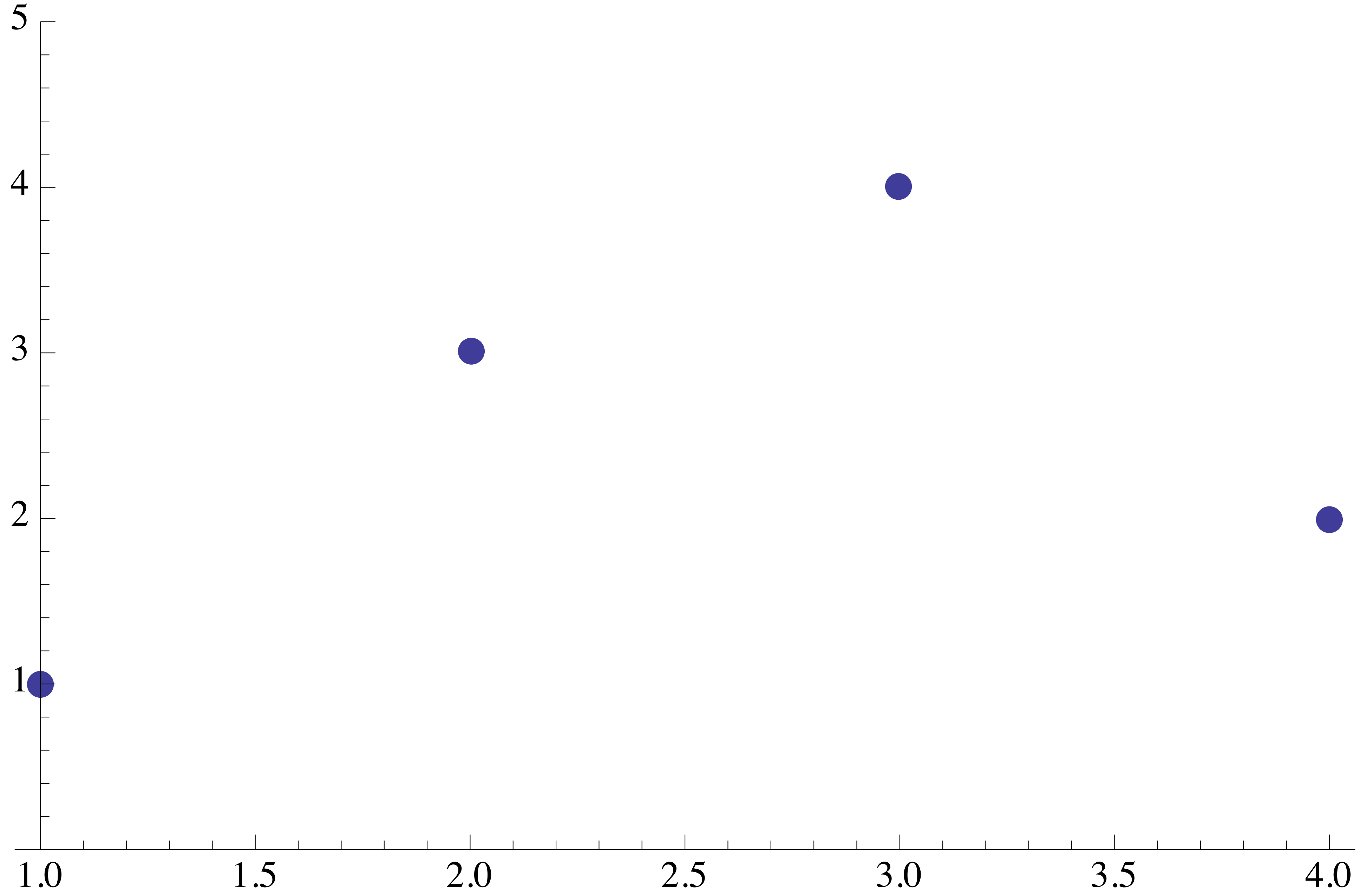
Interpolation, Curves, Surfaces

CS 355: Interactive Graphics and Image Processing

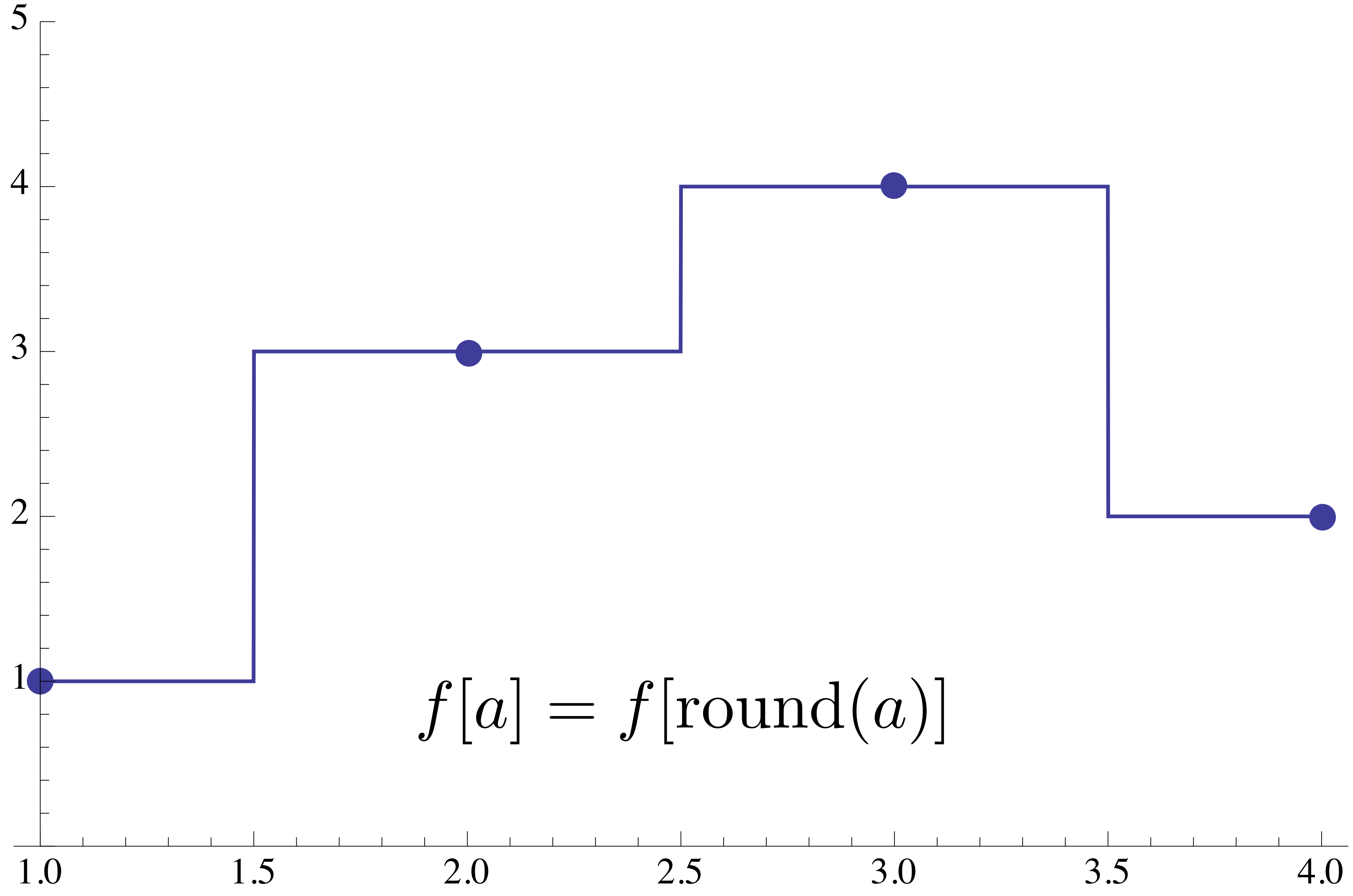
Interpolation

- Used to estimate a function between known sampled values
- Digital signals to analog
  - Audio playback
  - Image display
- Spatial image manipulation
  - Changing size
  - Rotation
  - Warping
- Smooth geometric models between defined points

