FX Options-Implied Probability Densities and Macroeconomic Surprises

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Abstract

A key result in the finance literature is that risk-neutral probability density functions (PDFs) of an asset's price can be extracted from the prices of its related options. In this paper, I extract risk-neutral PDFs for the JPY/USD exchange rate from 2018-2024 and examine how the moments of the options-implied distributions change in response to macroeconomic surprises. Specifically, I seek to verify three claims: (i) that the standard corollaries to the interest-rate parity principle hold, (ii) that the passing of an event should lower the options-implied variance of the underlying asset, and (iii) that the widespread use of the yen as a funding currency in the carry trade should result in the options-implied distribution being skewed towards yen appreciation. I find that, in general, interest parity holds, and that currency returns are distributed leptokurtotically. I find no evidence that the passing of an event affects the options-implied variance of the JPY/USD risk-neutral PDF. Finally, I find that the JPY/USD risk-neutral PDF is actually frequently skewed towards yen depreciation, and relate that finding to money managers' positions in the yen carry trade.

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1 Introduction

Asset prices are rich sources of information for policymakers, regulators, academics, investors, and market-makers. Since the "true" value of an asset is a function of what will happen in the future — such as the number of phones Apple will sell, or what the demand for lumber will be in a given month — the price of an asset reflects an aggregation of market participants' expectations about the future. The key idea is that, even when stakes are small, traders bid their true beliefs, and the collective wisdom of the crowd determines the price.

The extraction of expectations from asset prices is a useful tool in a range of settings. A central bank might check whether its forward guidance is effective in moderating inflation expectations by examining the breakeven rate implied by TIPS and Treasury yields (D'Amico et al., 2018). A STIR market-making desk might hedge its book in anticipation of rate cuts or rate hikes; in this task, looking at the expected path of the short rate implied by futures contracts would be a solid starting point. Wolfers and Zitzewitz (2004) describe how the U.S. Department of Defense started a project in which traders could speculate on geopolitical risk, allowing government officials to leverage the informational content contained in the prices of these risks. When it came to gauging win probabilities during the 2024 U.S. presidential election cycle, polling data took a backseat to event contracts. By October 24th, 2024 (about two weeks before the election), nearly $2.4 billion had been wagered on the election on the platform Polymarket, and the odds implied by Polymarket's event contract featured prominently on the news and social media.

The explosive growth of derivatives markets over the last half-century has been a boon for research in this area. Futures and options contracts exist on virtually every major currency pair and interest rate. At the end of June 2023, the Bank for International Settlements (2023) estimated that notional amounts outstanding of over-the-counter FX and interest rate derivatives stood at $700 trillion. If you include exchange-listed contracts in that tally, such as those offered by the Chicago Mercantile Exchange (CME) and the Chicago Board of Exchange, that number is even higher.

From a volume perspective, options in particular have become quite popular. They give one a view into the higher-order moments of an asset's possible prices. With an option, a trader can not only bet on the price of an asset, but also on the distribution associated with the range of price outcomes at the option's maturity, and the related moments of that distribution. Unsurprisingly, a longstanding and well-known result in the options literature is that the prices of these derivatives contain risk-neutral distributions for the future values of their underlying assets (Breeden and Litzenberger, 1978). The wide use of this information is indicative of its value. For example, until March 2021, the Bank of England published weekly estimates of the options-implied distributions of short sterling futures and the FTSE 100 (Clews et al., 2000). Starting in 2014 and continuing to this day, the Federal Reserve Bank of Minneapolis publishes options-implied distributions for underlying assets ranging from the consumer price index to soybean futures.

In this paper, I study these distributions in the dollar-yen currency market. I extract daily risk-neutral distributions implied by the prices of 3-month and 1-month Japanese yen - U.S. dollar (JPY/USD) options, measure the moments of the distributions, and examine how the moments change in response to macroeconomic surprises. The structure of the paper is as follows. Section 2 contains a literature review encompassing various methods of estimation and empirical analyses. In Section 3, I discuss the data and methodology used in this paper. Section 4 contains the results and discussion thereof, and Section 5 concludes.

2 Literature Review

Arrow and Debreu (1954) proved that a competitive equilibrium exists in a model of an economy subject to certain conditions. Their work lends two important ideas to this paper: Arrow-Debreu securities and state prices. An Arrow-Debreu security is a financial contract that pays exactly one unit of a currency if a specific state occurs at a specific time in the future and zero otherwise; the price of an Arrow-Debreu security is the state price. A market is considered complete if there exists an Arrow-Debreu security for every possible state of the world. In that case, Arrow-Debreu securities would form a complete basis for any contingent payoff structure, as you could replicate any of those structures with an Arrow-Debreu portfolio. Given a menu of states S, we can then express the price p of any asset a as a linear combination of the payoff structure $X_{S,a}$ and the state price π_S.

$$p_a = \sum_{S=1} X_{S,a}\pi_S \tag{1}$$

This also implies no arbitrage opportunities exist. If p_a was, for example, cheaper than the price of a portfolio of Arrow-Debreu securities that replicated the payoff structure $X_{S,a}$, then a trader could earn a riskless profit simply by selling the Arrow-Debreu portfolio and buying the asset (Varian, 1987). This no-arbitrage condition further implies that a risk-neutral probability q for each state S exists and is embedded in state prices (with appropriate discounting by the interest rate r).

$$q_S = \pi_S(1+r) \tag{2}$$

These probabilities are strictly positive and sum to one.[1]

$$q_S > 0,\ \sum_{S=1} q_S = 1 \tag{3}$$

Using Arrow-Debreu securities as building blocks, Breeden and Litzenberger (1978) were the first to show that one can recover an implied risk-neutral distribution from a set of options prices with a constant maturity across different strikes. First, consider an

[1] Negative probabilities — mathematical nonsense — indicate the existence of arbitrage opportunities.

Arrow-Debreu security paying \$1.00 at time t if the value of an underlying portfolio is S at time t and 0 otherwise. Assume that the value of the underlying portfolio can take discrete values of $S_t = [\$1.00, \$2.00, \$3.00, ..., \$n]$. The state prices must reflect traders' assessment of the probability of the underlying portfolio taking value S at time t. We can create this pure contingent claim (read: Arrow-Debreu security) with a portfolio of European call options.

Following Breeden and Litzenberger (1978), we denote the price of a European call option by the real-valued function $c(K,\, t)$, where K is its strike price and its expiry is time t. Now we can construct an Arrow-Debreu security — whose payoff is \$1.00 if and only if $S_t = K$ — by putting on a butterfly spread with these options:

- Long a call option $c(K-1, t)$
- Long a call option $c(K+1, t)$
- Short two call options $c(K, t)$

In practice, S_t can take values with much smaller step sizes. Denote the step size δ, and the price vector of the portfolio is:

$$[c(K-\delta, t) - c(K, t)] - [c(K, t) - c(K+\delta, t)] \tag{4}$$

In simplifying this equation, note that we are setting up a second-order central finite difference approximation:

$$c(K-\delta, t) + c(K+\delta, t) - 2c(K, t) \tag{5}$$

Ultimately, we would like to approximate the risk-neutral probability density function of S_t. Hull (2015) lays out how we can implement (2) in this task. The price of a European call option with strike K and expiry t is given by

$$c = e^{-rt} \int_{S_t=K}^{\infty} (S_t - K) g(S_t)\, dS_t \tag{6}$$

where g is the risk-neutral probability density function (PDF) of S_t and r is the risk-free rate. Differentiating twice with respect to K yields this function.

$$\frac{\partial c}{\partial K} = -e^{-rt} \int_{S_t=K}^{\infty} g(S_t)\, dS_t \tag{7}$$

$$\frac{\partial^2 c}{\partial K^2} = e^{-rt} g(K) \tag{8}$$

$$g(K) = e^{rt} \frac{\partial^2 c}{\partial K^2} \tag{9}$$

When our δ is small, the value of our payoff can also be calculated by integrating the payoff over $g(S_t)$.[2] Then we have the equivalence

$$e^{-rt} g(K)\delta^2 = c(K-\delta, t) + c(K+\delta, t) - 2c(K, t) \tag{10}$$

and our approximation of $g(K)$ is

$$g(K) = e^{rt} \frac{c(K-\delta, t) + c(K+\delta, t) - 2c(K, t)}{\delta^2} \tag{11}$$

2.1 Methods of Extraction

The methods used to generate risk-neutral distributions from options prices can be divided into two categories: parametric and non-parametric (Jackwerth, 1999).[3] Parametric methods assume the data is distributed according to a specific form — which is helpful when one does not have a lot of data to work with — while non-parametric methods fit the form of the distribution to the data, requiring a large dataset to be accurate.

Following Jackwerth (1999), there are three general categories of parametric methods used for estimating risk-neutral distributions. The first are expansion methods, which add correction terms to a normal or lognormal distribution, like the Taylor expansion of a function. Rockinger and Abadir (1997) use hypergeometric functions to estimate densities, while Abken et al. (1996) use Hermite polynomials with four correction terms. Corrado and Su (1996) use an Edgeworth expansion to fit the PDF of the

[2] This allows us to assume that $g(S_t) = g(K)$ in our payoff range. In other words, if the distance between our strike prices is small, we can assume that the probability of the asset price being anywhere in the range $K-\delta, K+\delta$ is about the same as the probability of the asset price being equal to K.

