Fundamental of Matrix Computation: Section 1.2

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June 22, 2021

1 Exercise 1.2.4

Prove that A^{-1} exists, there can be no nonzero y for which $Ay = 0$.

$$Ay = 0 \tag{1}$$
$$A^{-1}Ay = A^{-1}0 \tag{2}$$
$$y = 0 \tag{3}$$

2 Exercise 1.2.5

Prove that if A^{-1} exists, then $\det(A) \neq 0$.

First, we know that $\det(AB) = \det(A)\det(B)$ and $A^{-1}A = I$.

$$A^{-1}A = I \tag{4}$$
$$\det(A^{-1})\det(A) = \det(I) \tag{5}$$

If $\det(A) = 0$, then $\det(A^{-1})\det(A) = 0$. But, $\det(A^{-1})\det(A) \neq 0$, it equals the determinant of the identity matrix which is a hypercube of volume 1.

3 Exercise 1.2.6

Identify the system of equations that enables us to determine the voltages at each node.

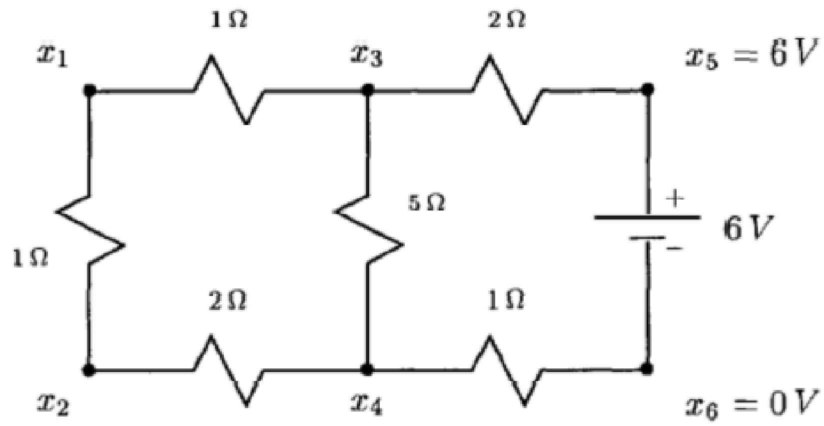

Fig. 1.1 Solve for the nodal voltages.

For x_1:

$$\begin{align}
x_1 - x_3 &= 1I_3 \tag{6}\\
I_3 &= x_1 - x_3 \tag{7}\\
&\dots \tag{8}\\
x_1 - x_2 &= 1I_2 \tag{9}\\
I_2 &= x_1 - x_2 \tag{10}\\
&\dots \tag{11}\\
I_2 + I_3 &= 0 \tag{12}\\
x_1 - x_2 + x_1 - x_3 &= 0 \tag{13}\\
2x_1 - x_2 - x_3 &= 0 \tag{14}
\end{align}$$

For x_2:

$$x_2 - x_1 = 1I_1 \tag{15}$$
$$I_1 = x_2 - x_1 \tag{16}$$
$$\dots \tag{17}$$
$$x_2 - x_4 = 2I_4 \tag{18}$$
$$I_4 = 0.5(x_2 - x_4) \tag{19}$$
$$\dots \tag{20}$$
$$I_1 + I_4 = 0 \tag{21}$$
$$x_2 - x_1 + 0.5(x_2 - x_4) = 0 \tag{22}$$
$$-x_1 + 1.5x_2 - 0.5x_4 = 0 \tag{23}$$

For x_3:

$$x_3 - x_1 = 1I_1 \tag{24}$$
$$I_1 = x_3 - x_1 \tag{25}$$
$$\dots \tag{26}$$
$$x_3 - x_4 = 5I_4 \tag{27}$$
$$I_4 = 0.2(x_3 - x_4) \tag{28}$$
$$\dots \tag{29}$$
$$x_3 - x_5 = x_3 - 6 = 2I_5 \tag{30}$$
$$I_5 = 0.5(x_3 - 6) \tag{31}$$
$$\dots \tag{32}$$
$$I_1 + I_4 + I_5 = 0 \tag{33}$$
$$x_3 - x_1 + 0.2(x_3 - x_4) + 0.5(x_3 - 6) = 0 \tag{34}$$
$$-x_1 + 1.7x_3 - 0.2x_4 = 3 \tag{35}$$

For x_4:

$$x_4 - x_2 = 2I_2 \tag{36}$$
$$I_2 = 0.5(x_4 - x_2) \tag{37}$$
$$\dots \tag{38}$$
$$x_4 - x_3 = 5I_4 \tag{39}$$
$$I_3 = 0.2(x_4 - x_3) \tag{40}$$
$$\dots \tag{41}$$
$$x_4 - x_6 = x_4 - 0 = I_6 \tag{42}$$
$$I_6 = x_4 - 0) \tag{43}$$
$$\dots \tag{44}$$
$$I_2 + I_3 + I_6 = 0 \tag{45}$$
$$0.5(x_4 - x_2) + 0.2(x_4 - x_3) + x_4 = 0 \tag{46}$$
$$-0.5x_2 - 0.2x_3 + 1.7x_4 = 0 \tag{47}$$

All of these manipulations resolve to the following system:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 1.5 & 0 & -0.5 \\ -1 & 0 & 1.7 & -0.2 \\ 0 & -0.5 & -0.2 & 1.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} \tag{48}$$