B.Sc. Engg. CSE 4th Semester (115) **15 January 2021**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

FINAL EXAMINATION **SUMMER SEMESTER, 2019-2020**

DURATION: 1.5 Hours **FULL MARKS: 75**

Math 4441: Probability and Statistics

You are allowed to assume any missing data.

If exact values are not available in the tables provided, then use closest values.

There are 6 **(six)** questions. Answer all of them.

Figures in the right margin indicate full marks.

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1. Consider a coin that has a probability p of getting heads when it is tossed. Assume that you tossed the coin repeatedly until you observe the pattern T, T, H. More specifically, you stop tossing the coin when you observe a sequence of two tails followed by a head. Let X denote the number of tosses made. Find the expected value of X. 10

2. First a point X is selected at random from the interval (0,1). Then another point Y is selected at random from the interval $(X, 1)$ in such a way so that $X + Y \geq 1$. Find the probability density function of Y. 10

3. A poll is taken to determine the fraction p of the students of IUT that would support a decision whether face-to-face classes will be started from the Winter semester of the next academic year.
   a) Assume $p = 0.5$. Use the central limit theorem to find the approximate probability that in a poll of 30 students at least 20 students support the decision of face-to-face class. 8
   b) With the assumption that both p and n (the number of random students polled) are unknown, let θ be the fraction of the polled students who support the decision. Find the value of n in order to have a 90% confidence that θ is within 0.01 of the true value of p. 7

4. Suppose a student collects measurements of packet delays from a network which uses a newly developed protocol at the MAC layer. The observations are $x_1, x_2, \dots, x_n$, and are the values of n IID random variables. Suppose the observed values follow a theoretical distribution, known as Rayleigh distribution, with parameters τ given by 10

$$f_X(x) = x\tau e^{-\frac{1}{2}\tau x^2}$$

   Find the maximum likelihood estimate for the parameter τ.

5. Suppose data is collected at the main gate of IUT measuring the inter arrival times of the students. Assume that the size of the data in a given day is 45 with the sample mean $\bar{x} = 5.0$ and the sample standard deviation is $s = 4.0$. Let the data follows a normal distribution.
   a) Find an 80% confidence interval for the mean μ. 7
   b) Find an 80% confidence interval for the variance. 8

6. An attempt was recently made to verify whether female students are more serious about their study as compared to the male students in IUT. To study this claim, the university appointed a researcher to obtain sample of per day study time of the fourth year students. The researcher found the following data about the per day study time for both the female and male students.

Female students		Male students	
Sample size	55	Sample size	72
Sample mean	10.80	Sample mean	12.20
Sample variance	0.90	Sample variance	1.1

a) What hypothesis should be tested? Mention both the null and alternative hypotheses. 4

b) Find the p value. 8

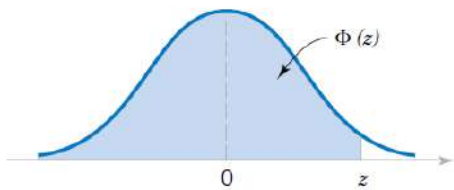
c) What would you conclude from the study? 3

