B.Sc. Engg. / HD CSE 2nd Semester (64)

11 August 2016 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

DURATION: 1 Hour 30 Minutes

SUMMER SEMESTER, 2015-2016

FULL MARKS: 75

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **4 (four)** questions. Answer any **3 (three)** of them.
Figures in the right margin indicate marks.

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1. a) Once upon a time there was a king named Ozymandias. He was a brave king and ruled a big kingdom. He was happy and relaxed until his only and beloved daughter fall ill. So ill that she took the death bed. The king became restless and ordered his men to bring all the famous physicians of that time. Many physicians came, but in vein. None of them could even diagnose the disease of the princess. Everyone lost their hope. Meanwhile an undistinguished and absurdly dressed physician came to the palace. He spent three days to diagnose the disease. After being sure he said, "I can treat the Princess to cure." Most of the people was doubtful about his capability. But he was their only chance. The weird physician treated her for 40 days. Proving peoples doubt wrong, the prince was on her feet again. The King was the happiest person among all. He said to the physician, "Ask for your reward. Whatever you wish will be granted." The physician was humble and straight. He asked for 3 gold coin for each days of diagnosis. But for the treatment period he gave a strange rule. The first day would cost 1 gold coin. For each of the other treatment days, the payment would be double of the previous day. People were laughing at him thinking he demanded very less. Being good at math, the physician knows how much he will get. 10
Calculate the total amount of the payment.

   b) Give a proof by contradiction of the theorem "If $3n+2$ is odd, then n is odd." 8

   c) For the following set of premises, what relevant conclusion or conclusions can be drawn? Mention the rules of inference used to obtain each conclusion from the premises. 7
   - "All foods that are healthy to eat do not taste good."
   - "Tofu is healthy to eat."
   - "You only eat what tastes good."
   - "You do not eat tofu."
   - "Cheeseburgers are not healthy to eat."

2. a) Express each of these statements using predicates and quantifiers. 12
   i. A passenger on an airline, qualifies as an elite flyer, if the passenger flies more than 25,000 miles in a year or takes more than 25 flights during that year.
   ii. A student must take at least 60 course hours, or at least 45 course hours and write a master's thesis, and receive a grade no lower than a B in all required courses, to receive a master's degree.
   iii. Whenever there is an active alert, all queued messages are transmitted.
   iv. Each participant on the conference call whom the host of the call did not put on a special list was billed.

   b) What is a Power Set? What is the power set of the set $\{\emptyset\}$? 5

c) There are three people (Alex, Brook, and Cody), one of whom is a knight, one a knave, and one a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth.
Alex says: "Cody is a knave."
Brook says: "Alex is a knight."
Cody says: "I am the spy."
Who is the knight, who the knave, and who the spy? Explain your answer.

3. a) What is the worst-case complexity of the insertion sort in terms of the number of comparisons made? The algorithm for insertion sort is given in figure 1,

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Procedure insertion sort(a1,a2,...,an: real numbers with n≥2)
For j:= 2 to n
       i := 1
       while aj >ai
              i := i+1
       m:= aj
       for k:= 0 to j-i-1
              aj-k := aj-k-1
       ai := m
{a1,...,an is in increasing order}
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Figure 1: Code listing for question 3.a)

b) Give big-O estimates for the following functions:
   i. $f(n) = \log(n!)$
   ii. $f(x) = (x+1)\log(x^2+1) + 3x^2$

4. a) Write short notes on:
   i. One-to-one function
   ii. Big theta
   iii. The Division Algorithm

b) Use the extended Euclidean algorithm to express gcd(144, 89) as a linear combination of 144 and 89.

c) Show that if a, b, k, and m are integers such that $k \geq 1$, $m \geq 2$, and $a \equiv b \pmod{m}$, then $a^k \equiv b^k \pmod{m}$.