ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2019-2020**

DURATION: 1 Hour **FULL MARKS: 45**

Math 4341: Linear Algebra

General Instructions

- Write your **Name**, **Student-ID** and **Course Code** on the top of the first page. Maintain a **serial number** on the Top-right corner of each page.
- Answer all the questions. Figures in the right margin indicate marks.
- Sit in proper position and maintain the environment as per the Guidelines.
- No examinee is allowed to scan the file unless 1 hour is finished.
- For any circumstances, follow the instructions of the invigilator.

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1 Suppose, a group of 4 students from IUT CSE'18 went for a tour. They bought a number of souvenirs from there. While packing, they found there were 4 different items. Suppose the items are named A, B, C & D. Everyone bought at least one item or more than that. Now, they tried to put this purchase information into a matrix (Purchase Matrix). So, the columns represent the number of a particular item they bought and the rows represent individual purchase information (number of items) of the students.

a. It was found out that the items were arranged in the columns in this order- D->C->B->A. How do you put it in the correct order(alphabetical) in the matrix without directly manipulating the columns? 5

b. The purchase details are as follows- 7

Items	Student 1	Student 2	Student 3	Student 4
A	1	0	3	1
B	0	2	0	5
C	2	0	4	0
D	0	5	0	2

Let's say, V_1=subspace generated by the second column of the purchase matrix
V_2=subspace generated by the fourth column of the purchase matrix
Find out V_1 & V_2(Draw). What is V_1 **U** V_2 ? Is it a vector subspace as well?

c. Mention the dimension of all four fundamental subspaces of the purchase matrix. What can be one good basis for the columnspace? 5

2 Time (in -th seconds) and number of errors done by a machine is given below:

Time (in -th second)	Number of errors
-3	3
1	5
4	9

a. Can you write these data points as a simple linear system? 2
b. Find a linear equation that fits these points by minimizing the error. 10
c. Can you predict the number of errors done by the machine on the 7^{th} second? 3

3 The following system is given- 6

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array}\right]$$

Find out the values of ***h*** and ***k*** so that the system becomes-

a. Inconsistent. [No solution]
b. Consistent with infinitely many solutions.
c. Consistent with a unique solution.

4 Short Questions-

a. If a matrix has an eigenvalue with $\boldsymbol{\lambda}$=0, what can you decide about the rank and columnspace of the matrix? Is it a matrix with full rank as column? Do the columns span the whole vectorspace? 4

b. If a matrix has two eigenvalues $\boldsymbol{\lambda}_1$=5 and $\boldsymbol{\lambda}_2$=-21, 3

   i. Find out the determinant of the matrix.
   ii. Find out the sum of the diagonal elements of the matrix.