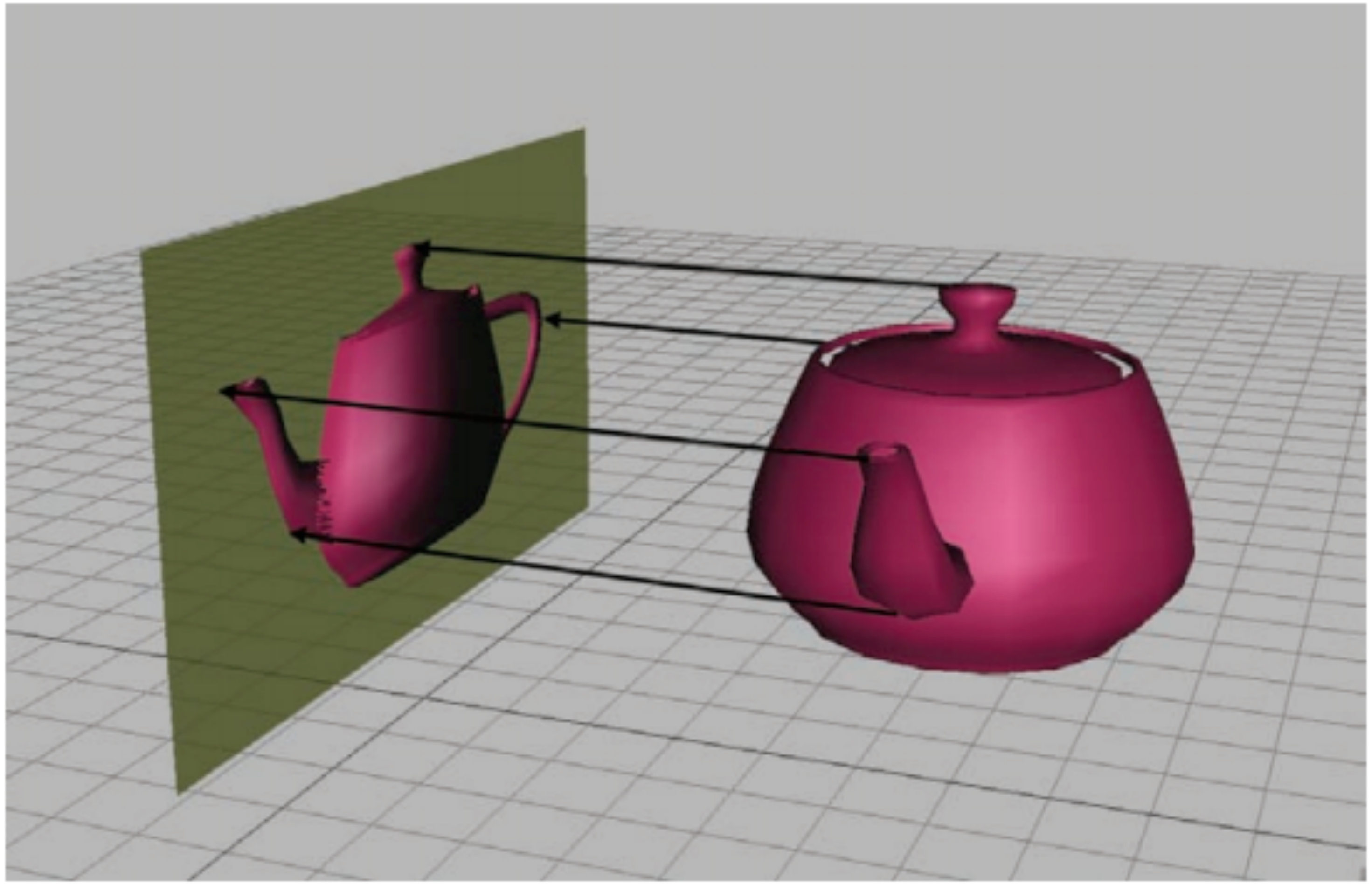
Cameras and Projection

CS 355: Interactive Graphics and Image Processing

Projection

- To get 2D pictures of a 3D world, you have to use *projection*
  - Orthographic
  - Perspective

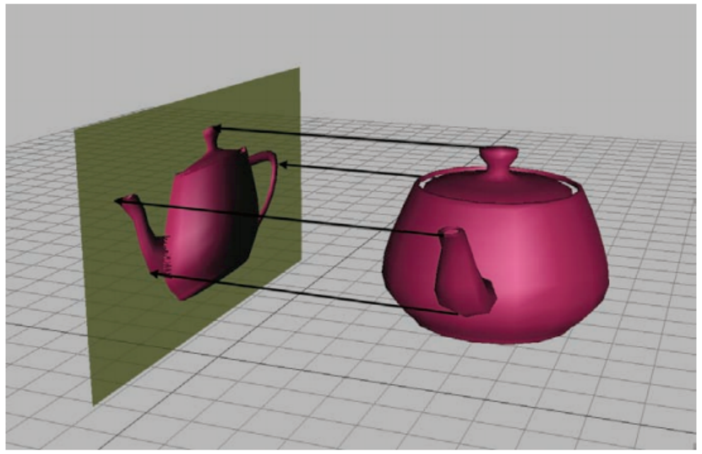
Orthographic Projection



Orthographic projection is just dropping a dimension.

Used in technical drawings, etc.

