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HW6 - bayes II
CS 470

1. (Adapted from Pearl (1988)). Three prisoners, A, B, and C, are locked in their cells. It is common knowledge that one of them is to be executed the next day and the others are to be pardoned. Only the Governor knows which one will be executed. Prisoner A asks the guard a favor: "Please ask the Governor who will be executed, and then take a message to one of my friends B or C to let him know that he will be pardoned in the morning." The Guard agrees, and comes back later and tells A that he gave the pardon message to B. What are A's chances of being executed, given this information? Show how you obtain this answer using Bayes rule.

Let
A = prisoner A is executed
B = prisoner B is executed
C = prisoner C is executed

P(A) = P(B) = P(C) = 1/3
(note the sum of the three events = 1)

b = prisoner A finds out that prisoner B is pardoned
P(b|A) = 1/2 -> the guard could tell A that B or C are being pardoned
P(b|B) = 0 -> the guard would never tell A that B is being pardoned if he will be executed
P(b|C) = 1 -> if C is being executed, the guard has no choice but to tell A that B will be pardoned

P(A|b) = P(b|A)*P(A) / P(b|A)P(A)+P(b|B)P(B)+P(b|C)P(C)
= P(b|A) / P(b|A)+P(b|B)+P(c|C)
= 1/2 / 1/2+0+1
= 1/2 / 3/2
= 1/3

2. Suppose that you were going to write a small program that simulates what message the Guard gives to prisoner A. What message would the Guard bring if prisoner C was being executed? If prisoner B was being executed? If prisoner A was being executed?

Probability of the two events (execution + message) occurring together:

Guard tells A:	Event: A is executed	Event: B is executed	Event: C is executed
B is pardoned	1/2	0	1
C is pardoned	1/2	1	0

When prisoner C is being executed, the guard must bring A the message that B was pardoned. When prisoner B is being executed, the guard must bring A the message that C

was pardoned. When prisoner A is being executed, the guard may bring one of two messages: that B or C are being pardoned.

3. Write a small program that randomly selects which of the three prisoners will be executed. (Hint, use tmp=rand(); if tmp<=0.33 A is executed; elseif tmp<0.67 B is executed; else C is executed.) Use the model of the Guard that you created in part 2 and report which message is delivered to prisoner A. Run the program several times, keeping track of which prisoner would actually be executed whenever the Guard delivers a message to prisoner A stating that prisoner B gets pardoned. What percentage of time was prisoner A the one who would have been executed? Prisoner B? Prisoner C?

My program outputted:

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A is executed: [0, 16658, 16877]
B is executed: [0,     0, 33212]
C is executed: [0, 33252,     0]
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Interpret this as follows: when A is executed, A was told A was pardoned 0 times, B was pardoned 16658 times, and C was pardoned 16877 times. This is the same format for when B and C are executed.

The code ran a total of 100,000 times. Each prisoner was executed ⅓ of the time, regardless of who A was told would be pardoned.