Appendix A: PMF/PDF and the expected values of some Random Variables

Distribution	PMF/PDF	Expected value	Variance
Bernoulli	$P_X(x) = \begin{cases} 1-p & x=0 \\ p & x=1 \\ 0 & otherwise \end{cases}$	$E[X] = p$	$Var[X] = p(1-p)$
Geometric	$P_X(x) = \begin{cases} p(1-p)^{x-1} & x \geq 1 \\ 0 & otherwise \end{cases}$	$E[X] = 1/p$	$Var[X] = (1-p)/p^2$
Binomial	$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 1, \dots, n \\ 0 & otherwise \end{cases}$	$E[X] = np$	$Var[X] = np(1-p)$
Pascal	$P_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{x-k} & x = k, k+1, \dots \\ 0 & otherwise \end{cases}$	$E[X] = k/p$	$Var[X] = k(1-p)/p^2$
Poisson	$P_X(x) = \begin{cases} \frac{(\lambda T)^x e^{-(\lambda T)}}{x!} & x \geq 0 \\ 0 & otherwise \end{cases}$	$E[X] = \alpha$ $\alpha = \lambda T$	$Var[X] = \alpha$
Uniform (discrete)	$P_X(x) = \begin{cases} \frac{1}{b-a+1}, & x = a, a+1, a+2, \dots, b \\ 0, & \text{otherwise} \end{cases}$	$E[X] = \frac{a+b}{2}$	$Var[X] = \frac{(b-a)(b-a+2)}{12}$
Exponential	$f_X(x) = \begin{cases} ae^{-ax} & x \geq 0 \\ 0 & otherwise \end{cases}$	$E[X] = 1/a$	$Var[X] = 1/a^2$
Gaussian	$f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} & \sigma > 0 \\ 0 & otherwise \end{cases}$	$E[X] = \mu$	$Var[X] = \sigma^2$
Uniform (Continuous)	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b \\ 0, & otherwise \end{cases}$	$E[X] = \frac{a+b}{2}$	$Var[X] = \frac{(b-a)^2}{12}$
Sample variance		$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$	
Sample mean		$\bar{x} = \frac{\sum_{i=1}^{n} Xi}{n}$	
Variance		$Var[X] = E[(X-\mu)^2] = E[X^2] - (E[X])^2$	
Standardization of Normal Random Variable		$Z = \frac{X-\mu}{\sigma}$	
Joint distribution of $\bar{X}$ and S^2		$(n-1)\frac{S^2}{\sigma^2} \sim {\chi^2}_{n-1}$	
Distribution of $\bar{X}$ with unknown σ^2		$\sqrt{n}\frac{(\bar{X}-\mu)}{S} \sim t_{n-1}$	
Joint PDF of X and Y		$f_{XY}(x,y) = f_{X\|Y}(x\|y) f_Y(y)$	
Joint PMF of X and Y		$P_{XY}(x,y) = P_{X\|Y}(x\|y) P_Y(y)$	

Appendix B: CDF of Standard Normal Distribution

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

Appendix C

t-table of *left tail* probabilities. (The tables show $P(T < t)$ for $T \sim t(df)$.)

t\df	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5628	0.5700	0.5729	0.5744	0.5753	0.5760	0.5764	0.5768	0.5770
0.4	0.6211	0.6361	0.6420	0.6452	0.6472	0.6485	0.6495	0.6502	0.6508
0.6	0.6720	0.6953	0.7046	0.7096	0.7127	0.7148	0.7163	0.7174	0.7183
0.8	0.7148	0.7462	0.7589	0.7657	0.7700	0.7729	0.7750	0.7766	0.7778
1.0	0.7500	0.7887	0.8045	0.8130	0.8184	0.8220	0.8247	0.8267	0.8283
1.2	0.7789	0.8235	0.8419	0.8518	0.8581	0.8623	0.8654	0.8678	0.8696
1.4	0.8026	0.8518	0.8720	0.8829	0.8898	0.8945	0.8979	0.9005	0.9025
1.6	0.8222	0.8746	0.8960	0.9076	0.9148	0.9196	0.9232	0.9259	0.9280
1.8	0.8386	0.8932	0.9152	0.9269	0.9341	0.9390	0.9426	0.9452	0.9473
2.0	0.8524	0.9082	0.9303	0.9419	0.9490	0.9538	0.9572	0.9597	0.9617
2.2	0.8642	0.9206	0.9424	0.9537	0.9605	0.9649	0.9681	0.9705	0.9723
2.4	0.8743	0.9308	0.9521	0.9628	0.9692	0.9734	0.9763	0.9784	0.9801
2.6	0.8831	0.9392	0.9598	0.9700	0.9759	0.9797	0.9823	0.9842	0.9856
2.8	0.8908	0.9463	0.9661	0.9756	0.9810	0.9844	0.9867	0.9884	0.9896
3.0	0.8976	0.9523	0.9712	0.9800	0.9850	0.9880	0.9900	0.9915	0.9925
3.2	0.9036	0.9573	0.9753	0.9835	0.9880	0.9907	0.9925	0.9937	0.9946
3.4	0.9089	0.9617	0.9788	0.9864	0.9904	0.9928	0.9943	0.9953	0.9961
3.6	0.9138	0.9654	0.9816	0.9886	0.9922	0.9943	0.9956	0.9965	0.9971
3.8	0.9181	0.9686	0.9840	0.9904	0.9937	0.9955	0.9966	0.9974	0.9979
4.0	0.9220	0.9714	0.9860	0.9919	0.9948	0.9964	0.9974	0.9980	0.9984

