B.Sc. Engg (CSE) 7th Semester **05 March 2019 (Afternoon)**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2018-2019**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

Math 4741: Mathematical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. a) In a sequence of independent flips of a biased coin(probability of a head is .6), let N denote the number of flips until there is a run of three consecutive heads. Find 6+4
   i. $P(N \leq 8)$
   ii. $P(N = 8)$

   b) Define the following terms: 1.5x4
   i. Accessible
   ii. Communicate
   iii. Transient State
   iv. Recurrent State

   c) Let the Markov chain consisting of the states 0, 1, 2, 3 have the transition probability matrix: 4

$$\mathbf{P} = \begin{Vmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{Vmatrix}$$

   Determine which states are transient and which are recurrent.

   d) Let the Markov chain consisting of the states 0, 1, 2, 3, 4 have the transition probability matrix: 5

$$\mathbf{P} = \begin{Vmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{Vmatrix}$$

   Determine which states are transient and which are recurrent.

2. For a given Bonus Malus system, let $s_i(k)$ denote the next state of a policyholder who was in state i in the previous year and who made a total of k claims in that year. If we suppose that the number of yearly claims made by a particular policyholder is a Poisson random variable with parameter λ, then the successive states of this policyholder will constitute a Markov chain with transition probabilities

$$P_{i,j} = \sum_{k:s_i(k)=j} e^{-\lambda}\frac{\lambda^k}{k!}, \quad j \geqslant 0$$

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Consider Table 1, which specifies a hypothetical Bonus Malus system having four states.

Table 1

		Next state if			
State	Annual Premium	0 claim	1 claim	2 claims	≥ 3 claims
1	200	1	2	3	4
2	250	1	3	4	4
3	400	2	4	4	4
4	600	3	4	4	4

Thus, for instance, the table indicates that $s_2(0) = 1$; $s_2(1) = 3$; $s_2(k) = 4$, $k \geq 2$. Consider a policyholder whose annual number of claims is a Poisson random variable with parameter λ. If a_k is the probability that such a policyholder makes k claims in a year, then

$$a_k = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k \geqslant 0$$

Considering $\lambda = .5$ determine the following:

a) Determine the transition matrix. 10

b) Draw the transition diagram. 5

c) If the process runs for a long time, determine the long term proportions of all of the states. 10

3. Assume a football game of penalty shootout where goals are scored with $\lambda = .6$/min. You will play the game for at least two minutes and if there is a goal scored within this interval, you will stop playing after two minutes. Otherwise, you will continue until there is at least a goal scored (no matter how long it takes past the first two minutes). Answer the following based on this scenario:

a) P(play for more than two minutes) 5

b) P(play for more than two minutes and less than five minutes) 5

c) P(scoring at least two goals) 5

d) E[number of fish] 5

e) E[total fishing time] 5

4. a) State the differences between Bernoulli process and Poisson process. 5

b) There are four light bulbs burning with Poisson rate $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. What is the expected time until the last light bulb burns out? Show necessary calculation with proper explanation. 8

c) Consider, two different color light bulbs are blinking with Poisson rate λ_1 and λ_2 respectively. A colorblind person observes the experiment and tells you that the blink came from the bulb with λ_1 rate. What is the probability that he is right? Show justification for your answer. 7

d) "An average family size is four and an average person comes from a family size of six" – is the quote contradictory? Show justification for your answer. 5