Interpolation

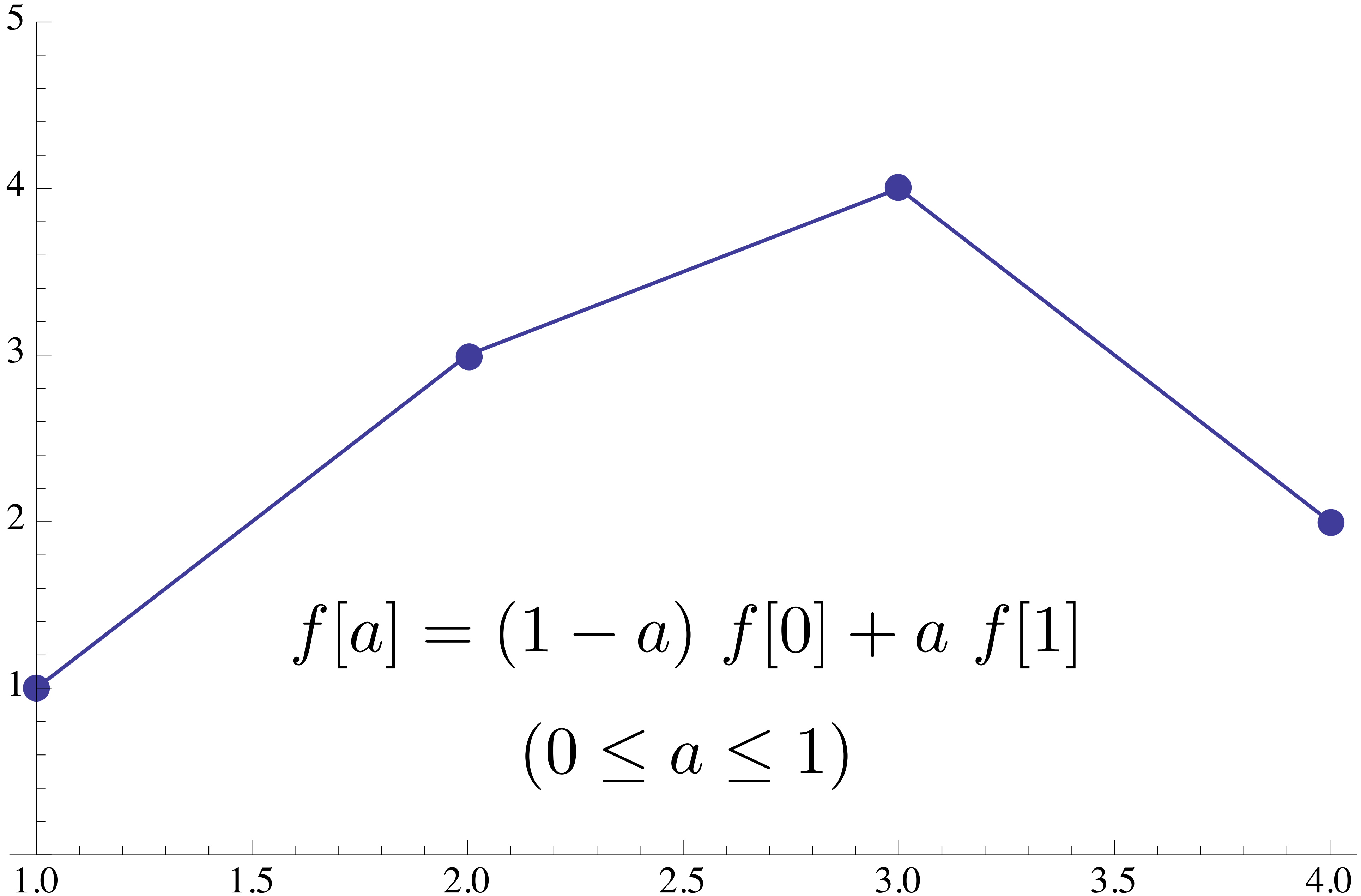


Nearest Neighbor



0th order

Linear Interpolation

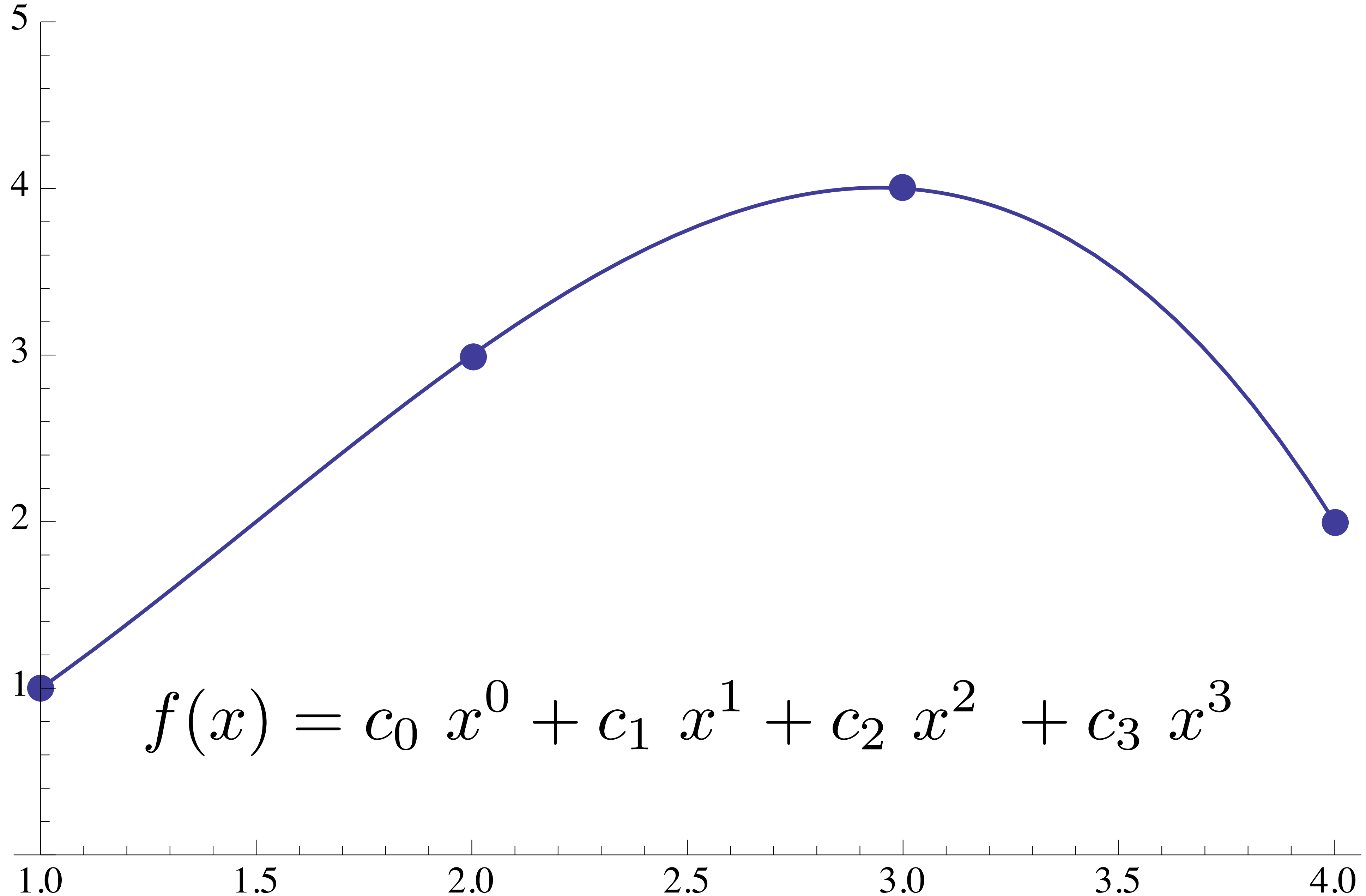


$$f[a] = (1-a)\ f[0] + a\ f[1]$$

$$(0 \leq a \leq 1)$$

1st order

Cubic Interpolation



$$f(x) = c_0\ x^0 + c_1\ x^1 + c_2\ x^2\ + c_3\ x^3$$

3rd order

Cubic Interpolation