[3] See Jackwerth (2004) for a comprehensive catalogue of research by method type.

option by successively adding higher-order moments. Corrado and Su (1996), however, use only one random variable; multi-asset options, or options derived from the value of multiple underlying assets, require the use of multiple random variables.[4] Arismendi (2014) extend their method to multi-asset options using an expansion of an infinite series over the derivatives of the known continuous time density, and later Arismendi and Prokopczuk (2016) derive a semi-parametric estimation of the risk-neutral density in the case of American multi-asset options. Rompolis and Tzavalis (2008) use a C-type Gram-Charlier expansion to estimate the risk-neutral density, with the main benefit being that their method guarantees positive values of the densities, as opposed to other parametric approaches that can result in negative values.[5]

The second category of parametric approaches are generalized distribution methods, which add parameters to the distribution beyond the two parameters of a normal or lognormal distribution. Jackwerth (1999) cites Posner and Milevsky (1998) and Sherrick et al. (1996) as examples. More recently, Figlewski (2010) uses a (non-parametric) cubic spline fitted to implied volatilities but constructs the tails using generalized extreme value distributions.

Finally, there are mixture of distribution methods. Like generalized distribution methods, these allow for the insertion of additional parameters, but do so by sourcing probabilities from several different distributions. Ritchey (1990), Bahra (1996), and Melick and Thomas (1997) use a mixture of univariate lognormal distributions to construct the risk-neutral distribution. Liu et al. (2007) obtain risk-neutral densities from a mixture of two lognormal densities and a generalized beta density; they find that the log-likelihoods of risk-neutral densities obtained in this way are significantly higher than those obtained by spline methods.

The three general types of non-parametric methods are kernel methods, maximum-entropy methods, and curve-fitting methods.[6]. Kernel methods are essentially "localized" nonlinear regressions in which a kernel measures the likelihood of a function passing through an observed data point. Aït-Sahalia and Lo (1998) use a non-parametric kernel

[4] Margrabe (1978) uses a multivariate approach to value exchange options, while Deng et al. (2008) use a multivariate approach to value spread options.

[5] Jondeau and Rockinger (1999) also use a Gram-Charlier expansion and force the coefficients to fall in a region such that the densities are positive.

[6] Garcia and Gençay (2000) do not fit neatly into one of these categories. They use a feedforward neural network and introduce a "homogeneity hint" in order to limit overfitting.

regression to estimate densities on S&P 500 options, noting that kernel approaches are attractive for finance research because they require few assumptions and because large samples of high quality data are generally available. Härdle and Yatchew (2001) use a non-parametric least squares procedure and impose various constraints on features such as the smoothness of derivatives and the monotonicity and convexity of the call function. Maximum-entropy methods involve choosing a prior distribution and then finding the risk-neutral distribution by maximizing the level of uncertainty associated with the subject variable's possible outcomes. Buchen and Kelly (1996) use uniform and lognormal distributions as priors, while Rockinger and Jondeau (2001) use normal, t, and generalized error distributions. Rompolis (2010) devised a new maximum-entropy method in which the risk-neutral moments are used as maximization constraints in the place of option prices.

In this paper, I opt to use a non-parametric curve-fitting method. Here there are two rough classifications: polynomial methods and spline methods. Polynomial methods fit a single curve to the implied volatility smile, while spline methods use multiple polynomials combined into a piecewise function such that the knots are smooth and continuous. Shimko (1993) was the first to fit a quadratic polynomial to implied volatilities. Brown and Toft (1999) adapt the approach of Shimko (1993) to a seventh-order cubic spline method. Malz (1997) fit a quadratic polynomial to deltas instead of strike prices. The benefit in his approach is that traders typically quote in deltas rather than strikes and the range of deltas is bound to $[0, 1]$, with the drawback being that translating from delta space to strike price can be unwieldy (Jackwerth, 2004). Gatheral and Jacquier (2014) fit stochastic volatility inspired models to S&P 500 implied volatilities with excellent fit qualities. Campa et al. (1998) compare the cubic spline and mixture of lognormals approaches and find both produce skewed and leptokurtotic distributions (as one might expect). More recently, Jiang et al. (2021) develop a "piecewise constant" method in which there are knots on each distinct strike price. They find that their approach outperforms not only the typical cubic spline method, but also a quartic B-spline approach and a parametric method based on an inverse normal Gaussian distribution.[7]

Jackwerth (2004) remarks that, while certain methods have marginal benefits over others, ultimately each method produces similar PDFs. As a result, it makes sense to

[7] As far as I know, Lee (2014) was the first to use a quartic B-spline to estimate densities.

use the "easiest and most stable methods" which "tend to be in the group of methods for curve-fitting" (Jackwerth, 2004). The caveat is that the methods often significantly differ in their respective estimations of the tails. This can be mitigated, however, by obtaining data with a very wide range of strikes.

2.2 Applications

Risk-neutral probability densities can be extracted for any underlying on which options exist. What exactly does "risk-neutral" mean? Imagine we play a game where a coin is flipped and you win \$1 if it is heads. The "true" probability of you winning the game is 50%; therefore, you should be willing to pay 50 cents to play the game. Suppose, however, you are only willing to pay 45 cents — maybe you don't like taking risk. By quoting a price of 45 cents, you have implicitly estimated the odds of the coin being heads to be 45% (which we know to be incorrect, assuming the coin is fair). We refer to the former probability (50%) as the $\mathbb{P}$-measure probability, and the latter probability (45%) as the $\mathbb{Q}$-measure probability.

The probabilities implied by options prices are $\mathbb{Q}$-measure probabilities. So, we have to proceed with caution in interpreting these densities. To translate $\mathbb{Q}$-measure probabilities to $\mathbb{P}$-measure probabilities, we have to estimate the marginal utility and resulting level of risk aversion of the representative investor.[8] One method is to use a stochastic discount factor, which is the ratio of the state prices (the price of a butterfly structure with infinitesimally close strikes) to actual probabilities.

As mentioned earlier, the expectations implied by the price of options are useful information for a country's monetary authorities. Campa et al. (1999) use options on BRL/USD to evaluate the credibility of Brazil's crawling-peg exchange rate regime. Their work is an instructive example in the usefulness of options-implied PDFs. For example, since the implied kurtosis of BRL/USD increased over the observation period (1994-1997), they can deduce that traders increasingly lowered their expectation of the exchange rate devaluing outside of the target zone, but increasingly expected that if BRL/USD *did* devalue outside of the target zone, then the devaluation would be relatively larger. Few other indicators can provide that level of insight. Further research

[8]See Meucci (2011).

on options-implied exchange rate expectations, mostly geared towards central bankers, has been done by Castrén (2005), Csávás (2008), and Özlü et al. (2010).

There are also applications to interest rates. There exists a large quantity of term structure research that endeavors to explain bond yields and swap rates. Since the prices of interest rate options reflect their nonlinear payoff functions, they can be used to uncover new information about term structure dynamics. Li and Zhao (2009) use a nonparametric technique to estimate PDFs of the (nominal) LIBOR rate. In doing so, they are able to both document the existence of stochastic volatility factors in interest rate derivatives markets, and conjecture that these factors are driven by prepayments in mortgage markets.[9] With the rise of a market for inflation-linked derivatives, Kitsul and Wright (2013) were able to estimate how traders' expectations of inflation change in response to macroeconomic news announcements. More specifically, they find that the implied probabilities of deflation are highly related to the size of a macroeconomic data surprise. Later, Wright (2016) uses options on TIPS to directly estimate implied probability density functions for real interest rates. In a discussion of Wright (2016), Swanson (2016) notes that the "paper introduces us to a new options dataset and provides us with the first market-implied risk-neutral PDFs for the U.S. real interest rate...This kind of information is difficult or impossible to get any other way."

Finally, these densities can also provide insight on how important "skin in the game" is for forecasts (Taleb, 2018). For example, do event contracts outperform polling data in election predictions? Options-implied PDFs can be used as an indirect method of answering this question. Gürkaynak and Wright (2023) give an interesting anecdote: on the day of Brexit, before voting had concluded, the 3-month probability density function for cable was skewed in the direction of pound depreciation, suggesting that traders were more concerned with the tail risk of a "Yes" vote than a "No" vote. Their observation diverged significantly from that of opinion polls leading up to the election, which showed a consistent voter preference towards a "No" vote (Financial Times, 2016). This example demonstrates how options-implied PDFs can contain a wider range of informational content than simple polling data (i.e., a look into the higher-order moments).

[9]This research relates to the "unspanned stochastic volatility puzzle." In dynamic term structure models, there is usually an assumption that the same set of risk factors drive bond yields and interest rate derivatives prices. Stochastic rate volatility is a risk factor that exists in interest rate derivatives markets but not in the bond market (hence "unspanned"). See Backwell (2021).

