B.Sc. Engg. CSE 8th Semester 23 October 2019

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2018-2019**
DURATION: 3Hours **FULL MARKS: 150**

CSE 4841: Introduction to Optimization

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any 6 **(six)** of them.

Figures in the right margin indicate marks.

---

1. Consider the following problem:

   Maximize $f = 2x_1 + 6x_2 + 9x_3$,
   subject to
   $x_1 \quad + x_3 \le 3$ (resource 1)
   $x_2 + 2x_3 \le 5$ (resource 2)
   and $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$.

   a) Construct the dual problem for this primal problem. 5
   b) Solve the dual problem graphically. Use this solution to identify the shadow prices for the resources in the primal problem. 6+6
   c) Confirm your results from part (b) by solving the primal problem by the simplex method and then identifying the shadow prices. 8

2. Consider the following problem:

   Maximize $f = 2x_1 + 7x_2 - 3x_3$,
   subject to
   $x_1 + 3x_2 + 4x_3 \le 30$
   $x_1 + 4x_2 - x_3 \le 10$
   and $x_1 \ge 0|, x_2 \ge 0, x_3 \ge 0$.

   By letting x_4 and x_5 be the slack variables for the respective constraints, the simplex method yields the following final set of equations:

   $$f \quad + x_2 + x_3 \quad + 2x_5 = 20,$$
   $$- x_2 + 5x_3 + x_4 - x_5 = 20$$
   $$x_1 + 4x_2 - x_3 \quad + x_5 = 10$$

   Now you are to conduct sensitivity analysis by independently investigating each of the following changes in the original model. For each change, use the sensitivity analysis procedure to revise this set of equations (in tableau form) and convert it to proper form from Gaussian elimination for identifying and evaluating the current basic solution. Then test this solution for feasibility and for optimality. If either test fails, reoptimize to find a new optimal solution.

   a) Introduce a new constraint $3x_1 + 2x_2 + 3x_3 \le 25$. 13
   b) Change the objective function to $f = x_1 + 5x_2 - 2x_3$. 12

3. a) Coach Night is trying to choose the starting lineup for the basketball team. The team consists of seven players who have been rated (on a scale of 1 = poor to 3 = excellent) according to their ball-handling, shooting, rebounding, and defensive abilities. The positions that each player is allowed to play and the player's abilities are listed in the following Table. 15

Player	Position	Ball Handling	Shooting	Rebounding	Defense
1	G	3	3	1	3
2	C	2	1	3	2
3	G-F	2	3	2	2
4	F-C	1	3	3	1
5	G-F	3	3	3	3
6	F-C	3	1	2	3
7	G-F	3	2	2	1

The five-player starting lineup must satisfy the following restrictions:

i. At least 4 members must be able to play guard (G), at least 2 members must be able to play forward (F), and at least 1 member must be able to play center (C).
ii. The average ball-handling, shooting, and rebounding level of the starting lineup must be at least 2.
iii. If player 3 starts, then player 6 cannot start.
iv. If player 1 starts, then players 4 and 5 must both start.
v. Either player 2 or player 3 must start.

Given these constraints, Coach Night wants to maximize the total defensive ability of the starting team. Formulate an IP that will help him choose his starting team.

b) A company is considering opening warehouses in four cities: New York, Los Angeles, Chicago, and Atlanta. Each warehouse can ship 100 units per week. The weekly fixed cost of keeping each warehouse open is \$400 for New York, \$500 for Los Angeles, \$300 for Chicago, and \$150 for Atlanta. Region 1 of the country requires 80 units per week, region 2 requires 70 units per week, and region 3 requires 40 units per week. The costs (including production and shipping costs) of sending one unit from a plant to a region are shown in the following Table. 10

From	To		
	Region 1	Region 2	Region 3
New York	20	40	50
Los Angles	48	15	26
Chicago	26	35	18
Atlanta	20	50	35

We want to meet weekly demands at minimum cost, subject to the preceding information and the following restrictions:

i. If the New York warehouse is opened, then the Los Angeles warehouse must be opened.
ii. At most two warehouses can be opened.
iii. Either the Atlanta or the Los Angeles warehouse must be opened.

Formulate an IP that can be used to minimize the weekly costs of meeting demand.

4. Consider the following IP: 25

Maximize $f = 14x_1 + 18x_2$

Subject to

$-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

The optimal tableau for this IP's linear programming relaxation is given in the following Table.

f	x_1	x_2	y_1	y_2	RHS
1	0	0	$\frac{56}{11}$	$\frac{30}{11}$	126
0	0	1	$\frac{7}{22}$	$\frac{1}{22}$	$\frac{7}{2}$
0	1	0	$-\frac{1}{22}$	$\frac{3}{22}$	$\frac{9}{2}$

Use the cutting plane algorithm to solve this IP.