Orthographic Projection



3D point in homogeneous coordinates

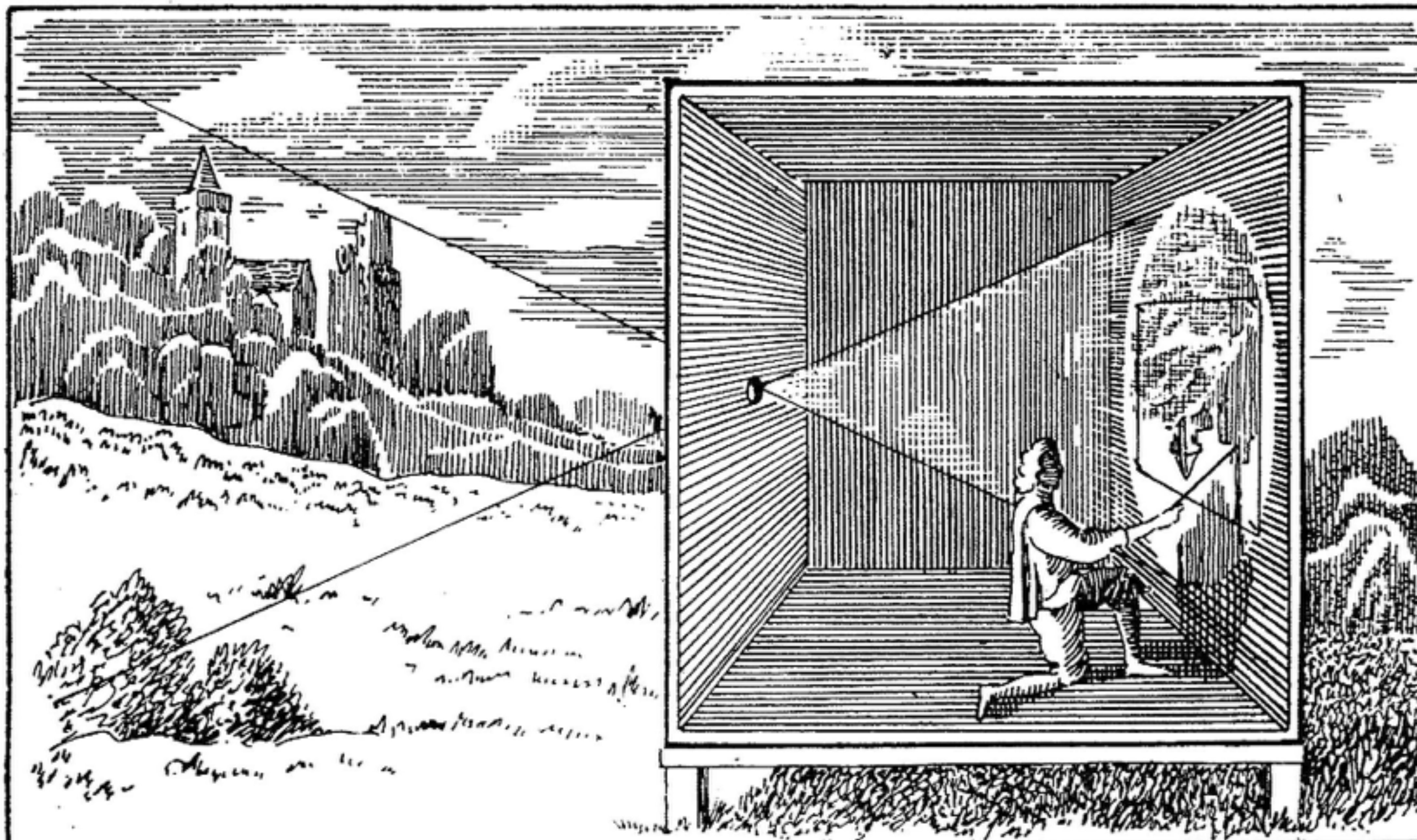
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Orthographic projection is just dropping a dimension.

