152

B.Sc. Engg. CSE/HD CSE 4th Semester

13 August 2018 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2017-2018**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

Math 4441: Probability and Statistics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. a) The post office of a certain small town has only one clerk to wait on customers. The probability that a customer will be served in any given minute is 0.6, regardless of the time that the customer has already taken. The probability of a new customer arriving is 0.45 regardless of the number of customers already in line. The chances of two customers arriving during the same minute are negligible (i.e., the probability is zero). Similarly, the chances of two customers being served in the same minute are negligible (i.e., the probability is zero). Suppose that we start with exactly two customers: one at the postal window and one waiting on line. After 4 minutes, find the probability that there will be exactly four customers: one at the window and three waiting on line. 10

   b) A *binary digit* or *bit* is a zero or one. A computer assembly language program translator, known as the assembler, translates an assembly language program into a sequence of zeros and ones independently. Assume that each of the translated bits is equally likely to be a zero or one. Let X be the number of independent random bits to be generated until both 0 and 1 are obtained.

      i. Find the probability mass function of X. 8

      ii. Find the minimum value of X, so that there will be a 95% chance that the first X bits will have at least one 0 and one 1. 7

2. a) To determine whether or not 100 peoples in a community have a certain disease, they are to have their blood tested. However, rather than testing each individual separately, it has been decided first to group the people in groups of 10. The blood samples of the 10 people in each group will be pooled and analyzed together. If the test is negative, one test will suffice for the 10 people. If the test is positive, each of the 10 people will also be individually tested. Suppose the probability that a person has the disease is 0.10 for all people independently from each other. Let X represent the number of groups with at least one people having the disease, and therefore, all the 10 people of those groups are individually tested. Find the probability mass function of X. 10

   b) Suppose that two players (A and B) play a series of games that ends when one of them has won i games. Suppose also that each game played is, independently, won by player A with probability p and by player B with probability $(1-p)$. Let N represent the number of games that are played when $i = 3$. Find the PMF of N. 15

3. a) The time it takes for a student to finish an aptitude test (in hours) has a probability density function of the form

$$f_X(x) = \begin{cases} c(x-1)(2-x), & 1 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

   i. Determine the value of the constant c. 5

ii. Calculate the cumulative distribution function of the time it takes for a randomly selected student to finish the aptitude test. 5

b) Let X, the marks obtained by a randomly selected students in a test of a probability course, be a normal random variable. The professor of the course first finds the average μ and standard deviation σ of the obtained marks of the students, and then assigns letter grades according to the following table: 15

Range of Marks	$X \geq \mu + \sigma$	$\mu \leq X < \mu + \sigma$	$\mu - \sigma \leq X < \mu$	$\mu - 2\sigma \leq X < \mu - \sigma$	$X < \mu - 2\sigma$
Grade	A	B	C	D	F

Determine the percentage of students who will get A, B, C, D, and F grades, respectively.

4. a) Two 4-sided fair dice are rolled. The sum of the outcomes is denoted by X and the absolute value of their difference is denoted by Y. 10
   i. Calculate the joint probability mass function of X and Y.
   ii. Find the marginal probability mass function of X.

b) Suppose that the joint probability density function of two random variables X and Y is given as follows:

$$f_{XY}(x,y) = \begin{cases} cx^2y, & x^2 \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

i. Determine the value of the constant c. 5

ii. Find the probability that the random variable X has a value greater than the random variable Y, i.e., $P[X > Y]$. 5

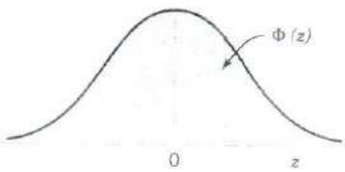
iii. Find the marginal PDF of random variable X, $f_X(x)$. 5

PMF/PDF, expected value and variance of some known Random Variables

Distribution	PMF/PDF	Expected value	Variance
Bernoulli	$P_X(x) = \begin{cases} 1-p & x=0 \\ p & x=1 \\ 0 & otherwise \end{cases}$	$E[X] = p$	$Var[X] = p(1-p)$
Geometric	$P_X(x) = \begin{cases} p(1-p)^{x-1} & x \geq 1 \\ 0 & otherwise \end{cases}$	$E[X] = 1/p$	$Var[X] = (1-p)/p^2$
Binomial	$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 1, \dots, n \\ 0 & otherwise \end{cases}$	$E[X] = np$	$Var[X] = np(1-p)$
Pascal	$P_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{x-k} & x = k, k+1, \dots \\ 0 & otherwise \end{cases}$	$E[X] = k/p$	$Var[X] = k(1-p)/p^2$
Poisson	$P_X(x) = \begin{cases} \frac{(\lambda T)^x e^{-(\lambda T)}}{x!} & x \geq 0 \\ 0 & otherwise \end{cases}$	$E[X] = \alpha$ $\alpha = \lambda T$	$Var[X] = \alpha$
Hyper Geometric	$P_X(x) = \frac{\binom{r}{x}\binom{g}{n-x}}{\binom{r+g}{n}}$	$E[X] = \frac{rn}{r+g}$	
Uniform (discrete)	$P_X(x) = \begin{cases} \frac{1}{b-a+1}, & x = a, a+1, a+2, \dots, b \\ 0, & \text{otherwise} \end{cases}$	$E[X] = \frac{a+b}{2}$	$Var[X] = \frac{(b-a)(b-a+2)}{12}$
Exponential	$f_X(x) = \begin{cases} ae^{-ax} & x \geq 0 \\ 0 & otherwise \end{cases}$	$E[X] = 1/a$	$Var[X] = 1/a^2$
Gaussian	$f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} & \sigma > 0 \\ 0 & otherwise \end{cases}$	$E[X] = \mu$	$Var[X] = \sigma^2$
Uniform (Continuous)	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b \\ 0, & otherwise \end{cases}$	$E[X] = \frac{a+b}{2}$	$Var[X] = \frac{(b-a)^2}{12}$

Appendix A: CDF of Standard Normal Distribution

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967