2.3 Event Studies and Ex-Ante Hypotheses

In a narrow sense, economists are in the business of devising models that explain the world and verifying those models with empirical analyses. In many cases, a convenient way to verify such a model is with an event study, which is a catch-all term for statistical methods that quantify the impact of an event on, for example, the value of a security. Event studies have long found immense popularity in finance, perhaps owing to the wide availability of data and the notion of market efficiency introduced by Eugene Fama.[10] Indeed, the first event study was published in 1933 and studied the effect of stock splits on stock price changes (MacKinlay, 1997). The event window of an event study is the period in which the asset price is examined. Short-horizon event studies — i.e., studies examining daily or intraday price data — are typically more popular than long-horizon event studies with weekly, monthly, or even yearly event windows. This is because the power of long-horizon event studies is generally much lower than the power of short-horizon event studies (Kothari and Warner, 2007). Researchers can increase the power of their event studies by shortening their event windows (MacKinlay, 1997).

Just what models am I seeking to verify with this event study? The first is the general principle of interest rate parity, or the idea that exchange rates must rise or fall to equilibrate the yield on deposits in different countries. A convenient way to illustrate this condition is by examining covered interest rate parity (CIP), which is essentially a no-arbitrage condition for the pricing of FX forward contracts.[11] It states that the forward rate of a currency pair should be equal to the difference in the interest rates between the two countries. The mathematical expression of this relationship is given by

$$1 + r^d_{t,t+1} = (1 + r^f_{t,t+1}) \frac{S_t}{F_{t,t+1}} \tag{12}$$

where $r^d_{t,t+1}$ is the $t+1$ tenor domestic swap rate at time t in currency d, $r^f_{t,t+1}$ is the $t+1$ tenor foreign swap rate at time t in currency f, S_t is the spot exchange rate at time t, and $F_{t,t+1}$ is the $t+1$ forward exchange rate at time t expressed as units of currency f per currency d. The corollary to this no-arbitrage condition is that, given a standard

[10] Event studies have also found wide use in marketing (Sorescu et al., 2017).

[11] Uncovered interest rate parity (UIP) refers to the scenario in which interest rate parity is satisfied without the use of a forward contract; i.e., where international investors leave themselves exposed to exchange rate risk. UIP is systematically violated in currency markets, which allows investors to take advantage of the ”carry trade,” expanded on later in this section.

central bank reaction function (e.g., the Taylor Rule), stronger-than-expected economic data releases in the domestic country imply that $r^d_{t,t+1}$ will rise, and therefore currency d should appreciate. As a result, I expect that stronger-than-expected macroeconomic news announcements in the U.S. (Japan) will cause the mean and skew of the options-implied JPY/USD distribution to decrease (increase).

There are a few caveats. Prior to the Great Financial Crisis (GFC), CIP was an iron-clad rule; it almost perfectly explained forward exchange rates, with some deviations opening up in very small windows of time for arbitrageurs to take advantage of (Akram et al., 2008). Since the GFC, large and persistent CIP deviations have been the norm (Levich, 2017). The difference between the theoretical (read: CIP-implied) forward rate and the observed market forward rate is referred to as "cross-currency basis." Du et al. (2018) document the existence of sizeable and persistent cross-currency basis and conjecture that the driving factors include rising costs of arbitrage and market-making activities for financial intermediaries induced by post-GFC regulation and inelastic demand for assets denominated in high-interest-rate currencies. Still, event study research examining the effects of macroeconomic news announcements on exchange rates largely confirm the basic conclusions of CIP. Faust et al. (2007), using high-frequency exchange rate data (and therefore increasing the power of their test), find that unexpectedly strong announcements in the United States cause the dollar to appreciate in the short-run. Almeida et al. (1998) articulate a standard reaction function for the Federal Reserve and Bundesbank (where, in general terms, both central banks prefer lower inflation) and find that the DEM/USD response to macroeconomic surprises comport with the expected policy reaction of the two central banks.

The second phenomenon I am seeking to verify is that the passing of a scheduled macroeconomic news release should reduce the implied variance of the JPY/USD distribution. The intuition is that scheduled news releases resolve information uncertainty, and therefore should lower the level of uncertainty (read: variance) contained in the distribution of the exchange rate outcomes. Ederington and Lee (1996) find evidence to support this hypothesis. They express the daily percentage change in volatility as follows:

$$\frac{\sigma_t^2 - \sigma_{t-1}^2}{\sigma_{t-1}^2} = \left(\frac{1}{T_t}\right)\left[\frac{\left(\sigma_{t-1}^2 - \sigma_{t,t-1}^2\right)}{\sigma_{t-1}^2} + \sum_{u=t+1}^{te} \frac{\left(\sigma_{u,t}^2 - \sigma_{u,t-1}^2\right)}{\sigma_{t-1}^2}\right]. \tag{13}$$

where σ_t^2 is the mean anticipated daily volatility over the life of an option on day t, T_t is the time to expiration of the option on day t, $\sigma_{u,t}^2$ is the anticipated variance on day u given all of the public information available on day t, and te is the option expiration date. Suppose an announcement is scheduled on day t. Since traders know on day $t-1$ that the announcement is scheduled for day t, they will anticipate that volatility on day t will be higher than the mean anticipated daily volatility over the remaining life of the option, implying that $\sigma_{t,t-1}^2 > \sigma_{t-1}^2$. Therefore, the options-implied variance should decline because the remaining life of the option will no longer include the relative-higher-volatility announcement day t. Jiang et al. (2012) examine the effect of macroeconomic announcements in the United States and Europe on various implied equity volatility indices and confirm the model of Ederington and Lee (1996).

Third, thanks to the widespread use of the yen as a funding currency in carry trades, I am seeking to verify that the JPY/USD options-implied distribution is typically skewed towards yen appreciation. The carry trade is a trading strategy in which an investor borrows in a low-yielding currency and invests in a high-yielding currency. Under uncovered interest rate parity (UIP), the exchange rate should theoretically adjust to equilibrate the returns between the two currencies' yields. In practice, however, UIP is systematically violated and thus the carry trade is usually profitable. The risk to carry trades is "crash risk," or the risk that the funding currency will quickly and dramatically appreciate against the investment currency. Brunnermeier et al. (2009) recall the old traders' adage that "exchange rates go up by the stairs and down by the elevator." This is why the carry trade is often colloquially referred to as "picking up pennies in front of a steamroller." A common measure of skewness in options markets is the price of a 25-delta risk reversal (25Δ RR), which is a trading strategy where an investor sells the 25-delta put and buys the 25-delta call. The price of the 25-delta risk reversal, R_{25}, is then

$$R_{25} = \sigma_{call,25} - \sigma_{put,25} \tag{14}$$

If R_{25} is negative then the implied volatility of the call is less than the implied volatility of the put, therefore implying a negatively skewed distribution. I expect that the JPY/USD distribution will be positively skewed (towards yen appreciation) and will later compare the options-implied skewness measure with the risk reversal measure. In this context, positive skewness suggests that the *slight* yen depreciation is most likely, but that *extreme* yen appreciation is more probable than *extreme* yen depreciation.

3 Data and Methodology

3.1 Data

The options data were obtained from the CME via Bloomberg. The contract specifications are European-style monthly options on JPY/USD futures expiring at 9:00am central time.[12] A complete time series of 3-month and 1-month put and call market-close prices across over fifty strikes was constructed from this dataset from January 1, 2018 to April 1, 2024. Additionally, JPY/USD spot and forward rates, U.S. and Japanese policy rates, and economic release data and forecasts came from Bloomberg.

I measure the size of an economic surprise as the difference between the expected value of a data release and its actual value, as is standard practice in the literature.[13]. Bloomberg's forecast data is extremely comprehensive — its forecaster database covers approximately 2,300 economic tickers across 180 economies. The firm maintains relationships with around 1,600 forecasters, including academics, traders, and portfolio managers, and the most popular data series have forecasts composed of up to eighty individual estimates. For each release, one can use Bloomberg's ECOS function to examine the performance of individual forecasters and observe the dispersion of their forecasts. Bloomberg uses the median estimate of each release as its survey value. This is all to say that Bloomberg's forecasts are quite good.

The CME FX options data is also reliable and accurate.[14] Although most of the trading volume in its options is focused in shorter-dated expiries, the two-way price quotes displayed by the CME are submitted by a plethora of bank and nonbank market makers who stand ready to buy and sell at those prices.[15] Further, the CME will only display a quote if one has been submitted by a market maker; otherwise, the market is displayed as empty. So, we can conclude that liquidity exists for every price quote contained in the dataset.

[12] Note that the quote convention for spot dollar-yen is USD/JPY. To convert the CME strike to the dollar-yen rate, simply divide 10,000 by the strike.

[13] For a discussion of the properties of this value, see Wójtowicz (2022)

[14] Thanks are due to Chris Povey, head of FX Options at the CME, for clarifying the process by which quotes are collected.

[15] In 2023, approximately 23% of CME FX options volume was concentrated in tenors less than or equal to one week, and 59% was concentrated in tenors less than or equal to 1 month. Trading rarely occurs in tenors longer than 3 months.

