



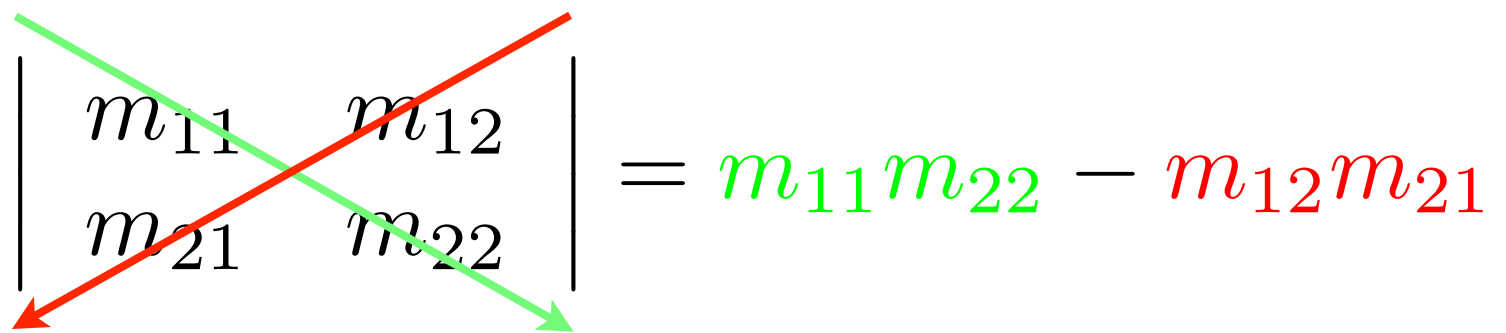
More Matrix Stuff

CS 355: Interactive Graphics and Image Processing

Determinant

$$|\mathbf{M}| = \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = m_{11}m_{22} - m_{12}m_{21}$$

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Determinant

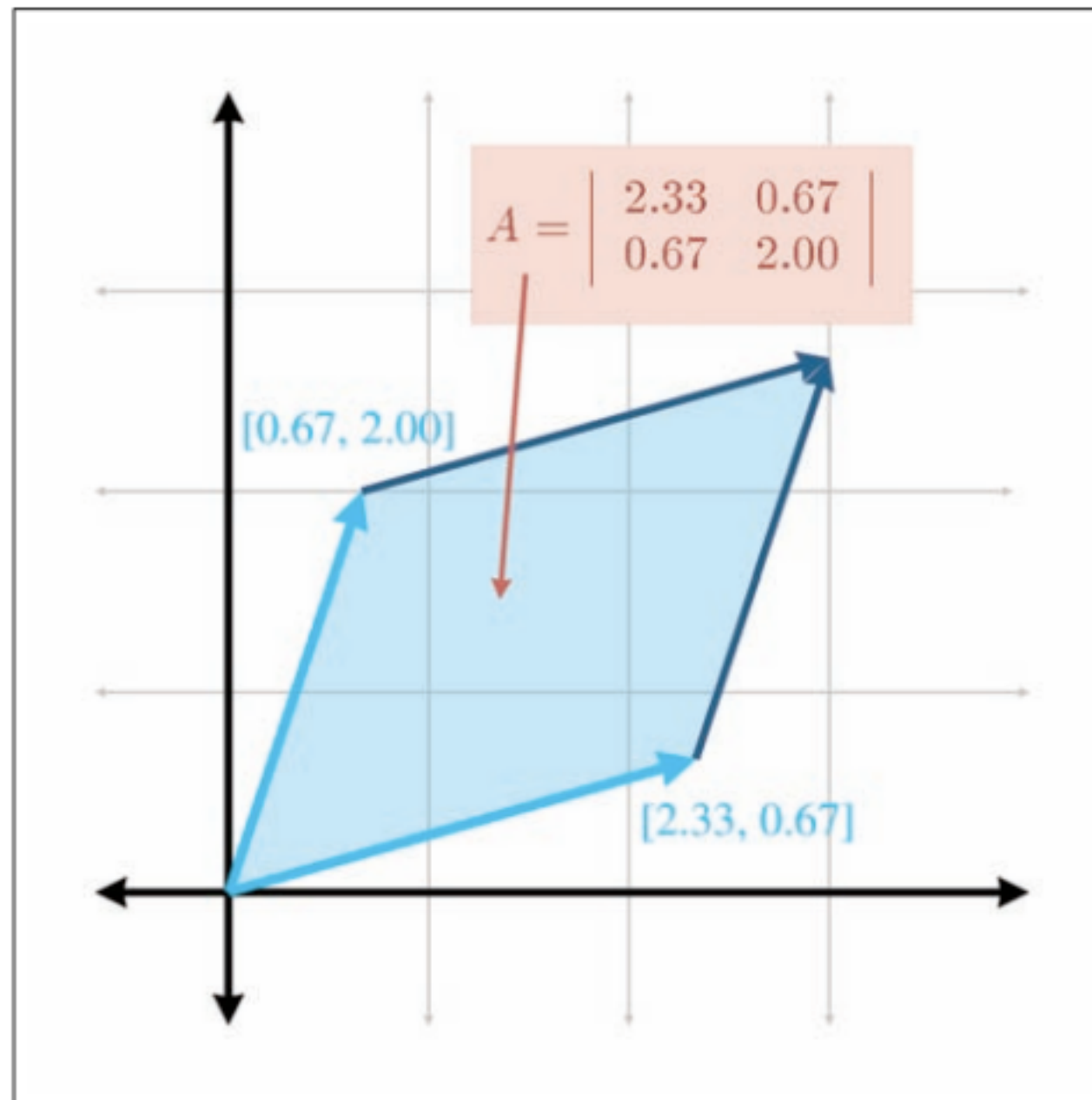
$$|\mathbf{M}| = \begin{vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{vmatrix}$$

Diagram illustrating the calculation of a 3x3 determinant using Sarrus' rule. The matrix elements are arranged in two columns of three rows each. Green arrows point down-right from the top-left to the bottom-right, and red arrows point up-right from the bottom-left to the top-right. The signs above the arrows are +, +, + for the green arrows and -, -, - for the red arrows.

