1. A camera is located at position (25, 20, 5) in the 3D world and is looking at the point (25, 40, 25) so that the direction [0, 1, 0] points (roughly!) up.

(a) Use the process we covered in class (a 3D variant of Gram-Schmidt orthonormalization using cross products) to calculate the camera's x, y, and z axis directions.

$$e_3 = p_{at} - p_{from} \,/\, \| p_{at} - p_{from} \| = diff \,/\, \| diff \|$$
$$diff = -(25, 20, 5) + (25, 40, 25) = <0, 20, 20>$$
$$\| diff \| = sqrt(20^2+20^2) = sqrt(800) = 20sqrt(2)$$
$$\mathbf{e_3 = <0, 1/sqrt(2), 1/sqrt(2)>}$$

$$e_1 = e_3 \times v_{up} \,/\, \| e_3 \times v_{up} \| = cross \,/\, \| cross \|$$
$$cross = <0, 1/sqrt(2), 1/sqrt(2)> \times <0, 1, 0> = <-1/sqrt(2), 0, 0>$$
$$\| cross \| = 1/sqrt(2)$$
$$\mathbf{e_1 = <-1, 0, 0>}$$

$$e_2 = e_1 \times e_3 \,/\, \| e_1 \times e_3 \| = cross \,/\, \| cross \|$$
$$cross = <-1, 0, 0> \times <0, 1/sqrt(2), 1/sqrt(2)> = <0, -1/sqrt(2), 1/sqrt(2)>$$
$$\| cross \| = 1$$
$$\mathbf{e_2 = <0, -1/sqrt(2), 1/sqrt(2)>}$$

(b) Write this camera's world-to-camera transformation as the composition of a rotation matrix and translation matrix. (You do not have to multiply out this matrix.)

$$p_c = RTp_w$$

$$T = \begin{bmatrix} 1 & 0 & 0 & -25 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/sqrt(2) & 1/sqrt(2) & 0 \\ 0 & 1/sqrt(2) & 1/sqrt(2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) What are the camera-space coordinates of the point p_w = (5, 6, 7)?

$$p_w = <5, 6, 7, 1>$$
$$Tp_w = <5+-25, 6+-20, 7+-5, 1> = <-20, -14, 2, 1>$$
$$Rtp_w = <-20, (14+2)/sqrt(2), (-14+2)/sqrt(2), 1> = <-20, 16/sqrt(2), -12/sqrt(2), 1>$$
$$= \mathbf{(-20, 11.3137, -8.4853)}$$

2. A camera is located at position (20, 5, −40) and oriented so that it is pointing parallel to the x-z plane at an angle of 30 degrees off the z axis. (This is the basic setup for Labs #4 and #5.)

(a) Write this camera's world-to-camera transformation using the composition of a 3D rotation matrix (around the y axis) and a translation matrix. (You also do not have to multiply out this matrix. You may also leave your answer in terms of trig functions.)

$p_c = RTp_w$

$$T = \begin{bmatrix} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 40 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos(30) & 0 & \sin(30) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(30) & 0 & \cos(30) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) What are the camera-space coordinates of the point p w = (5, 6, 7)?

$p_w = <5, 6, 7, 1>$

$Tp_w = <5-20, 6-5, 7+40, 1> = <-15, 1, 47, 1>$

$Rtp_w = <-15\cos(30)+47\sin(30), 1, 15\sin(30)+47\cos(30), 1>$

= **(10.5096, 1, 48.2032)**

3. A virtual camera has the following parameters:

- vertical field of view of 60 degrees
- aspect ratio of 16:9 (horizontal to vertical)
- near plane n = 10
- far plane f = 1000

$zoom_y = 1/\tan(fov/2) = 1/\tan(60/2) = -0.1561$

$zoom_x = zoom_y*16/9$

(a) What is the clip matrix for this camera?

$$\begin{bmatrix} -0.2775 & 0 & 0 & 0 \\ 0 & -0.1561 & 0 & 0 \\ 0 & 0 & 1.0202 & -20.2020 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) What are the clip-space coordinates of the camera-space point $p_c = (5, −5, 50)$?

$\mathbf{p}_c = <5, −5, 50, 1>$

$\mathbf{p}_{clip} = <5*-0.2775, -5*-0.1561, 50*1.0202-20.2020, 50>$

$\mathbf{p_{clip} = <-1.3877, 0.7806, 30.8080, w=50>}$

(c) Is this point p c = (5, −5, 50) within the view frustum of this camera? How can you tell without doing a devision?

It is in the view of the frustrum. Since each point will be between -w and w when it is in the view of the frustrum, and each point *is* between -w and w, it lies within the frustrum.

(d) What are the canonical coordinates of this point p c = (5, −5, 50)?

$p_{canonical} = p_{clip} / 50$

$\mathbf{p_{canonical} = (0.0277, 0.0156, 0.6161)}$

(e) If rendered to a high-definition display (1920 × 1080), what are the screen coordinates of this point?

$$T = \begin{bmatrix} 1920/2 & 0 & 1920/2 \\ 0 & -1080/2 & 1080/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{p}_{screen} = T * \langle 0.0277, 0.0156, 1 \rangle$$

$$\mathbf{p}_{screen} = (0.0277 + 1)1920/2, (-0.0156 + 1)1080/2, 1$$

$$\mathbf{p}_{screen} = \langle 986.592, 531.576, 1 \rangle$$

$$\mathbf{p_{screen} = (986.592, 531.576)}$$