3.2 Converting Prices to Probabilities

I use the parabolic volatility structure of Shimko (1993) to extract daily observations of the probability densities from the options price data.[16] First, I run a regression of the form:

$$Y_t = \beta_0 + \beta_1 K_t \tag{15}$$

where Y is the call price minus the put price, K is the strike price, and t is the date of observation. This regression yields the market-implied exchange rate and risk-free interest rate differential because of put-call parity, which states that prices of European call and put options are governed by the relationship:

$$c + Ke^{-rt} = p + S \tag{16}$$

where c is the price of a call, K is the strike discounted at the risk-free rate, p is the price of a put, and S is the spot price of the underlying. Any deviation from this linear relationship between call and put prices will allow for arbitrage opportunities. After running the regression, β_0 can be interpreted as the market-implied exchange rate, and β_1 can be interpreted as a the price of a bond whose implied yield for \$1 of face value is equivalent to the difference between the U.S. and Japanese risk-free rate with corresponding maturities. Then, with the Black and Scholes (1973) model for valuing options, I use the implied exchange rate, discount rate, and market call prices to solve for implied volatilities using the Newton-Raphson method, with the key difference being that the discount rate is the difference of the domestic and foreign interest rate with maturity t (Bisesti et al., 2005). For a European FX call option with strike K and maturity t, let S_t be the market-implied exchange rate, B_t be the discount factor, σ be volatility, and τ be years to expiry:[17]

$$c(K, t) = S_t \phi(\delta_1) - K B_t \phi(\delta_2) \tag{17}$$

[16] Code and data are publicly available at https://github.com/chofmann79/Senior-Thesis.
[17] B_t is the negative of our slope coefficient, β_1.

where

$$\delta_1 = \frac{\ln(\frac{S_t}{KB_t}) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}} \tag{18}$$

$$\delta_2 = \delta_1 - \sigma\sqrt{(\tau)} \tag{19}$$

Now it is possible to plot implied volatilities against strikes and estimate the volatility structure. The estimate is a quadratic equation of the form:

$$\sigma(K,\tau) = a(\tau) + b(\tau)K + c(\tau)K^2 \tag{20}$$

where a, b, c, are constant coefficients. Using this smoothed volatility structure, I calculate smoothed call prices, and differentiate those call prices by (8) to graph the probability density function. I then calculate its first four moments as below, where K_i is the ith strike price, $P(x_i)$ is the ith density, and δ is the constant distance between strikes.

$$\mu = \sum_{i=1}^{n} K_i P(x_i)\delta \tag{21}$$

$$\sigma^2 = \sum_{i=1}^{n} (K_i - \mu)^2 P(x_i)\delta \tag{22}$$

$$\text{Skew} = \sum_{i=1}^{n} \frac{(K_i - \mu)^3 P(x_i)\delta}{\sigma^3} \tag{23}$$

$$\text{Kurtosis} = \sum_{i=1}^{n} \frac{(K_i - \mu)^4 P(x_i)\delta}{\sigma^4} \tag{24}$$

I also capture percentile-based measures of the distribution's moments. For example, the change in variance can be approximated by the change in the difference between the 90th and 10th percentiles of the cumulative distribution function (CDF). The change in skew can be approximated by the change in Kelly's measure of skewness, which is given by:

$$\text{Kelly's Skewness} = P_{90} + P_{10} - (2P_{50}) \tag{25}$$

3.3 Illustrative Example

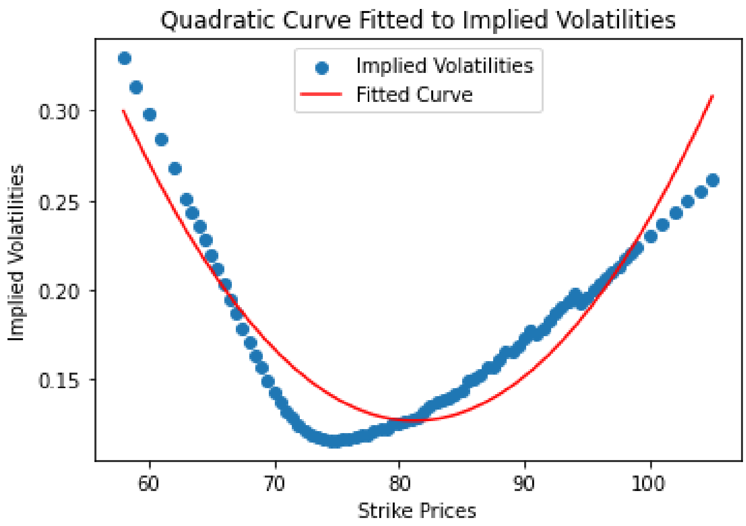
On December 20, 2022, the Bank of Japan (BOJ) made a surprise tweak to its yield curve control policy (YCCP), allowing the yield on its 10-year bonds to float in a range of ±50 basis points — a broadening from the previous range of ±25 basis points (Dooley, 2022). The market prices of 3-month put and call options on JPY/USD on December 19, 2022 are contained in Table 1.

Table 1: Call and Put Prices on December 19, 2022

Strike Price	Call Price	Put Price
58.0	15.7	0.005
59.0	14.71	0.005
⋮	⋮	⋮
74.5	1.14	1.8
75.0	0.97	2.12
75.5	0.82	2.46
⋮	⋮	⋮
104.0	0.01	29.9
105.0	0.01	30.89

We regress the difference in the call and put prices on the strike prices, yielding an intercept (or market-implied exchange rate) of 73.84 and a slope (negative of the implied bond price) of -1.0091. Next, we use the coefficients in the regression to solve for implied volatilities and plot them against strikes, and fit a quadratic curve to them. This is shown in Figure 1.

Figure 1: Implied Volatilities Plotted against Strikes



The equation for the quadratic polynomial is:

$$0.0003197159\mathbf{K}^2 - 0.0519412790\mathbf{K} + 2.2367416809 \tag{26}$$

This equation is used to calculate smoothed called prices to approximate the risk-neutral probability density function, $g(K)$. To plot $g(K)$, we use (11), which yields Figure 2.

Figure 2: 3-Month Options-Implied PDF for USD/JPY on 12/19/2022 and 12/20/2022

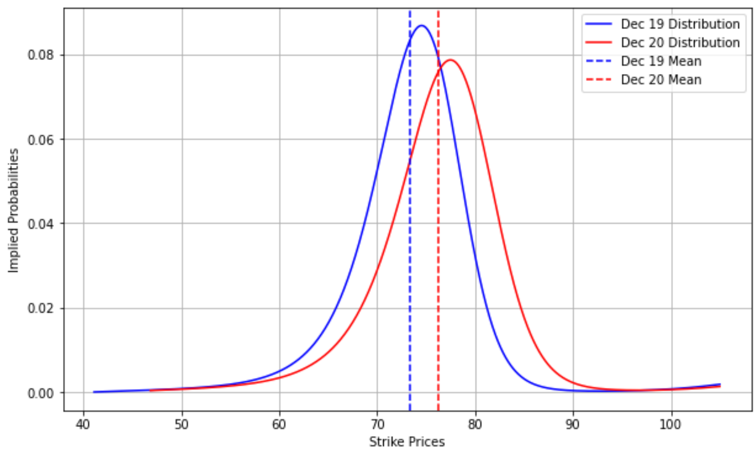


Figure 2 plots the options-implied PDFs and means before the YCCP tweak (12/19) and after (12/20). On December 19, the distribution has a mean of 73.40, a variance of 36.96, a skew of 0.06, and a kurtosis of 7.76. The distribution changes quite dramatically in response to the tweak — the mean has risen to 76.24, the variance has risen to 40.20, the skew has fallen to -0.34, and the kurtosis has fallen to 5.65. We have measured that, following this surprise, traders expected the yen to appreciate relatively more, expected a wider range of possible outcomes, assigned more weight in general to the probability of slight yen appreciation but a higher weight to extreme depreciation rather than extreme appreciation, and discounted the probability of extreme outcomes.

The change in the mean is consistent with interest parity framework outlined in section 2.3. Higher interest rates in Japan should, *ceteris paribus*, cause the yen to appreciate over the subsequent 3 months as Japanese bonds have become more attractive to foreign investors. That the yen became more skewed towards depreciation following the YCCP announcement likely reflects carry trade dynamics. Brunnermeier et al. (2009) document the phenomenon that for funding currencies, lower interest-rate differentials predict lower skewness. The idea here is that as the magnitude of the interest rate differential between two countries declines, the juice will no longer be worth the squeeze for some speculators, and they will exit their short yen positions.

The changes in the variance and kurtosis are harder to interpret and at first glance may seem contradictory. The YCCP tweak marked a surprising divergence from a monetary policy regime that had been mostly stable for the prior six years. It created a lot of uncertainty over the future path of interest rates in Japan, and by extension uncertainty over the value of the yen. In that sense, it follows that the implied variance of the distribution should rise. But why should the implied probability of extreme outcomes fall?

As it turns out, the secret ingredient might be the credibility of the BOJ. "Central bank credibility" is a rather fuzzy term. In general, the degree to which a central bank is credible is the degree to which people believe it will do what it says. It might be hard to quantify, but it certainly exists; for example, traders clearly "believe" in the Federal Reserve's statements much more than they believe in those of emerging-market central banks. High credibility brings benefits, too: in a survey conducted by Blinder (2000), central bankers and economists were in general agreement that a central bank is better positioned to defend its currency against a speculative attack when it is highly credible.

Is the BOJ credible? Blinder's respondents rated the following two components as the most important contributors to a central bank's credibility: (i) a history of living up to its word, and (ii) central-bank independence. While I lean towards giving the BOJ a thumbs up on (i), the honesty of a central bank is admittedly difficult to measure. The latter statement is quite easy to verify. The revision of the Bank of Japan Act in 1998 put significant checks on the ability of the Japanese government to interfere with BOJ operations, and thus far has stood the test of time. What's more, the BOJ is one of the largest central banks in the world, located in one of the largest economies in the world, and is in charge of one of the most popular currencies in the world — if the BOJ isn't credible, what central bank is?

