B.Sc. Engg. CSE 8th Semester 6 November 2019

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

FINAL SEMESTER EXAMINATION **SUMMER SEMESTER, 2018-2019**
DURATION: 3 Hours **FULL MARKS: 150**

CSE 4803: Graph Theory

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any **6 (six)** of them.

Figures in the right margin indicate marks.

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1. a) Define the followings with example and figure 2×5
   - i. Cocycle
   - ii. Matching
   - iii. Covering
   - iv. Residual Graph
   - v. Orientable Graph

   b) Prove that the number of vertices of odd degree in a graph is always even. 7

   c) Let G be a connected graph. Prove that G is orientable if and only if each edge of G is contained in at least one cycle. 8

2. a) Generate a labelled tree represented by the sequence (3, 3, 4, 5, 5, 6) 8

   b) In Figure 1, the subgraph in bold lines indicates a spanning tree T of the graph G.
   - i. What is the rank and nullity of graph G? 2
   - ii. What is the maximum distance between any two spanning trees in G? 3
   - iii. Draw a spanning tree T_2 that is at a distance of 4 from T. 6
   - iv. List all the fundamental circuits with respect to T_2. 6

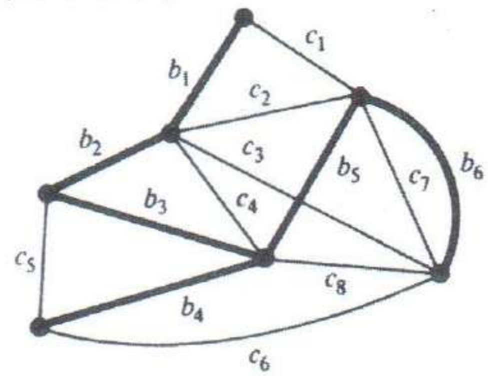

Figure 1: Graph for Question 2 (b)

3. a) Define DAG and Topological Ordering with example and figure. 6

   b) Discuss a real-life example of vertex coloring problem. 5

   c) Let G be a simple graph with n vertices and m edges. Use induction on m, together with the equation $P_G(k) = P_{G-e}(k) - P_{G/e}(k)$, to prove the following:
   - i. The coefficient of k^{n-1} is m. 7
   - ii. The coefficients of $P_G(k)$ alternates in sign. 7

4. a) Write down the observations that we can make from the incedence matrix of any graph G. 5

   b) Find the structure rank of the following matrix A, using Hungarian Algorithm. Moreover, determine the minimum number of lines (and the particular lines) to contain all 1's of the matrix. 20

$$A = \begin{matrix} 1&0&0&1&0&1&0&0&0 \\ 0&1&0&0&0&1&0&1&1 \\ 0&0&1&1&0&0&0&0&0 \\ 0&0&1&1&1&0&0&0&0 \\ 0&0&0&0&1&0&1&1&1 \\ 0&0&1&0&1&0&0&0&0 \\ 0&0&0&1&1&0&0&0&0 \\ 1&1&1&0&1&0&0&0&0 \\ 0&0&1&0&1&0&0&0&0 \end{matrix}$$

5. a) For each of the following graphs, compare the upper bound for the chromatic number given by Brook's theorem with the correct value. 5×3

i. K_{21} ii. $K_{11,21}$ iii. C_{23}

b) Find the chromatic polynomials of the following: 5×2

i. $K_{2,5}$ ii. C_5

6. a) Prove the following in the context of The Matching Algorithm (TMA): 3×3

i. TMA terminates within N^2+1 days.

ii. Everyone is married in TMA.

iii. TMA produces stable marriages.

b) Four students want internships in four different companies. The preference lists of the students and companies are given in the following tables. Use TMA to produce upto **TWO** stable matchings of Students to Companies. 16

Student preference

Student	**Companies**
Abdul	Dell, Bose, HP, Apple
Bari	Apple, Bose, Dell, HP
Choyon	HP, Dell, Apple, Bose
Didar	Dell, Apple, Bose, HP

Company preference

Company	**Students**
Apple	Didar, Abdul, Choyon, Bari
Bose	Abdul, Bari, Choyon, Didar
HP	Choyon, Didar, Abdul, Bari
Dell	Bari, Didar, Choyon, Abdul

7. Suppose you are playing a game with your friend. The game is a simplified version of "Nim" where there are two piles each with three sticks in it. On each turn, a player can take out any number of sticks from a particular pile and the player who takes the last stick(s) wins.

a) Explain why this is a perfect-information, finite game where someone will always in or lose. 4

b) Draw a directed graph describing the complete game indicating the possible moves and resulting states. Each state (vertex) should be described with an ordered pair of labels (x,y) indicating the number of sticks in the first and second pile respectively. 10

c) Label the states as "won" or "lost" assuming each player makes the best possible move at each stage. 6

d) Is there a strategy you can follow so that you always lose? Explain. 5

8. a) Prove that the number of vertices of odd degree in a graph is always even. 5

b) Show that a connected simple planar graph all of whose vertices has degree at most 5 must have at least 12 vertices. 8

c) Give an example of a connected planar graph in which "$e < 3n - 6$" 3

d) Define dual of a graph. A planar graph is called 'self dual' if it is isomorphic to its geometric dual. Prove that if G is a self-dual graph, then "$e = 2n - 2$" 2+7