B.Sc. Engg. CSE 2nd Semester 23 October 2018

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2018-2019**

DURATION: 3 Hours **FULL MARKS: 150**

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (Eight)** questions. Answer any **6 (Six)** of them.

Figures in the right margin indicate marks.

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1. a) *Scenario 1*

"Among the board members of a company, the President can influence the Director of Research and Development, the Director of Marketing, and the Director of Operations; the Director of Research and Development can influence the Director of Operations; the Director of Marketing can influence the Director of Operations; and no one can influence, or be influenced by, the Chief Financial Officer ."

Scenario 2

"In a round-robin tournament the Tigers beat the Blue Jays, the Tigers beat the Cardinals, the Tigers beat the Orioles, the Blue Jays beat the Cardinals, the Blue Jays beat the Orioles, and the Cardinals beat the Orioles. Model this outcome with an appropriate graph."

Considering the above scenarios answer the following:

i. What is a graph model? What types of graph models are required for each of the above scenarios? 1+2

ii. Draw the associated graphs for each of the scenarios. 8

b) If $m \in \mathbf{Z}^+$, $a, b \in \mathbf{Z}$ and $a \equiv b (\mathbf{mod}\ m)$ then prove that, 2×3

i. $(a+b)\mathbf{mod}\ m \equiv \big((a\ \mathbf{mod}\ m) + (b\ \mathbf{mod}\ m)\big)\ \mathbf{mod}\ m$

ii. $(ab)\mathbf{mod}\ m \equiv \big((a\ \mathbf{mod}\ m)(b\ \mathbf{mod}\ m)\big)\ \mathbf{mod}\ m$

c) Show the step by step computation of the summation and multiplication of the following numbers $(110011)_2$ and $(110111)_2$ using respective algorithms, 4+4

2. a) Mr. Luke Skywalker is a leader of the resistance against the empire. He is stranded on a distant planet and now needs help from the resistance for rescuing him. Luke wants to send the message "SABOTAGE EMPIRE" to his friend R2D2 far away in planet Nebula so that the Empire does not understand the message he is sending. Luke has with him his robot friend C-3PO who is able to encrypt and send the message. Your task is to help C-3PO encrypt the message using a hybrid encryption involving public key cryptography with key value of 4 first and then Transposition Cipher with the following details in mind, 10

i. $\sigma = \{1, 2, 3, 4\}$

ii. $\sigma(1) = 4,\ \sigma(2) = 3, \sigma(3) = 1, \sigma(4) = 2$

b) Use mathematical induction to prove that $2n < n!$ is valid for every integer n with $n \geq 4$. 9

c) The most commonly used procedure for generating *pseudorandom* numbers is the *linear congruential method*. We choose four integers: the modulus m, multiplier a, increment c, and seed x_0, with $2 \leq a < m, 0 \leq c < m$, and $0 \leq x_0 < m$. We generate a sequence of pseudorandom numbers $\{x_n\}$, with $0 \leq x_n < m$ for all n, by successively using the recursively defined function 6

$$x_{n+1} = (ax_n + c) \bmod m.$$

Now, based on the following information and considering $m = 9,\ a = 7,\ c = 4,\ x_0 = 3$ find the sequence of pseudorandom numbers generated by the *linear congruential method.*

3. a) Give a recursive algorithm for finding the reversal of a bit string. Using that algorithm find the reverse of the bit string 101101. Show step by step execution of your algorithm. 5+4

b) Give a recursive algorithm for computing the **GCD** (*Greatest Common Divisor*) of two nonnegative integers a and b with $a < b$. Using that algorithm find the **GCD** of the numbers $(25, 49)$. Show step by step execution of your algorithm. 5+4

c) If x is a real number then prove that, $\lfloor 2x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{2}{5} \right\rfloor$. 7

4 Consider the following adjacency matrix of a town map:

$\because$	a	b	c	d	e	f	g
a	0	1	0	0	2	0	0
b	1	0	1	0	0	2	0
c	0	1	0	1	2	0	0
d	0	0	1	0	1	0	2
e	2	0	2	1	0	0	0
f	0	2	0	0	0	0	0
g	0	0	0	2	0	0	0

