Do (Non-)Regularized Partial Correlation Networks Generalize? Re-evaluating Network Generalizability

Jeongwon Choi Alexander Christensen

Vanderbilt University

Introduction

- **Gaussian Graphical Models (GGM)** represent conditional (in)dependence of multiple variables (nodes) using (regularized) partial correlations (edges). In such models, community detection algorithms identify groups of connected nodes, revealing underlying structures and dimensions in the data that may not be directly observable.
- **Graphical LASSO (GLASSO)** aims to reduce overfitting by creating sparser networks with the goal of enhancing model generalizability. Williams and Rodriguez (2022) demonstrated that regularization may not actual reduce overfitting and potentially could lead to decreases in generalizability of inferences.
- This study re-evaluates the generalizability and predictability of (non-)regularized partial correlation networks. The present research aims to address previous limitations by examining generalizability using simulated and empirical data, implementing categorical prediction, and using evaluation metrics that are more appropriate for categorical data.

Methods

- First, a Monte Carlo simulation was performed to evaluate the (non-)regularized GGMs ability to generalize predictive inference and community detection to out-of-sample data.
- Second, empirical examples used in Williams and Rodriguez (2022)'s study were re-evaluated using categorical prediction and updated evaluation metrics.

Manipulated Factors	Options
Data type	Dichotomous, 5-point Likert, Continuous
Sample size (N)	100, 250, 500, 1000
Correlation (CORF)	0.3, 0.6
Loading (LOAD)	0.4, 0.55, 0.7
Number of factors (NFAC)	2, 4, 6
Skewness (SKEW)	0, 0.5, 1

- We evaluated the performance of four distinct network models—graphical LASSO (GLASSO), nonregularized (NONREG), fully connected (FULL), and empty (EMPTY) networks.
- For data split, the dataset was partitioned into training and testing subsets, with 80% for model training and the remaining 20% for testing model predictions.

Data Categories	Metrics
Dichotomous	Accuracy & Kappa
5-point Likert	Krippendorff's α & Linear Weighted Kappa
Continuous	R^2 & Root Mean Square Error (RMSE)
Communities	Total Entropy Fit Index (TEFI) & Adjusted Rand Index (ARI)

Results

- **Figure 1:**
  - GLASSO performs well in large sample sizes and high loadings, often achieving the highest testing accuracy and demonstrating better generalizability than other methods.
  - GLASSO consistently shows lower TEFI at larger sample sizes and higher loadings, and NONREG shows the highest TEFI, indicating substantial errors due to overfitting.
  - GLASSO achieves high ARI compared to NONREG, especially in large sample sizes and high loadings, indicating better clustering.
- **Figure 2:**
  - Krippendorff's alpha and linear kappa generally decrease for both GLASSO and NONREG as the number of factors and sample size increase, with GLASSO showing a steeper decline.
  - TEFI values become more negative with increases in both the number of factors and the sample size, indicating a worse model fit. GLASSO consistently shows more negative TEFI values than NONREG.
  - ARI improves for both methods as the number of factors and sample sizes increase, with GLASSO generally outperforming NONREG.
- **Figure 3:**
  - GLASSO shows higher Linearly Weighted Kappa and Krippendorff's Alpha for smaller training sample sizes (100 or 200) compared to NONREG.
  - TEFI values are much higher for NONREG method compared to GLASSO.
  - GLASSO has a higher ARI than NONREG.

Conclusion

Overall, GLASSO shows superior accuracy, model fit, and clustering accuracy, particularly with larger sample sizes and higher loadings, compared to other methods. In our findings, across 12 conditions characterized by different sample sizes and loading, GLASSO often outperformed NONREG in test data. These results are inconsistent with the result of Experiment 2 in Williams and Rodriguez (2022). This inconsistency suggests that the broader scope of our study provides additional insights that may have been overlooked in studies with limited data conditions. This study demonstrates the advantage of regularization methods not only in enhancing generalizability but also in dimensionality within partial correlation networks.

