1 Intro

Here are the self-consistent equations:

$$f_i = -\log \sum_j \frac{\exp[-u_i(x_j)]}{\sum_k N_k \exp[f_k - u_k(x_j)]}$$

$$c_i = \sum_j \frac{q_i(x_j)}{\sum_k N_k c_k^{-1} q_k(x_j)}$$

2 Exp Space

Let

$$Q_{ij} = q_i(x_j)$$

and

$$R_{ij} = Q_{ij} N_i$$

Then

$$c_i = \sum_j \frac{Q_{ij}}{\sum_k R_{kj} c_k^{-1}}$$

3 Log Space

Let's work out an optimized form of the log-space calculation.

Let

$$\mu_{ij} = \log(N_i) - u_i(x_j)$$

Thus

$$f_i = -\log \sum_j \frac{\exp[-u_i(x_j)]}{\sum_k \exp[f_k + \mu_{kj}]}$$

$$f_i = -\log \sum_j \exp[-u_i(x_j) - \log \sum_k \exp[f_k + \mu_{kj}]]$$

Thus, we need to perform two logsumexp calculations:

$$s_j = -\log \sum_k \exp[f_k + \mu_{kj}]$$

$$f_i = -\log \sum_j \exp[-u_i(x_j) + s_j]$$