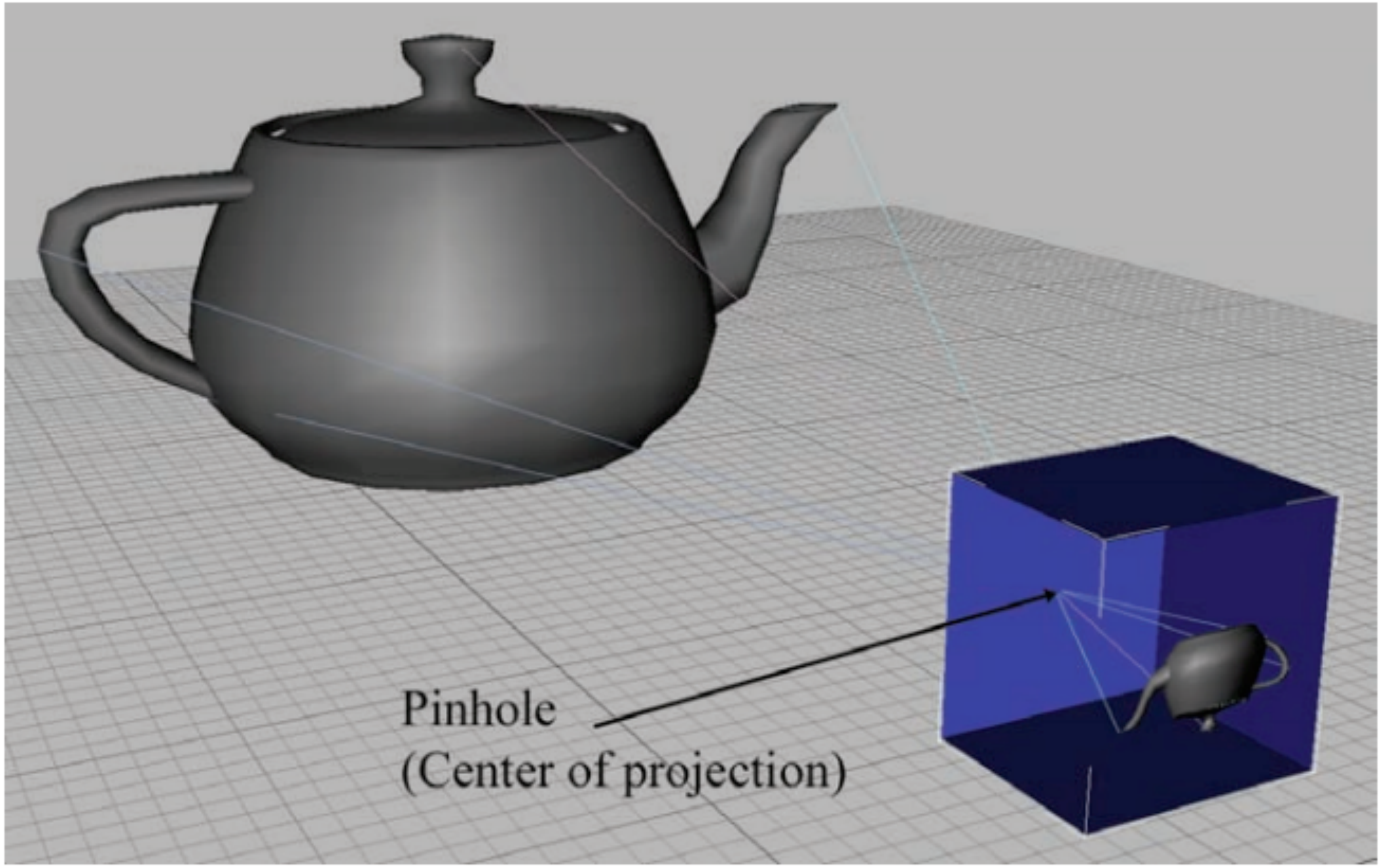
Lacking Perspective

Perspective Projection

Camera Obscura

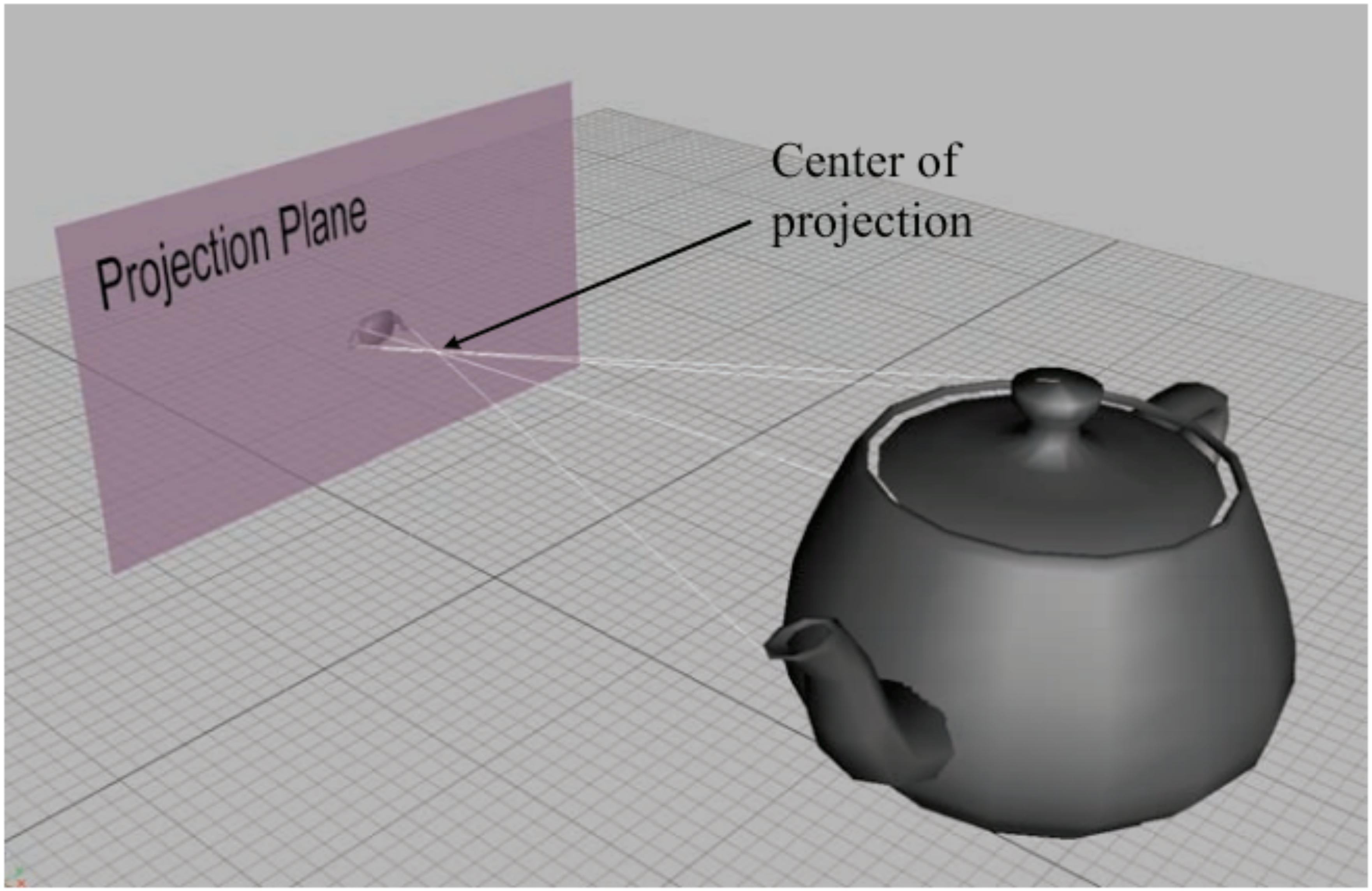


