(X, d) a metric space, $A \subset X$

$x \in A \subset X$ is an interior point of A if $\exists r > 0$ s.t.$B_r(x) \subset A$.

$x \in A \subset X$ is an exterior point if A if $\exists r > 0$ s.t $B_r(x) \subset A^c$

$x \in A \subset X$ is a boundary point of A if every ball $B_r(a)$ contains points of A and A^c

Prop. $X = intA \cup \partial A \cup extA$ (pairwise disjoint)

Ext A $= intA^c$, $intA = ext(A^c)$

$intA \subset A$, $extA \subset A^c$

TODO: Diagram.

Definition 0.1 *$x \in X$ is a limit point of A if every ball $B_r(x)$ contains points of A other than X. X may or may not be an element of A.*

E.g. (0,1]

0 is a limit point, 1 is a limit point. $0 \notin A$, $1 \in A$

$x \in A$ is an isolated point of A if $\exists r > 0$ s.t. $B_r(x) \cup A = \{x\}$

Closure of A $\bar{A} = A \cup \{limit\ points\ of\ a\}$

Proposition 0.2

1. *A limit point of L of need not be an element of A*
2. *If x is a limit point of A then every open ball $B_r(x)$ contains infinitely many points of A.*
3. $A \subset \bar{A}$
4. *Every point of $\bar{A}$ is either a limit point of A or an isolated point of A, but not both.*
5. *$x \in \bar{A}$ iff every ball $B_r(x)$ contains a point fo A.*

Proof: 2) If finitely many points, just draw a ball of smaller distance -¿ isolated point, not limit point by contradiction. ■

Let $A = [0, 1] \cup 3 \subset \mathbb{R}$

$intA = (0, 1)$ $\partial A = \{0, 1, 3\}$

3 is an isolated point

Limit points of $A = [0, 1]$

Let $A = [0, 1] \cup 3 \subset\ [0, 2] \cup \{3\}$

$intA = [0, 1)$

Theorem 0.3 $\bar{A} = (extA)^x$

$\bar{A} = intA \cup \partial A$

$\bar{A} = A \cup \partial A$

Proof: 1) $x \in \bar{A}$ iff every ball B_r(x) contains at least one element of A.

2) WEe have $X = intA \cup \partial A \cup extA$, but these are pairwise disjoint. It follws that $\bar{A} = intA \cup \partial A$

3) Let $x \in A \cup \partial A$

$x \in \partial A \implies x \in intA \cup \partial A = \bar{A}$

■

0.0.1 Special case $A = B_r(x) \subset \mathbb{R}^n$

$intA = A$

$extA = \{y : d(y, x) > r\}$

$\partial A = y : d(y, x) = r$

$\bar{A} = \{y : d(y, x) \leq r\}$

0.0.2 General case

$intA = A$

$extA \supset \{y : d(y, x) > r\}$

$\partial A \subset y : d(y, x) = r$

$\bar{A} \subset \{y : d(y, x) \leq r\}$

Ex. $X = 0, 1$

d(0,0) = 0 d(1,1) = 0 d(0,1) - d(1,0) = 1

A = $B_1(0) = \{x : d(x, 0) < 1\} = 0$

$intA = A$

$extA = \{1\}$

$\partial A = \emptyset$

$\bar{A} = A$