t\df	10	11	12	13	14	15	16	17	18	19
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5773	0.5774	0.5776	0.5777	0.5778	0.5779	0.5780	0.5781	0.5781	0.5782
0.4	0.6512	0.6516	0.6519	0.6522	0.6524	0.6526	0.6528	0.6529	0.6531	0.6532
0.6	0.7191	0.7197	0.7202	0.7206	0.7210	0.7213	0.7215	0.7218	0.7220	0.7222
0.8	0.7788	0.7797	0.7804	0.7810	0.7815	0.7819	0.7823	0.7826	0.7829	0.7832
1.0	0.8296	0.8306	0.8315	0.8322	0.8329	0.8334	0.8339	0.8343	0.8347	0.8351
1.2	0.8711	0.8723	0.8734	0.8742	0.8750	0.8756	0.8762	0.8767	0.8772	0.8776
1.4	0.9041	0.9055	0.9066	0.9075	0.9084	0.9091	0.9097	0.9103	0.9107	0.9112
1.6	0.9297	0.9310	0.9322	0.9332	0.9340	0.9348	0.9354	0.9360	0.9365	0.9370
1.8	0.9490	0.9503	0.9515	0.9525	0.9533	0.9540	0.9546	0.9552	0.9557	0.9561
2.0	0.9633	0.9646	0.9657	0.9666	0.9674	0.9680	0.9686	0.9691	0.9696	0.9700
2.2	0.9738	0.9750	0.9759	0.9768	0.9774	0.9781	0.9786	0.9790	0.9794	0.9798
2.4	0.9813	0.9824	0.9832	0.9840	0.9846	0.9851	0.9855	0.9859	0.9863	0.9866
2.6	0.9868	0.9877	0.9884	0.9890	0.9895	0.9900	0.9903	0.9907	0.9910	0.9912
2.8	0.9906	0.9914	0.9920	0.9925	0.9929	0.9933	0.9936	0.9938	0.9941	0.9943
3.0	0.9933	0.9940	0.9945	0.9949	0.9952	0.9955	0.9958	0.9960	0.9962	0.9963

t\df	20	21	22	23	24	25	26	27	28	29
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5782	0.5783	0.5783	0.5784	0.5784	0.5785	0.5785	0.5785	0.5785	0.5786
0.4	0.6533	0.6534	0.6535	0.6536	0.6537	0.6537	0.6538	0.6538	0.6539	0.6540
0.6	0.7224	0.7225	0.7227	0.7228	0.7229	0.7230	0.7231	0.7232	0.7233	0.7234
0.8	0.7834	0.7837	0.7839	0.7841	0.7842	0.7844	0.7845	0.7847	0.7848	0.7849
1.0	0.8354	0.8357	0.8359	0.8361	0.8364	0.8366	0.8367	0.8369	0.8371	0.8372
1.2	0.8779	0.8782	0.8785	0.8788	0.8791	0.8793	0.8795	0.8797	0.8799	0.8801
1.4	0.9116	0.9119	0.9123	0.9126	0.9128	0.9131	0.9133	0.9136	0.9138	0.9139
1.6	0.9374	0.9377	0.9381	0.9384	0.9387	0.9389	0.9392	0.9394	0.9396	0.9398
1.8	0.9565	0.9569	0.9572	0.9575	0.9578	0.9580	0.9583	0.9585	0.9587	0.9589
2.0	0.9704	0.9707	0.9710	0.9713	0.9715	0.9718	0.9720	0.9722	0.9724	0.9725
2.2	0.9801	0.9804	0.9807	0.9809	0.9812	0.9814	0.9816	0.9817	0.9819	0.9820
2.4	0.9869	0.9871	0.9874	0.9876	0.9877	0.9879	0.9881	0.9882	0.9884	0.9885
2.6	0.9914	0.9916	0.9918	0.9920	0.9921	0.9923	0.9924	0.9925	0.9926	0.9927
2.8	0.9945	0.9946	0.9948	0.9949	0.9950	0.9951	0.9952	0.9953	0.9954	0.9955
3.0	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9971	0.9972	0.9973