While the YCCP tweak may have added uncertainty to the path of Japanese interest rates, thanks to the BOJ's credibility, the market may also have viewed it as a strong, credible move by the central bank, scaring off would-be carry-trade speculators. In this case, it follows that the implied variance would increase, but the implied kurtosis — the implied risk of a big crash — would decrease.

3.4 Model Specification

To quantify the effect of macroeconomic surprises on the moments of the market-implied distribution, I run a regression of the following form:

$$\Delta Y_t = \beta_0 + \beta_1 X_{1t} D_{1t} + \beta_2 X_{2t} D_{2t} + \ldots + \beta_{10} X_{10t} D_{10t} + \epsilon_t \tag{27}$$

where ΔY_t is the change in a moment of the risk-neutral distribution (e.g., the mean), β_0 is an intercept, $X_1 ... X_{10}$ are the ten types of macroeconomic surprises under study, $D_1 ... D_{10}$ are dummy variables that are 1 if the corresponding release occurred on that day and 0 otherwise, and ϵ_t are heteroskedasticity-robust standard errors. This regression design allows us to accommodate days in which there are multiple releases.

The ten release types are listed in Table 2. The magnitude of the surprise is measured as the actual value of the release minus the expected value. For example, a consumer price index (CPI) release of 2.5% against an expectation of 2.0% is measured as a 50 basis point surprise. Since the outcomes of Federal Open Market Committee (FOMC) and BOJ meetings are typically expected, I use the log change in ten-year Treasury and Japanese government bond (JGB) futures, respectively, as a proxy for the magnitude of the surprise. Most of the Treasury futures data are intraday; i.e., they capture the change in price from 10 minutes before the FOMC announcement to 20 minutes afterwards. A fraction of the Treasury dataset and all of the JGB dataset are daily changes.

In an effort to quantify the effect of an event's passing on the options-implied variance, I also run a regression where the dummy variables are not interacted with the size of the macroeconomic surprises.

$$\Delta Y_t = \beta_0 + \beta_1 D_{1t} + \beta_2 D_{2t} + \ldots + \beta_{10} D_{10t} + \epsilon_t \tag{28}$$

where ΔY_t is a measure of the change in variance, $D_1 ... D_{10}$ are dummy variables that are 1 if the specific release occurred on that day and 0 otherwise, and ϵ_t are heteroskedasticity-robust standard errors.

4 Results

In this section, I will determine whether the three models discussed in Section 2.3 are supported by or are in conflict with my results. Regression results are reported in Tables 2, 3, and 4 in this section and in Tables 5 and 6 in Appendix B. Time series of each moment from January 1, 2018 to April 1, 2024 are contained in Appendix A.

4.1 Interest Rate Parity and other Stylized Facts

Tables 2 and 3 contain the results of the regression in (26) for 1-month and 3-month PDF moments, respectively, on macroeconomic surprises.[18] To verify the general principles of interest rate parity, I first examine the coefficients on the mean of the options-implied distribution. Stronger-than-expected Federal Open Market Committee (FOMC) meeting results and U.S. CPI releases have a statistically significant inverse relationship with the mean of the 1-month and 3-month distributions. This is consistent with interest rate parity; stronger-than-expected economic releases in the U.S. imply higher U.S. interest rates and therefore yen depreciation. The signs on the rest of the coefficients (those that are not statistically significant) also comport with interest rate parity, except for the Japanese Producer Price Index (PPI) and Japanese Gross Domestic Product (GDP).

It's not clear that there should be any linear relationship between the size of a macroeconomic surprise and the change in the options-implied variance or kurtosis; as a result, I hesitate to ascribe any causal mechanism to the statistically significant coefficients.[19] The interpretation of the skewness coefficients are complicated by the carry trade phenomenon outlined in Section 2.3. Brunnermeier et al. (2009) demonstrate, with a simple linear regression, the stylized fact in currency markets that investment currencies in the carry trade typically have negative skewness and funding currencies (of which the yen is the most popular) typically have positive skewness. The magnitude of the negative skewness is related to the size of the interest rate differential between the funding currency country and other countries. It would then make sense for any

[18] I also took alternative, percentile-based measurements of the moments, which are contained in Tables 4 and 5 in Appendix B.

[19] That being said, it seems that stronger than expected data in either the U.S. or Japan reduce the variance of the JPY/USD distribution. See, for example, the variance coefficients on U.S. GDP in Table 2 and the variance coefficients on U.S. GDP, Japanese PPI, and U.S. Unemployment in Table 3.

economic surprises that imply a higher *interest rate differential* between the U.S. and Japan to increase the skewness of the JPY/USD distribution. As the interest rate differential increases, more speculators pile into the carry trade, and the price of crash risk increases. The statistically significant skewness coefficients somewhat confirm this. A positive U.S. GDP surprise increases the skewness of the JPY/USD distribution (Table 2), while a positive Japanese PPI surprise decreases the skewness of the JPY/USD distribution (Table 3). In this framework, the sign of the coefficient on Japanese GDP (Table 3) is puzzling; however, it is only significant at the 10% level.

Table 2: Regression of the 1-Month PDF Moments on Macroeconomic Surprises

Release Type	**Mean**	**Variance**	**Skewness**	**Kurtosis**
FOMC Meetings	-7.59***	-15.31*	1.15	18.10
	(2.78)	(8.45)	(2.10)	(12.46)
BOJ Meetings	33.30	290.64**	13.14	15.41
	(56.89)	(124.95)	(13.53)	(132.10)
U.S. CPI	-110.11**	-88.13	-49.18	25.30
	(54.14)	(115.95)	(40.04)	(156.96)
Japanese CPI	-52.13	168.34	12.20	282.66
	(47.56)	(180.76)	(75.02)	(368.81)
U.S. PPI	-8.72	29.51	4.84	19.10
	(15.81)	(42.45)	(22.93)	(82.58)
Japanese PPI	-10.04	17.19	-2.10	26.66
	(18.36)	(53.35)	(10.38)	(54.37)
U.S. GDP	0.60	-89.42***	43.27**	237.16***
	(16.01)	(34.78)	(18.01)	(83.56)
Japanese GDP	-2.58	-10.07	-0.31	8.04
	(4.35)	(11.90)	(5.02)	(16.21)
U.S. Unemployment	-64.07	7.26	-1.26	-6.11
	(90.38)	(18.39)	(5.97)	(41.73)
Japanese Unemployment	49.12	-195.53*	114.22	451.06
	(44.17)	(113.61)	(80.46)	(345.99)
Intercept	-0.14	-0.03	0.03	0.21*
	(0.17)	(0.07)	(0.02)	(0.12)

Notes: Statistical significance at the ten, five, and one percent levels are denoted by one, two, and three asterisks, respectively.

Only a handful of the coefficients are statistically significant, indicating that a 1-day event window might be too large to capture meaningful effects. Intraday data should capture the surprise with more precision; daily-level data seems to "miss the move." For example, in a conversation with an FX options market maker, it was relayed to me that on July 11th, 2024, when the yen rallied sharply off the back of cooler-than-expected

U.S. CPI data, intraday volatility spiked, but settled quickly.[20] Further, this person stated that this was the norm in options markets when release surprises occurred. This aspect of options markets is unsurprising. Short-term uncertainty driven by surprises can give rise to significant, market-moving order flow, creating spikes in implied volatility and altering the portfolio greeks of bank- and nonbank-dealers.[21] These large market participants will move quickly to adjust their hedges, effectively causing volatility spikes to settle quickly. In a related example, I observed this dynamic personally when watching intraday changes of the Treasury-TBA basis, a spread that is largely determined by changes in interest-rate volatility. Therefore, for better results, it is advisable to use tick-level data, as in Andersen and Bollerslev (1998), Andersen et al. (2003), and Faust et al. (2007).

Table 3: Regression of the 3-Month PDF Moments on Macroeconomic Surprises

Release Type	**Mean**	**Variance**	**Skewness**	**Kurtosis**
FOMC Meetings	-8.48***	-10.88	-0.33	0.72
	(2.47)	(11.09)	(0.945)	(2.63)
BOJ Meetings	403.41	-58.60	-1.73	56.47
	(450.68)	(157.67)	(8.86)	(62.38)
U.S. CPI	-151.28***	42.82	12.90	24.58
	(58.67)	(169.27)	(14.27)	(48.75)
Japanese CPI	79.73	116.89	13.66	113.83***
	(76.32)	(139.04)	(14.89)	(38.53)
U.S. PPI	-20.73	2.30	-1.81	-6.13
	(17.33)	(57.75)	(4.00)	(11.71)
Japanese PPI	-33.3	-155.16**	-4.53*	-5.30
	(23.20)	(77.56)	(2.66)	(12.91)
U.S. GDP	-21.36	-52.07*	-1.79	6.60
	(31.99)	(27.13)	(3.54)	(9.96)
Japanese GDP	-2.96	-11.74	1.97*	4.37
	(4.86)	(19.47)	(1.06)	(4.14)
U.S. Unemployment	12.43	43.96***	-0.01	-2.33
	(7.88)	(11.27)	(1.32)	(3.67)
Japanese Unemployment	-666.02	-60.37	-14.22	-89.07
	(730.77)	(144.47)	(14.21)	(71.21)
Intercept	0.15	0.04	0.00	0.06**
	(0.26)	(0.10)	(0.01)	(0.03)

Notes: Statistical significance at the ten, five, and one percent levels are denoted by one, two, and three asterisks, respectively.