$$f(x) = ax^3 + bx^2 + cx + d$$

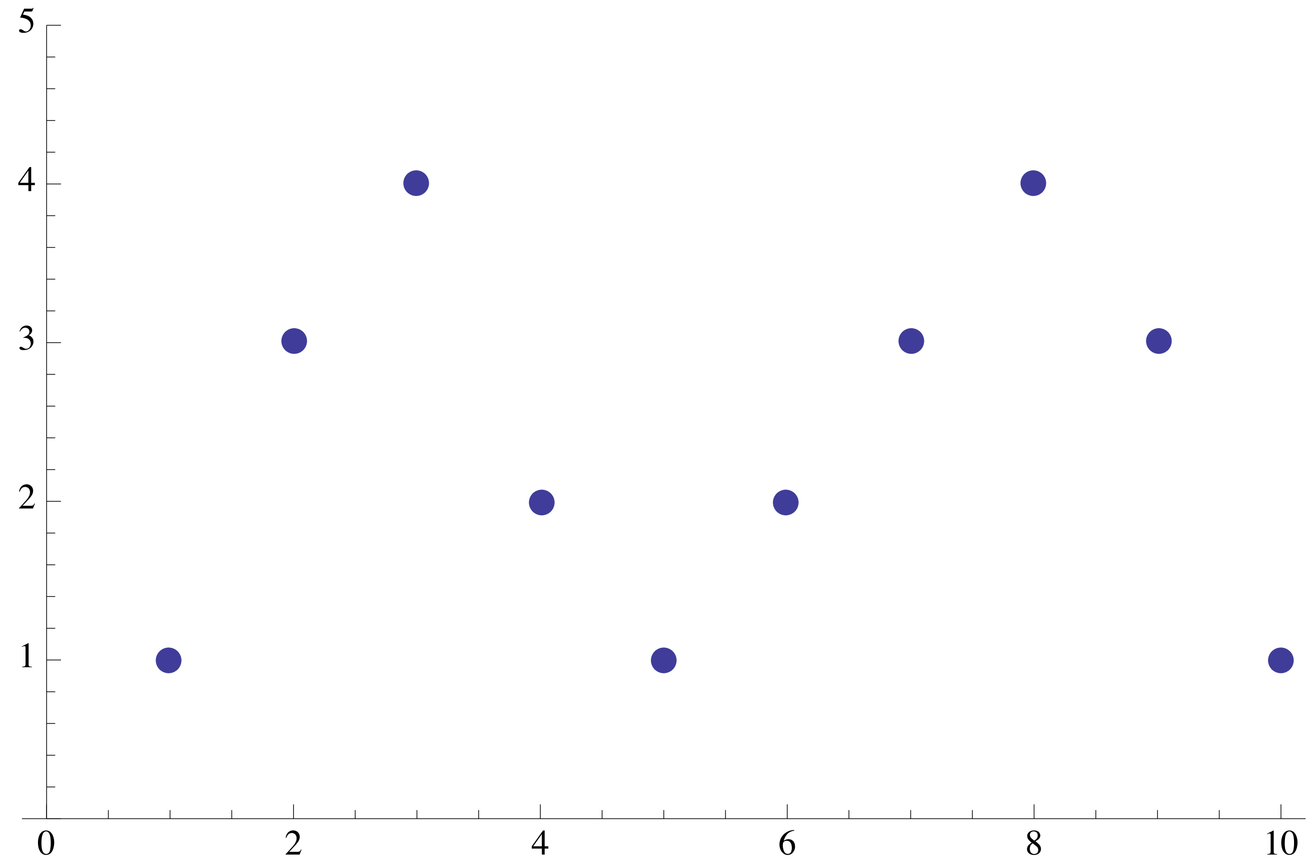
$$(x_1 \le x \le x_2)$$

$$\begin{aligned}
f(x_0) &= a\ x_0^3 + b\ x_0^2 + c\ x_0 + d \\
f(x_1) &= a\ x_1^3 + b\ x_1^2 + c\ x_1 + d \\
f(x_2) &= a\ x_2^3 + b\ x_2^2 + c\ x_2 + d \\
f(x_3) &= a\ x_3^3 + b\ x_3^2 + c\ x_3 + d
\end{aligned}$$

