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Durable-goods oligopoly with secondary markets: the case of automobiles

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We study the effects of durability and secondary markets on equilibrium firm behavior in the car market. We construct a dynamic oligopoly model of a differentiated product market to incorporate the equilibrium production dynamics that arise from the durability of the goods and their active trade in secondary markets. We derive an econometric model and estimate its parameters using data from the automobile industry over a 20-year period. Our estimates are used to provide a measure of the competitive importance of the secondary market.

1. Introduction

■ In many durable-goods industries, used products are traded in decentralized secondary markets that are not directly controlled by the producers of new goods: the automobile industry is perhaps the most prominent example. In this article, we seek to understand the effects of durability and secondary markets on equilibrium production behavior in this industry. In the context of a dynamic equilibrium model, we model explicitly how product durability and trade in secondary markets affects equilibrium producer behavior in the automobile market.

The durability of cars and the existence of a secondary market have important competitive implications for new-car producers. The secondary market introduces, in the form of used cars, a large number of (imperfect) substitutes to the new cars produced each period, which limits the market power of each producer. In turn, rational firms recognize that their current production will reach the secondary market in the future and, by lowering prices in those markets, will erode future profits. A monopolist fully internalizes this effect by curtailing current production. In an oligopoly, however, each producer internalizes only the effect this has on its own future profits but not the detrimental effect it has on its rivals' future profits.[1] Indeed, each oligopolistic producer derives an indirect benefit from increases in current production if this causes its rivals to lower their future production levels; in equilibrium, therefore, a firm may choose to overproduce today if these indirect benefits outweigh the costs of more vigorous competition tomorrow.

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[1] This dynamic effect was also identified in Carlton and Gertner (1989).

Moreover, the presence of a secondary market also introduces an additional component—the resale value—to consumers' valuations of new cars. This dependence of new-car valuations on expected future prices introduces an intertemporal linkage between a firm's current profits and its own future behavior as well as the future behavior of its competitors. Given these linkages, the firm wishes to commit to low levels of production in the future to increase the expected resale value. Such behavior, however, would not be time consistent because, once the future arrives, the firm no longer cares about its past profits and is tempted to increase its production. Rational consumers will anticipate the firm's future actions and expect low resale prices, thus curbing current demand.

The intertemporal linkages between each firm's current profits and its own current, past and future production, as well as the current, past and future production of its rivals, makes for a rich dynamic game. In this article, we examine the equilibrium dynamics of this game within the context of the automobile industry. First, we construct a dynamic oligopoly model of a differentiated-product market that incorporates durability of the goods and their active trade in secondary markets. Second, we use data from the automobile market to estimate a tractable linear-quadratic version of the model. While the empirical model is quite stylized and incorporates restrictive assumptions, it represents (as far as we are aware) a first attempt at structural estimation of a dynamic durable goods model for this industry.

□ **Background and existing literature.** As the discussion above emphasized, durability and secondary markets introduce dynamics into both producers' output decisions and consumers' purchase decisions in the automobile market, which creates challenges for both theoretical and empirical work. In this article, we overcome these challenges by constructing a dynamic equilibrium model of the car market in which tractability is provided by its linear-quadratic structure.[2] Our model captures four key characteristics of the car industry: (i) oligopolistic time-consistent multiproduct automobile producers; (ii) an active, decentralized secondary market; (iii) differentiated products and (iv) depreciation schedules that differ across the competing car models.

However, we make some restrictive assumptions in deriving the linear-quadratic model: (a) consumers face no transactions costs in buying or selling cars, which makes the secondary market active by increasing the substitutability between new and used cars; (b) the automobile market is vertically differentiated, which places strong restrictions on the substitutability between cars in consumers' choice sets; and (c) there is perfect information, so we abstract away from adverse selection issues.[3] While the resulting model is quite stylized, our empirical results demonstrate the feasibility of estimating a dynamic durable-goods model for this industry and, we hope, encourage future progress.

Since the seminal work of Coase (1972), a large theoretical literature has analyzed how durability erodes market power for a monopoly producer.[4] Coase conjectured that a monopolist producing an infinitely durable good may lose all of its market power due to its inability to commit to high prices (or low production) in the future. Stokey (1981), Gul, Sonnenschein, and Wilson (1986) and Ausubel and Deneckere (1989) showed how Coase's conjecture can arise as an equilibrium limiting result in models where the time lag between the monopolist's price offers shrinks to zero.[5] In the presence of Coasian commitment problems, Liang (1999) shows that a secondary market can reduce the monopolist's temptation to increase future output because it reduces competition with the secondary market by selling more slowly to consumers, thus nearing the commitment solution.

[2] See Kydland (1975) for a description of discrete-time linear-quadratic dynamic games, and Judd (1996) for an application to dynamic oligopoly models where firms set both prices and quantities. Also in a linear-quadratic setting Kahn (1986) analyzes the effects of increasing costs in an infinitely durable-goods monopoly.

[3] Adverse selection has been a concern in the literature on secondary markets (Akerlof, 1970). See Hendel and Lizzeri (1999) and House and Leahy (2000) for recent contributions to this literature and Bond (1982) for related empirical work.

[4] See Waldman (2003) for a recent survey.

[5] See also Bulow (1982) for a treatment of the durable-goods monopolist problem within a two-period model.

The implications of durability and secondary markets on the dynamics of car demand have not been ignored in the literature. Berkovec (1985), Rust (1985a) and Stolyarov (2002) focus on dynamic consumer demand in a durable goods-market with primary, secondary and scrappage market segments. Adda and Cooper (2000) employ the optimal decision rules from a dynamic discrete-choice model to explore the effects of scrappage subsidies on car demand, where cars are held until scrapped and, hence, are not actively traded in the secondary market. Finally, Eberly (1994) and Attanasio (2000) consider (s, S) models of automobile demand in which idiosyncratic shocks lead consumers to change their stock of cars. In all these articles, the focus is on the timing of consumer purchases, so that automobile prices are assumed to evolve exogenously, and firms' automobile production decisions are not explicitly modeled. In our article, we model firms' equilibrium production decisions in a dynamic oligopoly model but abstract away from consumer transactions costs in order to ensure the tractability of the model.

Our emphasis on the equilibrium dynamics due to durability and secondary markets also distinguishes our work from existing market-level empirical studies of demand and supply in the automobile market. Bresnahan (1981), Berry, Levinsohn, and Pakes (1995) and Goldberg (1995) and Petrin (2002) have employed static models to quantify the degree of market power and the welfare effects of new product introductions in the car industry. These articles have focused on accommodating multiple dimensions of consumer heterogeneity in modeling the demand for automobiles. While some of these authors have allowed consumers to substitute between new and used cars in their models, they have not accommodated the intertemporal link between primary and secondary markets (i.e., that new cars today become used cars in the future), which is a crucial feature of our model. However, in order to maintain tractability in the dynamic oligopoly model, we restrict ourselves to a single-dimensional model of consumer heterogeneity.

Several articles have considered the empirical implications of durability and monopoly power. Suslow (1986) estimated a structural model of Alcoa's aluminum monopoly, taking into account the competition from the recycled aluminium sector. Iizuka (2007) and Chevalier and Goolsbee (2005) studied producer and consumer behavior in the academic-textbook market. For the automobile industry, Ramey (1989) estimated a durable-goods monopoly model to explain aggregate trends in car prices, and Porter and Sattler (1999) tested empirical predictions on the volume of trade in secondary car markets using a durable-goods monopoly model with transactions costs. There have been fewer articles on durable-goods oligopoly. Carlton and Gertner (1989) analyzed the effects of mergers among oligopolistic durable-goods producers, and Esteban (2002) characterizes the equilibrium production dynamics in a durable-goods oligopoly with homogeneous products.

The article proceeds as follows. In Section 2, we introduce the model and derive the Markov perfect equilibrium of the dynamic game. Subsequently, we derive a linear-quadratic specification of this model that is convenient for the empirical illustration. In Section 3, we describe the data and discuss the empirical implementation of the model. We also present our estimation results and conduct some counterfactual experiments. We conclude in Section 4.

2. A model of a durable-goods oligopoly with secondary markets

■ We consider a dynamic quantity-setting game among oligopolistic producers of differentiated durable goods (which, for convenience, we call "cars"). On the demand side, we assume that consumers are forward looking, so that durability and secondary markets introduce investment considerations into their car consumption decisions. On the supply side, we assume that new-car producers are quantity-setting oligopolists that recognize both the intertemporal effect of current production on future profits due to the secondary market as well as the dependence of current profits on past, present and expected future production. Several institutional features support a quantity-setting assumption. First, an implicit assumption of the Bertrand price-setting model is flexible capacity, and capacity does not appear easily adjustable in car production (see Bresnahan and Ramey, 1994). Second, in the car market, prices seem to adjust to clear the market at given quantity levels, as in the quantity-setting case. For example, rebates are a common way of adjusting new-car prices to clear the inventories at the end of the model year. Finally, dealer behavior limits the manufacturers' ability to control prices.

Because the model we derive here is linear-quadratic, we focus on a deterministic version of the model because the certainty equivalence property of linear-quadratic models ensures that the same equilibrium decision rules would obtain in its stochastic counterpart. For simplicity and tractability, we assume a stationary market environment and do not consider the entry and exit of car models from the market.

Following Esteban (1999), we assume that the available cars are vertically differentiated. We refer to cars in their first period of life as *new* cars and, thereafter, as *used* cars. Throughout, we assume that used cars are transacted in competitive and decentralized secondary markets, so that new-car producers can manipulate market outcomes in the secondary market only indirectly, through their production of new cars.

Each period, N firms produce new cars. We let $\mathcal{N}$ denote the set of firms, where $N \equiv |\mathcal{N}|$. Each firm $j \in \mathcal{N}$ produces L_j distinct models, where $L_j \equiv |\mathcal{L}_j|$ and $\mathcal{L}_j$ is the set of all models produced by firm j. Then $\mathcal{L} \equiv \cup_j \mathcal{L}_j$ is the set of all models produced by all firms and $L \equiv |\mathcal{L}|$ is their total number. We index models by $i = 1, \ldots, L$.