[20] In options markets, prices are quoted in implied volatility. If a trader quoted the price of an option with an implied volatility of, e.g., 15, he would say "15 vols."

[21] The term "greeks" refers to the various partial derivatives of an option's price with respect to its underlying parameters.

Another stylized fact in the currency market is that currency returns are distributed with significant leptokurtosis ((Friedman and Vandersteel, 1982), (Boothe and Glassman, 1987), (Koedijk et al., 1990)). The time series of the kurtosis of the JPY/USD options-implied distribution, which are contained in Figure 7 in Appendix A, confirm this phenomenon. 1-month kurtosis in consistently higher than 3-month kurtosis, and both 1-month and 3-month distributions are significantly leptokurtotic. That short-dated kurtosis is higher than long-dated kurtosis is a reflection of the fact that traders are more worried about extreme events in the short-term — where returns are more sensitive to one-off disasters — than over a longer time horizon, where the impact of disasters on returns in smoothed out.

4.2 Volatility and the Passing of Events

Table 4 contains the results of the dummy-only regression in (27) for measures of the 1-month and 3-month variance on the passing of a macroeconomic release. To confirm the model of Ederington and Lee (1996), I would expect to see negative, statistically-significant coefficients across the board. I find no such evidence to support their model; on the contrary, the only statistically significant coefficient indicates that the passing of the monthly Japanese unemployment rate data release actually serves to *raise* the variance of the distribution.

Table 4: Regression of the 1-Month and 3-Month Variance on the Passing of an Event

Release Type	**1m Var**	**1m** $P_{90} - P_{10}$	**3m Var**	**3m** $P_{90} - P_{10}$
FOMC Meetings	0.37	0.09	-0.01	-0.07
	(0.41)	(0.14)	(0.61)	(0.15)
BOJ Meetings	0.19	0.03	0.54	0.06
	(0.29)	(0.09)	(0.44)	(0.14)
U.S. CPI	0.20	0.04	0.27	-0.06
	(0.29)	(0.08)	(0.57)	(0.13)
Japanese CPI	-0.33	-0.04	0.22	-0.02
	(0.26)	(0.07)	(0.39)	(0.10)
U.S. PPI	0.01	-0.01	0.45	-0.01
	(0.22)	(0.07)	(0.33)	(0.08)
Japanese PPI	-0.04	-0.03	-0.02	-0.07
	(0.27)	(0.08)	(0.51)	(0.11)
U.S. GDP	0.06	-0.04	0.11	-0.08
	(0.24)	(0.07)	(0.33)	(0.10)
Japanese GDP	0.26	0.06	0.16	-0.02
	(0.27)	(0.08)	(0.43)	(0.11)
U.S. Unemployment	0.34	0.12	0.29	-0.11
	(0.35)	(0.13)	(0.50)	(0.15)
Japanese Unemployment	0.51**	0.14	0.20	0.07
	(0.26)	(0.08)	(0.36)	(0.12)
Intercept	-0.18	-0.09	-0.23	0.03
	(0.24)	(0.07)	(0.39)	(0.10)

Notes: $P_{90} - P_{10}$ is the ninetieth percentile of the distribution minus the tenth percentile of the distribution. Statistical significance at the ten, five, and one percent levels are denoted by one, two, and three asterisks, respectively.

While these results are inconsistent with their model, they are not necessarily inconsistent with the empirical tests that Ederington and Lee (1996) conducted. They found much stronger evidence for options-implied variance falling after an event passes in the interest rate markets than in currency markets. This is consistent with their observation that macroeconomic releases have a much larger effect on the price of interest rate futures than on the price of currency futures. Additionally, one event, considered in isolation, is small in the life of a 1-month or 3-month option. On average, considering only the ten events used in this paper, an event will occur 9 times over the life of a 1-month option and 27 times over the life of a 3-month option. If one considers *all* of the events relevant to JPY/USD, such as the release of current account statistics, central bank press conferences, and various sentiment indices, there is virtually an event every day. My results signal that the passing of an event does little to resolve information uncertainty in the market for JPY/USD.

4.3 The Carry Trade

Thanks to its long-standing negative or near-zero interest rates, the Japanese yen is a popular funding vehicle for carry trades. As such, the crash risk inherent in carry trades should induce a positive (negative) skew in the JPY/USD (USD/JPY) distribution. To sanity check my measure of skew, Figures 3 and 4 plot a time series of the 1-month and 3-month PDF skew and 25Δ risk-reversal prices (from Bloomberg), respectively. Note that the Bloomberg risk-reversal prices are based off of USD/JPY options, while my PDF measure is based off of JPY/USD options.

Figure 3: 1-Month PDF Skewness and 25Δ RR, 1/2018-4/2024

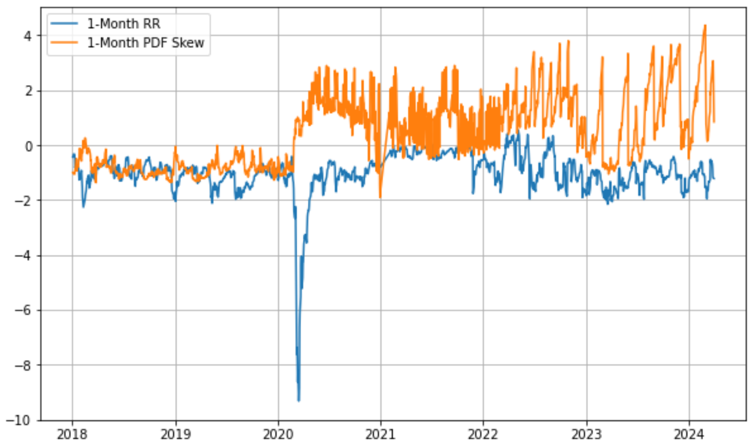
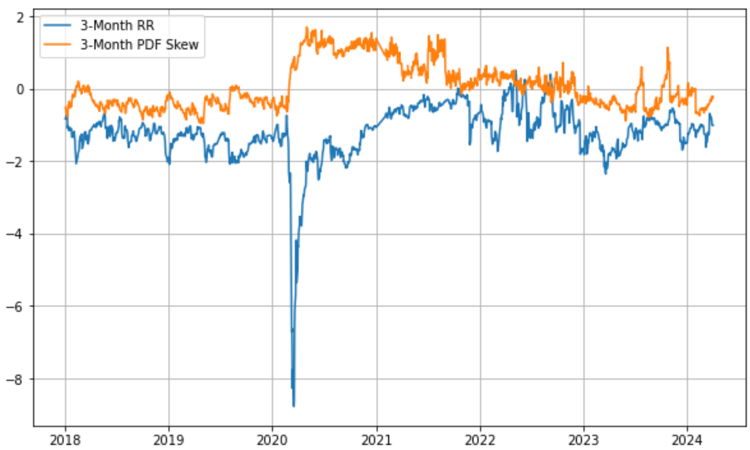


Figure 4: 3-Month PDF Skewness and 25Δ RR, 1/2018-4/2024



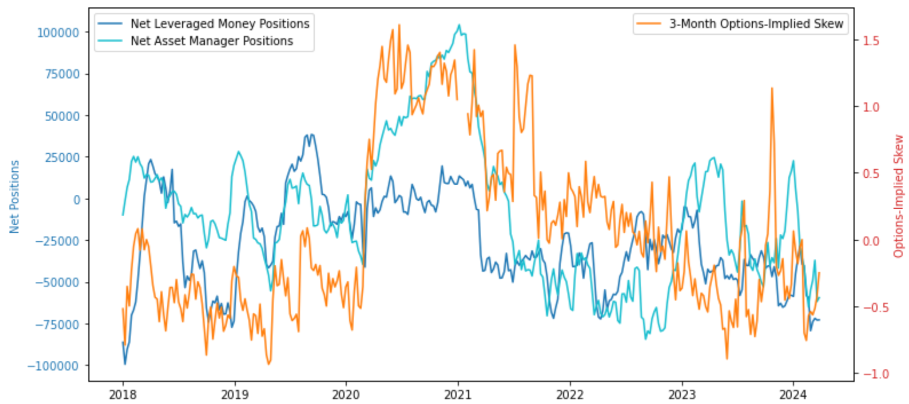
Immediately it is clear that these measures are substantively different, which is both puzzling and concerning, as their interpretation leads to virtually opposite conclusions.

The PDF measure, which mostly moves inversely with the risk-reversal measure (as expected), is sometimes slightly negative for sustained periods of time, indicating a skew towards yen depreciation. On the other hand, Bloomberg's risk-reversal measure shows a consistent skew towards appreciation of the yen. Thus, the Bloomberg measure is consistent with my ex-ante hypothesis, while the PDF measure is not. Both the PDF and risk-reversal measures are risk-neutral, so there is no $\mathbb{P}$-measure versus $\mathbb{Q}$-measure argument to be considered. Even if there were such an argument to be considered, Brunnermeier et al. (2009) document a close relationship between physical and risk-neutral skewness in carry trade currencies.