$$\begin{bmatrix} x_0^0 & x_0^1 & x_0^2 & x_0^3 \\ x_1^0 & x_1^1 & x_1^2 & x_1^3 \\ x_2^0 & x_2^1 & x_2^2 & x_2^3 \\ x_3^0 & x_3^1 & x_3^2 & x_3^3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \\ f(x_3) \end{bmatrix}$$

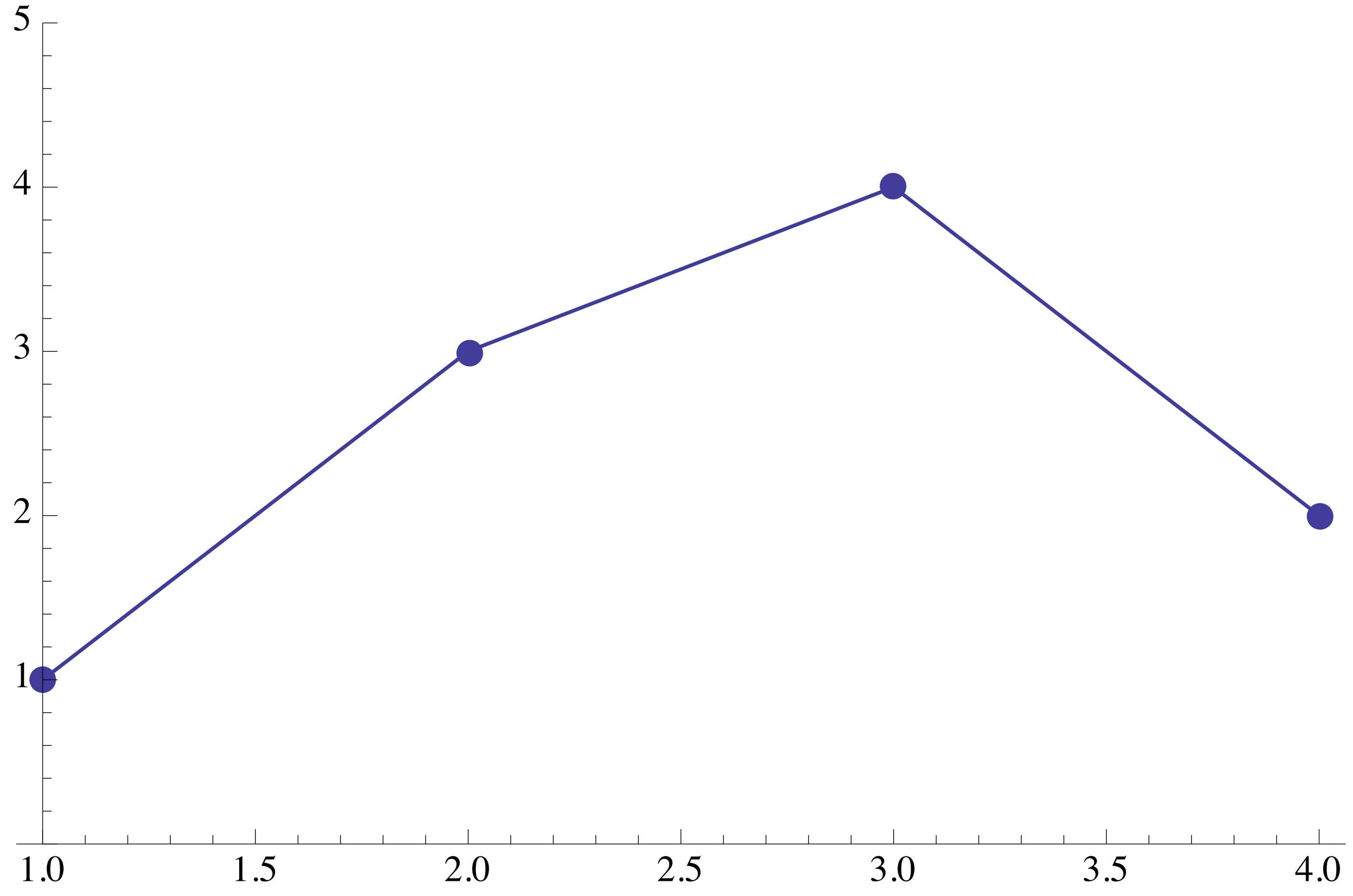
Solve for a, b, c, d and plug into function

Longer Sequences



0th - nearest neighbor
1st - use one sample on both sides
3rd - use two samples on both sides