New cars differ in quality and durability. For each model $i \in \mathcal{L}$, we let $q_{i,h}$ denote its quality at age h, where $h = 1, \ldots, T_i$ indexes its age and $T_i < \infty$ denotes the number of periods it lasts. Then each car (new or used) is completely described by the pair (i, h), and the set of all distinct cars transacted is given by $\mathcal{K} \equiv \{(i, h) \mid i \in \mathcal{L}, h = 1, \ldots, T_i\}$, and $K \equiv |\mathcal{K}|$ is their total number. Consumers have the opportunity to choose not only which model $i \in \mathcal{L}$ they drive but also the vintage $h \in \{1, \ldots, T_i\}$. In the rest of this article, we use the term "model year" to denote the elements of $\mathcal{K}$, which are the set of choices available to consumers each period. As we remarked before, we assume that the market is stationary, in the sense that the set of available models, $\mathcal{L}$, and model years, $\mathcal{K}$, do not change over time.

Next, we define a mapping $\omega : \mathcal{K} \mapsto \{1, \ldots, K\}$, which ranks cars from highest to lowest quality as follows:

$$\forall\, (i, h) \in \mathcal{K}, \quad q_{i,h} > q_{i',h'} \Rightarrow \omega(i, h) < \omega(i', h').$$

Hence, a ranking of 1 denotes the highest quality car and a ranking of K the lowest quality. Given this ranking, we define a quality ladder as follows.

Definition. A vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_K, 0]'$ is a quality ladder representing the quality structure of this problem if $\alpha_k \equiv \{q_{i,h} \mid k = \omega(i, h)\}$.

To facilitate the subsequent exposition, it is convenient to define a depreciation schedule for each car model as follows. Given a quality ladder $\boldsymbol{\alpha}$, we define a second mapping, $\upsilon : \{1, \ldots, K\} \mapsto \{1, \ldots, K\}$, which tracks the position that a car currently in position k in the ladder occupies after one period of depreciation. Then $\upsilon(\omega(i, 1)) = \omega(i, 2)$ and, more generally,

$$\upsilon^{h-1}(\omega(i, 1)) \equiv \underbrace{\upsilon(\upsilon(\ldots\upsilon(}_{h-1 \text{ times}}\omega(i, 1)))) = \omega(i, h), \qquad \text{for } h = 2, \ldots, T_i - 1.$$

Cars that die (i.e., all cars (i, h), where $h > T_i$) are given a ranking of $K + 1$ (because $\alpha_{K+1} = 0$), so that $\upsilon^{T_i}(\omega(i, 1)) = K + 1$, for all $i \in \mathcal{L}$ and $\upsilon(K + 1) = K + 1$. The mapping $\upsilon(\cdot)$ depends only on a model year's rank and only indirectly on the model i or vintage h.

Finally, we let $\eta(i) \equiv \omega(i, 1)$, for all $i \in \mathcal{L}$, denote the position that a model i car takes in the quality ladder when new. Then we represent the depreciation schedule of a model i car by the sequence

$$\eta(i), \upsilon(\eta(i)), \upsilon^2(\eta(i)), \ldots, \upsilon^{T_i}(\eta(i)),$$

which indicates the positions in the ladder that a model i car occupies as it ages. We now turn to the demand side of our model.

□ **Demand in a vertically differentiated market.** Our consumer population is a continuum of infinitely-lived agents in which heterogeneity in consumers' taste for quality generates demand for each type of car. In each period t, each consumer determines her optimal consumption choice among the K available cars and the option of not consuming a car at all (which we index $K+1$) to maximize her discounted utility function.

We assume that consumers face no transaction costs in any primary or secondary market, which is a key assumption to obtain the linear-quadratic structure of the model.[6] Without transaction costs, the secondary market is very active, which may overstate the substitutability between new and used automobiles for consumers. In deriving consumers' optimal car choice rules, we assume that consumers have rational expectations about future production decisions for all firms in all car markets.

Heterogeneity among consumers is parameterized by a scalar type $\theta \in [0, \overline{\theta}]$ (where $\overline{\theta} < \infty$) and is distributed in the population according to the cumulative distribution $F(\cdot)$. To obtain the tractable linear-quadratic specification that we take to the data, we assume that θ is uniformly distributed along its support, so that $F(\theta) = \theta/\overline{\theta}$, for $\theta \in [0, \overline{\theta}]$.

The population has size M. A consumer of type θ chooses a sequence of car choices to maximize her discounted lifetime utility,

$$U^{\theta} \equiv \sum_{t=1}^{\infty} \delta^{t-1} U_t^{\theta}. \tag{1}$$

Her period t utility flow is assumed to be quasilinear in income and given by

$$U_t^{\theta} \equiv \alpha_k \theta + m_t^{\theta} - p_t^k, \tag{2}$$

where δ denotes the discount factor common across all consumers and firms, θ measures this consumer's willingness-to-pay for quality and m_t^{θ} denotes consumer θ's income at the beginning of period t, which can include the resale price for a used car if the consumer is endowed with one in t.[7]

As is well known (e.g., Berkovec, 1985; Rust, 1985a), the assumptions of quasilinearity and no transaction costs imply that a consumer's optimal consumption decision in any period does not depend on her past and future choices because her decisions are independent of income.[8] Then it is easy to verify that consumer θ's optimal car choice in period t is determined by simply comparing the utility gains

$$UG_t^k(\theta) \equiv \alpha_k \theta - p_t^k + \delta p_{t+1}^{\upsilon(k)} \tag{3}$$

across all choices $k = 1, \ldots, K+1$. Each utility gain is just the difference of $\alpha_k \theta$, the flow of services consumer θ obtains from car k in period t and $\rho_t^k \equiv (p_t^k - \delta p_{t+1}^{\upsilon(k)})$, the implicit rental price paid for those services, where $\delta p_{t+1}^{\upsilon(k)}$ is the discounted resale price in tomorrow's secondary market. In every period, therefore, the optimal decision rule dictates that consumer θ chooses the car $k = 1, \ldots, K+1$ that maximizes the utility gain given in equation (3).

Deriving the demand functions. The consumers' preferences specified above imply that competing cars are vertically differentiated, in the sense that, if all cars were priced identically, all consumers would choose the highest quality car. Following the literature on oligopoly models of vertically differentiated product markets (e.g., Prescott and Visscher, 1977; Bresnahan, 1981; Berry, 1994), we derive the period-t demand functions as follows.[9]

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[6] Without this assumption, the individual-level dynamic demand functions would be involved, as the decision for each consumer would depend on her past and future purchases; see, for instance, Eberly (1994). See also footnote 8.

[7] Because we normalize α_{K+1} to zero, $p_t^{K+1} = 0 \; \forall \, t$.

[8] Allowing for transaction costs would complicate the derivation of the dynamic demand functions because, in that case, consumers' utility gains are state dependent (Andreson and Ginsburgh, 1994; Porter and Sattler, 1994; Stolyarov, 2002).

[9] The recent empirical literature on the car industry (Berry, Levinsohn, and Pakes, 1995, for example) has allowed

Given prices $p_t^k, p_{t+1}^{\upsilon(k)}$ for $k = 1, \ldots, K$ and quality levels $\alpha_1, \ldots, \alpha_K$ and given equilibrium inequalities ensuring that each car model has positive demand (see the discussion at the end of this section), we find K cutoff values, $\widetilde{\theta}_t^1, \ldots, \widetilde{\theta}_t^K$, such that

$$\overline{\theta} \geq \widetilde{\theta}_t^1 \geq \widetilde{\theta}_t^2 \geq \widetilde{\theta}_t^3 \geq \ldots \geq \widetilde{\theta}_t^K \geq 0, \tag{4}$$

and all consumers with preference parameter $\theta \in [\widetilde{\theta}_t^1, \overline{\theta}]$ consume car 1, all consumers with preference parameter $\theta \in [\widetilde{\theta}_t^2, \widetilde{\theta}_t^1]$ consume car 2, etc. Finally, all consumers with preference parameter $\theta \in [0, \widetilde{\theta}_t^K]$ do not consume a car. These cutoff values solve the indifference conditions

$$\begin{aligned} \alpha_k \widetilde{\theta}_t^k - p_t^k + \delta p_{t+1}^{\upsilon(k)} &= \alpha_{k+1} \widetilde{\theta}_t^k - p_t^{k+1} + \delta p_{t+1}^{\upsilon(k+1)}, \qquad \text{for } k = 1, \ldots, K-1, \\ \alpha_k \widetilde{\theta}_t^k - p_t^k &= 0, \qquad \text{for } k = K. \end{aligned} \tag{5}$$

Then by letting x_t^k denote the demand for car k in period t, we find that

$$\begin{aligned} x_t^1 &= M(1 - F(\widetilde{\theta}_t^1)) = \frac{M}{\overline{\theta}}(\overline{\theta} - \widetilde{\theta}_t^1), \\ x_t^k &= M(F(\widetilde{\theta}_t^{k-1}) - F(\widetilde{\theta}_t^k)) = \frac{M}{\overline{\theta}}(\widetilde{\theta}_t^{k-1}\widetilde{\theta}_t^k), \qquad \text{for } k = 2, \ldots, K, \\ x_t^{K+1} &= M(F(\widetilde{\theta}_t^K)) = \frac{M}{\overline{\theta}}(\widetilde{\theta}_t^K). \end{aligned} \tag{6}$$

Substituting these demand functions recursively in equation (6), we can write the K cutoff values as

$$\widetilde{\theta}_t^k = \overline{\theta}\left(1 - \frac{1}{M}\sum_{r=1}^{k} x_t^r\right), \qquad \text{for } k = 1, \ldots, K. \tag{7}$$

Substituting these expressions for the cutoff values into the indifference conditions given in equation (5), we obtain the inverse demand functions for each of the cars sold. In particular, a new-car model $i \in \mathcal{L}$ with depreciation schedule $\eta(i), \upsilon(\eta(i)), \ldots, \upsilon^{T_i}(\eta(i))$ has inverse demand function

$$p_t^{\eta(i)} = \overline{\theta}(\alpha_{\eta(i)} - \alpha_{\eta(i)+1})\left(1 - \frac{1}{M}\sum_{r=1}^{\eta(i)} x_t^r\right) + \delta p_{t+1}^{\upsilon(\eta(i))} + p_t^{\eta(i)+1} - \delta p_{t+1}^{\upsilon(\eta(i)+1)}, \tag{8}$$

where the prices $p_{t+1}^{\upsilon(\eta(i))}$, $p_t^{\eta(i)+1}$ and $p_{t+1}^{\upsilon(\eta(i)+1)}$ have analogous expressions. In deriving the demand equations, we assume that the primary and secondary markets are not in excess supply, so that the nonnegativity price constraints never bind in any market. This assumption, made in order to maintain tractability for our model, rules out the possibility that a firm could choose high production levels to bring a primary or secondary market into excess supply, making the price of a car in this market (and the prices of all cars ranked below it in the quality ladder) equal to zero.[10]

By substituting prices recursively into equation (8), we obtain the inverse demand function for new model $i \in \mathcal{L}$ as

$$\begin{aligned} p_t^{\eta(i)} = \overline{\theta}&\left(\alpha_{\eta(i)}\left(1 - \sum_{r=1}^{\eta(i)} \frac{1}{M} x_t^r\right) - \sum_{r=\eta(i)+1}^{K} \alpha_r \frac{1}{M} x_t^r\right) \\ &+ \overline{\theta}\sum_{h=1}^{T_i - 1} \delta^h \left(\alpha_{\upsilon^h(\eta(i))}\left(1 - \sum_{r=1}^{\upsilon^h(\eta(i))} \frac{1}{M} x_{t+h}^r\right) - \sum_{r=\upsilon^h(\eta(i))+1}^{K} \alpha_r \frac{1}{M} x_{t+h}^r\right), \end{aligned} \tag{9}$$

which is linear in current and future production levels.