Upon closer inspection, however, I am inclined to believe that the PDF measure of skewness is in fact *more accurate* than the Bloomberg risk-reversal measure. For one thing, the risk-reversal measure of skew is essentially a back-of-the-envelope measure; it accounts for only two distinct points on the volatility surface, while the PDF measure captures the entire surface. Even so, I identified the 25-delta call and put prices in the CME JPY/USD options price dataset, and found that these risk-reversal prices were frequently negative, too. This suggests that Bloomberg's USD/JPY risk-reversal measure may not be accurate.[22] To illustrate the accuracy of my PDF measure, I plot the net leveraged money and asset manager yen futures positions against the 3-month PDF skew in Figure 5. Note that the positions displayed in the chart are on the CME's JPY/USD futures contracts.

[22]Bloomberg collects quotes from interdealer brokers, such as TP ICAP, BGC, and Tradition. In many cases, it can take days for specific risk-reversal quotes to be refreshed by interdealer brokers, and the existence of a certain quote does not imply that a trade was ever completed at the level of that quote. It only means that a dealer has provided such a quote to the broker. In rare cases, dealers will provide outlandish quotes to a broker simply to 'test the waters.'

Figure 5: Yen Futures Positions and 3-Month PDF Skewness, 1/2018-4/2024



Source: Author's calculations and the Commodity Futures Trading Commission.
The 'net positions' are the number of futures contracts of JPY/USD 12,500,000.
Negative values indicate that money managers are net short the yen.

Fundamentally, money managers who put on carry trades are betting on slow and consistent yen depreciation. Money managers will hedge their yen carry trade positions by going short USD/JPY futures (equivalently, long JPY/USD futures) to offset the risk of sharp yen appreciation. Therefore, as the yen becomes more skewed towards appreciation, money managers should be increasing their short USD/JPY futures positions. As evident from Figure 5, the PDF skew closely tracks money manager positions. Indeed, regressing the sum of the leveraged money and asset manager positions on the PDF skew yields a positive coefficient significant at the 1% level. As the yen becomes increasingly skewed towards appreciation (read: the JPY/USD skew increases), money managers increase the size of their long JPY/USD positions.

Put simply, the yen is, in fact, *not* consistently skewed towards appreciation; rather, it is heavily influenced by the contemporaneous magnitude of the carry trade. If money managers are net long the yen — indicating that the carry trade is diminished — the yen can actually be slightly skewed towards depreciation. This result diverges from the observation of Brunnermeier et al. (2009), as well as conventional wisdom in markets, and would be imperceptible to a practitioner or researcher whose only source of skew information was Bloomberg's risk reversal time series. Such is the power of options.

5 Conclusion

In this paper, I have extracted market-implied risk-neutral probability density functions for the JPY/USD exchange rate going back to 2018, and examined how the moments of those distributions change in response to macroeconomic surprises. First, I find that generally, surprises that imply higher U.S. rates cause traders to increasingly price in dollar appreciation and yen depreciation (and vice versa), confirming the simple principles of interest parity. Second, I find no evidence to support the hypothesis that the passing of an event should lower the implied variance of JPY/USD, suggesting that the passing of an event does little to resolve information uncertainty in the dollar-yen market. Finally, I find that the JPY/USD distributions are not uniformly skewed towards yen appreciation, but instead are frequently skewed towards depreciation of the yen and are clearly linked to the magnitude of money managers' carry trade positions.

The broader motivation of this research is to demonstrate the utility of asset prices, and in particular derivative prices, as signals of information. Friedman (1953) wrote that "The canons of formal logic alone can show whether a particular language is complete and consistent...Factual evidence alone can show whether the categories of the "analytical filing system" have a meaningful empirical counterpart, that is, whether they are useful in analyzing a particular class of concrete problems." He continues: "A hypothesis is important if it "explains" much by little, that is, if it abstracts the common and crucial elements from the mass of complex and detailed circumstances surrounding the phenomena to be explained and permits valid predictions on the basis of them alone." The conclusion of his famous essay is that economic theory should be judged only on its ability to predict outcomes rather than on the supposed 'realism' of its assumptions.

The theory of finance and asset prices do exactly what Milton Friedman describes. Financial markets — and especially foreign exchange markets — are impossibly complex and and subject to a countably infinite variety of idiosyncratic shocks and confounding variables. Indeed, as a trader once poetically explained to me, exchange rates are determined by oceans of capital crashing into each other. Yet theory allows us to abstract away the noisy movement of these oceans, and extract vital informational content from the chaotic global *agorá*. This is reminiscent of the German mathematician Gottfried Wilhelm Leibniz, who, in a 1689 essay attempting to explain the causes of celestial motions, wrote:

> ”Truly it has been proposed by us, and the matter will be taught to explain the laws of motion themselves more clearly, because it is by far too deep to be investigated. And since somehow in that manner a light shall shine forth for us, and the question may be seen to be successful both very conveniently and naturally, in that I have given hope of the true causes of the motions of the heavens to be approached by us.”

To conclude, by allowing investors to hedge (de-risk) and speculate (take risk) in a long list of derivative contracts, governments and regulators are doing themselves and the rest of us a favor. As such, efforts should be made to responsibly expand and innovate in these markets in a measured fashion. Civilization has been shunted forward over generations by all those willing to take and bear risk; to grease the wheels of progress, the purchase and sale of risk should be encouraged and augmented.

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A Time Series of the Moments

