B.Sc.Engg. CSE/SWE 3rd Semester

24 February 2020 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION — **WINTER SEMESTER, 2019-2020**

DURATION: 1 Hours 30 Minutes — **FULL MARKS: 75**

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer **3 (three)** **including Question no. 1.**

Figures in the right margin indicate marks.

Mandatory

1. a) Draw vectors u, v, w so that their combinations $cu + dv + fw$ fill only a line. Draw vectors u, v, w so that their combinations $cu + dv + fw$ fill only a plane. 3+3

   b) Describe geometrically (line, plane, or all of $\mathbb{R}^3$) all linear combinations of: 6

   i. $\begin{bmatrix}1\\2\\3\end{bmatrix}$ *and* $\begin{bmatrix}3\\6\\9\end{bmatrix}$ ii. $\begin{bmatrix}1\\0\\0\end{bmatrix}$ *and* $\begin{bmatrix}1\\1\\0\end{bmatrix}$ iii. $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}4\\5\\6\end{bmatrix}$ *and* $\begin{bmatrix}7\\8\\9\end{bmatrix}$

   c) Suppose **A** is the matrix, 13

$$\mathbf{A}=\begin{bmatrix}0 & 1 & 2 & 2\\0 & 3 & 8 & 7\\0 & 0 & 4 & 2\end{bmatrix}$$

   Find all special solutions to $Ax = 0$ showing all necessary steps and describe in words the whole nullspace of A.

2. a) For the vectors $v = (3,4)$ and $w = (4,3)$ test the Schwarz inequality on $v \cdot w$ and the triangle inequality on $\|v + w\|$. Find the angle between v and w. 7

   b) Choose a coefficient **b** that makes this system singular. Then choose a right-side **g** that makes it solvable. Find two solutions in that singular case. 6

$$3x + by = 16$$
$$6x + 16y = g$$

   c) Suppose **A** is the matrix, 12

$$\mathbf{A}=\begin{bmatrix}1 & 2\\0 & 9\\5 & 1\end{bmatrix}$$

   i. Describe the column space of this particular matrix **A**. "All combinations of the four columns" is not a sufficient answer.

   ii. Explain in words how knowing all solutions to $Ax = b$ decides if a given vector b is in the column space of **A**.

   iii. Is the vector $b = \begin{bmatrix}8\\28\\14\end{bmatrix}$ in the columnspace of **A**?

3. a) E_{21} subtracts row 1 from row 2 and then P_{23} exchanges rows 2 and 3. What matrix $M = P_{23}E_{21}$ does both steps at once? 5

b) Find A^{-1} and B^{-1} *(if they exist)* by Gauss-Jordan method starting with [A I] and [B I]: 7+7

$$A=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 3\\ 0 & 0 & 1\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{bmatrix}$$

c) What subspace of 3 by 3 matrices is spanned (take all combinations) by 6

i. the invertible matrices?
ii. the rank one matrices?
iii. the identity matrix?

4. a) Fill in the blanks: 6

The largest possible rank of a 6 by 4 matrix is _____. Then there is a pivot in every _____ of U and R. The solution to $Ax = b$ always exists/is unique (mention the right choice). The nullspace of A is _____. An example is A = _____.

b) Find the complete solution to: 15

$$\begin{bmatrix}1 & 3 & 1 & 2\\ 2 & 6 & 4 & 8\\ 0 & 0 & 2 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ t\end{bmatrix}=\begin{bmatrix}1\\ 3\\ 1\end{bmatrix}$$

c) Suppose A is the matrix, 4

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\end{bmatrix}$$

i. The columnspace and the rowspace are in ___ and ____ vector-spaces respectively.
ii. Mention the rank of all four fundamental subspaces of matrix A.