The determinant is calculated as the sum of the products of the elements along the green arrows, minus the sum of the products of the elements along the red arrows:

$$m_{11}m_{22}m_{33} + m_{12}m_{23}m_{31} + m_{13}m_{21}m_{32} - m_{13}m_{22}m_{31} - m_{12}m_{21}m_{33} - m_{11}m_{23}m_{32}$$

Geometric Interpretation



Properties of Determinants

$$|\mathbf{I}| = 1$$

$$|\mathbf{AB}| = |\mathbf{A}| |\mathbf{B}|$$

$$|\mathbf{M}^T| = |\mathbf{M}|$$

$$|\mathbf{M}^{-1}| = \frac{1}{|\mathbf{M}|}$$

Linear Independence

A set of vectors is said to be *linearly dependent* if at least one of them can be expressed as a linear combination (weighted sum) of the others:

$$\mathbf{v}_j = \sum_{i \neq j} w_i \mathbf{v}_i$$

If not linearly *dependent*, then linearly *independent*

Singular Matrices

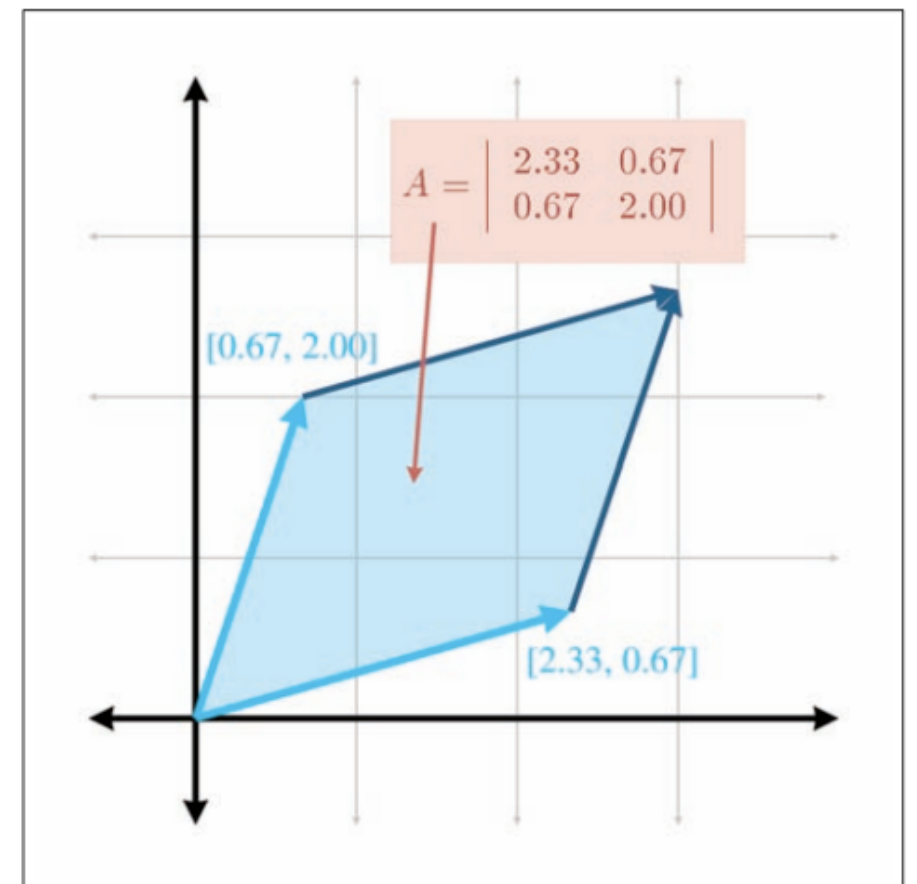
$$|\mathbf{M}^{-1}| = \frac{1}{|\mathbf{M}|}$$

But what if $|\mathbf{M}| = 0$?

A matrix whose determinant is zero
has no inverse and is said to be *singular*

Singular Matrices

- What does a singular matrix mean geometrically?
- *The rows are linearly dependent*



Matrix Rank

- The *rank* of a matrix is the number of linearly independent rows
- When used as transforms, matrices with full rank transform to the full space
- Singular matrices have *insufficient rank* and collapse to a corresponding subspace

Orthogonal Matrices

- Two (square) matrices are said to be orthogonal iff

$$\mathbf{M}\mathbf{M}^T = \mathbf{I}$$

- Implies rows are orthonormal vectors

Orthogonal Matrices

- Orthogonal matrices are also easily invertible:

$$\mathbf{M}^{-1} = \mathbf{M}^T$$

- Implies

$$|\mathbf{M}| = |\mathbf{M}^{-1}| = 1$$

Orthogonal Matrices

- All rotation matrices are orthogonal

AND

- All orthogonal matrices are rotations!

Coming up...

- Viewing transformations (revisited)
- Transformation hierarchies
- 3D!