t\df	30	31	32	33	34	35	36	37	38	39
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5786	0.5786	0.5786	0.5786	0.5787	0.5787	0.5787	0.5787	0.5787	0.5787
0.4	0.6540	0.6541	0.6541	0.6541	0.6542	0.6542	0.6542	0.6543	0.6543	0.6543
0.6	0.7235	0.7236	0.7236	0.7237	0.7238	0.7238	0.7239	0.7239	0.7240	0.7240
0.8	0.7850	0.7851	0.7852	0.7853	0.7854	0.7854	0.7855	0.7856	0.7857	0.7857
1.0	0.8373	0.8375	0.8376	0.8377	0.8378	0.8379	0.8380	0.8381	0.8382	0.8383
1.2	0.8802	0.8804	0.8805	0.8807	0.8808	0.8809	0.8810	0.8811	0.8812	0.8813
1.4	0.9141	0.9143	0.9144	0.9146	0.9147	0.9148	0.9150	0.9151	0.9152	0.9153
1.6	0.9400	0.9401	0.9403	0.9404	0.9406	0.9407	0.9408	0.9409	0.9411	0.9412
1.8	0.9590	0.9592	0.9594	0.9595	0.9596	0.9598	0.9599	0.9600	0.9601	0.9602
2.0	0.9727	0.9728	0.9730	0.9731	0.9732	0.9733	0.9735	0.9736	0.9737	0.9738
2.2	0.9822	0.9823	0.9824	0.9825	0.9826	0.9827	0.9828	0.9829	0.9830	0.9831
2.4	0.9886	0.9887	0.9888	0.9889	0.9890	0.9891	0.9892	0.9892	0.9893	0.9894
2.6	0.9928	0.9929	0.9930	0.9931	0.9932	0.9932	0.9933	0.9933	0.9934	0.9935
2.8	0.9956	0.9956	0.9957	0.9958	0.9958	0.9959	0.9959	0.9960	0.9960	0.9960
3.0	0.9973	0.9974	0.9974	0.9974	0.9975	0.9975	0.9976	0.9976	0.9976	0.9977

t\df	40	41	42	43	44	45	46	47	48	49
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5788	0.5788	0.5788	0.5788	0.5788	0.5788	0.5788	0.5788	0.5788	0.5788
0.4	0.6544	0.6544	0.6544	0.6544	0.6545	0.6545	0.6545	0.6545	0.6545	0.6546
0.6	0.7241	0.7241	0.7241	0.7242	0.7242	0.7242	0.7243	0.7243	0.7243	0.7244
0.8	0.7858	0.7858	0.7859	0.7859	0.7860	0.7860	0.7861	0.7861	0.7862	0.7862
1.0	0.8383	0.8384	0.8385	0.8385	0.8386	0.8387	0.8387	0.8388	0.8388	0.8389
1.2	0.8814	0.8815	0.8816	0.8816	0.8817	0.8818	0.8819	0.8819	0.8820	0.8820
1.4	0.9154	0.9155	0.9156	0.9157	0.9157	0.9158	0.9159	0.9160	0.9160	0.9161
1.6	0.9413	0.9414	0.9415	0.9415	0.9416	0.9417	0.9418	0.9419	0.9419	0.9420
1.8	0.9603	0.9604	0.9605	0.9606	0.9606	0.9607	0.9608	0.9609	0.9609	0.9610
2.0	0.9738	0.9739	0.9740	0.9741	0.9742	0.9742	0.9743	0.9744	0.9744	0.9745
2.2	0.9832	0.9833	0.9833	0.9834	0.9834	0.9835	0.9836	0.9836	0.9837	0.9837
2.4	0.9894	0.9895	0.9895	0.9896	0.9897	0.9897	0.9898	0.9898	0.9898	0.9899
2.6	0.9935	0.9935	0.9936	0.9936	0.9937	0.9937	0.9938	0.9938	0.9938	0.9939
2.8	0.9961	0.9961	0.9962	0.9962	0.9962	0.9962	0.9963	0.9963	0.9963	0.9964
3.0	0.9977	0.9977	0.9977	0.9978	0.9978	0.9978	0.9978	0.9978	0.9979	0.9979

Appendix D

Table of χ^2 critical values (right-tail)

The table shows $c_{df,p}$ = the $1-p$ quantile of $\chi^2(df)$.

