-x-4 & -4\le x\le -1\\ 3x & -1\le x\le 1\\ -x+4 & 1<x\le 4\end{cases}$ then graph $f(x)$ and $|f(x)|$. 8

   c) i. Graph the function $f(x)=-(x-2)^2(x-1)^2(x+3)$ by showing necessary steps. 17.33

   ii. Find the inverse of the function $f(x)=2e^{x+2}-1$ and graph the f and f^{-1} in the same plane.

4. a) Define Limit of a function. If $f(x)=\frac{3x+|x|}{7x-5|x|}$, does the limit $\lim_{x\to 0} f(x)$ exist? 8

   b) Find the Limit of the following functions 9

   i. $\lim_{x\to 2}\frac{1}{(x-2)^4}$ ii. $\lim_{x\to\infty}\frac{x^2+2x-x^3}{3x^4-7}$ iii. $\lim_{x\to 2}\ln|x-2|$

c) If $f(x)=\begin{cases} x^2-1 & -1\le x<0 \\ 2x & 0<x<1 \\ 1 & x=1 \\ -2x+4 & 1<x<2 \\ 0 & 2<x<3 \end{cases}$, then graph the function. Answer the following questions: 9

i. Does *f(-1)* and *f(-1)* exist? ii. Does Limit $\lim_{x\to 0} f(x)$, $\lim_{x\to 1} f(x)$ and $\lim_{x\to 2} f(x)$ exist? If so what is it? If not, explain why?

d) A raindrop forms in the atmosphere and begins to fall to earth. If we assume air resistance is proportional to the speed of the raindrop, then the drop's velocity as a function of time is given by $v(t)=\frac{mg}{k}\left(1-e^{-kt/m}\right)$ where m is the mass of the raindrop, g is acceleration of gravity and k is a positive constant. Find $\lim_{t\to\infty} v(t)$ and draw the graph of *v(t)*. 7.33