Continuity



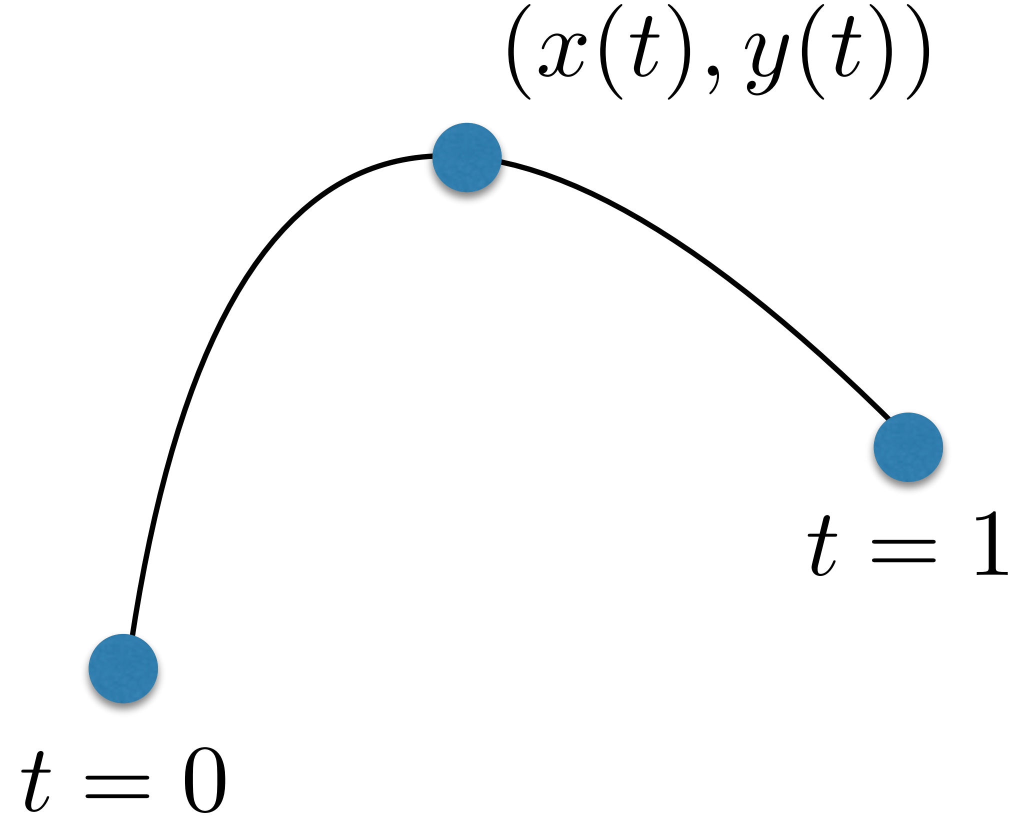
0th order: function is continuous

1st order: slope is continuous

nth order: nth derivative is continuous

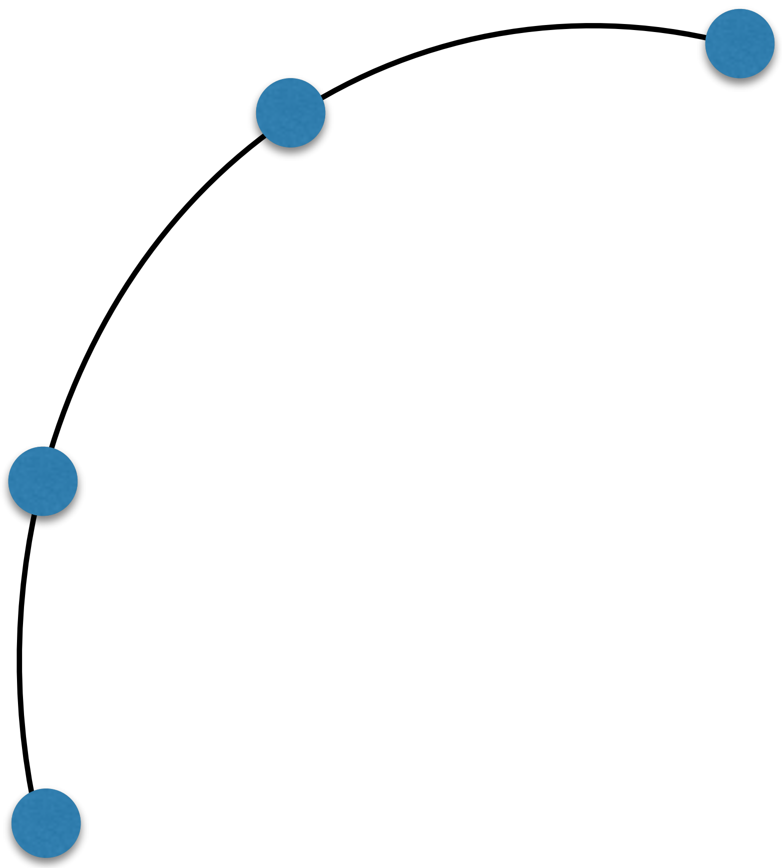
2D Curves

- Parametric curves
  - Parameter *t* traces the curve
  - One function for each dimension



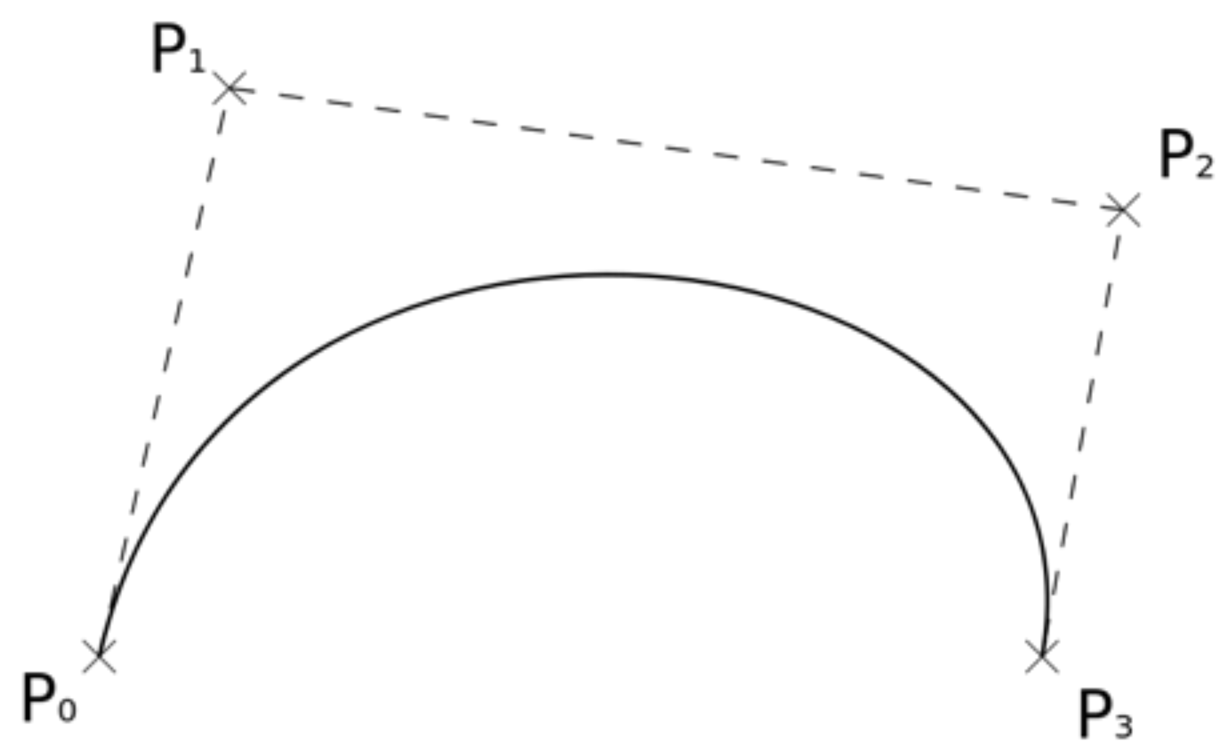
Cubic Curves

- Lots of types
  - Exact-fit curves (splines, etc.)
  - Bezier
  - B-splines
  - …



