1. Write the transformation matrix that rotates around the origin by a counterclockwise angle of $\theta = \pi/6$ radians. If I applied this to the point (10, 20), what are the (x, y) coordinates of the resulting point?

```
R(π/6) =
     |  cos(π/6)  -sin(π/6)  |
     |  sin(π/6)   cos(π/6)  |

P    = |  10  20  |^T

P'   = RP
P'   = 10*cos(π/6) + 20*-sin(π/6) , 10*sin(π/6) + 20*cos(π/6)
     = (-1.3397, 22.3205)
```

2. Write the transformation matrix that translates by an offset of (30, −50). If I applied this to the point (10, 20), what are the (x, y) coordinates of the resulting point?

```
T(30, -50) =
     |   30  |
     |  -50  |

P    = |  10  20  |^T

P'   = P + T
     = 10 + 30 , 20 + -50
     = (40, -30)
```

3. Write the transformation matrix that scales uniformly by a factor of 3. If I applied this to the point (10, 20), what are the (x, y) coordinates of the resulting point?

```
S(3) =
     |  3  0  |
     |  0  3  |

P    = |  10  20  |^T
P'   = SP
     = 10*3 + 20*0 , 10*0 + 20*3
     = (30, 60)
```

4. Write the transformation matrix that scales nonuniformly by a factor of 2 horizontally and 5 vertically. If I applied this to the point (10, 20), what are the (x, y) coordinates of the resulting point?

```
S(2, 5) =
     |  2  0  |
     |  0  5  |

P    = |  10  20  |^T
P'   = SP
P'   = 10*2 + 20*0 , 10*0 + 20*5
P'   = (20, 100)
```

5. Write the transformation matrix that will apply a shearing transform where x = x and y = 3x + y. If I applied this to the point (10, 20), what are the (x, y) coordinates of the resulting point?

```
Sh(x, 3x+y) =
     |  1  0  |     =    |   x    |
     |  3  1  |          |  3x+y  |

P    = |  10  20  |^T
P'   = ShP
     = 10*1 + 20*0 , 10*3 + 20*1
     = (10, 50)
```

6. Rotation around an arbitrary center of rotation c can be done by applying a translation by an offset of −c, rotating by the desired angle, and then translating by an offset of +c. Write an equation that would give you a single linear transformation rotates counterclockwise by an angle of $\theta = \pi/4$ radians around the point 40, 50. You do not have to actually calculate this matrix—write it as the product of multiple other matrices. If I applied this sequence to the point (45, 50), what are the (x, y) coordinates of the resulting point?

```
T(40, 50) =
     |  40  |
     |  50  |

R(π/4) =
     |  cos(π/4)  -sin(π/4)  |
     |  sin(π/4)   cos(π/4)  |

P'   = (R(P-T))+T

P    = |  45  50  |^T

P'   = (R[5, 0]^T)+T
     = ([5*sqrt(2)/2, 5*sqrt(2)/2]^T)+T
     = |  5/2*sqrt(2)+40  5/2*sqrt(2)+50  |^T
     = (43.5355, 53.5355)
```

7. A square of length 20 on each side is centered at (100, 80) in world coordinates and tilted counterclockwise by an angle of $\pi/4$ radians. Write an equation that would give you a single transformation matrix that maps points in object coordinates to world coordinates. Where is the upper right corner of the square ((10, 10) in object space) in world coordinates?

$$T(100, 80) = \begin{vmatrix} 100 \\ 80 \end{vmatrix}$$

$$R(\pi/4) = \begin{vmatrix} \cos(\pi/4) & -\sin(\pi/4) \\ \sin(\pi/4) & \cos(\pi/4) \end{vmatrix}$$

$$P^w = (RP)+T$$

$$= \begin{vmatrix} \cos(\pi/4) & -\sin(\pi/4) & 100 \\ \sin(\pi/4) & \cos(\pi/4) & 80 \\ 0 & 0 & 1 \end{vmatrix}$$

$$P = \begin{vmatrix} 10 & 10 & 1 \end{vmatrix}^T$$

$$P^W = \begin{vmatrix} 10\cos(\pi/4)-10\sin(\pi/4)+100 & 10\sin(\pi/4)+10\cos(\pi/4)+80 & 1 \end{vmatrix}^T$$
$$= \begin{vmatrix} 100 & 10*\mathrm{sqrt}(2)+80 \end{vmatrix}^T$$
$$= (100, 94.1421)$$

8. Using the same square as in Question 7, write an equation that gives you a single transformation matrix that maps points in world coordinates to object coordinates. Use this to determine whether the point (90, 90) is in the square.

$$P^O = R^{-1}(P-T)$$
$$= R^{-1}P + R^{-1}(-T)$$

$$P^o = \begin{vmatrix} \cos(\pi/4) & \sin(\pi/4) & -100\cos(\pi/4)-80\sin(\pi/4) \\ -\sin(\pi/4) & \cos(\pi/4) & 100\sin(\pi/4)-80\cos(\pi/4) \\ 0 & 0 & 1 \end{vmatrix}$$

$$P = \begin{vmatrix} 90 & 90 & 1 \end{vmatrix}^T$$

$$P^O = \begin{vmatrix} 90\cos(\pi/4)+90\sin(\pi/4)-100\cos(\pi/4)-80\sin(\pi/4) , \\ -90\sin(\pi/4)+90\cos(\pi/4)+100\sin(\pi/4)-80\cos(\pi/4) , \\ 90*0+90*0+1*1 \end{vmatrix}$$
$$= \begin{vmatrix} 0 & 14.1421 & 1 \end{vmatrix}$$
$$= (0, 14.1421)$$

Because the point is more than 10 away from the origin in the Y direction, the point is not in the square.

