Mock Mid Semester Exam

1. You are the king of the country USB. There are n cities and m bidirectional train routes connecting the cities in USB. Each route has a unique cost associated with it. Any two cities in the country have at least one path between them consisting of the train routes. Being a kind king, you want to reduce the cost of traveling from one city to another. Specifically, you want to keep only those routes that have a cost less than x. But you still want to keep at least one path between any pair of cities. $18 + 7$
   Propose an efficient algorithm to determine the largest x and analyze its running time.

2. You are given an array A containing n integers. Consider an increasing subsequence of array indices $B = (b_0, b_1, \ldots, b_{m-1})$ where $0 \leq b_0 < b_1 < \cdots < b_{m-1} < n$. Your task is to find out the maximum value of the following function if the indices are picked optimally: 5×5

$$\sum_{i=0}^{m-1} (-1)^i A[b_i] = A[b_0] - A[b_1] + A[b_2] - A[b_3] + \ldots$$

   Propose a dynamic programming solution to the problem. You need to define a set of subproblems, relate the subproblems recursively, provide base cases, construct a solution from the subproblems, and analyze the running time.

3. *Seed for Need* is a racing video game set in Fortune City where the player needs to carry seeds to the farmers by driving their cars. There are N towns in Fortune City. The towns are connected by M roads. Each town has a positive integer difficulty level. When you go from town u to town v, you will face obstacles if the difficulty level of town u is strictly less than the difficulty level of town v. You are given the map of Fortune City containing the difficulty level of each town and the length of each road.

   Consider that you are in town X and you need to go to town Y carrying the seeds as fast as you can. Your car travels along the roads at a constant speed S. However, when you enter a town that has obstacles, your car will be delayed by a fixed amount of time D.
   Your goal is to find a path to go from town X to town Y as quickly as possible.

   a) Construct the graph associated to the problem. 10

   b) Describe and justify the graph algorithm applied to plan your route. $6 + 6$

   c) State the running time of your algorithm in terms of the nodes and edges in your graph. 3