Bezier Curves

- Specify end points and a set of *control points* in between
- Curve doesn't (usually) pass through control points
- Control points determine slopes at certain points
- Common in 2D drawing programs

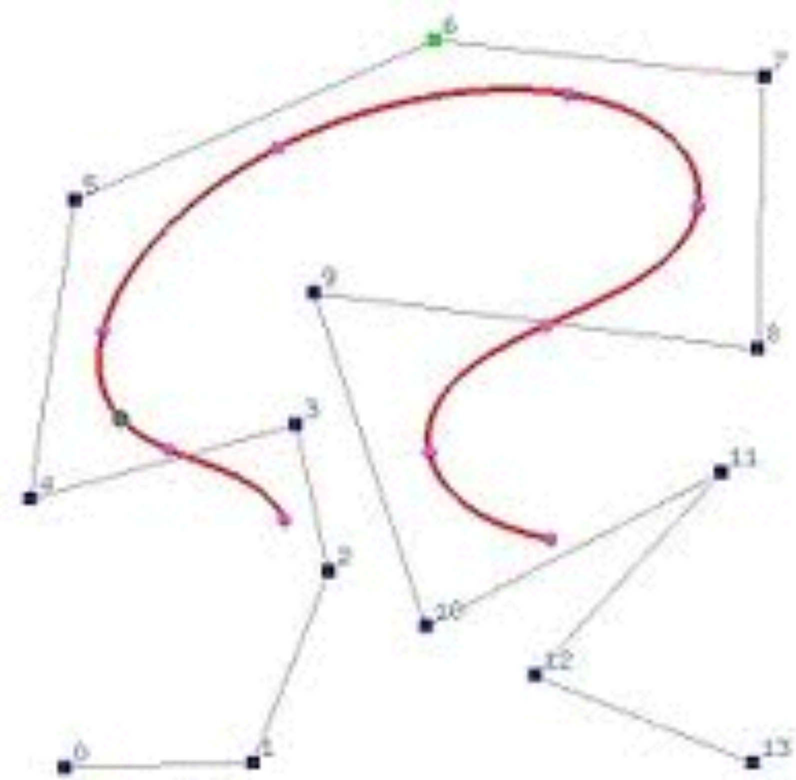


Complicated Curves

- Piecewise cubic segments
  - Can get 0th-order continuity by sharing endpoints
  - Can get 1st-order continuity by aligning control points
- Use higher-degree polynomials
  - Can make an nth-order Bezier curve out of two end points and *n-1* control points
  - Cumbersome and slow for long curves
  - Non-local control!

B-Splines

- More common to use *basis splines* (B-splines)
- Each local portion of the curve is a *weighted blend* of nth-order basis functions (usually cubic)



2D Images

- Extend ideas of single-value interpolation
  - Nearest neighbor
  - Linear
  - Cubic
  - …

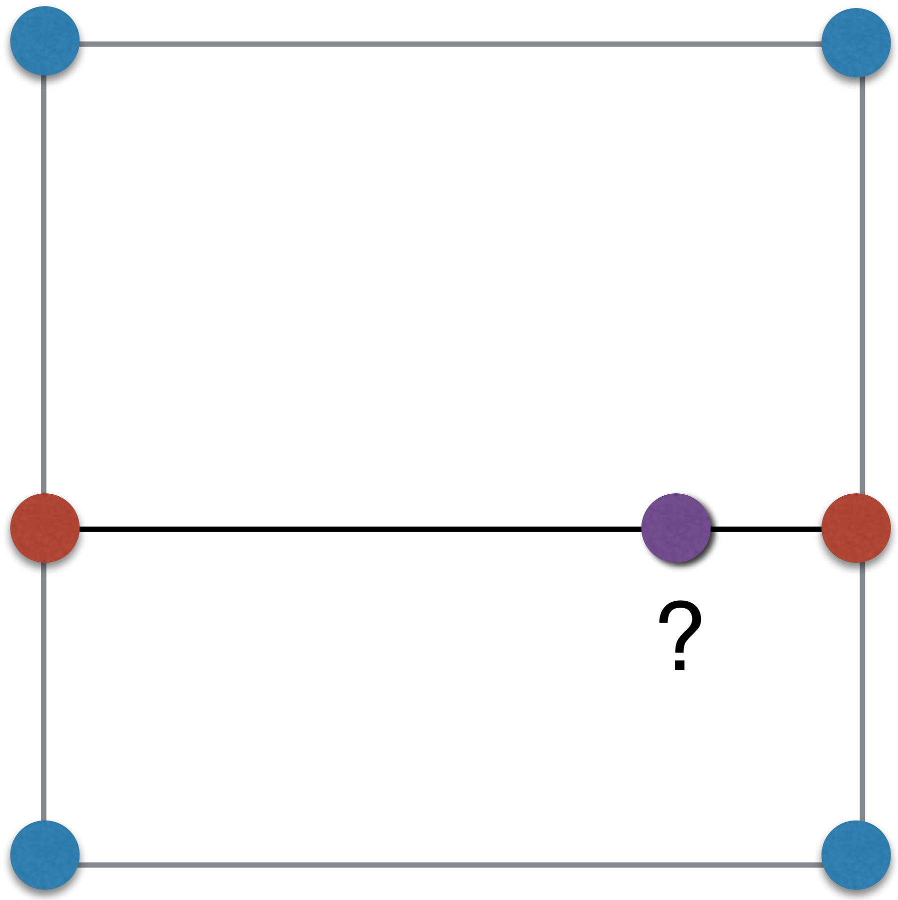
Nearest Neighbor

- Same idea as in 1-d:
  Round off to nearest pixel
- Also called “pixel replication”
- Big “blocky” pixels