Figures

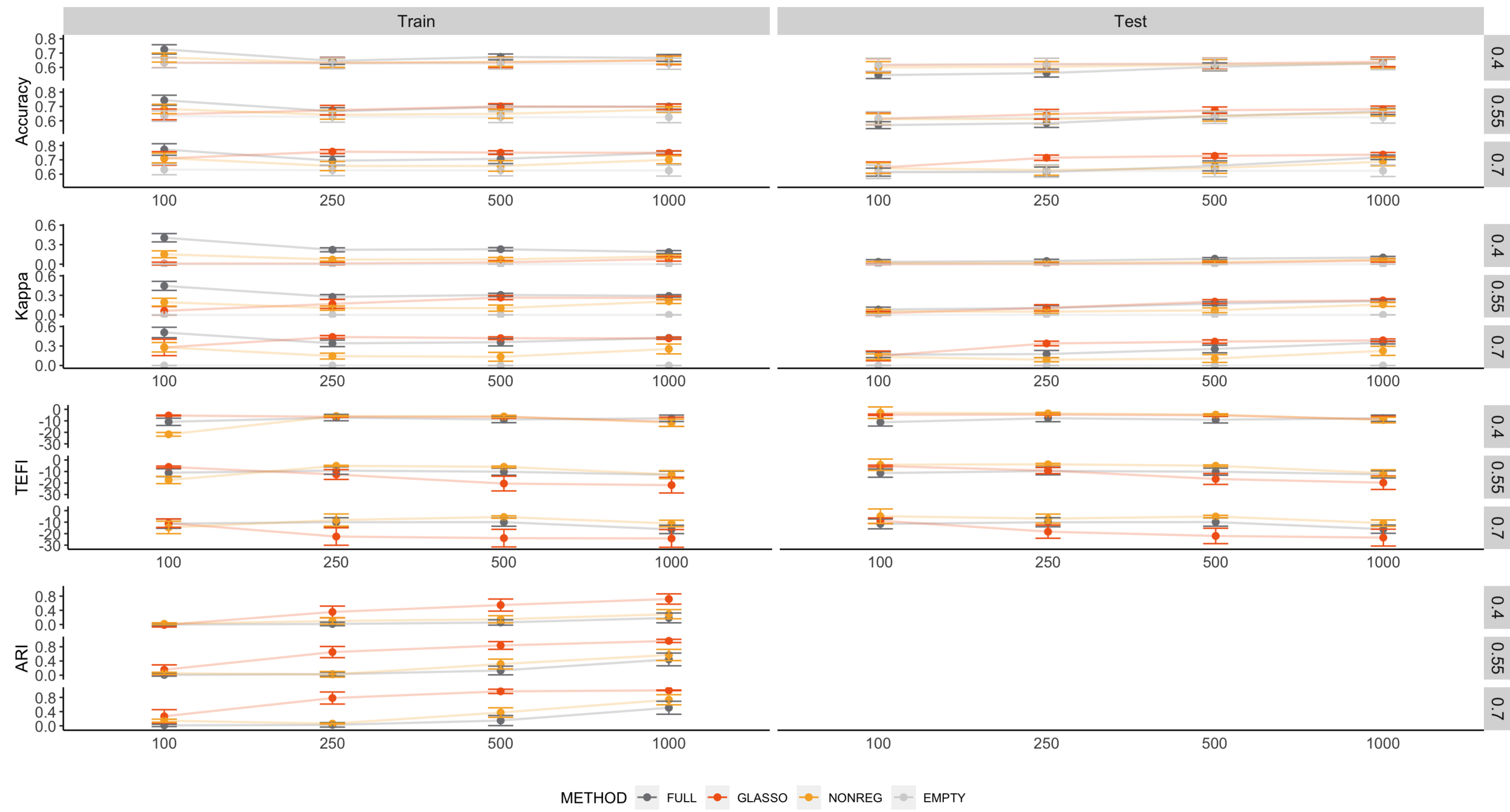

Figure 1. Dichotomous Data, by Loading and Sample Size

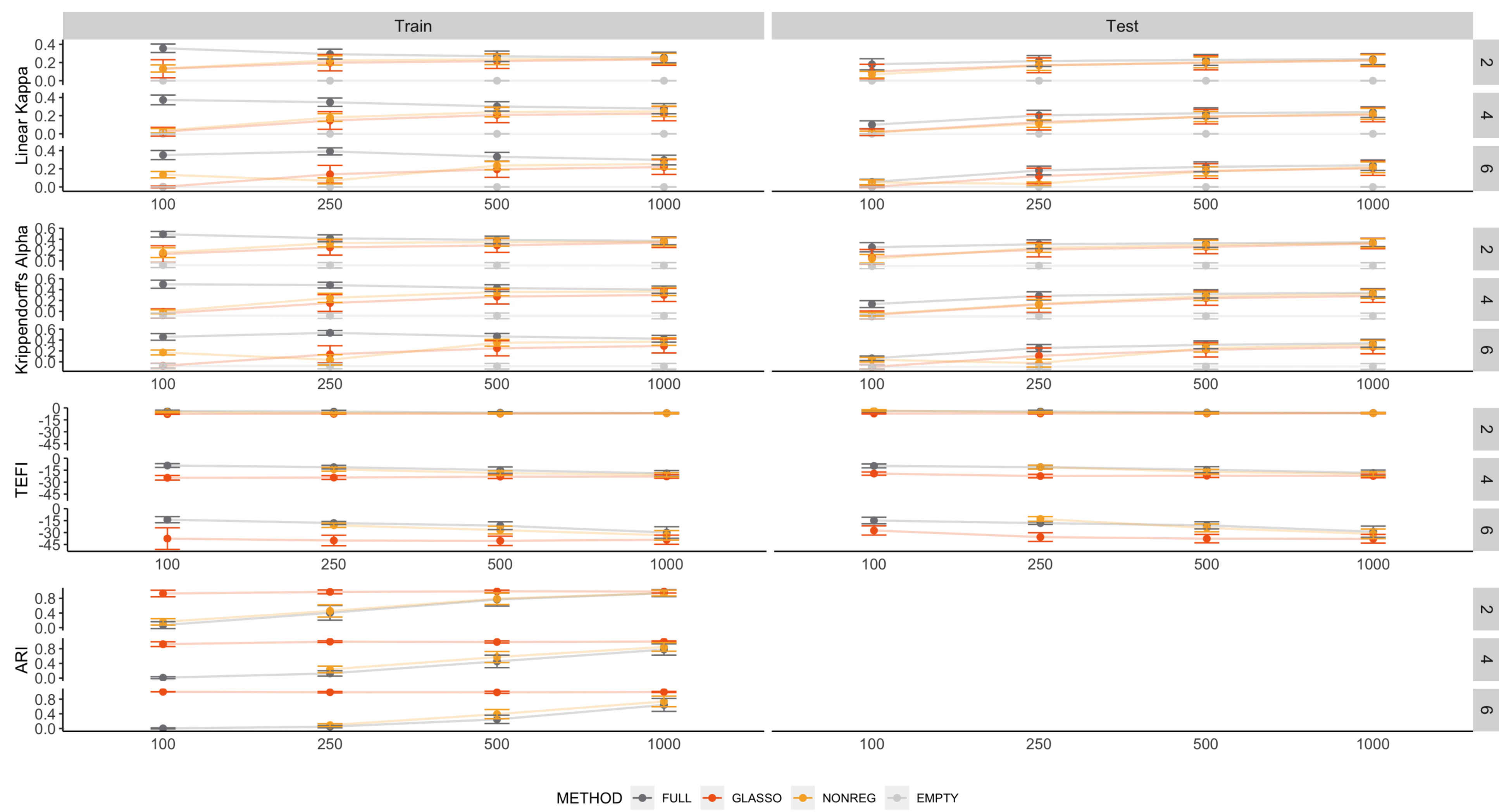