From the above information do the following:

a) Draw the graph that is represented by the adjacency matrix. 3

b) What is an Euler path? Does the above graph have an Euler path or circuit? Explain your answer logically. 2+5

c) If this graph has an Euler path then find an Euler path for travelling from town a to town e. 3

d) With the help of Dirac's and Ore's Theorem find whether the graph has a Hamilton circuit or a Hamilton path or both. 6

e) From the above graph prove that "*an undirected graph has an even number of vertices of odd degree*". 6

5 a) Consider the following adjacency matrix for a directed graph

$\because$	a	b	c	d	e
a	1	1	1	0	1
b	0	0	0	1	0
c	1	1	1	0	0
d	0	1	0	0	0
e	1	0	1	1	0

From the above information do the following:

i. Draw the graph represented by the matrix. 2

ii. Mathematically show that $\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$. 3

iii. Does this graph have a Hamilton path and a circuit? Justify your answer. If there is any Hamilton path and/or circuit then write down the Hamilton path and/or circuit. 5+2

b) Determine whether the pair of graphs in *Figure* 1 are isomorphic. 10

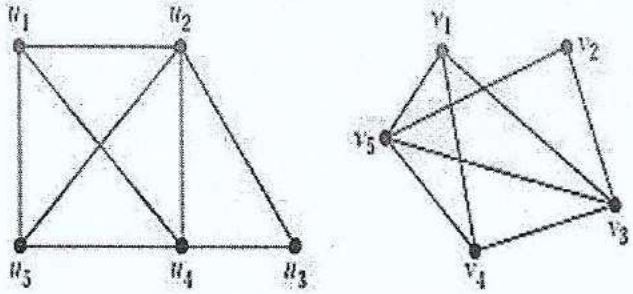

Figure: 1

c) Draw the following graphs Q_3 , $K_{3,3}$, W_8 and determine their chromatic number χ. 3

6 a) 3×3

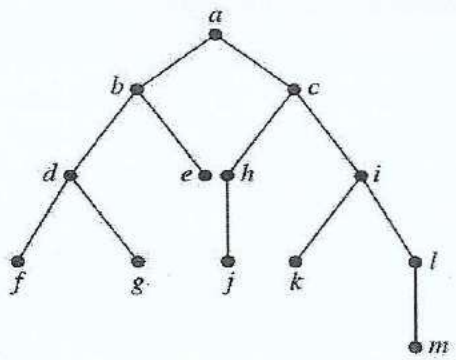

Figure : 1

From *Figure* 2 generate the sequence of nodes in the following methods of tree traversal:

i. Pre-order traversal
ii. In-order traversal
iii. Post-order traversal

b) Considering *Figure* 2, find out the following: 2×2

i. Descendants of nodes b and i.
ii. Ancestors of nodes m and j

c) Consider the following premises:
"It is not sunny this afternoon and it is colder than yesterday."
"We will go swimming only if it is sunny."
"If we do not go swimming, then we will take a canoe trip." and
"If we take a canoe trip, then we will be home by sunset."
Where,
p : *"It is sunny this afternoon."*
q : *"It is colder than yesterday."*
r : *"We will go swimming."*
s : *"We will take a canoe trip."*

Answer the following questions:

i. Express the premises using propositional logic. 4
ii. Using rules of inference show that the premises lead to the conclusive proposition, 8
t : *"We will be home by sunset."*

7 a) Using rules of inference show that the premises 7
"A student in this class has not read the book," and *"Everyone in this class passed the first exam"* imply the conclusion *"Someone who passed the first exam has not read the book."*

b) Determine whether each of the functions $f(a) = a \; \boldsymbol{div} \; d$ and $g(a) = a \; \boldsymbol{mod} \; d$, where d is a fixed positive integer, from $\mathbb{Z} \to \mathbb{Z}$, is one-to-one, and determine whether each of these functions is onto 8

c) Consider that f and g are the functions from the set of integers to the set of integers defined by
$f(x) = \frac{2x+3}{x+2}$ and $g(x) = \frac{3x+2}{x-3}$. Answer the following based on this information.

i. Find $fog(x)$ and $gof(x)$. 3+3

ii. Find the inverse of $f(x)$ 4

8 a) A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning. 10

b) Devise an algorithm for computing the quotient and remainder in a division operation. 7

c) Using the algorithm in 8(b) find the quotient and remainder of divisor is 5 and dividend is -21. You have to show step by step operation of the algorithm. 8