5. Consider the following optimization problem: 25

   Minimize $f = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$

   If a base simplex is defined by the vertices

   $$\boldsymbol{X_1} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, \quad \boldsymbol{X_2} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}, \quad \boldsymbol{X_3} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

   Find a sequence of four improved vectors using reflection, expansion, and/or contraction. Assume the values of the necessary parameters as required.

6. a) Consider the following optimization problem:

   Minimize $f(x_1, x_2) = x_1 - x_2$

   subject to

   $$3x_1^2 - 2x_1x_2 + x_2^2 - 1 \le 0$$

   i. Consider the bounds on x_1 and x_2 as $-2 \le x_1 \le 2$ and $-2 \le x_2 \le 2$ and solve the LP with only constraints given by the bounds. 5

   ii. Generate the approximating LP problem at the vector found as the solution of part a(i). 10

   b) Perform two iterations of the steepest descent method to minimize the following function: 10

   $$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

   starting from the point $\boldsymbol{X_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

7. a) Weenies and Buns is a food processing plant which manufactures hot dogs and hot dog buns. They grind their own flour for the hot dog buns at a maximum rate of 200 pounds per week. Each hot dog bun requires 0.1 pound of flour. They currently have a contract with Cowland, Inc., which specifies that a delivery of 800 pounds of beef product is delivered every Monday. Each hot dog requires $\frac{1}{4}$ pound of beef product. All the other ingredients in the hot dogs and hot dog buns are in plentiful supply. Finally, the labor force at Weenies and Buns consists of 5 employees working full time (40 hours per week each). Each hot dog requires 3 minutes of labor, and each hot dog bun requires 2 minutes of labor. Each hot dog yields a profit of \$0.20, and each bun yields a profit of \$0.10. 12

   Weenies and Buns would like to know how many hot dogs and how many hot dog buns they should produce each week so as to achieve the highest possible profit. Formulate a linear programming model for this problem.

   b) Oxbridge University maintains a powerful mainframe computer for research use by its faculty, Ph.D. students, and research associates. During all working hours, an operator must be available to operate and maintain the computer, as well as to perform some programming services. Beryl Ingram, the director of the computer facility, oversees the operation. 13

   It is now the beginning of the fall semester, and Beryl is confronted with the problem of assigning different working hours to her operators. Because all the operators are currently enrolled in the university, they are available to work only a limited number of hours each day, as shown in the following table.

Operators	Wage Rate	Maximum Hours of Availability				
		Mon.	Tue.	Wed.	Thu.	Fri.
K. C.	$10.00/hour	6	0	6	0	6
D. H.	$10.10/hour	0	6	0	6	0
H. B.	$ 9.90/hour	4	8	4	0	4
S. C.	$ 9.80/hour	5	5	5	0	5
K. S.	$10.80/hour	3	0	3	8	0
N. K.	$11.30/hour	0	0	0	6	2

There are six operators (four undergraduate students and two graduate students). They all have different wage rates because of differences in their experience with computers and in their programming ability. The above table shows their wage rates, along with the maximum number of hours that each can work each day.

Each operator is guaranteed a certain minimum number of hours per week that will maintain an adequate knowledge of the operation. This level is set arbitrarily at 8 hours per week for the undergraduate students (K. C., D. H., H. B., and S. C.) and 7 hours per week for the graduate students (K. S. and N. K.).

The computer facility is to be open for operation from 8 A.M. to 10 P.M. Monday through Friday with exactly one operator on duty during these hours. On Saturdays and Sundays, the computer is to be operated by other staff.

Because of a tight budget, Beryl has to minimize cost. She wishes to determine the number of hours she should assign to each operator on each day.

Formulate a linear programming model for this problem.

8. Maximize $\quad f = 3x_1 + 4x_2 + 2x_3$

subject to

$$x_1 + x_2 + x_3 \leq 20$$
$$x_1 + 2x_2 + x_3 \leq 30$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

Let x_4 and x_5 be the slack variables for the respective functional constraints. Starting with these two variables as the basic variables for the initial BF solution, you now are given the information that the simplex method proceeds as follows to obtain the optimal solution in two iterations: (1) In iteration 1, the entering basic variable is x_2 and the leaving basic variable is x_5; (2) in iteration 2, the entering basic variable is x_1 and the leaving basic variable is x_4.

Identify the set of defining equations for each of the three solutions (including the initial one) obtained by the simplex method. Then use the revised simplex method to find the three solutions.