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for multiple dimensions of consumer heterogeneity, but there are difficult conceptual and computational issues in extending such models to a dynamic equilibrium framework (see Berry and Pakes, 1999).

[10] Esteban (1999, 2006) showed that, for the durable-goods monopoly case, the secondary market will not be in excess supply in equilibrium. The result might not generalize for the more general model presented in this article.

In equilibrium, prices will satisfy the inequalities

$$\frac{\rho_t^{k-1}-\rho_t^k}{\alpha_{k-1}-\alpha_k} \geq \frac{\rho_t^k-\rho_t^{k+1}}{\alpha_k-\alpha_{k+1}} \geq \frac{\rho_t^{k+1}-\rho_t^{k+2}}{\alpha_{k+1}-\alpha_{k+2}} \geq 0, \qquad k=2,\ldots,K,\ \forall\, t, \tag{10}$$

which imply that the demand for each car is positive given the assumption that the markets are not in excess supply. For the empirical work, given actual prices, these inequalities function as restrictions on the feasible values that the α parameters can take. Because these restrictions are nonlinear in prices, it is difficult to impose them in estimating the parameters. We do not impose these restrictions in estimation, but rather examine *ex post* how many of these inequalities are satisfied for the parameter estimates we obtain.

□ **The producers' dynamic problem.** Having derived the inverse-demand functions for each car transacted, we now turn to the supply side and analyze the dynamic optimization problem faced by producers. The assumption that no market will be in excess supply (i.e., that the price given in equation (8) is strictly positive) implies that the quantity demanded will equal the quantity supplied in all markets. Consequently, for each car model $i \in \mathcal{L}$, volumes in the secondary market evolve according to

$$x_{t+h-1}^{\upsilon^{h-1}(\eta(i))} \equiv x_t^{\eta(i)}, \qquad \text{for } h=2,\ldots,T_i. \tag{11}$$

That is, the current production of model i, $x_t^{\eta(i)}$, becomes the supply in secondary market $\upsilon(\eta(i))$ during period $t+1$ and becomes the supply in secondary market $\upsilon^2(\eta(i))$ during period $t+2$ and so on.[11]

Let $\boldsymbol{y}_t$ denote the vector of all cars in use (both new and used) in period t, defined as[12]

$$\boldsymbol{y}_t \equiv [1, x_t^1, \ldots, x_t^K]'. \tag{12}$$

Let $\boldsymbol{d}_t$ denote the L-dimensional vector of all new cars ($L \times 1$) produced in period t as

$$\boldsymbol{d}_t \equiv \left[x_t^{\eta(1)}, x_t^{\eta(2)}, \ldots, x_t^{\eta(L)}\right]'.$$

Then, given these definitions, the law of motion of the cars-in-use vector $\boldsymbol{y}_t$ is

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}\boldsymbol{d}_t, \tag{13}$$

where $\boldsymbol{B}$ and $\boldsymbol{A}$ are matrices that, respectively, place new-car models in the quality ladder and shift cars within the quality ladder as they age. Specifically, $\boldsymbol{B}$ is a $(K+1)\times L$ matrix with entries

$$\boldsymbol{B}(k+1,i) \equiv \begin{cases} 1, & \text{if } \eta(i)=k \\ 0, & \text{otherwise} \end{cases}, \qquad \text{for } i=1,\ldots,L,\ k=1,\ldots,K, \tag{14}$$

and $\boldsymbol{A}$ is a $(K+1)\times(K+1)$ matrix with entries[13]

$$\boldsymbol{A}(k'+1,k+1) \equiv \begin{cases} 1, & \text{if } k'=k=0 \\ 1, & \text{if } \upsilon(k)=k' \\ 0, & \text{otherwise} \end{cases}, \qquad \text{for } k,k'=1,\ldots,K. \tag{15}$$

[11] Our model can accommodate exogenous depletion of the stocks by setting $x_{t+h-1}^{\upsilon^h(\eta(i))} \equiv \delta_{i,h} x_t^{\eta(i)}$, where $\delta_{i,h} \in [0,1]$ is the exogenous probability that car i be scrapped at age h. This is done in our empirical work below.

[12] As is standard in the matrix formulation of linear-quadratic problems, we set the first entry of $\boldsymbol{y}_t$ equal to 1 identically across all t.

[13] The first entry in $\boldsymbol{A}$ is a 1, to be consistent with the first entry in the $\boldsymbol{y}_t$ vector.

Next, we let $C_j(x_t^{\eta(i)}; \forall\, i \in \mathcal{L}_j)$ denote the total cost function for firm j. We assume the total costs of production are quadratic in output and independent across car models, so that

$$C_j(x_t^{\eta(i)};\ \forall\, i \in \mathcal{L}_j) = \sum_{i \in \mathcal{L}_j} \left[\beta_{0i} + \beta_{1i} x_t^{\eta(i)} + \beta_{2i} (x_t^{\eta(i)})^2\right]. \tag{16}$$

Given the linear inverse-demand functions and quadratic cost function, the period-t profit function for firm j will be a quadratic function in current and future production,

$$\begin{aligned} \pi_t^j &= \sum_{i \in \mathcal{L}_j} p_t^{\eta(i)} \cdot x_t^{\eta(i)} - C_j(x_t^{\eta(i)}; \forall\, i \in \mathcal{L}_j) \\ &= \sum_{i \in \mathcal{L}_j} \left[\sum_{h=1}^{T_i} \delta^{h-1} \boldsymbol{y}_{t+h-1}' \boldsymbol{R}_{\omega(i,h)} \boldsymbol{y}_t\right] - \boldsymbol{y}_t' \boldsymbol{C}_j \boldsymbol{y}_t \\ &\equiv \sum_{i \in \mathcal{L}_j} \Pi^i(\boldsymbol{y}_{t+\tau}; \tau = 0, \ldots, T_i - 1), \end{aligned} \tag{17}$$

where (i) $\boldsymbol{C}_j$ is the $(K+1) \times (K+1)$ matrix of cost-function coefficients for firm j and (ii) the matrices $\boldsymbol{R}_1, \ldots, \boldsymbol{R}_K$ contain the linear coefficients from the inverse demand functions for cars $k = 1, \ldots, K$, respectively, in equation (9). Specifically, for each car model $i \in \mathcal{L}$, $\boldsymbol{R}_{\omega(i,h)}$, for $h = 1, \ldots, T_i$, are $(K+1) \times (K+1)$ matrices with 0's everywhere except for the $\eta(i)$th column (the column that corresponds to the quality ranking of a new model i). This column is set to

$$\left[\alpha_{\omega(i,h)}\overline{\theta},\ \underbrace{-\alpha_{\omega(i,h)}\frac{\overline{\theta}}{M}, \ldots, -\alpha_{\omega(i,h)}\frac{\overline{\theta}}{M}}_{\text{entries } 2, \ldots, \omega(i,h)},\ \underbrace{-\alpha_{\omega(i,h)}\frac{\overline{\theta}}{M}, -\alpha_{\omega(i,h)+1}\frac{\overline{\theta}}{M}, \ldots, -\alpha_K\frac{\overline{\theta}}{M}}_{\text{entries } \omega(i,h)+1, \ldots, K+1}\right]'.$$

From the expression for π_t^j in equation (17), we see that firm j's profits in period t depend not only on its own current, past and future production, but also on the current, past and future production of all its rivals. The latter dependence arises only in a durable-goods oligopoly. In this dynamic setting, therefore, firms' production strategies at a given period t can become unwieldy because they can depend on the entire production history of all firms prior to period t. An appealing and natural assumption here is to allow firms' production choices today to depend only on cars produced in the past that still actively trade in secondary markets today.

This corresponds to a standard Markov assumption that firms' strategies depend only on past variables that affect current (period t) profits.[14] In our dynamic setting, these "payoff-relevant" variables are $\boldsymbol{A}\boldsymbol{y}_{t-1}$, the vector of the stock of cars produced prior to period t that are still actively traded in secondary markets.[15] Hence, we focus on production strategies of the form

$$x_t^{\eta(i)} = g_i(\boldsymbol{A}\boldsymbol{y}_{t-1}), \qquad \forall\, i \in \mathcal{L}_j,\ \forall\, j \in \mathcal{N}. \tag{18}$$

In order to obtain the linear-quadratic specification, we assume that production rules are linear in the state vector, so that

$$x_t^{\eta(i)} = \boldsymbol{g}_i \cdot \boldsymbol{A}\boldsymbol{y}_{t-1},$$

where $\boldsymbol{g}_i$ is a $K+1$-vector of linear coefficients.

[14] See Fudenberg and Tirole (1991) for a discussion.

[15] Note that the state vector cannot be $\boldsymbol{y}_{t-1}$ because this vector contains $x_{t-1}^{\upsilon^{T_i-1}(\eta(i))}$, the cars that have died between periods $t-1$ and t, which cannot affect period t profits directly.

