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B.Sc. Engg. / HD CSE 2nd Semester (64)

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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION
SUMMER SEMESTER, 2015-2016
DURATION: 3 Hours
FULL MARKS: 200

Math 4241: Integral Calculus and Differential Equations

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **8 (eight)** questions. Answer any **6 (six)** of them.
Figures in the right margin indicate marks.

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1. a) What is the physical meaning of $\int_a^b f(x)dx$? Find the area under the curve represented by the following data: 12

X :	5	10	15	20	25	30
Y :	1.5	5.12	4.25	6.65	5.75	2.45

b) Evaluate and sketch the region whose area is represented by the integral $\int_{-a}^{a}\sqrt{a^2-x^2}\,dx$ and then verify it using appropriate formula from geometry. 12

c) Find the total area between the curve $y=1-x^2$ and the x-axis over the interval [0, 2] by using anti-derivative method. 9.33

2. a) Write the properties of improper integral with examples. Evaluate the integrals and state whether they are divergent or convergent: 15

i. $\int_0^1 \frac{1}{\sqrt{x}\ (x+1)}dx$, ii. $\int_{-1}^{\infty}\frac{x}{1+x^2}dx$

b) Define Beta and Gamma functions. Find the relationship between them. 10

c) Evaluate $\int_0^\infty e^{-x^2}\,dx$ and hence show that $\left|\frac{1}{2}\right. = \sqrt{\pi}$ 8.33

3. a) Sketch the region enclosed by the curves $y^2=4x$ and $y=2x-4$ then find its area. 13.33

b) Find the volume of the solid that is obtained when the regions between the curves $f(x) = x^2+2$, and $g(x) = x$ over the interval [1, 3] is revolved about x-axis. 20

4. a) Define the arc length for a curve and for parametric equations. Find the circumference of a circle of radius 15 meters from the parametric equations $x = 15\,cos\theta$ and $y = 15\,sin\theta$, $0\le\theta\le 2\pi$. 13.33

b) Find the area of the surface that is generated by revolving the portion of the curve $y=x^2$ for $0\le x\le 1$ about x-axis and for $0\le y\le 3$ about y-axis. 20

5. a) Define linear and nonlinear ordinary differential equations (DE) with examples. Find the differential equations corresponding to the family of curves $y=k(x-k)^2$, where k is an arbitrary constant. 13.33

b) Determine the constant A such that the given DE $(Ax^2y+2y^2)dx + (x^3+4xy)dy=0$ is an exact and then solve it. 20

6. a) What is integrating factor? Consider the DE $(y^2+2xy)\,dx + x^2dy = 0$, find the integrating factor of the form y^n for which the given DE transformed into an exact DE, where n is an integer. 13.33

b) Solve the following differential equations:

i. $(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$

ii. $(2xy + 3y^2)dx - (2xy + x^2)dy = 0$

7. a) Define Bernoulli's DE. State in what conditions the Bernoulli's DE reduces to a first order DE, explain with examples.

b) Solve the following initial value problems:

i. $x\frac{dy}{dx} - 2y = 2x^4, \quad y(2) = 8$

ii. $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}, \quad y(1) = 2$

8. a) Define partial differential equation and solve the following PDE:

i. $(y + z)\frac{\partial z}{\partial x} + (z + x)\frac{\partial z}{\partial y} = x + y$

ii. $(x^2 - yz)\frac{\partial z}{\partial x} + (y^2 - zx)\frac{\partial z}{\partial y} = z^2 - xy$

b) Find the integral surface of the linear partial differential equation $x(y^2 + z)\frac{\partial z}{\partial x} - y(x^2 + z)\frac{\partial z}{\partial y} = (x^2 - y^2)z$, which passes through the curve $xz=a^3$, $y=0$.