Artificial Intelligence

CS 470-1

Uncertainty and Utility Theory Homework

Name:_____

Instructions: *Do the following problems.*

1. Consider the problem of detecting an incoming missile given an alarm. The alarm is either on (corresponding to percept $x = 1$) or off (corresponding to percept $x = 0$). The world is such that there is either a missile (corresponding to state $\theta = 1$) or no missile (corresponding to state $\theta = 0$). You know the following probabilities:

$P(x = 0\|\theta = 0) = 0.9$	$P(x = 0\|\theta = 1) = 0.2$
$P(x = 1\|\theta = 0) = 0.1$	$P(x = 1\|\theta = 1) = 0.8$
$P(\theta = 0) = 0.7$	$P(\theta = 1) = 0.3$

An alarm occurs. What is the most likely explanation? (To get full credit, you must show your work.)

2. Consider a problem similar to the missile detection problem above. Suppose that you make an observation $x = 1$ (the alarm has gone off) and determine that the probability of an incoming missile given the observation is $P(\theta = 1|x = 1) = 0.01$, and the probability of a false alarm given the observation is $P(\theta = 0|x = 1) = 0.99$. You have two actions that you can take: u_1 is the option of launching a defensive weapon to defuse the missile, and u_0 is the option of doing nothing. You have constructed the following utilities: $\mathcal{U}(u_0; \theta = 0) = 0.5$, $\mathcal{U}(u_1; \theta = 0) = 0.4$, $\mathcal{U}(u_0; \theta = 1) = 0.0$, and $\mathcal{U}(u_1; \theta = 1) = 8.0$. For example, $\mathcal{U}(u_0; \theta = 0)$ is the utility of doing nothing when no missile is coming in. What is the optimal solution? (Hint, you must show how you derive the utility of both options to get credit.)

3. Using the *preferences among lotteries* technique, build a utility function over the set of alternatives $\{A, B, C, D\}$ given the following constraints:

- $A \succ B \succ C \succ D$
- $U(A) = 200$
- $U(D) = 40$
- $B \sim [\frac{4}{5}, A; \frac{1}{5}, C]$
- $C \sim [\frac{1}{4}, B; \frac{3}{4}, D]$

Please carefully read the lotteries and note that these constraints state that we are indifferent between B and a lottery in A and C (not in A and D), and indifferent between C and a lottery in B and D (not in A and D).