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CS 470 – HW9 - Decisions

Utility: [9 points] If I am slightly risk averse which do I prefer in each of the following cases. Give a mathematical justification for your conclusion:

1. a=[0.5,$900;0.5,$800] or b=[0.1,$8750;0.9,$0]

The average value of a is:

0.5*900 + 0.5+800 = $850

The average value of b is:

0.1*8750 + 0.9*0 = $875

Over time, b pays out more than a. However, if an agent is slightly risk averse, the agent will almost definitely prefer a over b because 90% of the time b pays nothing.

2. a=[0.6,$100;0.4,$90] or b=[0.6001,$100;0.3999,$90]

The average value of a is:

0.6*100 + 0.4*90 = $96

The average value of b is:

0.6001*100 + 0.3999*90 = $96.001

Over time, b pays out more than a. There isn't really much to calculate here. The agent would see that b pays more than a, then assess risk. B actually is less risky than a, because the probability of receiving the *higher* payout *increase*s. Thus, b pays more and is less risky. (Though the differences are minute.)

3. a=[0.5,$110;0.5,$90] or b=[0.5001,$90;0.4999,$150]

The average value of a is:

0.5*110 + 0.5*90 = $100

The average value of b is:

0.5001*90 + 0.4999*150 = $119.994

Over time, b pays more than a. The agent would see that b is a more valuable choice than a, but then see that b is also riskier. The agent may then choose either choice based on how risky/safe it wants to play it.

There is a 0.5 probability that the agent will receive $90 in both cases, so this acts neutrally in the agent's decision-making process.

In b, there is a 0.0001 *additional* chance of the agent receiving $90 instead of $150 *versus* a's 0.5 chance of receiving $90. In a, the agent stands to gain $20 versus b, where the agent stands to lose $60, 0.01% of the time. This would be the risk that the agent must weigh: a 0.0001 chance of gaining/losing $80.

In b, there is a 0.4999 chance of it receiving $150 *instead of* receiving $110 in a.

Thus, when the agent makes a decision, it must decide whether or not the 0.0001 probability of gaining $80 (choosing a over b) is superior to the 0.4999 probability of gaining $40 (choosing b over a). The choice should be clear to readers and agents alike by now - 0.4999*40 >>> 0.001*80. Thus, the agent should always choose b regardless of how "safe" it wants to play it. The outcome of this problem would be significantly different if the probabilities were more separated.

4. Compute the Expected Utilities and state what choice you would make.

When D = 1:

U(D=1) = 400*0.2 + 2*0.8 = 81.6

When D = 2:

U(D=2) = 20*0.2 + 100*0.8 = 84

D = 2 provides the higher utility, so I would choose that.

5. Compute posterior probabilities.

P(x|Y=1) = P(Y=1|x)P(x)/P(Y=1) = (0.2)(0.2)/(0.52) = 0.077

P(x|Y=2) = P(Y=2|x)P(x)/P(Y=2) = (0.4)(0.2)/(0.32) = 0.250

P(x|Y=3) = P(Y=3|x)P(x)/P(Y=3) = (0.4)(0.2)/(0.16) = 0.500

6. Use posterior probabilities to compute the posterior expected utilities.

E(U(D=A,X | Y=B)) => E(U(D=A,X) | Y=B) => U(D=A|x)P(x|Y=B) + U(D=A|!x)P(!x|Y=B)

A=1, B=1 => 400*0.077 + 2*0.923 = 32.646

A=2, B=1 => 20*0.077 + 100*0.923 = 93.840

A=1, B=2 => 400*0.250 + 2*0.750 = 101.500

A=2, B=2 => 20*0.250 + 100*0.750 = 80.000

A=1, B=3 => 400*0.500 + 2*0.500 = 201.000

A=2, B=3 => 20*0.500 + 100*0.500 = 60.000

7. What choice would you make in each of the following cases? What utility would you expect in each case given your choice? Use these three conditional expected utilities in the next section.

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Y=1 => D=2 (93.84)

Y=2 => D=1 (101.5)

Y=3 => D=1 (201)
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8. Compute the following probabilities: (I did this on scratch paper)

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P(Y=1) = 0.04 + 0.48 = 0.52

P(Y=2) = 0.08 + 0.24 = 0.32

P(Y=3) = 0.08 + 0.08 = 0.16
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What is the Expected Posterior Utility?

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P(Y=1) * U(Y=1, D=2) = 0.52 * 93.84 = 48.7968

P(Y=2) * U(Y=2, D=1) = 0.32 * 101.5 = 32.48

P(Y=3) * U(Y=3, D=1) = 0.16 * 201 = 32.16
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9. What is the Expected Value of Sample Information?

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EVSI(Y=1, D=2) = 93.84 - 48.7968 = 45.0432

EVSI(Y=2, D=1) = 101.5 - 32.48 = 69.02

EVSI(Y=3, D=1) = 201 - 32.16 = 168.84
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10. Did I do any of this right?

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Nobody knows.
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