We can then write each firm's maximization problem as a dynamic programming problem with state variable $A\boldsymbol{y}_{t-1}$. The Bellman equation for this problem is

$$V_j(\boldsymbol{A y}_{t-1}) = \max_{x_t^{\eta(i)},\ \forall\ i \in \mathcal{L}_j} \sum_{i \in \mathcal{L}_j} \Pi^i(\boldsymbol{y}_t, \boldsymbol{y}_{t+1}, \dots, \boldsymbol{y}_{t+T_i-1}) + \delta V_j(\boldsymbol{A y}_t). \tag{19}$$

Given our assumptions so far, the value function $V_j(\cdot)$ will be a quadratic function of the state vector. Subsequently, each firm's dynamic programming problem in equation (19) is a linear-quadratic problem in the state vector $\boldsymbol{A y}_{t-1}$:[16]

$$\boldsymbol{y}_{t-1}'\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A y}_{t-1} = \max_{x_t^{\eta(i)},\ \forall\ i \in \mathcal{L}_j} \left\{ \sum_{i \in \mathcal{L}_j} \left[\sum_{h=1}^{T_i} \delta^{h-1} \boldsymbol{y}_{t+h-1}' \boldsymbol{R}_{\omega(i,h)} \boldsymbol{y}_t \right] \right\} - \boldsymbol{y}_t' \boldsymbol{C}_j \boldsymbol{y}_t + \delta \boldsymbol{y}_t' \boldsymbol{A}' \boldsymbol{S}_j \boldsymbol{A y}_t, \tag{20}$$

where, for $h = 1, \dots, T_i - 1$,

$$\boldsymbol{y}_{t+h} = \boldsymbol{A y}_{t+h-1} + \boldsymbol{B d}_{t+h}, \tag{21}$$

and

$$\boldsymbol{d}_{t+h} = \boldsymbol{G A y}_{t+h-1}. \tag{22}$$

In these equations, (i) $\boldsymbol{S}_j$ is the $(K+1) \times (K+1)$ matrix of coefficients in firm j's value function, which is quadratic in $\boldsymbol{A y}_t$; and (ii) $\boldsymbol{G} = [\boldsymbol{g}_1, \dots, \boldsymbol{g}_L]'$ is a matrix containing the coefficients of the linear equilibrium production rules. Hence, for our linear-quadratic dynamic durable-goods oligopoly game, a Markov perfect equilibrium specifies linear decision rules, summarized by the coefficient matrix $\boldsymbol{G}$, and value functions $V_j(\cdot)$, as summarized by the matrix $\boldsymbol{S}_j$, for $j \in \mathcal{N}$, such that these solve the dynamic programming problems given by equations (20)–(22).

We note that the production strategies that solve the dynamic programming problem in (19) are time consistent, in the sense that firms correctly anticipate their own future optimal behavior. Time consistency is equivalent to the principle of optimality for dynamic programming problems. With our payoff function, firms can obtain a higher discounted profit stream by committing (at time t) to future production paths $\{x_{t+\tau}^{\eta(i)}, \forall\ i \in \mathcal{L}_j\}_{\tau=0}^{\infty}$, that solve

$$\max_{\{x_t^{\eta(i)},\ i \in \mathcal{L}_j\}_{t=0}^{\infty}} \sum_{\tau=0}^{\infty} \sum_{i \in \mathcal{L}_j} \delta^t \pi_{t+\tau}^j, \tag{23}$$

subject to the law-of-motion in equation (21). The solution to this problem, however, would be time inconsistent because, once period t passes, the firm no longer internalizes the effect of period $t+1$ production on her period t profits and will choose to revise its production plan.

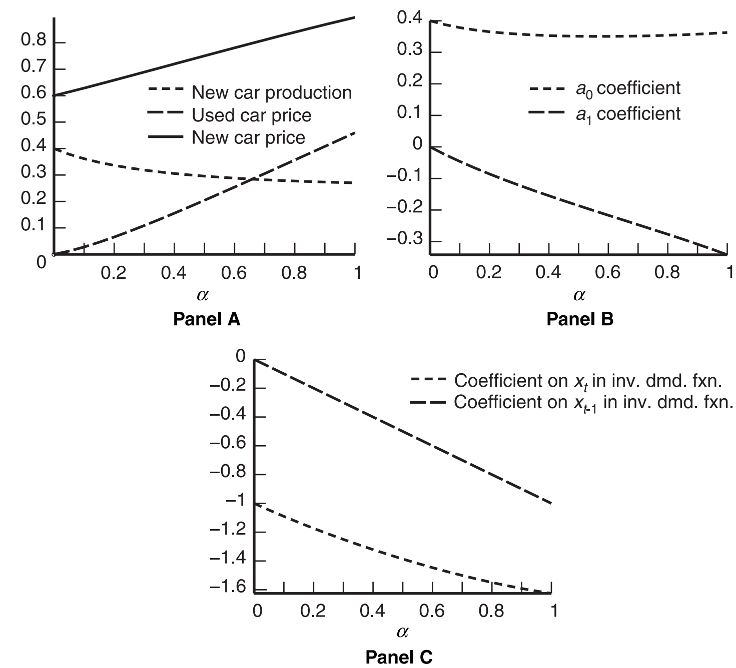
In the Appendix, we complete the derivation of the linear-quadratic Markov-perfect equilibrium for our problem and also describe how we compute it using a value function iteration procedure. For our empirical work below, we extend the linear-quadratic model to accommodate imported automobiles, which are assumed to be exogenously produced from the Big 3 manufacturers' (i.e., General Motors, Ford, Chrysler) point of view. Details of this extension are given in the Appendix.

□ **Illustrative simulations from simplified linear-quadratic model.** In our empirical work to follow, we will use time series on automobiles prices (both new and used) and quantities over a 20-year period to estimate the model parameters, using the linear inverse demand and supply relations (equations (9) and (A1)) as estimating equations. In order to gain some intuition for the variation in the data identifying the model parameters, we consider a simple version of the model, in which a monopolist produces new cars that last for two periods. We normalize the quality of a new car to 1 and let α (< 1) denote the quality of a used automobile, where α also measures

[16] Judd (1996) uses a similar approach to derive a linear-quadratic dynamic oligopoly model.

FIGURE 1

SIMULATION OF DURABLE-GOODS MONOPOLY MODEL: CHANGES IN α



the substitutability between a new and a used car. We assume the marginal cost of production is constant. (For more detail on the workings of this model and proofs for these results, see Esteban (1999, 2006).)

For this more tractable setup, the inverse demand function given in equation (9), which relates the price of a new car with current, lagged and future production, is given by

$$p_t^N = (1-\alpha)(1-x_t) + \alpha(1-x_t-x_{t-1}) + \delta\alpha(1-(a_0+a_1x_t)-x_t),$$

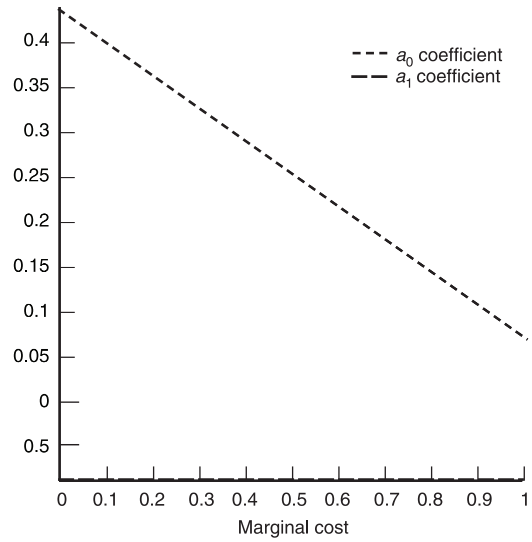
and the supply-equilibrium decision in equation (A1) is the slope coefficient and a_1 captures the degree of serial dependence in the production process. The coefficients in these equations (and thus the size and magnitude of the comovements among prices and current and lagged output) depend on the values of α, c and δ.

We first consider the effects of different values of α. As the top left graph in Figure 1 shows, larger values of α, which imply more substitutability between primary and secondary markets, increase both new and used prices and decrease new-car production. Furthermore, the top right graph in Figure 1 shows that increases in α raise the magnitude of a_1 (which is negative in value), so that current production becomes more sensitive to previous production. The bottom graph shows that increases in α decrease the coefficients for current and lagged production (making them larger in magnitude because these coefficients are negative) in the inverse demand equation. Hence, as α increases, the (negative) association between past production and current price becomes relatively stronger.

Next, we look at the effects of different values of the marginal cost parameter c. Not surprisingly, larger values of c lower new-car production and increase primary and secondary market prices. However, Figure 2 shows that changes to c leave the a_1 coefficient unchanged, which implies that these changes do not affect the serial dependence of new-car production across time and thus do not affect the comovements of prices. These results suggest that levels of prices and output identify the cost parameters, while across-time dependence in these variables and their levels help identify the α parameters. Along the dynamic equilibrium path of the deterministic model that

FIGURE 2

SIMULATION OF DURABLE-GOODS MONOPOLY MODEL: CHANGES IN MARGINAL COST



we have described up to now, prices and output vary only across time during the convergence to the steady state; once the steady state is reached, prices and output remain constant thereafter. For the purposes of our empirical work, we allow for exogenous shocks to firms' marginal costs that drive movements in prices and output over time.

3. Empirical illustration: the automobile market 1971–1990

■ In this section, we describe our empirical implementation of the model presented above, using annual data for the automobile industry from 1971 to 1990 inclusive. For new cars, we use data on list prices and quantities collected from past issues of *Ward's Automotive Yearbook*.[17] We manually compiled time series on secondary market retail prices at the model-vintage level from back issues of the *Kelley Blue Book* (western U.S. edition).[18]

We aggregate each domestic (U.S.) manufacturer's car production up to the segment level and assume that each manufacturer produces three "composite" goods each period. By modeling competition between cars at the manufacturer-segment level, we are able to abstract away from issues regarding the entry and exit of individual car models, which our model does not address; furthermore, it reduces the dimensionality of the state space of the dynamic programming problem, which is convenient for computational reasons. Specifically, consumers choose from the following 10 composite cars each period:

Chrysler: subcompact (SC), compact (C), mid/full size (MF)

Ford: SC, C, MF

GM: SC, C, MF

Import[19]

The *Kelly Blue Book* contains used car prices up to 7-year-old vintages. For this reason, we assume that cars are available in seven vintages (new, 1-year old, 2-years old, . . . , 6-years old).

