Theorem 0.1 *(Galois Solvability Theorem) Let F be a field of char 0. Then an extension is solvable by radicals if and only if L is a subextension of a Galois extension E/F with a solvable Galois group.*

Proof: ($\Longrightarrow$) Given $F = F_0 \subset F_1 \subset F_2 \subset \ldots F_m$. s.t. $F_i = F_{i-1}[\alpha_i]$, $\alpha \in F_i$. $\alpha_i^{r_i} \in F_{i-1}$.

$L \subset F_m$.

Assume L splitting field of a polynomial f.

Let $n = r_1 \ldots r_m$, let Ω be a field containing F_m and a primitive n^{th} root of 1.

Let $\tilde{E}$ be the Galois closure of $F_m[\zeta]$ in Ω.

■