In R notation $c_{df,p}$ = `qchisq(1-p, df)`.

df\p	0.010	0.025	0.050	0.100	0.200	0.300	0.500	0.700	0.800	0.900	0.950	0.975	0.990
1	6.63	5.02	3.84	2.71	1.64	1.07	0.45	0.15	0.06	0.02	0.00	0.00	0.00
2	9.21	7.38	5.99	4.61	3.22	2.41	1.39	0.71	0.45	0.21	0.10	0.05	0.02
3	11.34	9.35	7.81	6.25	4.64	3.66	2.37	1.42	1.01	0.58	0.35	0.22	0.11
4	13.28	11.14	9.49	7.78	5.99	4.88	3.36	2.19	1.65	1.06	0.71	0.48	0.30
5	15.09	12.83	11.07	9.24	7.29	6.06	4.35	3.00	2.34	1.61	1.15	0.83	0.55
6	16.81	14.45	12.59	10.64	8.56	7.23	5.35	3.83	3.07	2.20	1.64	1.24	0.87
7	18.48	16.01	14.07	12.02	9.80	8.38	6.35	4.67	3.82	2.83	2.17	1.69	1.24
8	20.09	17.53	15.51	13.36	11.03	9.52	7.34	5.53	4.59	3.49	2.73	2.18	1.65
9	21.67	19.02	16.92	14.68	12.24	10.66	8.34	6.39	5.38	4.17	3.33	2.70	2.09
10	23.21	20.48	18.31	15.99	13.44	11.78	9.34	7.27	6.18	4.87	3.94	3.25	2.56
16	32.00	28.85	26.30	23.54	20.47	18.42	15.34	12.62	11.15	9.31	7.96	6.91	5.81
17	33.41	30.19	27.59	24.77	21.61	19.51	16.34	13.53	12.00	10.09	8.67	7.56	6.41
18	34.81	31.53	28.87	25.99	22.76	20.60	17.34	14.44	12.86	10.86	9.39	8.23	7.01
19	36.19	32.85	30.14	27.20	23.90	21.69	18.34	15.35	13.72	11.65	10.12	8.91	7.63
20	37.57	34.17	31.41	28.41	25.04	22.77	19.34	16.27	14.58	12.44	10.85	9.59	8.26
21	38.93	35.48	32.67	29.62	26.17	23.86	20.34	17.18	15.44	13.24	11.59	10.28	8.90
22	40.29	36.78	33.92	30.81	27.30	24.94	21.34	18.10	16.31	14.04	12.34	10.98	9.54
23	41.64	38.08	35.17	32.01	28.43	26.02	22.34	19.02	17.19	14.85	13.09	11.69	10.20
24	42.98	39.36	36.42	33.20	29.55	27.10	23.34	19.94	18.06	15.66	13.85	12.40	10.86
25	44.31	40.65	37.65	34.38	30.68	28.17	24.34	20.87	18.94	16.47	14.61	13.12	11.52
30	50.89	46.98	43.77	40.26	36.25	33.53	29.34	25.51	23.36	20.60	18.49	16.79	14.95
31	52.19	48.23	44.99	41.42	37.36	34.60	30.34	26.44	24.26	21.43	19.28	17.54	15.66
32	53.49	49.48	46.19	42.58	38.47	35.66	31.34	27.37	25.15	22.27	20.07	18.29	16.36
33	54.78	50.73	47.40	43.75	39.57	36.73	32.34	28.31	26.04	23.11	20.87	19.05	17.07
34	56.06	51.97	48.60	44.90	40.68	37.80	33.34	29.24	26.94	23.95	21.66	19.81	17.79
35	57.34	53.20	49.80	46.06	41.78	38.86	34.34	30.18	27.84	24.80	22.47	20.57	18.51
40	63.69	59.34	55.76	51.81	47.27	44.16	39.34	34.87	32.34	29.05	26.51	24.43	22.16
41	64.95	60.56	56.94	52.95	48.36	45.22	40.34	35.81	33.25	29.91	27.33	25.21	22.91
42	66.21	61.78	58.12	54.09	49.46	46.28	41.34	36.75	34.16	30.77	28.14	26.00	23.65
43	67.46	62.99	59.30	55.23	50.55	47.34	42.34	37.70	35.07	31.63	28.96	26.79	24.40
44	68.71	64.20	60.48	56.37	51.64	48.40	43.34	38.64	35.97	32.49	29.79	27.57	25.15
45	69.96	65.41	61.66	57.51	52.73	49.45	44.34	39.58	36.88	33.35	30.61	28.37	25.90
46	71.20	66.62	62.83	58.64	53.82	50.51	45.34	40.53	37.80	34.22	31.44	29.16	26.66
47	72.44	67.82	64.00	59.77	54.91	51.56	46.34	41.47	38.71	35.08	32.27	29.96	27.42
48	73.68	69.02	65.17	60.91	55.99	52.62	47.34	42.42	39.62	35.95	33.10	30.75	28.18
49	74.92	70.22	66.34	62.04	57.08	53.67	48.33	43.37	40.53	36.82	33.93	31.55	28.94