[17] This is the same dataset employed in Berry, Levinsohn, and Pakes (1995).

[18] We thank Bruce Hamilton for providing these old issues.

[19] We do not break down imports into additional categories because, during the sample period, most of the imported automobiles were in the subcompact segment.

Furthermore, we assume that owners of 6-year-old cars scrap their cars at the end of the period and obtain a "scrappage value" equal to the price of a 7-year-old model car.

Because the stock of used automobiles decreases significantly over a 7-year life span, we assume that a certain proportion of cars "die" exogenously over time, due to accidents, mechanical failures, etc. Let ζ_i denote the proportion of vintage i cars that die (i.e., $1 - \zeta_i$ denotes the survival rate of vintage i cars). In our work, we take $\zeta_1 = \zeta_2 = 0$, $\zeta_3 = 0.025$, $\zeta_4 = 0.042$, $\zeta_5 = 0.065$, $\zeta_6 = 0.101$. For data-availability reasons, we assume that these survival rates are identical across all car models.[20]

In the interests of space, summary statistics for the prices and output of the model years are not included here, but we summarize some salient characteristics.[21] Across years and models, prices for older vintages are monotonically lower, as we would expect. Some car models—especially the Chrysler models—experienced declining new production: the amount of new production fell sharply between the mid-1960s (the years for which we initialized the used car stocks for 1970, our initial sample year) and 1990 (the end of our sample period). Hence, for these car models, the stocks of used vintages can exceed new production in most years of the sample, resulting in higher (across-year) average stocks for used vintages than for new production. In addition, there are missing values for some prices, even during years where the quantity of the model year is not equal to zero. This appears to be due mainly to irregularities in how the *Kelly Blue Book* reported used car prices: in some years, certain models or vintages were simply not included in the book. Below, we describe how we accommodate these missing prices econometrically. Additional details on the construction of the data variables are given in the Appendix.

In each year, then, there are 70 (= 10 composites $*$ 7 vintages) model years in consumers' choice sets (in addition to the outside good of no purchase). Given this large number, we parameterize the α's as log-linear functions of model-year characteristics (similar to Bresnahan, 1981). The empirical results below employ the following parameterization:

$$\alpha_k = \exp(\mathbf{z}'\boldsymbol{\Gamma}),$$

where

$$\begin{aligned}\mathbf{z}'\boldsymbol{\Gamma} = {} & \gamma_1 * CHR_k + \gamma_2 * FORD_k + \gamma_3 * GM_k + \gamma_4 * IMP_k \\ & + (\gamma_5 * C_k + \gamma_6 * MF_k + \gamma_7 * SC_k) * (1 - IMP_k) \\ & + (\gamma_8 * CHR_k + \gamma_9 * FORD_k + \gamma_{10} * GM_k + \gamma_{11} * IMP_k) * AGE_k \\ & + (\gamma_{12} * C_k + \gamma_{13} * MF_k + \gamma_{14} * SC_k) * (1 - IMP_k) * AGE_k \\ & + \gamma_{15} * \mathbf{1}(1979 \le yr \le 1982) * IMP_k + \gamma_{16} * \mathbf{1}(1983 \le yr \le 1986) * IMP_k \\ & + \gamma_{17} * \mathbf{1}(1987 \le yr \le 1990) * IMP_k.\end{aligned}$$

In the above, CHR, $FORD$, GM and IMP are dummy variables for whether model year k is a Chrysler, Ford, GM or imported car, and SC, C and MF are dummy variables for whether it is a subcompact, compact or mid/full-size car. Note that we do not distinguish between different sizes of imported cars because most imports during the sample period were subcompacts. AGE_k denotes the age of model year k, with $AGE_k = 0$ for new cars. Finally, we also allow the quality of imported cars to change over time, to accommodate the possibility that foreign producers have improved the quality of their offerings over time, as captured by the coefficients $\gamma_{15} - \gamma_{17}$. We allowed the α's for the imports to vary for each 4-year block in our sample period (but we assumed that the import α's were constant from 1971 to 1978 because there were relatively low import volumes during these years).[22]

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[20] These values were derived using data from the Polk Corporation. We thank Darrel Cohen for providing this data; see Cohen and Greenspan (1996) for more details on these data.

[21] Interested readers can find a table of summary statistics online at www.econ.jhu.edu/people/shum/res.html.

[22] We also attempted to allow the effects of size to vary across years by estimating separate coefficients for C_k, SC_k

mss # Esteban & Shum; art. # 3; RAND Journal of Economics vol. 38(2)

□ **Empirical model.** Although the important demand and supply relations in this market are given by linear equations (equations (8) and (A1)), least-squares estimation of the reduced-form equations will not allow us to recover easily the structural parameters, which are implicit functions of the reduced-form regression coefficients. For this reason, we undertake direct structural estimation of the model parameters via a nested generalized method of moments (GMM) procedure where a value iteration procedure to compute the equilibrium production rules is nested inside an outer loop that searches over parameter values matching the predicted population moments of the data-generating process (which are functions of the parameters) to their sample counterparts. In the rest of this section, we discuss the derivation of these moment conditions.

Up to this point, we have not introduced structural errors—factors observed by the agents in the model but unobserved by the econometrician—into the model. An important property of linear-quadratic problems is the certainty equivalence property (see Sargent, 1987), which implies that the equilibrium decision rules derived above are unchanged if we introduce additive shocks to demand or/and production costs. Therefore, we introduce shocks to firms' cost of production so that the total variable cost of producing $x_t^{\eta(i)}$ is $x_t^{\eta(i)}(\beta_{1i} + \beta_{2i}x_t^{\eta(i)} + \varepsilon_{it})$. If we assume that the vector of cost shocks $\boldsymbol{\epsilon}_t \equiv [\varepsilon_{1t}, \ldots, \varepsilon_{Lt}]'$ is a zero-mean vector that is i.i.d. across all periods t, then the certainty-equivalence property of linear-quadratic games implies that the stochastic vector of optimal production rules in the presence of cost shocks is

$$\boldsymbol{d}_t = \boldsymbol{G}\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{w}_t, \tag{24}$$

where $\boldsymbol{w}_t$ is a vector of linear functions of the cost shocks $\varepsilon_{1t}, \ldots, \varepsilon_{Lt}$ with zero mean and G is the equilibrium–decision-rule coefficients derived in equation (A2).[23] Therefore, the decision rules with cost shocks (24) are equal to the decision rules without cost shocks (in equation (22)) plus an additive component that is stochastic from the econometrician's point of view, but with mean zero (and, furthermore, uncorrelated with $\boldsymbol{y}_{t-1}$) and independent over time.

In the presence of cost shocks, the linear inverse-demand functions in period t become

$$p_t^{\eta(i)} = (\alpha_{\eta(i)} - \alpha_{\eta(i)+1})\left(1 - \frac{1}{M}\sum_{r=1}^{\eta(i)} x_t^r\right) + \delta E[p_{t+1}^{\upsilon(\eta(i))} \mid \Omega_t] + p_t^{\eta(i)+1} - \delta E[p_{t+1}^{\upsilon(\eta(i)+1)} \mid \Omega_t], \tag{25}$$

where Ω_t denotes consumers' information sets as of period t. This is the stochastic analogue of equation (8). Given the linearity of this equation and our stochastic assumptions regarding the cost shocks, we derive that

$$E\left[p_t^{\eta(i)} - (\alpha_{\eta(i)} - \alpha_{\eta(i)+1})F^{-1}\left(1 - \frac{1}{M}\sum_{r=1}^{\eta(i)} x_t^r\right) + \delta p_{t+1}^{\upsilon(\eta(i))} + p_t^{\eta(i)+1} - \delta p_{t+1}^{\upsilon(\eta(i)+1)} \middle| \Omega_t\right] = 0. \tag{26}$$

Thus, equations (24) and (26) are the main estimating equations for our model.[24]

Details. The structural parameters of the model are (i) $\alpha_1, \ldots, \alpha_K$, the qualities of the competing cars; and (ii) β_{1i}, β_{2i}, $i = 1, \ldots, L$, the marginal cost parameters for the new cars. As is usual in empirical dynamic models, the discount factor δ is not estimated, but rather fixed (at 0.95, in our case). Furthermore, the upper bound of the consumer heterogeneity distribution, $\overline{\theta}$, cannot be identified separately from the scale of the α's and is fixed equal at 3 in the econometric implementation.

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and MF_k for the years 1981–1990. However, in our estimation, these parameters did not move at all from their starting values, indicating that they were not well identified.

[23] The certainty equivalence property also implies that stochastic production rules (24) will still hold if we also allowed for additive i.i.d. demand shocks in the inverse demand functions (equation (6)). However, we do not do this because allowing demand shocks would rule out the use of current market shares and prices as instruments (see below).

[24] In the presence of cost shocks, the inequalities in equation (10) hold for the *expected* rental prices $p_t^k - E_t\delta p_{t+1}^{\upsilon(k)}$.

For the supply side, the estimating equations are the equilibrium production rules linking current production of new cars to stock of used cars in the market (equation (24)). The sample moment conditions are

$$\frac{1}{T}\sum_{t}[\boldsymbol{d}_t - (\boldsymbol{GA})\boldsymbol{y}_{t-1}] * \boldsymbol{y}_{t-1}, \tag{27}$$

where past production $\boldsymbol{y}_{t-1}$ is an appropriate instrument orthogonal to the error term $\boldsymbol{w}_t$ in equation (24).