Figure 6: 1-Month and 3-Month Options-Implied Mean, 1/2018-4/2024

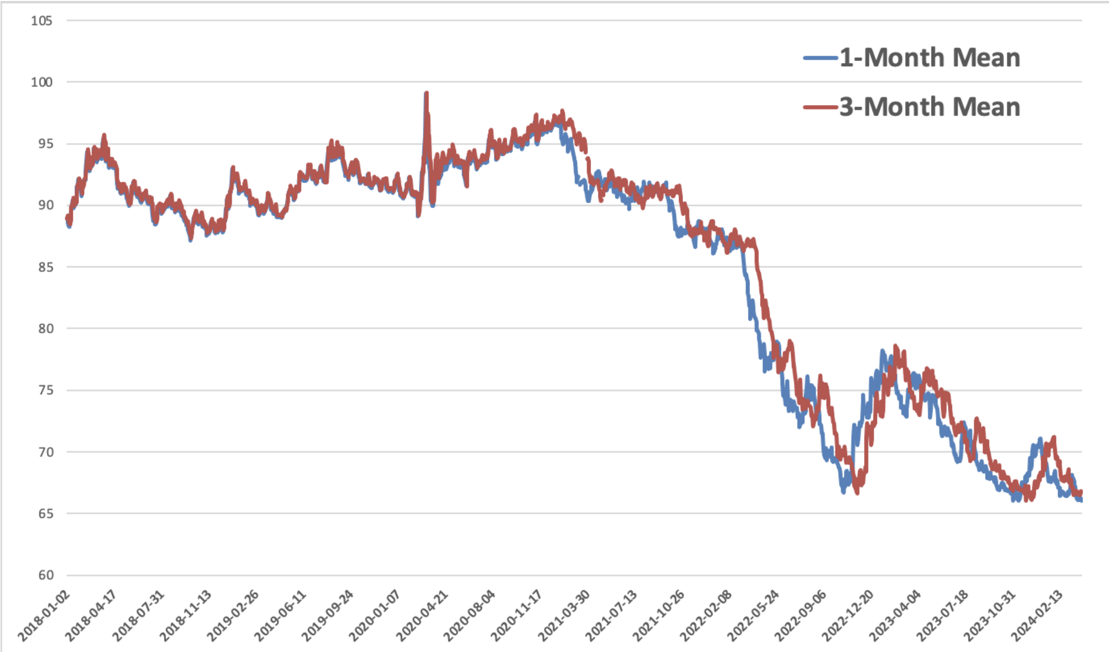


Figure 7: 1-Month and 3-Month Options-Implied Variance, 1/2018-4/2024

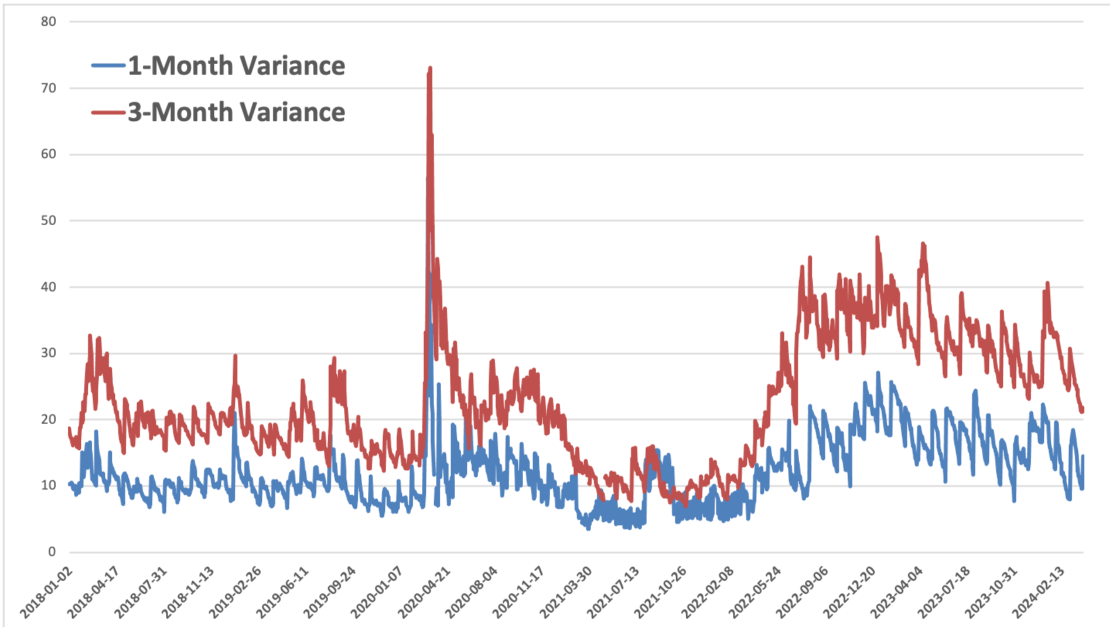


Figure 8: 1-Month and 3-Month Options-Implied Skew, 1/2018-4/2024

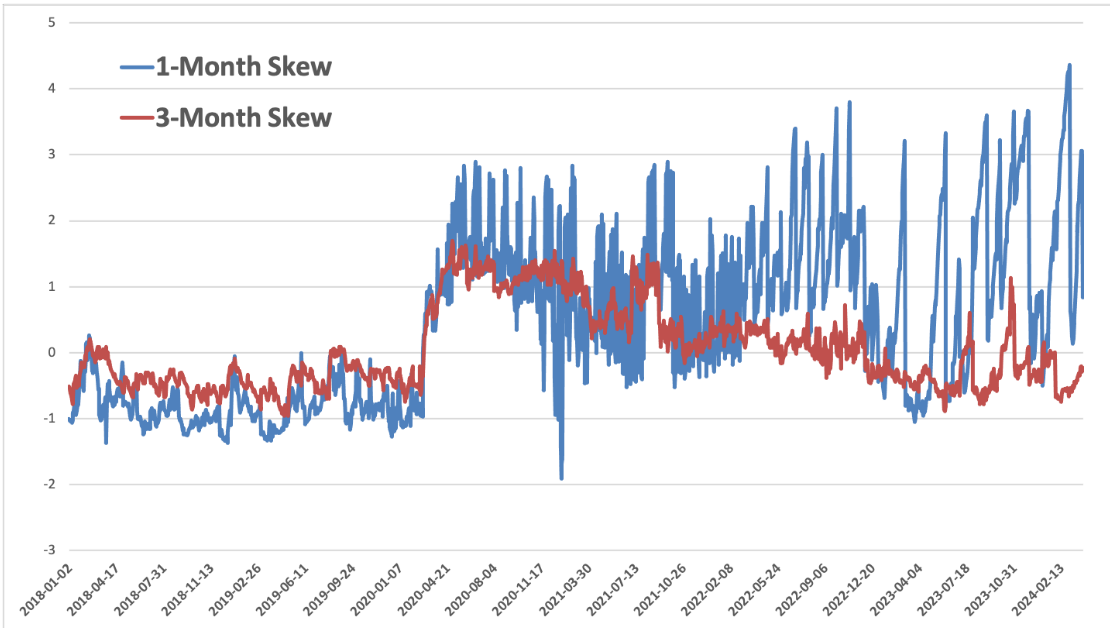
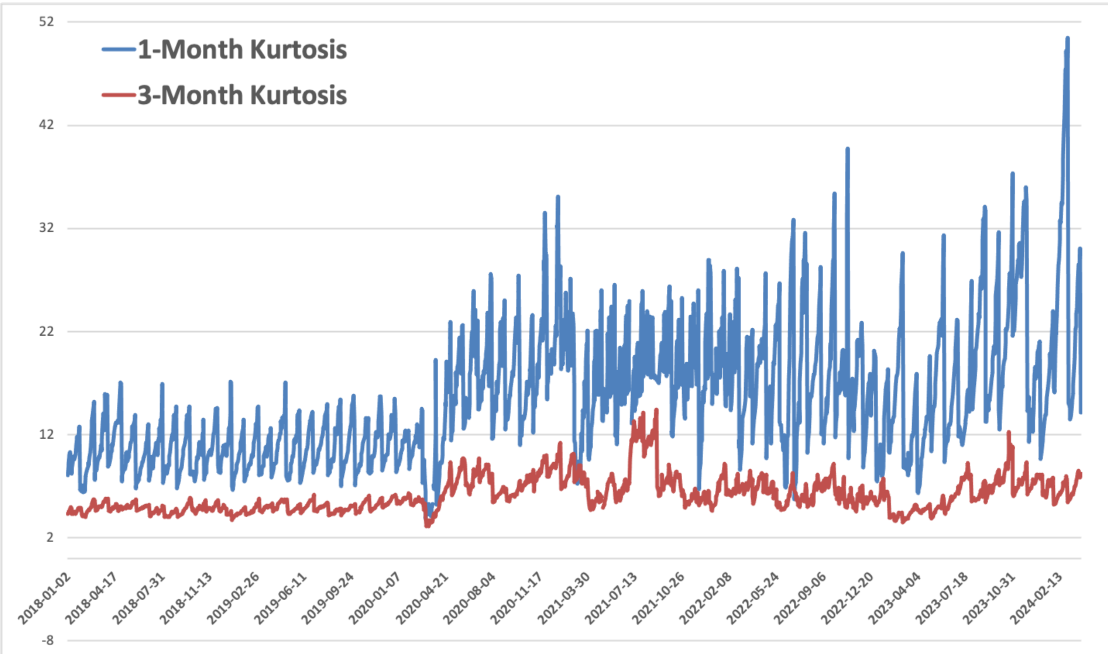


Figure 9: 1-Month and 3-Month Options-Implied Kurtosis, 1/2018-4/2024



B Percentile Regression Results

Table 5: Regression of the 1-Month PDF Percentiles on Macroeconomic Surprises

Release Type	P_{10}	P_{90}	$P_{90} - P_{10}$	**Kelly's Skewness**
FOMC Meetings	-5.04	-10.21***	-5.17	0.00
	(3.32)	(3.23)	(3.22)	(1.39)
BOJ Meetings	8.72	62.03	53.31	23.77***
	(37.39)	(81.94)	(52.23)	(6.96)
U.S. CPI	-109.84**	-97.26	12.58	3.42
	(51.10)	(59.41)	(28.07)	(14.56)
Japanese CPI	52.75	52.92	0.17	6.56
	(49.95)	(52.22)	(37.49)	(17.47)
U.S. PPI	-11.74	-2.49	9.25	1.82
	(16.77)	(17.83)	(14.49)	(6.58)
Japanese PPI	-8.29	-15.59	-7.30	-3.54
	(18.07)	(21.40)	(13.22)	(5.99)
U.S. GDP	12.34	-12.22	-24.57***	7.76
	(13.12)	(18.48)	(6.98)	(4.75)
Japanese GDP	-1.64	-3.25	-1.61	-0.17
	(3.55)	(5.54)	(3.25)	(1.75)
U.S. Unemployment	-61.84	-67.49	-5.64	-2.88
	(87.70)	(93.52)	(10.26)	(3.25)
Japanese Unemployment	78.88	13.16	-65.72*	29.02*
	(50.96)	(45.13)	(39.40)	(17.44)
Intercept	-0.12	-0.17	-0.05**	0.00
	(0.17)	(0.18)	(-0.02)	(0.01)

Notes: Statistical significance at the ten, five, and one percent levels are denoted by one, two, and three asterisks, respectively.

Table 6: Regression of the 3-Month PDF Percentiles on Macroeconomic Surprises

Release Type	P_{10}	P_{90}	$P_{90} - P_{10}$	**Kelly's Skewness**
FOMC Meetings	-7.70***	-9.65***	-1.87	-0.83
	(2.18)	(3.26)	(2.36)	(1.56)
BOJ Meetings	402.27	399.93	-4.55	-6.48
	(441.68)	(459.38)	(36.52)	(8.28)
U.S. CPI	-144.48**	-149.98**	-5.92	16.77
	(58.13)	(63.44)	(31.61)	(20.63)
Japanese CPI	82.55	67.14	-16.00	1.58
	(68.48)	(82.60)	(26.19)	(16.28)
U.S. PPI	-22.92	-15.65	7.89	-1.08
	(18.12)	(18.53)	(11.55)	(3.54)
Japanese PPI	-24.06	-46.65*	-22.64	-11.11
	(22.41)	(27.84)	(17.21)	(9.48)
U.S. GDP	-14.43	-29.62	-15.00*	-3.49
	(32.61)	(31.99)	(8.29)	(2.52)
Japanese GDP	-1.73	-4.03	-2.33	-0.09
	(3.66)	(6.65)	(4.42)	(1.97)
U.S. Unemployment	-8.65	16.04*	7.44**	2.36
	(7.44)	(8.59)	(3.28)	(2.00)
Japanese Unemployment	-637.63	-690.14	-49.55	4.38
	(703.87)	(757.27)	(61.79)	(12.22)
Intercept	0.16	0.15	0.00	0.01
	(0.25)	(0.27)	(0.03)	(0.01)

Notes: Statistical significance at the ten, five, and one percent levels are denoted by one, two, and three asterisks, respectively.