

## ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

## ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION

SUMMER SEMESTER, 2018-2019

DURATION: 3 Hours

FULL MARKS:200

**Math 4241: Integral Calculus and Differential Equations****Programmable calculators are not allowed. Do not write anything on the question paper.**

There are **8 (eight)** questions. Answer any **3 (three)** from each sections. Use **Separate answer script for each section**. Give figures where necessary.  
 Figures in the right margin indicate marks.

**Section-A**

(Answer Any Three)

1. a) Write the physical significance of  $\int_a^b f(x)dx$ . Explain that the definite integral can be written as limits of finite sum. 15.33  
 b) Find  $dy/dx$  of  $y = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}$  18
  - i. By using Fundamental theorem
  - ii. By evaluating the integral and then differentiating the result
2. a) Find the total area and net signed area between the region and the x-axis formed by the curve  $y = 3x^2 - 3$ ,  $-2 \leq x \leq 2$ . 20  
 b) Find the area of the regions enclosed by the curves  $x - y^2 = 0$  and  $x + 2y^2 = 3$  13.33
3. a) Find the volume of the solid generated by revolving the regions bounded by the given curve and the lines  $y = 4 - x^2$ ,  $y = 2 - x$  about x-axis. 15  
 b) Define length of a curve and the area of the surface generated by revolving the axis. Find the surface area of the cone frustum generated by revolving the line segment  $y = \left(\frac{x}{2}\right) + \left(\frac{1}{2}\right)$ ,  $1 \leq x \leq 3$ , about the y- axis. Check your result with the geometry formula : 18.33  
 Frustum surface area =  $\pi (r_1 + r_2) \times \text{slant height}$ .
4. a) Using Trapezoidal and Simpson's rules with  $n = 4$ , find the approximate value of  $\int_0^3 \sqrt{x+1} dx$ . Compare your results with true value and comments on it. Suppose, for this problem, the magnitude of error is considered less than 0.0001, what will be the minimum number of subintervals needed? 20  
 b) Define improper integrals with examples. Check the following integrals are convergent or divergent : 13.33
  - i.  $\int_{-1}^1 -x \ln|x| dx$ ,
  - ii.  $\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}} dx$

## Section-B

(Answer Any Three)

- 5. What is solution of a differential equation? Find the solution of the following differential equations: 33.33

a)  $\cos^2 x \frac{dy}{dx} + y = \tan x$   
b)  $(3x^2 y^2 - y^3 + 2x)dx + (2x^3 y - 3xy^2 + 1)dy = 0, \quad y(2) = 1$   
c)  $xy^2 dy - (x^3 + y^3)dx = 0, \quad y(1) = 0$

6. a) Derive the differential equation for the motion of the mass on a spring. 13

- b) A 12-lb weight is placed upon the lower end of a coil spring suspended from the ceiling. 20.33  
The weight comes to rest in equilibrium position, thereby stretching the spring 1.5 in. the weight is then pull down 2 in. below its equilibrium position and released from rest at  $t=0$ . Find the displacement of the weight as a function of the time. Determine the Amplitude and period of the resulting motion and graph the displacement as a function of the time. [Neglecting the resistance of the medium and assuming that no external force are present.]

- 7. Solve the following Differential Equations: 33.33

a)  $\frac{d^2 y}{dx^2} + y = 3x^2 - 4 \sin x \quad y(0) = 0, \quad y'(0) = 1$   
b)  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \sin(\ln x^2)$

- 8. a) Define Partial Differential Equation (PDE), Degree and Order of Differential Equation with an example. 9

- b) Using the method of separation of variables, Solve the Heat equation 24.33

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2} \frac{\partial u}{\partial t}$$

Over  $0 < x < 3, \quad t > 0$  for the boundary conditions

$$u(0, t) = u(3, t) = 0$$

and initial condition

$$u(x, 0) = 5 \sin 4\pi x$$