189

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION   SUMMER SEMESTER, 2018-2019
DURATION: 1 Hour 30 Minutes   FULL MARKS: 75

Math 4441: Probability and Statistics

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **4 (Four)** questions. Answer any **3 (Three)** them. Figures in the right margin indicate
marks. Programmable calculators are not allowed. Do not write on this question paper. The symbols
used have their usual meaning.

1. a) Define Sample, Population, Sample Space and Event Space   5

   b) Monitor three consecutive phone calls going through a telephone switching office.   12
   Classify each one as a voice call (v) if someone is speaking, or a data call (d) if the call
   is carrying a modem or fax signal. Your observation is a sequence of three letters (each
   letter is either v or d). For example, two voice calls followed by one data call corresponds
   to vvd. Write the elements of the following sets:

   (1) A_1 = {first call is a voice call}   (2) B_1 = {first call is a data call}
   (3) A_2 = {second call is a voice call} (4) B_2 = {second call is a data call}
   (5) A_3 = {all calls are the same}   (6) B_3 = {voice and data alternate}

   For each pair of events A_1 and B_1, A_2 and B_2, and so on, identify whether the pair of
   events is either mutually exclusive or collectively exhaustive or both.

   c) The odds against 'A' solving a problem are 13 to 7 and the odds in favour of B solving   8
   the problem are 15 to 8. What is the probability that if both of them try, the problem will
   be solved?

2. a) A circuit system is given below. Assume the components fail independently.   10

   (i) What is the probability that the entire system works?

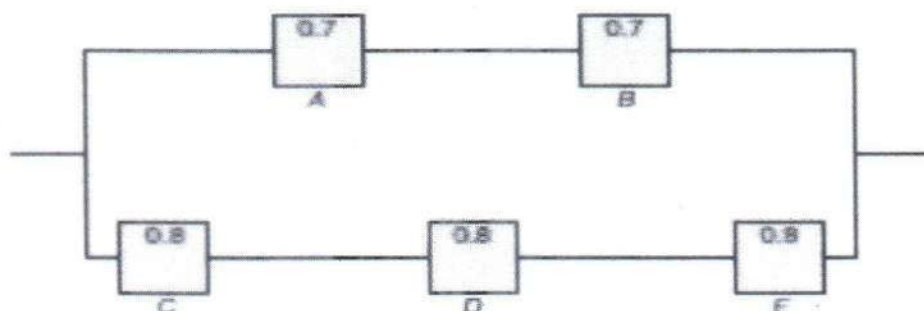   (ii) Given that the system works, what is the probability that the component A is not
   working?

Figure 1: Figure for question 2.a.

b) One prominent physician claims that 80% of those with lung cancer are chain smokers. If his assertion is correct, find the probability that of 20 such patients recently admitted to a hospital fewer than half are chain smokers?   7

c) A wholesale gift shop supplies gift items to the customers. The manager has the standing instruction to the sales person to remove the price tag while wrapping the gift boxes. It is observed that the sales persons inadvertently forgot to remove the tag in 2 cases out of every 1000. The gift boxes are supplied in bundles of 10. Calculate the number of bundles containing boxes with (i) no price tag, (ii) one price tag, (iii) at most 2 price tag in a consignment of 10,000 bundles.   8

3. a) The Random Variable X has the CDF: 15

$$F_X(x) = \begin{cases} 0, & \text{if } x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & \text{if, } x > b \end{cases}$$

(i) Find $\underline{P}\left(X \le \dfrac{2a+b}{3}\right)$. (ii) If $a = -1, b = 3$ find $\underline{P}\left(|X| \le \dfrac{2}{3}\right)$

b) Suppose that the number of cars X that pass through a car wash between 4:00 P.M. and 5:00 P.M. on any sunny Friday has the following probability distribution: 10

x	4	5	6	7	8	9
$P(X=x)$	1/12	1/12	1/4	1/4	1/6	1/6

Let $g(X) = 2X-1$ represent the amount of money, in dollars, paid to the attendant by the manager. Find the attendant's expected earnings for this particular time period.

4. a) The IQ's of 7 years old is assumed to be normal random variable. It is known that 15% of the children have IQ's under 90 and 2% exceed 135, what percentage of children have IQ's between 100 and 120? (Necessary chart 1 is attached).   10

b) Roll two fair four-sided dice. Let X_1 and X_2 denote the number of dots that appear on die 1 and die 2, respectively. Let A be the event $X_1 \ge 2$. What is P[A]? Let B denote the event $X_2 > X_1$. What is P[B]? What is P[A|B]?   7

c) Continuous random variable X has PDF   8

$$f_X(x) = \begin{cases} \dfrac{1}{4}, & -1 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

Define the random variable Y by $Y = h(X) = X^2$.
(i) Find E[X], Var[X], (ii) Find h(E[X]) and Var[Y].