Perspective Projection



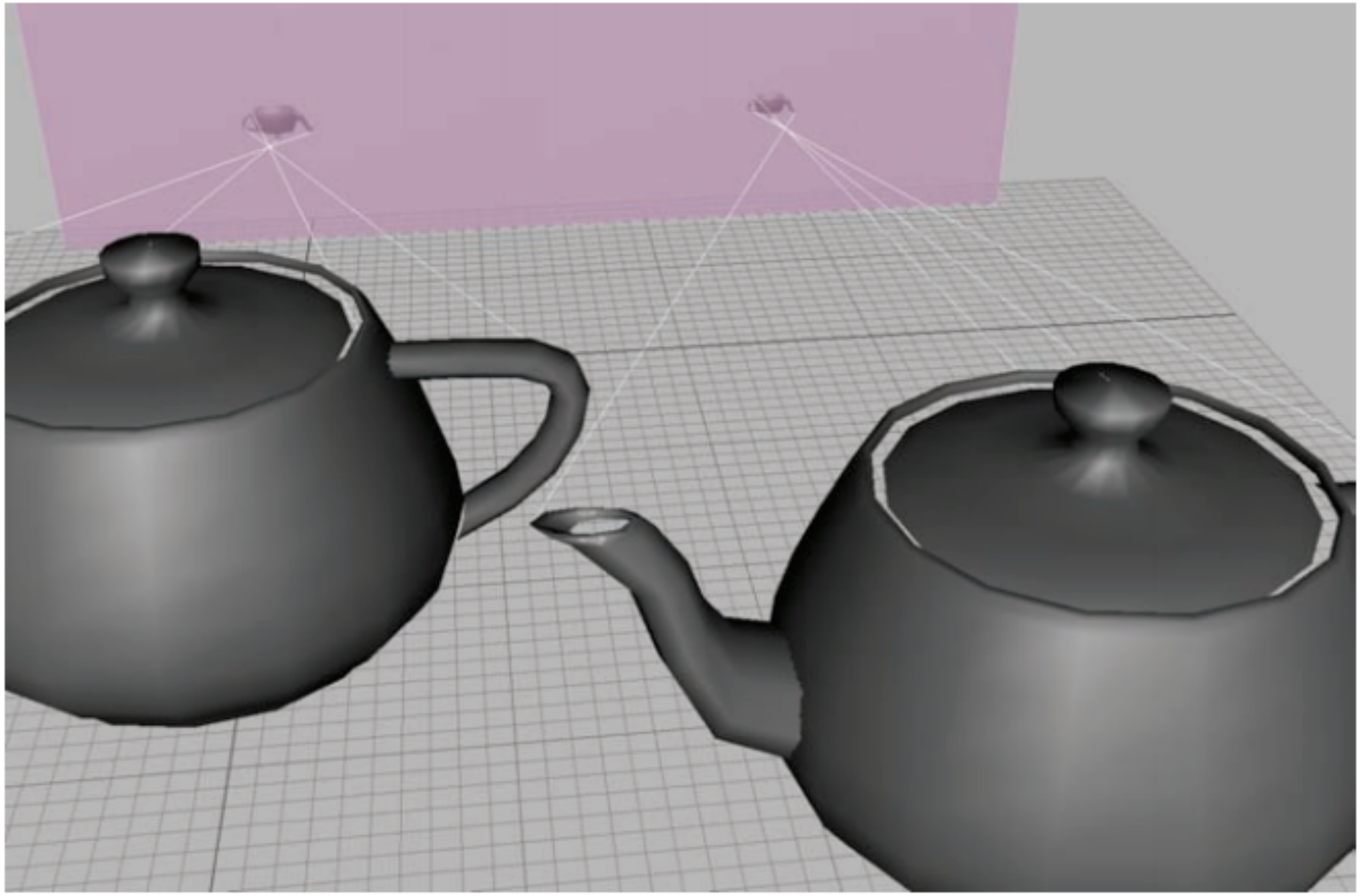
Many graphics systems assume a simple pinhole camera model