9. Suppose that the user zooms into the graphics "world" by a factor of 200% and then scrolls so that the point (50, 60) in world coordinates appears at the upper left corner of the display (i.e., at the origin of the viewing coordinates). Write an equation that would give you a single transformation matrix that maps points in the world to their on-screen coordinates in view space. Where does the world-coordinate point (200, 300) fall on the screen?

$$S(2) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$

$$T(50, 60) = \begin{vmatrix} 50 & 60 \end{vmatrix}^T$$

$$P^W = \begin{vmatrix} 200 & 300 & 1 \end{vmatrix}^T$$

$$\begin{aligned} P^S &= S(P^W - T) \\ &= SP^W - ST \end{aligned}$$

$$= \begin{vmatrix} 2 & 0 & -50*2 \\ 0 & 2 & -60*2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{aligned} P^S &= \begin{vmatrix} 200*2+300*0+1*-100 & 200*0+300*2+1*-120 & 1 \end{vmatrix}^T \\ &= (300, 480) \end{aligned}$$

10. Assuming the same zooming and scrolling as in Question 9, Write an equation that would give you a single transformation matrix that maps points from their on-screen viewing coordinates to their corresponding location in world coordinates. What world-coordinate point falls at position (60, 70) on the screen?

$$P^W = S^{-1}P^S + T$$

$$= \begin{vmatrix} 1/2 & 0 & 50 \\ 0 & 1/2 & 60 \\ 0 & 0 & 1 \end{vmatrix}$$

$$P^S = (60, 70)$$

$$\begin{aligned} P^W &= \begin{vmatrix} 60/2+70*0+50*1 & 60*0+70/2+60*1 & 1 \end{vmatrix}^T \\ &= (80, 95) \end{aligned}$$

11. Assuming the same zooming and scrolling as in Question 9, write an equation that would give you a single transformation matrix that maps points in the object coordinates for the square in Question 7 to the screen. Where would the upper right corner of the square in Question 7 fall on the screen? (Hint: write your answer in terms of the matrices you constructed for Questions 7 and 9.)

$$P^S = S((R^O P^O + T^O) - T^S)$$

$$= \begin{vmatrix} 2 & 0 & -50*2 \\ 0 & 2 & -60*2 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \cos(\pi/4) & -\sin(\pi/4) & 100 \\ \sin(\pi/4) & \cos(\pi/4) & 80 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \mathrm{sqrt}(2) & -\mathrm{sqrt}(2) & 100 \\ \mathrm{sqrt}(2) & \mathrm{sqrt}(2) & 40 \\ 0 & 0 & 1 \end{vmatrix}$$

$$P^S = \begin{vmatrix} 10 & 10 & 1 \end{vmatrix}^T$$

$$= \begin{vmatrix} 10\mathrm{sqrt}(2)-10\mathrm{sqrt}(2)+100 & 10\mathrm{sqrt}(2)+10\mathrm{sqrt}(2)+40 & 1 \end{vmatrix}^T$$

$$= (100,\ 68.2843)$$

12. Assuming the same zooming and scrolling as in Question 9, suppose that the user clicks on the screen at position (90, 70) in viewing coordinates. Does this fall within the square in Question 7? (Hint: write your answer in terms of the matrices you constructed for Questions 8 and 10.)

$$P^O = R^{-1}(S^{-1}P^S + T^S - T^W)$$

$$= \begin{vmatrix} 1/2 & 0 & 50 \\ 0 & 1/2 & 60 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \cos(\pi/4) & \sin(\pi/4) & -90\mathrm{sqrt}(2) \\ -\sin(\pi/4) & \cos(\pi/4) & 10\mathrm{sqrt}(2) \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \mathrm{sqrt}(2)/4 & \mathrm{sqrt}(2)/4 & -45\mathrm{sqrt}(2)+50 \\ -\mathrm{sqrt}(2)/4 & \mathrm{sqrt}(2)/4 & 5\mathrm{sqrt}(2)+60 \\ 0 & 0 & 1 \end{vmatrix}$$

$$P^S = \begin{vmatrix} 90 & 70 & 1 \end{vmatrix}^T$$

$$P^O = \begin{vmatrix} 45\mathrm{sqrt}(2)/2+35\mathrm{sqrt}(2)/2-45\mathrm{sqrt}(2)+50 \\ -45\mathrm{sqrt}(2)/2+35\mathrm{sqrt}(2)/2+5\mathrm{sqrt}(2)+60 \\ 1 \end{vmatrix}$$

$$P^O = (42.9289,\ 60)$$