B.Sc. Engg. CSE 3rd Semester   4 March 2019 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION   WINTER SEMESTER, 2018-2019
DURATION: 1 Hour 30 Minutes   FULL MARKS: 75

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **4 (four)** questions. Answer any **3 (three)** of them.
Figures in the right margin indicate marks.

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1. a) Consider the following two systems of equations. Solve both systems simultaneously by applying Gauss-Jordan reduction to an appropriate 3×5 matrix.   12

$$x + \ y + \ z = 7 \qquad\qquad x + \ y + \ z = 6$$
$$x + 2y + 2z = 10 \qquad\qquad x + 2y + 2z = 11$$
$$2x + 3y - 4z = 3 \qquad\qquad 2x + 3y - 4z = 3$$

   b) Which three elimination matrices E_{21}, E_{31}, E_{32} put A into its upper triangular form $E_{32}E_{31}E_{21}A = U$? Multiply E_{32}^{-1}, E_{31}^{-1} and E_{21}^{-1} to factor A into LU format.   13

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} \qquad L = E_{32}^{-1} \ E_{31}^{-1} \ E_{21}^{-1}$$

2. a) For the given matrix B, show by elimination that (b_1, b_2, b_3) is in the column space if $b_3 - 2b_2 + 4b_1 = 0$. Also find out what combination of the rows of B gives the zero row?   8

$$B = \begin{bmatrix} 1 & 3 & 1 \\ 3 & 8 & 2 \\ 2 & 4 & 0 \end{bmatrix}$$

   b) Define vector space and subspaces with appropriate examples. Mention the vector subspaces of $\mathbf{R}^2$ and $\mathbf{R}^3$ space.   7

   c) Generally when we multiply two matrices by hand (satisfying dimensions), we multiply rows with column. How can we get the same result by multiplying in the other way around i.e. column times row? Mention the special property of the sub-matrix that you get by multiplying a single column with a single row.   10

3. a) For a matrix of dimension $(m \times n)$ if rank $\Gamma = m = n$. Mention four important properties of this matrix. Also comment on the solution space of such matrix.   6

   b) Suppose that A is the matrix below. Explain in words how knowing all solutions to $Ax = b$ decides, if a given vector b is in the column space of A.   6

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 2 & 4 \end{bmatrix}$$

Is the vector $b = \begin{bmatrix} 8 \\ 28 \\ 14 \end{bmatrix}$ in the column space of A?

c) Find out the complete solution to the following linear system. Your solution should contain   13
both particular solution and special solution in null space.

$$x + 3y + 3z = 1$$
$$2x + 6y + 9z = 5$$
$$-x - 3y + 3z = 5$$

4. a) Invert the following matrix M by the Gauss-Jordan method starting with $[A \quad I]$ 5

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

b) For which right sides (find a condition on $b_1, b_2,$ and b_3) are these systems solvable? Also find   12
out the rank of the linear systems given.

i. $$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

ii. $$\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

c) Reduce the following matrix to its ordinary echelon form and then identify its free variables   8
and pivot variables.

$$\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$