Perspective Projection

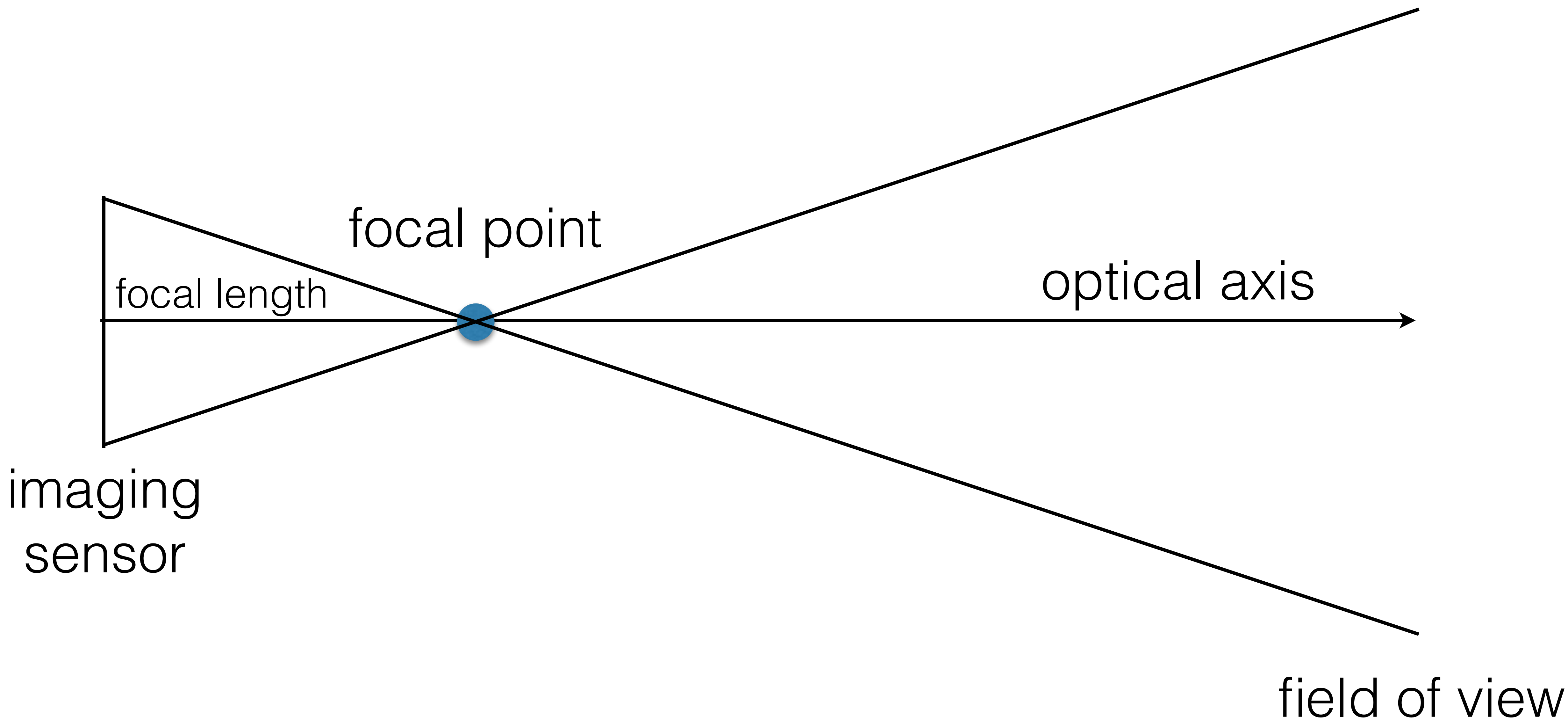


Pinhole camera model

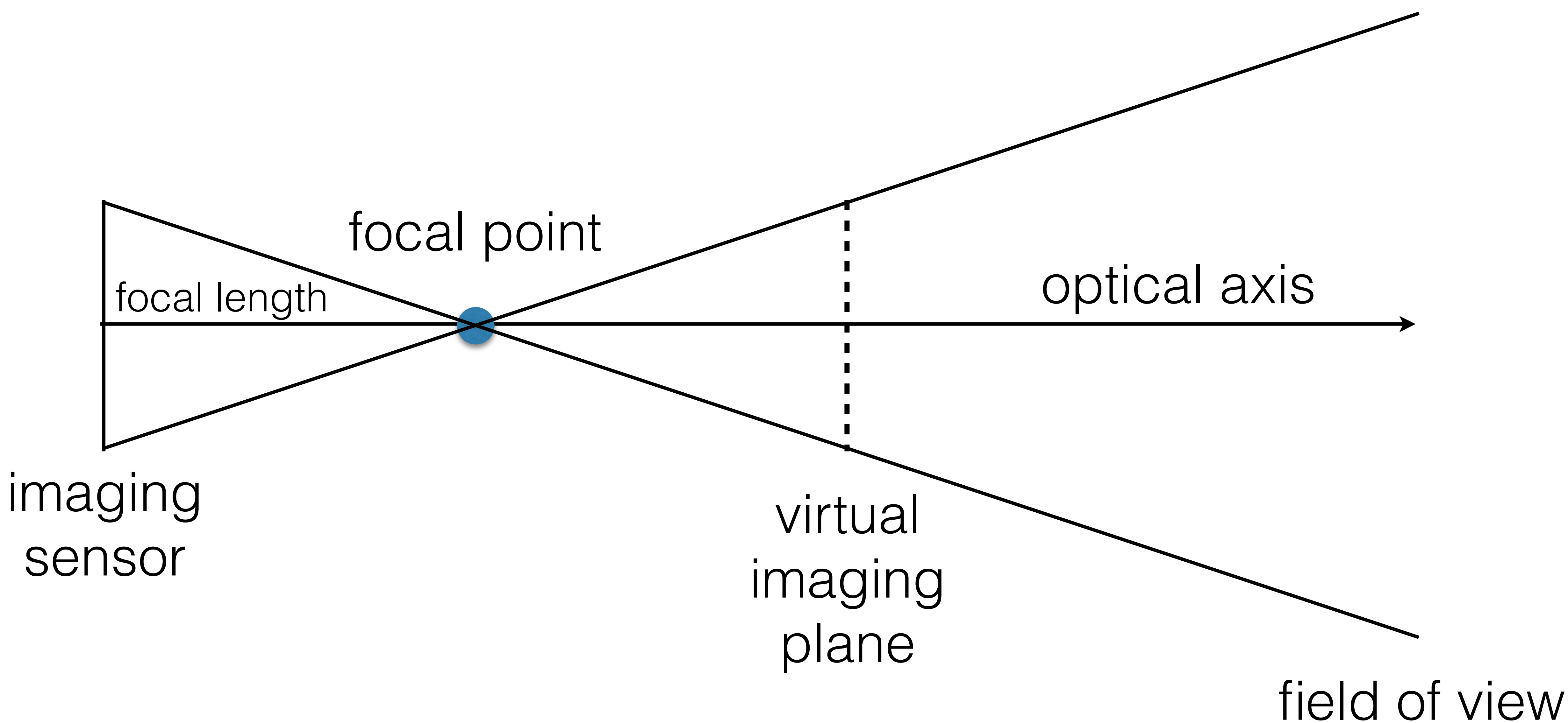
Perspective Projection



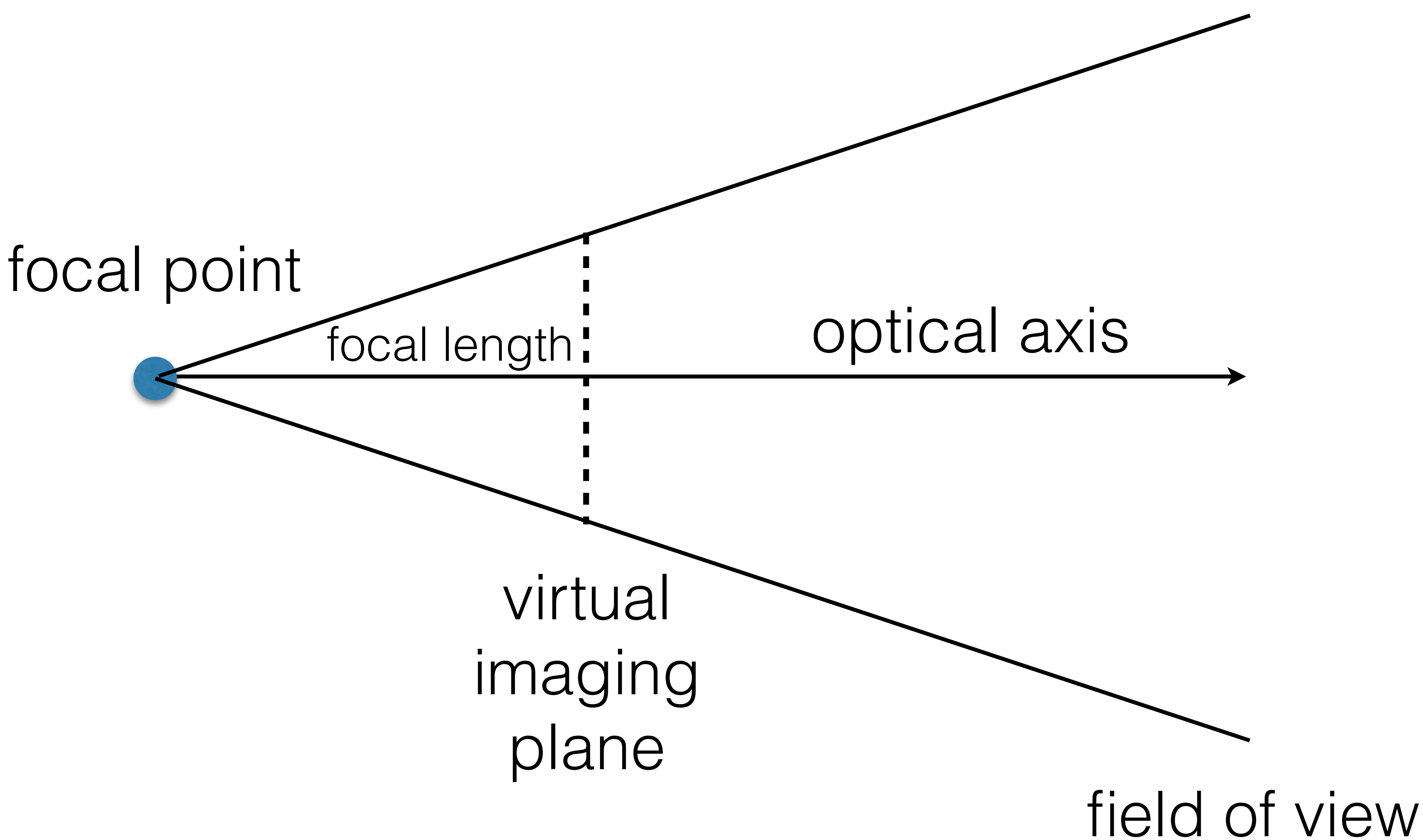
Pinhole Cameras



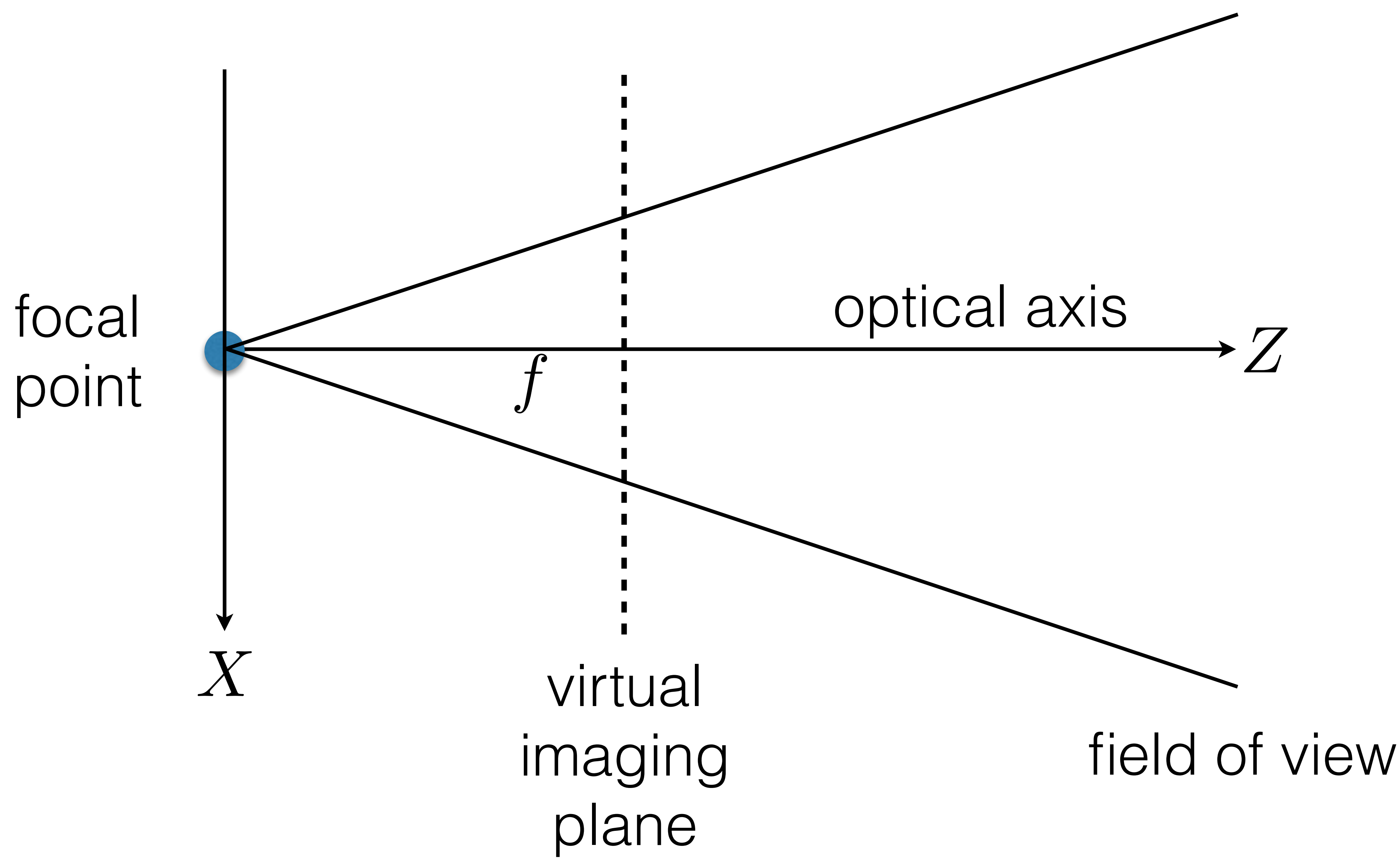
Geometric Model