Bilinear

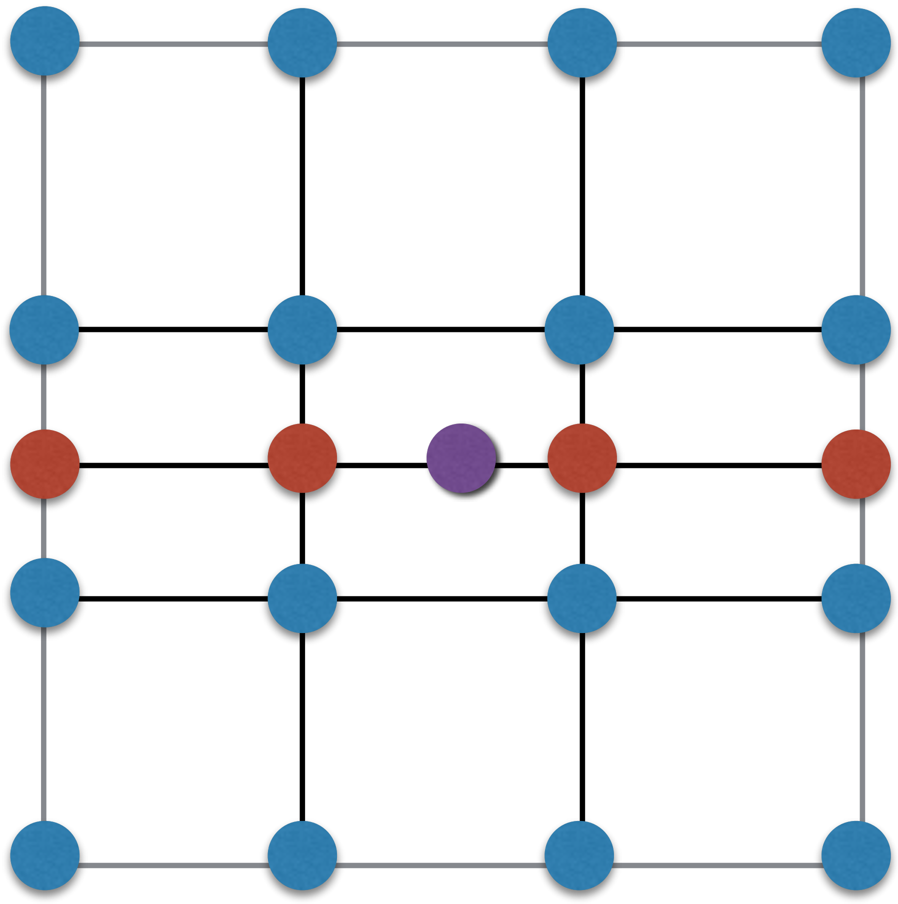
- Very common approach:
  - Interpolate vertically
  - Interpolate horizontally

  (or vice versa)
- For linear interpolation, this is called *bilinear*



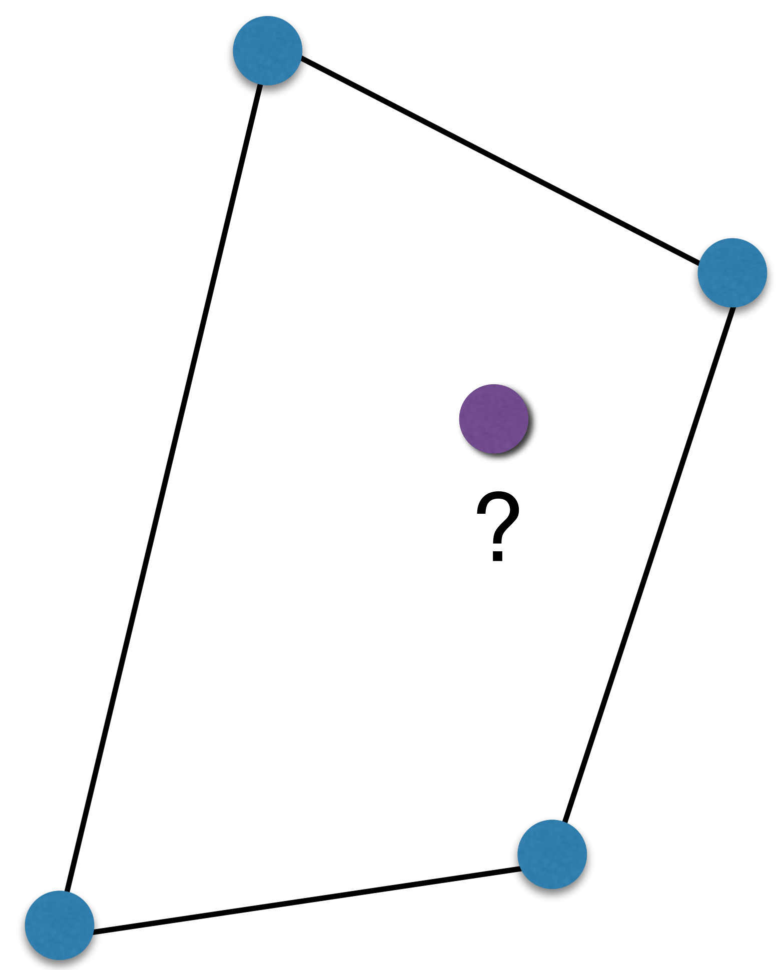
Bicubic

- Same idea as bilinear but using a 4 x 4 grid of neighbors
- Tends to produce sharper results
- More computationally intensive



Generalizing Bilinear

- Corner points don't have to lie on a square or rectangle
- Can be any quadrilateral



Generalized Bilinear

- General form:

$$f(x) = ax + by + cxy + d$$

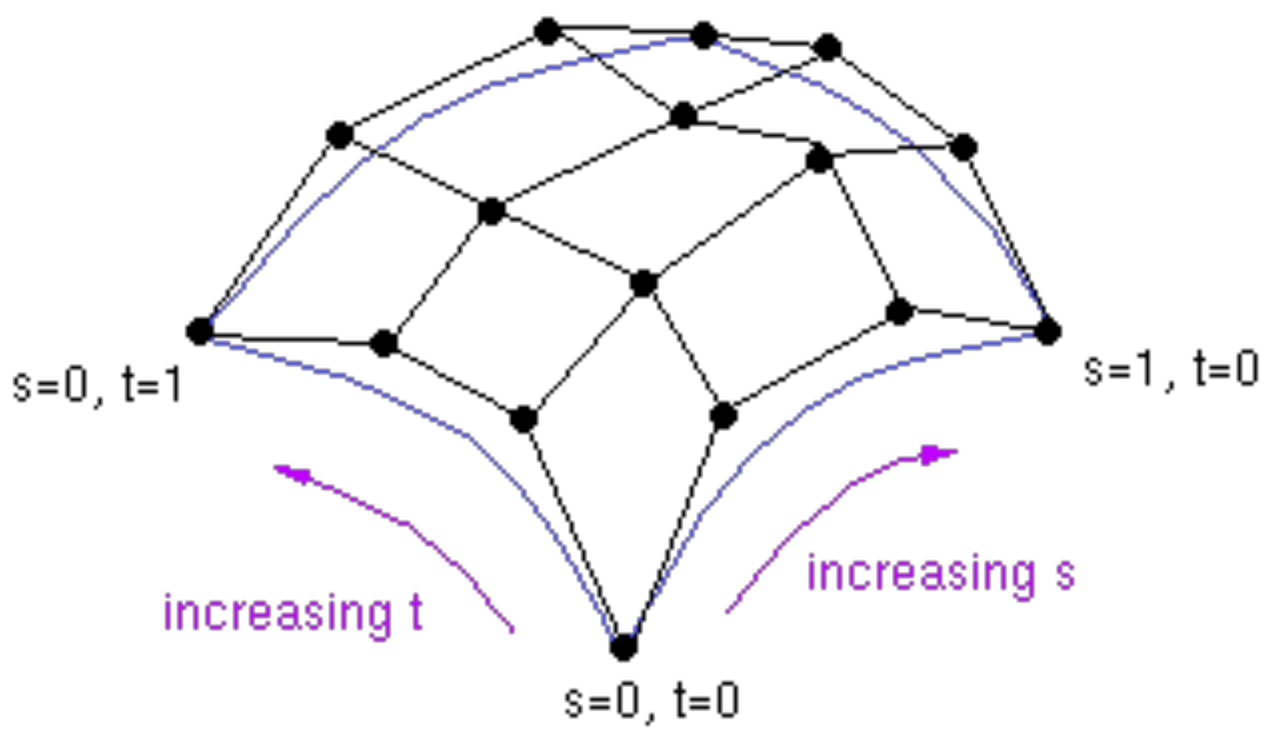
- Use similar strategy as with curve fitting:

$$\begin{aligned} f(x_1) &= ax_1 + by_1 + cx_1y_1 + d \\ f(x_2) &= ax_2 + by_2 + cx_2y_2 + d \\ f(x_3) &= ax_3 + by_3 + cx_3y_3 + d \\ f(x_4) &= ax_4 + by_4 + cx_4y_4 + d \end{aligned}$$

Solve for a, b, c, d and plug into function

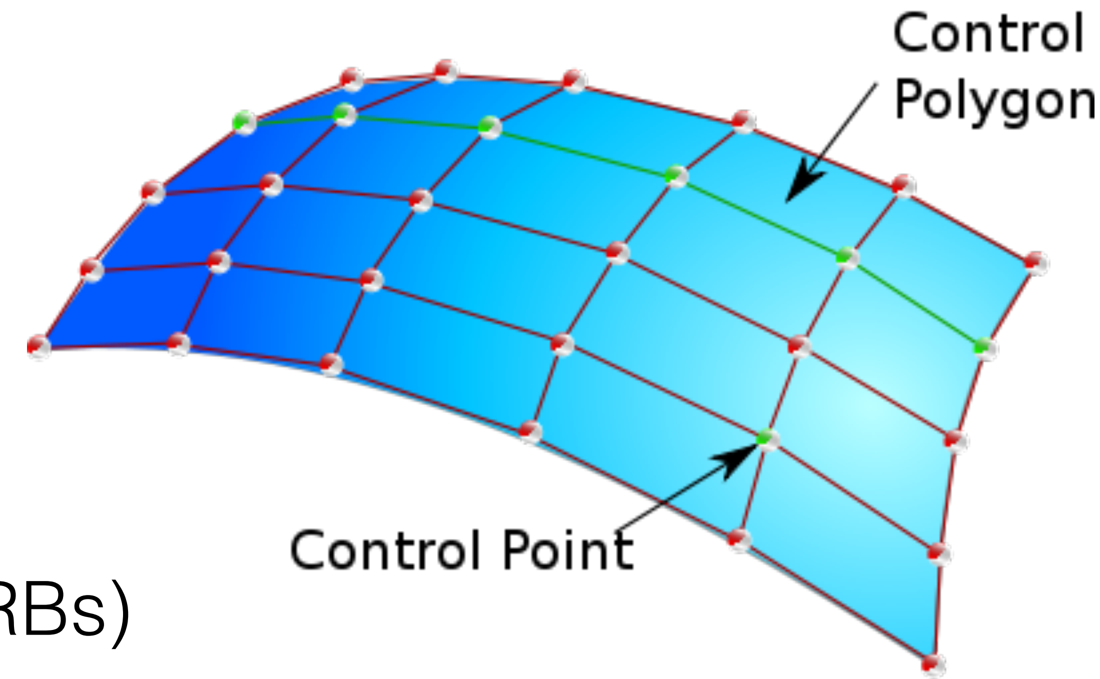
3D Surfaces

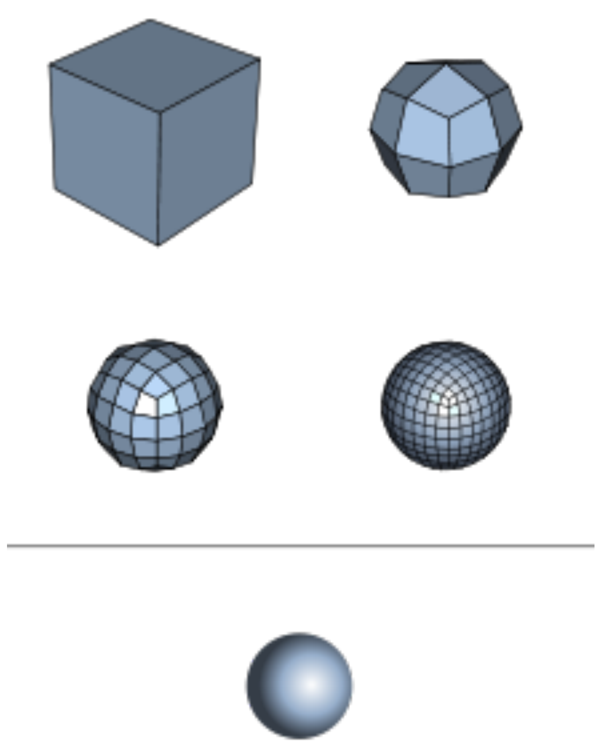
- Extend idea of Bezier or B-spline curves to a *mesh* or *grid* of control points
- Two parameters across surface: *s*, *t*
- Similar in principle to bicubic interpolation



3D Surfaces

- Bezier
- B-spline
- Non-Uniform Rational B-splines (NURBs)
- Catmull-Rom and other spline variations
- Subdivision surfaces





See CS 455 and CS 557 for more

Summary

Functions	$f(x)$
Images	$I(x,y)$ or $I(x,y,z)$
Parametric Curves	$x(t)$, $y(t)$, $z(t)$
Parametric Surfaces	$x(s,t)$, $y(s,t)$, $z(s,t)$

Coming up…

- Geometric operations:
  - Resizing
  - Rotating
  - Warping