The population moment restrictions for the demand side are given in equation (26). Let z_t denote a vector of instruments, which are elements of Ω_t, the information set of consumers for period t. In our specifications, z_t consists of the constant 1, current and lagged market shares, and current and lagged prices as instruments. Therefore, the sample analog of the demand-side moment restrictions for production of car $\eta(i)$ take the form

$$\frac{1}{T}\sum_{t=1}^{T}\left[p_t^{\eta(i)} - (\alpha_{\eta(i)} - \alpha_{\eta(i)+1})\left(1 - \frac{1}{M}\sum_{r=1}^{\eta(i)} x_t^r\right) + \delta p_{t+1}^{\upsilon(\eta(i))} + p_t^{\eta(i)+1} - \delta p_{t+1}^{\upsilon(\eta(i)+1)}\right] * z_t, \tag{28}$$

where the sample moment conditions are evaluated, for each period t, at the realized prices in periods t and $t+1$. We obtain estimates of the structural parameters $\boldsymbol{\psi}$, via GMM, by minimizing a quadratic form in the sample moment conditions given in equations (28) and (27).[25]

We accommodate the missing prices in a straghtforward manner. The missing prices do not affect the supply-side estimating equation (27), but we amend the demand-side estimating equation (28) as

$$\frac{1}{T}\sum_{t=1}^{T}\left[p_t^{\eta(i)} - (\alpha_{\eta(i)} - \alpha_{\eta(i)+1})\left(1 - \frac{1}{M}\sum_{r=1}^{\eta(i)} x_t^r\right) + \delta p_{t+1}^{\upsilon(\eta(i))} + p_t^{\eta(i)+1} - \delta p_{t+1}^{\upsilon(\eta(i)+1)}\right] * z_t * m_{kt}, \tag{29}$$

where m_{it} be an indicator variable that equals one if none of the prices $p_t^{\eta(i)}$, $p_{t+1}^{\upsilon(\eta(i))}$, $p_t^{\eta(i)+1}$, and $p_{t+1}^{\upsilon(\eta(i)+1)}$ are missing. We assume that prices are missing at random, in the sense that this amended sample analog converges to zero (at the true parameter values), similar to the unamended sample analog in equation (28).

□ **Empirical results.** Before discussing the results, we reiterate that our model is stylized and simplifies many aspects of the automobile market. Indeed, at the reported parameter values, only half (50.8%) of the equilibrium inequalities (in equation (10)) are satisfied, which captures the difficulty of the theoretical model in generating price patterns similar to those observed in the data.[26] Thus, we feel that a primary value of this empirical exercise is to demonstrate the feasibility of estimating a dynamic durable-goods model for this industry.

The quality ladder parameters (the α's) are reasonably precisely estimated. For convenience, rather than reporting the individual estimates of each α,[27] we have graphed these α estimates for the nine car models produced by the Big 3 companies, as well as the imports, in Figure 3. We see that, generally, car size is positively related to quality, with midsize cars offering the highest quality, followed by compact and subcompact cars. Brand effects are prominent, with Ford and GM cars offering higher quality than Chrysler cars. The extent of depreciation also differs across brands, with Ford cars depreciating relatively slowly and Chrysler cars depreciating relatively quickly.

As illustrated in the bottom graph of Figure 3, our results also suggest that the quality of

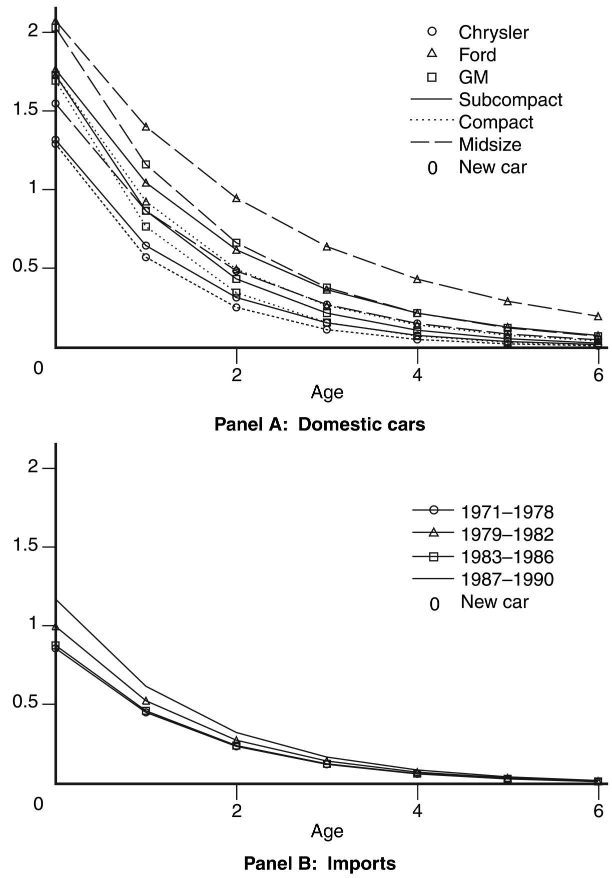
[25] For the results reported in this article, we employ a diagonal weighting matrix, in which each moment condition is weighted by the inverse of its marginal sample variance.

[26] In calculating the inequalities, we assumed that $E_t\delta p_{t+1}^{\upsilon(k)} = p_t^{\upsilon(k)}$, which holds in steady state.

[27] These estimates and standard errors are available on the web at www.econ.jhu.edu/people/shum/res.html.

FIGURE 3

GRAPHS OF ESTIMATED CAR QUALITY α PARAMETERS



imports improved over time. This supports anecdotal evidence that importers (notably Japanese manufacturers) improved the quality of their cars in response to the import quotas that the United States levied against them. However, this improvement was not monotonic; the import α's for the years 1983–1986 lie below those for 1979–1982.

To gauge how well our model fits the data, we simulated the model using our parameter estimates and present the steady-state output and price levels for each of the nine domestic composites in Table 1. A comparison of the actual to simulated values shows that the simulated production levels correspond reasonably closely to the actual production levels. However, the simulated prices fit poorly and are generally higher than the average prices in the data: for example, the actual average price of a Ford MF model is \$9,958, but the analogous value is \$14,609 in the simulated steady state of the dynamic model.

Furthermore, we also simulated the equilibrium profits for each car model on a year-by-year basis, to see how they change over time. These results (not reported for brevity's sake) indicate that the profits for all nine domestic car models fell over the 20-year period, which is perhaps not surprising given that the Big 3 new sales share fell over this time period relative to imports. However, these simulations also show that the profits of the Chrysler models fell most sharply (by 8.4%, 11.3% and 10.8%, respectively, for subcompact, compact and midsize/full cars), followed by Ford and then GM. While this could be consistent with the historical fact that Chrysler was a failing company for much of the 1970s, it also reflects the decreasing trend in the market share of Chrysler cars during the sample period.

The cost function parameters (reported in Table 2) are not estimated precisely. One reason

TABLE 1 **Simulated Steady-State in the Dynamic Model**

		Actual Values		Simulated Values		
Manufacturer	Segment	Quantity[a] (millions)	Price ('000s)	Quantity ('000s)	Price ('000s)	Marginal Cost ('000s)
Chrysler	SC	.176	5.344	.158	6.254	6.137
Chrysler	C	.379	7.923	.400	6.798	6.667
Chrysler	MF	.170	11.822	.170	8.837	8.692
Ford	SC	.322	6.098	.326	9.258	8.956
Ford	C	.267	7.457	.298	10.547	10.197
Ford	MF	.628	9.958	.637	14.609	14.093
GM	SC	.427	6.725	.376	7.956	7.535
GM	C	.526	8.530	.475	9.051	8.572
GM	MF	1.430	10.253	1.263	12.054	11.416

[a] Equilibrium values of endogenous variables evaluated at the steady state of the dynamic model, using the parameter values illustrated in Figure 3. In the calculations, we used the α's for the import cars set at their 1987–1990 values. Market shares are computed for the primary market and exclude imports, which are exogenous. We fixed the production of imports at 1.10 million cars, reflecting the average import volume during the sample period.

for this may be the relatively short ($T = 20$) time series for production quantities in the dataset. In the third column of Table 3, we report the markups corresponding to our marginal cost-parameter estimates. The markups are noticeably small, with all of them lying below 6%. This is perhaps the most puzzling of the results that we obtained, because these markup estimates are much lower than the markup estimates derived in previous empirical work.

One possible factor in the low markups is that we assume that consumers face no transactions costs, which increases transactions in all markets. While it is beyond the scope of this article to consider a model in which consumers face transactions costs in buying used cars, we performed

TABLE 2 **Estimation Results: Nonquality Ladder Parameters**

Manufacturer	Segment	Parameter	Estimate	Standard Error
Chrysler	SC	β_1	1.956	16.198
Chrysler	SC	β_2	13.206	56.502
Chrysler	C	β_1	3.315	8.359
Chrysler	C	β_2	4.192	11.234
Chrysler	MF	β_1	4.522	13.948
Chrysler	MF	β_2	12.286	44.353
Ford	SC	β_1	2.780	14.028
Ford	SC	β_2	9.468	21.007
Ford	C	β_1	3.293	22.293
Ford	C	β_2	11.588	32.309
Ford	MF	β_1	5.849	37.141
Ford	MF	β_2	6.470	30.667
GM	SC	β_1	.924	7.990
GM	SC	β_2	8.799	14.891
GM	C	β_1	1.505	12.695
GM	C	β_2	7.440	13.240
GM	MF	β_1	4.444	18.312
GM	MF	β_2	2.760	7.785
# moment restrictions	395[a]			

Note: Marginal costs ($'000): specification is $MC(x) = \beta_1 + 2\beta_2 x$.

[a] We used 5 demand instruments for each of the demand residuals corresponding to 70 model years, and 5 supply instruments for each of the supply function residuals corresponding to the 9 composite new-car models.

TABLE 3 Simulated Markups When Consumer Population Size Is Reduced

Manufacturer	Segment	Baseline[a]	75% pop'n	50% pop'n	25% pop'n
Chrysler	SC	1.860	2.270	2.952	4.2634
Chrysler	C	1.926	2.312	2.931	3.9894
Chrysler	MF	1.641	1.830	2.020	1.6065
Ford	SC	3.263	3.935	5.032	7.1601
Ford	C	3.314	3.905	4.801	6.1524
Ford	MF	3.533	4.029	4.697	5.2537
GM	SC	5.301	6.639	8.977	14.291
GM	C	5.294	6.558	8.709	13.281
GM	MF	5.296	6.410	8.188	11.313

[a] Computed using the marginal costs reported in Table 2. Baseline population value is 72.95 million households. We fixed the production of imports at 1.10 million cars, reflecting the average import volume during the sample period.

some simulations to gauge its potential effects by reducing the population scaling parameter M, which, as is seen from equation (6), shifts down the demand functions faced by the firm, which might approximate (imperfectly) some of the implications of accommodating transaction costs, which is to reduce the size of the consumer population actively trading in car markets.

