B.Sc. Engg. CSE 7th Semester 1788 19 March 2018 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2017-2018**
DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

Math 4707: Probability and Stochastic Processes

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. A computer network consists of several stations connected by various media (usually cables). There are certain instances when no message is being transmitted (i.e., the channel is free). At such "suitable instances," each station will send a message with probability p independently of the other stations. However, if two or more stations send messages in a single "suitable instance," a collision will corrupt the messages and they will be discarded. These messages will be retransmitted until they reach their destination. Note that the probability p of sending a message by a station is assumed irrespective of the fact that the packet can be either a new one or a retransmitted one. Suppose that the network consists of N stations.
   a) Find the probability that at a "suitable instance" a message is initiated by one of the stations and will go through without a collision. 10
   b) Show that, to maximize the probability of a message going through with no collisions, exactly one message, on average, should be initiated at each "suitable instance." 10
   c) Find the limit of the maximum probability obtained in Question 1(b) as the number of stations of the network grows to infinity (∞). 5

2. The simplest error detection mechanism used in data communication is *parity checking*. Usually messages sent consist of characters, each character consisting of a number of bits (a *bit* is the smallest unit of information and is either 1 or 0). Assume that the number of bits in a character is 7. In parity checking, a 1 or 0 is appended to the end of each character at the transmitter to make the total number of 1's even (and the parity checking mechanism is known as even parity).

   The receiver checks the number of 1's in every character received, and if the number of 1's is odd it signals an error. Suppose that each bit in a character is received correctly with probability 0.999, independently of other bits of the characters.

   a) Find the probability that character is received in error, but the error is not detected by the parity check mechanism. 10
   b) Find the probability that the parity check mechanism detects the error, if one or more bits are incorrectly received. 8
   c) Suppose that a message consisting of six characters is transmitted. Find the probability that the message is erroneously received (at least one character is erroneously received), but none of the errors is detected by the parity check mechanism. 7

3. a) Suppose that the loss in a certain investment, in thousands of dollars, is a continuous random variable X with the following probability density function
   $$f_X(x) = \begin{cases} k(2x - 3x^2), & -1 < x < 0 \\ 0, & \text{otherwise.} \end{cases}$$
   Calculate the value of k and find the probability that the loss is at most \$500. 10
   b) A point is selected at random on a line segment of length l. Find the probability that the longer segment is at least twice as long as the shorter segment. 8

c) Every week the average number of wrong number phone calls received by a certain office is 7. If the number of wrong phone calls received is Poisson distributed, find the probability that it will receive 7

i. Exactly two wrong phone calls tomorrow.
ii. At least one wrong call by tomorrow.

4. a) Let X be a random variable with probability density function 10

$$f_X(x) = \begin{cases} \frac{1}{2}e^{-|x|}, & -\infty < x < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $E[X]$ and $Var[X]$.

b) Voice calls cost 20 cents each and data calls cost 30 cents each. Let C is the cost of one telephone call. The probability that a call is voice call is $P[V] = 0.6$, and the probability that a call is data call is $P[D] = 0.4$. Find $P_C(c)$ and $E[C]$. 7

c) The probability mass function of a discrete random variable X is given by 8

$$P_X(x) = \begin{cases} \frac{x}{15}, & x = 1, 2, \dots, 5 \\ 0, & \text{Otherwise.} \end{cases}$$

Find the expected value and the variance of the derived random variable $Y = X(6 - X)$.

Appendix A: PMF/PDF and the expected values of some Random Variables

Distribution	PMF/PDF	Expected value	Variance
Bernoulli	$P_X(x) = \begin{cases} 1-p & x=0 \\ p & x=1 \\ 0 & otherwise \end{cases}$	$E[X] = p$	$Var[X] = p(1-p)$
Geometric	$P_X(x) = \begin{cases} p(1-p)^{x-1} & x \geq 1 \\ 0 & otherwise \end{cases}$	$E[X] = 1/p$	$Var[X] = (1-p)/p^2$
Binomial	$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 1, \dots, n \\ 0 & otherwise \end{cases}$	$E[X] = np$	$Var[X] = np(1-p)$
Pascal	$P_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{x-k} & x = k, k+1, \dots \\ 0 & otherwise \end{cases}$	$E[X] = k/p$	$Var[X] = k(1-p)/p^2$
Poisson	$P_X(x) = \begin{cases} \frac{(\lambda T)^x e^{-(\lambda T)}}{x!} & x \geq 0 \\ 0 & otherwise \end{cases}$	$E[X] = \alpha$ $\alpha = \lambda T$	$Var[X] = \alpha$
Uniform (Discrete)	$P_X(x) = \begin{cases} \frac{1}{b-a+1}, & x = a, a+1, a+2, \dots, b \\ 0, & \text{otherwise} \end{cases}$	$E[X] = \frac{a+b}{2}$	$Var[X] = \frac{(b-a)(b-a+2)}{12}$
Exponential	$f_X(x) = \begin{cases} a e^{-ax} & x \geq 0 \\ 0 & otherwise \end{cases}$	$E[X] = 1/a$	$Var[X] = 1/a^2$
Gaussian	$f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} & \sigma > 0 \\ 0 & otherwise \end{cases}$	$E[X] = \mu$	$Var[X] = \sigma^2$
Uniform (Continuous)	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b \\ 0, & otherwise \end{cases}$	$E[X] = \frac{a+b}{2}$	$Var[X] = \frac{(b-a)^2}{12}$