Figure 2. Polytomous Data, by Number of Factors and Sample Size

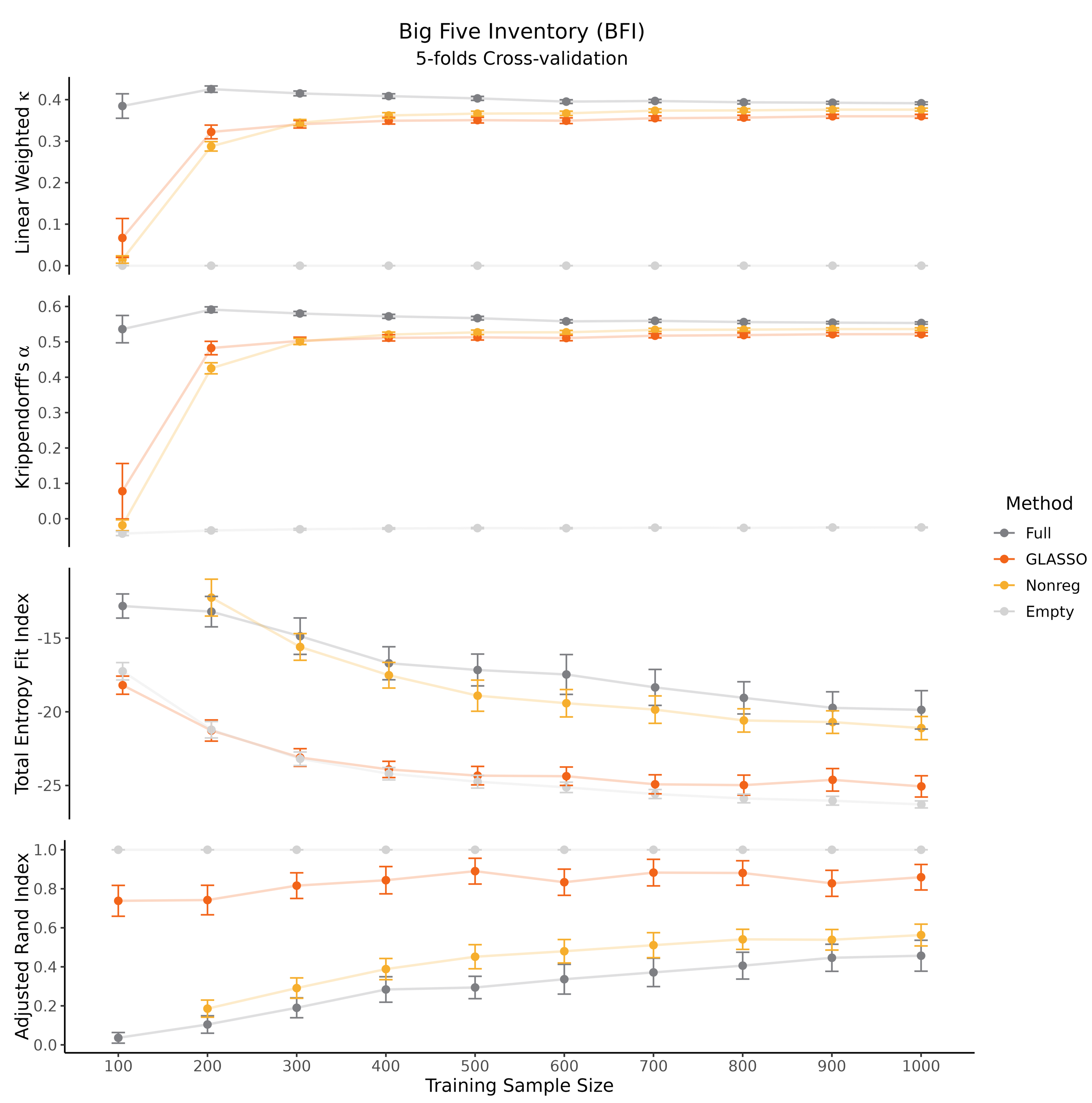

Figure 3. BFI: K-Fold Cross Validation

References

- Williams, D. R., Rodriguez, J. E. (2022). Why overfitting is not (usually) a problem in partial correlation networks. Psychological Methods, 27(5), 822–840.

 jeongwon.choi@vanderbilt.edu