We simulated the model, using the estimation results discussed above, for counterfactual population values equal to 75%, 50% and 25% of the actual average in-sample population value (of 73 million households). The counterfactual simulated markups are reported in Table 3. The results indicate that markups would increase: for example, for GM, markups would increase to 9.0%, 8.7% and 8.2% for the subcompact, compact and midsize/full models, respectively, if the population were halved. While this increase in markup appears uniform across all car models, it is smallest in magnitude for the Chrysler car models. These simulations suggest that the predicted markups may appear more reasonable if the effective population were reduced, which can imply that, by not accommodating transactions costs, we are overstating the effective consumer population in our econometric model.

Counterfactual experiments. Because an important difference between this article and previous empirical work on the automobile industry is the explicit modeling of the intertemporal links between the primary and secondary markets, we conclude the article with counterfactual simulations that quantify the effects of the secondary market on new-car production. In these counterfactuals, we simulated the effects of a temporary elimination of the secondary market by computing the change in production (relative to the steady-state production levels given in Table 1) if producers faced empty secondary markets for one period. This elimination of the secondary market lasts only one period because, in subsequent periods, the secondary markets will once again be active as new cars produced today age. The extent to which output adjusts in the period in which the stock is eliminated provides a measure of how much the secondary market affects the primary market after accounting for the intertemporal link between primary and secondary markets, which stems directly from the dynamic model derived in this article because it is measured by the constant coefficient in equation (A2).

In Table 4, we report the simulation results broken down by manufacturer, market segment and total primary market. As we would expect, we find that the elimination of the secondary market would lead firms to increase output, although market shares for each firm and segment show only small changes. The disaggregated results show that the temporary disappearance of the secondary market would increase Chrysler's total production by 20.11%, which is twice the corresponding increases for Ford (10.13%) and GM (10.47%). One possible reason for this is that we estimate Chrysler's cars to have the lowest quality when new (cf., the top graph in Figure 3), so that these cars would substitute most readily with used cars, and thereby benefit the most from the elimination of the secondary market. (Ford and GM experience smaller changes in output.) A similar effect appears when percentage changes are aggregated by market segment, where we

TABLE 4 **Changes in Output with the Temporary Elimination of the Secondary Market**

Manufacturer	Segment	Output, Baseline (millions)	Big 3 Mkt Share, Baseline (%)	Output, with Zero Used Stock (millions)	Big 3 Mkt Share, with Zero Used Stock	% Δ Output[a] (%)
Chrysler	SC	.158	3.86	.186	4.04	17.3
Chrysler	C	.400	9.74	.488	10.62	22.10
Chrysler	MF	.170	4.14	.200	4.36	18.05
Ford	SC	.326	7.95	.366	7.95	12.13
Ford	C	.298	7.26	.330	7.18	10.80
Ford	MF	.637	15.53	.693	15.08	8.80
GM	SC	.376	9.16	.417	9.07	10.98
GM	C	.475	11.58	.524	11.40	10.37
GM	MF	1.263	30.79	1.394	30.31	10.35
			Total by manufacturer			
Chrysler		.728	17.74	.874	19.01	20.11
Ford		1.261	30.74	1.389	30.21	10.13
GM		2.113	51.52	2.335	50.78	10.47
			Total by segment			
	SC	.860	20.97	.968	21.06	12.58
	C	1.173	28.58	1.342	29.20	14.48
	MF	2.070	50.45	2.287	49.74	10.51
			Total primary market			
		4.102		4.598		12.08

[a] Changes in output are computed for the period when the stock of used cars in the entire secondary market is eliminated relative to the baseline steady-state output (as reported in column 3) and using the α for the import cars set at its 1987–1990 level. Baseline population value is 72.95 million households. We fixed the production of imports at 1.10 million cars, reflecting the average import volume during the sample period. The market share is computed within the Big 3, which excludes imports. Output when the stock of used cars equals zero is equal to the constant coefficient in the equilibrium decision rule in equation (A2).

find that the output of cars in the highest ranked segment (MF) increases more modestly than the outputs of lower ranked segments (C, SC). Overall, we find that aggregate new-car production would increase by 12.08% for the 1987–1990 time frame were the secondary market to disappear temporarily.

4. Conclusions

■ In this article, we develop a model of dynamic oligopoly to understand the intertemporal links that arise from durability of the product and its trade in secondary markets. We use a tractable linear-quadratic specification of the model to obtain estimates of the structural parameters and calculate each producer's equilibrium decision rule. While the empirical model is stylized, it represents (as far as we are aware) a first attempt at structural estimation of a dynamic durable goods oligopoly model for the automobile industry.

While the linear-quadratic structure has facilitated the modeling of the effects of durability and secondary markets in the automobile industry, we plan to explore alternative models that may allow us to incorporate additional features that have been shown in the existing literature to be important in the automobile industry, such as transaction costs, asymmetric information and additional consumer heterogeneity. This may also resolve some of the problems in the current estimates. Incorporating these features in the context of a dynamic oligopoly model with secondary markets may involve substantial modeling and computational difficulties, which we plan to tackle in future work.

Appendix

■ Details on the derivation of the Markov-perfect equilibrium of the linear-quadratic game, a description of how the model could be extended to accommodate imports, and details on the construction of the data variables used in estimation follow.

□ **Detailed derivation of the linear-quadratic Markov-perfect equilibrium.** In this section, we complete the derivation of the linear-quadratic Markov-perfect equilibrium of this problem and derive $\boldsymbol{G}$, the matrix of coefficients for the equilibrium decision rule, as a function of the underlying model parameters. By substituting equation (22) into equation (21), we obtain that

$$\boldsymbol{y}_{t+h} = (\boldsymbol{I} + \boldsymbol{B}\boldsymbol{G})\boldsymbol{A}\boldsymbol{y}_{t+h-1}.$$

By iterating on this, we can write the law of motion as

$$\boldsymbol{y}_{t+h} = [(\boldsymbol{I} + \boldsymbol{B}\boldsymbol{G})\boldsymbol{A}]^h\, \boldsymbol{y}_t, \qquad \text{for } h = 1, \ldots, T_i - 1.$$

Then substituting the above into equation (20), we rewrite firm j's dynamic programming problem as

$$\begin{aligned}
\boldsymbol{y}'_{t-1}\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A}\boldsymbol{y}_{t-1} &= \max_{x_t^{\eta(i)}, \forall\, i\in\mathcal{L}_j} \boldsymbol{y}'_t \left\{ \left[\sum_{i\in\mathcal{L}_j} \sum_{h=1}^{T_i} (\boldsymbol{A}')^{h-1}[(\boldsymbol{I}+\boldsymbol{B}\boldsymbol{G})']^{h-1}\delta^{h-1}\boldsymbol{R}_{\omega(i,h)} \right] - \boldsymbol{C}_j + \delta[\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A}] \right\} \boldsymbol{y}_t \\
&\equiv \max_{x_t^{\eta(i)}, \forall\, i\in\mathcal{L}_j} \boldsymbol{y}'_t\boldsymbol{Q}_j\boldsymbol{y}_t \equiv \max_{x_t^{\eta(i)}, \forall\, i\in\mathcal{L}_j} \frac{1}{2}\boldsymbol{y}'_t(\boldsymbol{Q}_j + \boldsymbol{Q}'_j)\boldsymbol{y}_t \\
&= \max_{x_t^{\eta(i)}, \forall\, i\in\mathcal{L}_j} \frac{1}{2}(\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}\boldsymbol{d}_t)'(\boldsymbol{Q}_j + \boldsymbol{Q}'_j)(\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}\boldsymbol{d}_t).
\end{aligned}$$

The second equality in the second line of the above display is a symmetrization of the quadratic form that leaves the value of the objective function unchanged. Let $\boldsymbol{b}_i$ denote the ith column of matrix $\boldsymbol{B}$. Then the first-order condition for firm j's new production of model $i \in \mathcal{L}_j$ is

$$\boldsymbol{b}'_i(\boldsymbol{Q}_j + \boldsymbol{Q}'_j)(\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}\boldsymbol{d}_t) = 0.$$

Therefore, by stacking the first-order conditions for all of the models $i \in \mathcal{L}_j$ produced by firm j, we obtain

$$\boldsymbol{B}'_j(\boldsymbol{Q}_j + \boldsymbol{Q}'_j)\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}'_j(\boldsymbol{Q}_j + \boldsymbol{Q}'_j)\boldsymbol{B}\boldsymbol{d}_t = 0,$$

where the matrix $\boldsymbol{B}_j$ denotes the $(K+1) \times L_j$ matrix formed by extracting the columns of $\boldsymbol{B}$ corresponding to all the models $i \in \mathcal{L}_j$.

Define the matrix $\boldsymbol{W}_j \equiv \boldsymbol{B}'_j(\boldsymbol{Q}_j + \boldsymbol{Q}'_j)$ for each firm j and $\boldsymbol{W} \equiv [\boldsymbol{W}_1, \ldots, \boldsymbol{W}_N]'$. We stack the systems of first-order conditions for all N firms as

$$\boldsymbol{W}\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{W}\boldsymbol{B}\boldsymbol{d}_t = 0$$

and write the industrywide system of equilibrium decision rules as

$$\boldsymbol{d}_t = -(\boldsymbol{W}\boldsymbol{B})^{-1}(\boldsymbol{W}\boldsymbol{A})\boldsymbol{y}_{t-1}, \tag{A1}$$

which take the form of the equilibrium decision rule given by equation (22), with

$$\boldsymbol{G} \equiv -(\boldsymbol{W}\boldsymbol{B})^{-1}\boldsymbol{W}. \tag{A2}$$

In the present problem, we solve for the Markov-perfect equilibrium production strategies using a value iteration procedure. We consider a long but finite-horizon version of the game and, starting from the terminal period, solve recursively for each firm's optimal production strategies via backward induction. Specifically, for all firms $j \in \mathcal{N}$, we begin with initial guesses for $\boldsymbol{S}_j^0 \equiv [0]$, $j \in \mathcal{N}$ and $G^0 \equiv [0]$ for their respective matrices of value function and decision rule coefficients. Using these matrices, we calculate recursively, for $\tau = 1, 2, 3, \ldots,$

$$\begin{aligned}
\boldsymbol{Q}_j^\tau &\equiv \boldsymbol{A}'\boldsymbol{S}_j^\tau\boldsymbol{A}, \qquad j \in \mathcal{N}, \\
\boldsymbol{W}_j^\tau &\equiv \boldsymbol{B}'_j(\boldsymbol{Q}_j^\tau + \boldsymbol{Q}_j^{\tau\prime}), \\
\boldsymbol{W}^\tau &\equiv [\boldsymbol{W}_1^\tau, \ldots, \boldsymbol{W}_N^\tau]', \\
\boldsymbol{G}^{\tau+1} &\equiv (\boldsymbol{W}^\tau\boldsymbol{B})^{-1}\boldsymbol{W}^\tau.
\end{aligned}$$