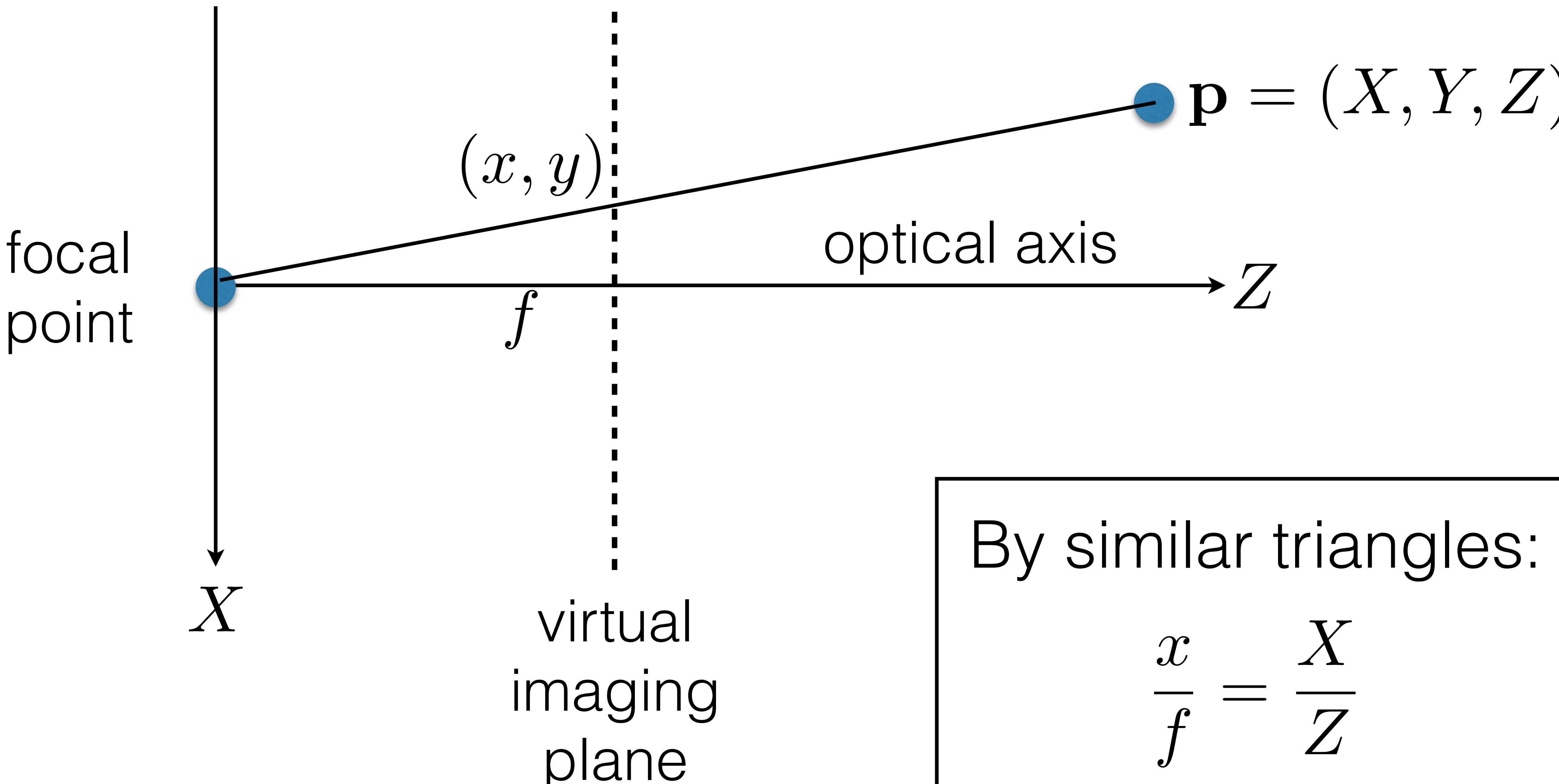
Geometric Model



Camera Coordinates



Projection



By similar triangles:

$$\frac{x}{f} = \frac{X}{Z}$$

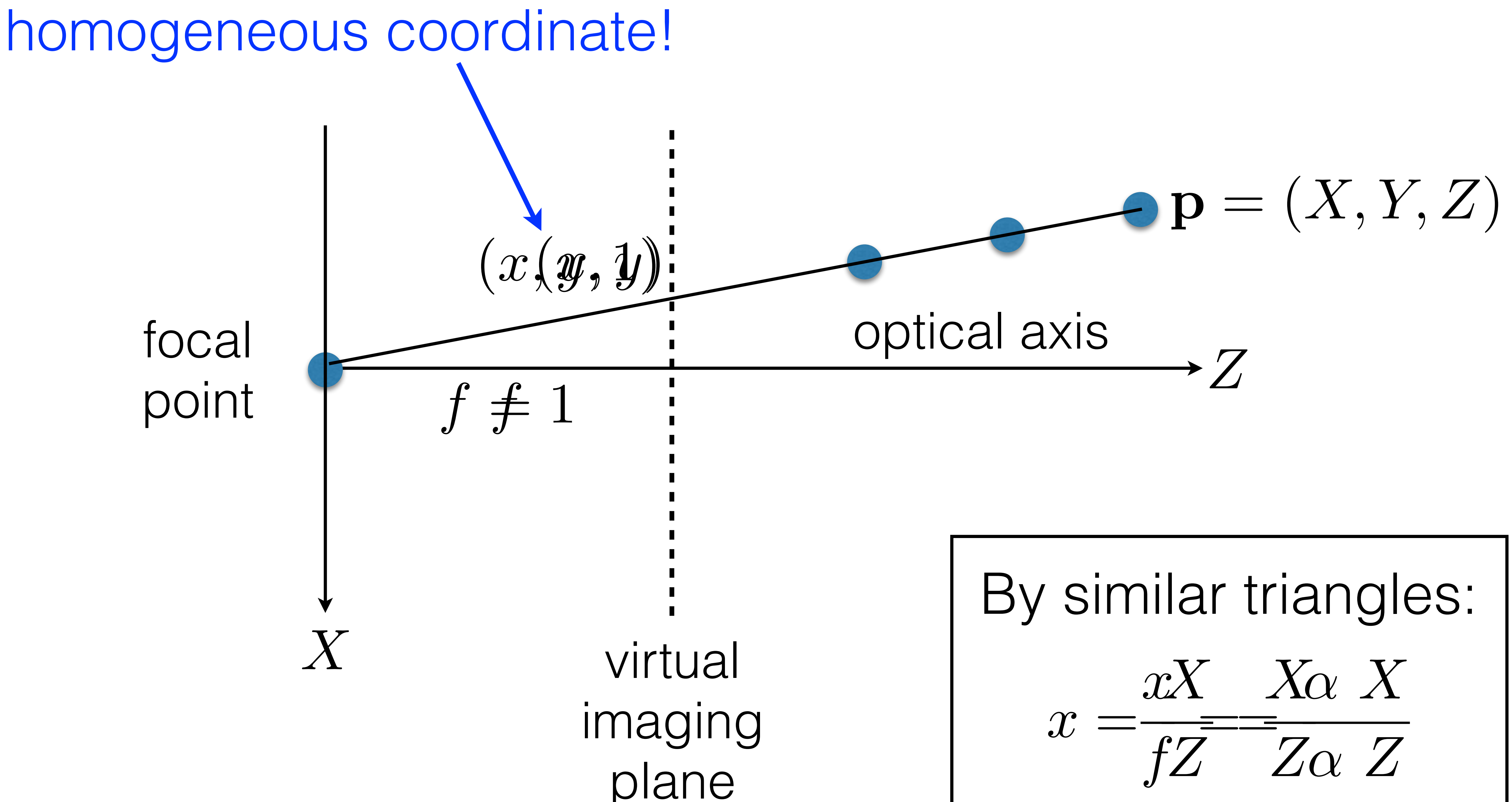
Projection

$$\frac{x}{f} = \frac{X}{Z} \qquad\qquad \frac{y}{f} = \frac{Y}{Z}$$

$$(x, y) = \left(\frac{fX}{Z}, \frac{fY}{Z} \right)$$

Note: this is the projected coordinate in real-world units.
To get actual pixel location, have to scale by pixel density
and apply offset to image origin (more on this later...)

Projection



By similar triangles:

$$x = \frac{fX}{Z} = \frac{\alpha X}{Z}$$

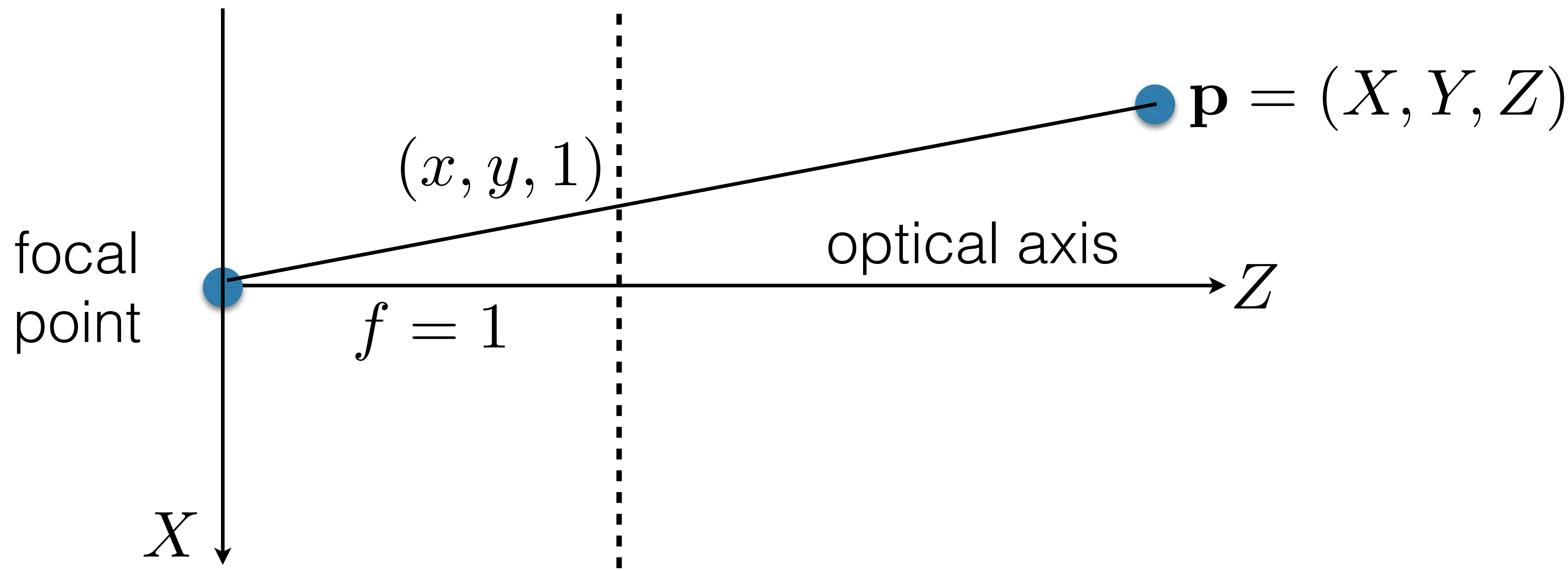
$$x = \frac{X}{Z}$$

Homogenous Coordinates

- Homogeneous coordinates are used to represent all 3D points along the ray that falls on the same 2D projection:

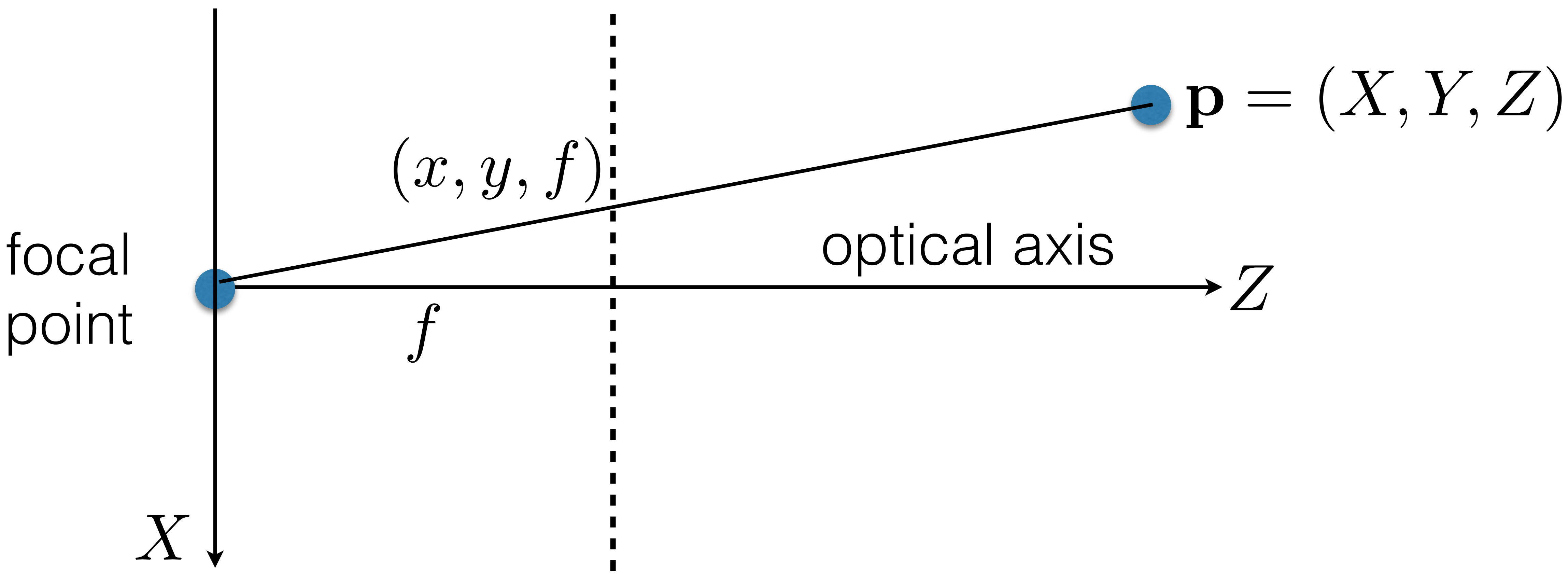
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} \alpha\ x \\ \alpha\ y \\ \alpha \end{bmatrix}$$

Perspective Projection



$$\begin{bmatrix} \boxed{\begin{matrix} x \\ y \\ 1 \end{matrix}} \\ 1 \end{bmatrix} = \begin{bmatrix} X/Z \\ Y/Z \\ 1 \\ 1 \end{bmatrix} \sim \begin{bmatrix} X \\ Y \\ Z \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \boxed{\begin{matrix} X \\ Y \\ Z \\ 1 \end{matrix}} \end{bmatrix}$$

Perspective Projection



$$\begin{bmatrix} \boxed{\begin{matrix} x \\ y \\ f \end{matrix}} \\ 1 \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \\ f \\ 1 \end{bmatrix} \sim \begin{bmatrix} X \\ Y \\ Z \\ Z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \boxed{1/f} & 0 \end{bmatrix} \boxed{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}$$

Alternative Form

One way (some implementation advantages):

$$\begin{bmatrix} x \\ y \\ \cancel{f} \\ 1 \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \\ \cancel{f} \\ 1 \end{bmatrix} \sim \begin{bmatrix} X \\ Y \\ \cancel{Z} \\ Z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Another way (some conceptual advantages):

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \\ 1 \end{bmatrix} \sim \begin{bmatrix} X \\ Y \\ Z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Coming up...

- World to camera transformations
- Specifying camera pose
- Clipping space
- Screen transformation