N-BODY SIMULATION TECHNIQUES:

THERE'S A RIGHT WAY AND A WRONG WAY...

Neal Katz

Testing Validity of Results

- Check energy, linear momentum and angular momentum conservation.
- Try running on problems with known solutions. A necessary but not sufficient condition.
- Try increasing the number of particles. Is the result the same? A necessary but not sufficient condition.
- Try decreasing the timestep. Is the result the same? A necessary but not sufficient condition.
- Try altering the gravitational softening.
- Try comparing with other codes.
- Follow the constitution!

THE N-BODY CONSTITUTION

Preamble

We, the people of the N–body community, in order to form more perfect N–body simulations, establish justice, insure domestic tranquility, provide for the common defense, promote the general welfare, and secure the blessings of scientific accuracy to ourselves and our posterity do ordain and establish this Constitution for the conduct of numerical N–body simulations.

Article I.

On the Gravitational Softening Length

Section 1—The gravitational Softening length should be chosen large enough to minimize the effects of two body relaxation.

The softening length should be chosen such that there is at least 8 particles in each softening volume in objects of interest.

Article II.

On the Size of Time steps

Section 1—Time steps should be chosen to be small enough to eliminate the effects of two body scattering introduced through integration errors.

The time step must satisfy

$$\Delta \mathrm{t} < \frac{1}{10}\left(\frac{1}{G\rho}\right)^{1/2}.$$

For an isothermal sphere this is equivalent to

$$\Delta \mathrm{t} < .38\frac{\epsilon_{grav}}{v_{max}}$$

for a standard Plummer softening. For a typical cosmological simulation

$$N_{steps} = \frac{\mathrm{T}_{\mathrm{HUBBLE}}}{\Delta \mathrm{t}} \sim 4500\frac{10\mathrm{kpc}}{\epsilon_{grav}}.$$

Spline kernel softening requires 33% more time steps. One might also include the additional constraint

$$\Delta \mathrm{t} < .38 = \sqrt{\frac{\epsilon_{grav}}{a_{max}}}$$

Article III.

On the Number of Particles

Section 1—The number of particles should be large enough that random potential fluctations do not dominate the dynamics that one is trying to study.

For example, to correctly model the internal dynamics of a galaxy halo requires at least one million particles.

Article IV.

On the Accuracy of Forces

Section 1—Forces should be calculated with a maximum absolute and relative error.

The maximum relative error, $\frac{\Delta a}{a_i}$, and the maximum absolute error, $\frac{\Delta a}{a_{rms}}$, in the calculated acceleration should always be less than 0.5% and be less than the errors introduced by discreteness effects.

Article V.

On the Accuracy of the Integrator

Section 1—The integrator must retain at least second order accuracy at all times, be time symmetric, and avoid any correlated higher order error terms.

Retaining second order accuracy is particularly important when the length of the time step in changed during the calculation. However, one must be careful not to introduce correlated third order errors when correcting to second order accuracy. If the integrator is not time symmetric then energy need not be conserved.

Article VI.

On the Size of the Simulation Volume

Section 1—The simulation volume must be large enough to correctly model all important non–linear effects.

There are mode couplings between large and small scales that increase power on small scales. These couplings manifest themselves as filaments and greatly effect the gravitational evolution at small scales. The diameter of the simulation for a CDM spectrum with $\sigma_8 = 0.7$ must be at least $40\mathrm{h}^{-1}$

Mpc. Models with more large scale power will require larger volumes. This may be related to the scale where the slope of the power spectrum becomes less steep than n^{-1}.

Section 2—No one object should dominate the evolution of the simulated volume.

To prevent objects from tidally influencing themselves and to prevent objects from artificially lowering the mean background density, the simulation volume must be large enough to ensure that no virialized object contains more than 1/10 the total simulated mass.

Section 3—The fundamental mode should remain well within the linear regime.

The simulated volume should be chosen to be large enough to ensure that the amplitude in the fundamental mode $\delta_1 < 0.01$ when the simulation is concluded.

Article VII.

On the Starting Redshift of the Simulation

Section 1—The simulation must start at a redshift high enough to ensure that all represented mass scales are still in the linear regime.

In particular, if the initial conditions are generated using the Zel'dovich approximation, the starting redshift must be sufficiently high to ensure that the absolute maximum $|\delta| = 1$. For example, a CDM simulation with $\sigma_8 = 0.7$ in a cubic volume $40\mathrm{h}^{-1}$ Mpc on a side simulated with 128^3 particles would require a starting redshift of at least $z = 50$.

Article VIII.

What Ratification Shall Establish Constitution

Section 1—This constitution must be ratified by the representatives of at least two different research groups.

The effects of violating the above articles are not known. Some violations will introduce unphysical dissipative effects, others will introduce errors that act like an artificial heat source, while the effects of other violations are not clear but equally unphysical. Hence, we the undersigned insist that to ensure scientific accuracy all N–body simulations should meet the above guidelines.

Done in convention by the unanimous consent of the summer school members present the Third day of July in the year of our Lord one thousand nine hundred and ninety eight. In witness whereof we have hereunto subscribed our names.