In each iteration, we update the coefficient matrix for the value functions via

$$\boldsymbol{S}_j^{\tau+1} = \left\{ \left[\sum_{i \in \mathcal{L}_j} \sum_{h=1}^{T_i} (\boldsymbol{A}')^{h-1} [(\boldsymbol{I} + \boldsymbol{B}\boldsymbol{G}^{\tau+1})']^{h-1} \delta^{h-1} \boldsymbol{R}_{\omega(i,h)} \right] - \boldsymbol{C}_j + \delta[\boldsymbol{A}'\boldsymbol{S}_j^{\tau}\boldsymbol{A}] \right\}, \qquad \text{for each } j \in \mathcal{N}.$$

We iterate this procedure until the sequence of matrices $\boldsymbol{S}_j^{\tau}$ and $\boldsymbol{G}^{\tau}$ converges. The converged values of these matrices are the coefficients of the equilibrium value functions and production rules, respectively. Under certain conditions, there is a unique feedback equilibrium, a sequence of production decision rules and value function coefficients, which converges to a Markov-perfect equilibrium of the infinite-horizon game.[28]

□ **Accommodating imports.** In this section, we describe how the linear-quadratic framework presented in the main text can be extended to accommodate imported automobiles. Because the focus of the present article is on competition among the Big 3 American producers from 1971 to 1990, we assume that the production of imports is an exogenous process.[29] In each period t, all domestic firms observe the quantity of new imported cars supplied to the primary market and choose their optimal level of output. We let m_t denote the vector of imports and assume that $m_t = m_{t-1} + \varepsilon_t$, where ε_t is i.i.d. across periods with zero mean and variance σ^2. Therefore, from the firms' perspective, the import production process is a random walk, with $E_t(m_{t+h}) = m_t$ for all t and $h > 0$.

We define $z_t \equiv Ay_{t-1} + Dm_t$ to be the state vector in t and $\widetilde{z}_t \equiv E_t z_{t+1} = Ay_t + Dm_t$ to be the expected state vector for $t + 1$ at t. (Thus, the matrix D ranks the vector of imports within the quality ladder and m_t can be recovered from y_t by $m_t = D' y_t$.)

We conjecture that the equilibrium decision rule and value function take the form

$$x_t = Gz_t \tag{A3}$$

and

$$E_t V(z_{t+1}) = \widetilde{z}_t' W \widetilde{z}_t. \tag{A4}$$

The law of motion for used cars is given by

$$y_t = Ay_{t-1} + Bx_t + Dm_t = z_t + Bx_t$$

or

$$y_t = z_t + BGz_t = (I + BG)z_t$$

after substituting in equation (A1).

We define $\Gamma \equiv (I + BG)A$. Then, extending the above equation for future periods and taking the expectation at time t, we find that

$$E_t y_{t+k} = \Gamma^{k+1} y_{t-1} + \sum_{i=0}^{k} \Gamma^i (I + BG)Dm_t.$$

By inspection, the definition of the $\boldsymbol{A}$ and $\boldsymbol{D}$ matrices implies that $AD = [0]$, so that $(I + BG)AD = [0]$. Similarly, $BGD = [0]$. Then we can write the above equation as

$$E_t y_{t+k} = \Gamma^{k+1} y_{t-1} + Dm_t.$$

Next, we consider the per-period profit function for a product. From equation (20),

$$\pi_i = \sum_{h=1}^{T_i} y_{t+h-1}' R_{\omega(i,h)} y_t,$$

where y for future periods must be written in expectations, $E_t y_{t+k} = \Gamma^k y_t + Dm_t$. Then the revenue for product i is

$$\pi_i = y_t \sum_{h=1}^{T_i} \Gamma^{h-1'} \delta^{h-1} R_{\omega(i,h)} y_t + m_t' D' \left(\sum_{h=2}^{T_i} \delta^{h-1} R_{\omega(i,h)} \right) y_t$$

[28] See Başar and Olsder (1982) and Kydland (1975) for more details on these conditions for linear-quadratic games.

[29] However, as noted in the main text, we do allow the quality of the imported automobiles to change over the sample period.

$$= y_t \underbrace{\sum_{h=1}^{T_i} \left(\Gamma^{h-1} \delta^{h-1'} R_{\omega(i,h)} + DD' \left(\sum_{h=2}^{T_i} \delta^{h-1} R_{\omega(i,h)} \right) \right)}_{\equiv \Upsilon_i} y_t$$

using $m_t = D' y_t$. To write the profit function, firm j produces car models $i \in \mathcal{L}_j$. Thus,

$$\pi_j = y_t' \underbrace{(\sum_{i \in \mathcal{L}_j} \Upsilon_i - C_j)}_{\equiv \Upsilon} y_t'. \tag{A5}$$

Then by substituting y_t for the law of motion above, we find that

$$\begin{aligned}\pi_j &= (Ay_{t-1} + Bx_t + Dm_t)'(\Upsilon - C_j)(Ay_{t-1} + Bx_t + Dm_t) \\ &= (Ay_{t-1} + Dm_t)'(I + BG)'\Upsilon(I + BG)(Ay_{t-1} + Dm_t).\end{aligned}$$

Taking expectations at $t - 1$,

$$E_{t-1}\pi_j = (Ay_{t-1} + Dm_{t-1})'(I + BG)'\Upsilon(I + BG)(Ay_{t-1} + Dm_{t-1}) + b\sigma^2,$$

where b is a constant and σ^2 is the variance of the import innovation.

We now verify our conjecture for the value function and decision rule. The total discounted profit at t is

$$\begin{aligned}&z_t'(I + BG)'\Upsilon_j(I + BG)z_t + \delta\widetilde{z}_t' W_j \widetilde{z}_t = \\ &= z_t'(I + BG)'\Upsilon_j(I + BG)z_t + \delta z_t'(I + BG)'(A + DD')' W_j (A + DD')(I + BG)z_t.\end{aligned}$$

Taking expectations at $t - 1$,

$$\widetilde{z}_{t-1}'((I + BG)'\Upsilon_j(I + BG) + \delta(I + BG)'(A + DD')'W_j(A + DD')(I + BG))\widetilde{z}_{t-1} = \widetilde{z}_{t-1}' W_j \widetilde{z}_{t-1},$$

which yields our conjecture in equation (A4).

The equilibrium decision rule is verified as follows. From equation (A5) and our conjectured decision rule (equation (A3)), the firm solves

$$\max\, y_t'\Upsilon_j y_t + \widetilde{z}_t' W_j \widetilde{z}_t,$$

where $\widetilde{z}_t = Ay_t + Dm_t$. Recall that $m_t = D'y_t$, which implies $\widetilde{z}_t = Ay_t + DD'y_t = (A + DD')y_t$, and allows us to write the problem for the firm as

$$\max\, y_t'\Upsilon_j y_t + y_t'(A + DD')'W_j(A + DD')y_t = \max\, y_t' Q_j y_t.$$

Then we follow the same steps as previously to obtain

$$G = -(WB)^{-1}W,$$

where $W = [W_1, \ldots, W_N]'$ and $W_j = B_j'(Q_j + Q_j')$.

□ **Data appendix.** Here we list additional assumptions made in constructing the dataset used in estimating the model.

The market size M is set equal to the total number of households in the United States. In the empirical work, in order to control for population growth over time, we normalize the market size M to be constant over time and equal to its value in the first sample year 1971 (in which the number of households was 72.96 million). We correspondingly adjust the quantities in each year to reflect this market size normalization, via the formula $\widetilde{q}_t^i = 72.96 * s_t^i$, where $\widetilde{q}_t^i$ denotes the adjusted quantity and s_t^i the observed market share for a model i car in year t.

Prices were deflated using a 1983 dollar as the base. The prices for each of the 10 composite goods were computed as quantity-weighted indices of the prices of several representative car models in each composite (size classifications of the included car models were taken from *Ward's Automotive Yearbook*). For each firm and size segment, we chose the representative car models by, first, including the top-selling car in each sample year for this firm and segment. Second, we included additional car models that were among the top 10 selling car models during any sample year. Finally, for each car model included by the first two criteria, we also included any car models that were stylistic antecedents or successors to this model (for example, the Ford Taurus was a top seller in the latter 1980s; thus, we included not only the Ford Taurus, but also the Ford LTD, which it replaced). In using these criteria, our aim was not only to have at least one representative car model for each firm and segment during each sample year, but also to include top sellers for each year. Finally, by the third criterion, we tried to ensure uniformity in cars over time, so that we would not be comparing the prices of very different cars across years.

Following these criteria, the representative car models included in each composite (and the model years for which we had data) are given here:

Chrysler subcompact: Colt (1971–1990).

Chrysler compact: Dart (1964–1976), Volare (1976–1980), Aries (1981–1989), Reliant (1981–1989).

Chrysler mid/full size: New Yorker (1971–1973, 1978–1979, 1981–1990).

Ford subcompact: Pinto (1971–1980), Escort (1981–1990).

Ford compact: Maverick (1970–1977), Fairmont (1978–1983), Tempo (1984–1990).

Ford mid/full size: Fairlane (1964–1970), LTD (1969–1986), Torino (1970–1976), Granada (1975–1980, 1982), Taurus (1986–1990).

GM subcompact: Vega (1971–1977), Chevette (1976–1987), Cavalier (1982–1990).

GM compact: Skylark (1964–1990), Malibu (1977–1983), Citation (1980–1985), Corsica (1987–1990), Nova (1964–1979, 1985–1988).

GM mid/full size: Cutlass (1965–1990), Delta (1969–1988), Grand Prix (1969–1990), Monte Carlo (1970–1988), Century (1973–1990), Impala (1975–1985), Regal (1979–1990), Celebrity (1982–1990).

Imports: Honda Accord (1976–1990), Datsun 210 (1977–1979, 1981–1982), Nissan Sentra (1984–1990), Hyundai Excel (1986–1990), Honda Civic (1974–1990), Camry (1983–1990).

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