USER-FRIENDLY SOFTWARE
FOR BAYESIAN ANALYSIS OF MEDICAL DATA

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Sampling theory

Bayesian theory

Simple example question:
do treatment A and treatment B have different effects on tumour increase X?

group A

group B

Very few data!

X = % tumour–volume increase after 6 months

You are probably familiar with the typical conclusions from sampling theory:

" Assuming that the two full populations:
- are gaussian and independent
- have identical variances (F-test, $p = 0.44$)

*It's actually slightly more probable that the variances are **different***

and that the sample sizes where decided
before running the experiment, then:

The hypothesis that the population means are equal
has a p-value 0.00043 (two-tail t-test: +4.0)

*This p-value is wrong
If the sample sizes 18 & 10
were **decided not in advance**
but by some other rule!*

What does the number "0.00043" really mean?

The sample-mean difference of X is 17.6
The 95% confidence interval is [8.6, 26.7] "

*This does **not** mean that $8.6 < X < 26.7$ with 95% probability!
It means that this technique to construct the interval
contains the true value in 95% of all imaginary datasets.
But what's the best interval that contains X for **our** dataset?*

An intuitive understanding of how Bayesian probability theory works:

*First, it evaluates **all distributions for future patients** in the two groups, given the data*

these distributions are all weighted according to their data fit and combined

and infinitely many more

No assumptions of gaussianity anywhere

Main result: estimates of full distributions and uncertainty of estimates.

New data is unlikely to modify our predictions beyond these regions

- estimate distribution A
- 75% uncertainty
- estimate distribution B
- 75% uncertainty

full-population frequency distribution

*These curves are **predictions** of the treatment effect **on the next future patient***

*We can estimate any quantity we want
For example the mean and variance of each population and their differences*

*These detailed probabilities can be used in a **cost/benefit analysis** (decision theory)*

"Treatment A costs $, gives a% benefit.
Treatment B costs $$, gives b% benefit.
Which treatment should we choose?"

estimated difference between variances = −14
−240 < difference < 130 (95% probability)
difference > 0 with 46% probability

(variance group A) – (variance group B)

estimated difference between means: 8.3
−0.41 < difference < 19 (95% probability)
difference > 0 with 94% probability

(mean group A) – (mean group B)

" Assuming that the two full-population distributions are probably smooth, we predict:
- The distributions of **future treatment outcomes** will be as in the plots, within uncertainties shown
- Future patients under treatment A will have on average $X=66$, and $60<X<72$ with 95% probability
- Future patients under treatment B will have on average $X=57$, and $50<X<65$ with 95% probability
- The difference between the treatment means will be within [−1, 17] with 95% probability
- Variance of X under treatment B will be **larger** than under treatment A with 54% probability
- Average X under treatment B will be **smaller** than under treatment A with 94% probability

These results say that treatment B is very probably better than A at slowing down tumour growth.
However, there is still a ~5% probability that A could be better than B.
If we want to reduce this uncertainty to 1% or less, then we need more data. "

## Compare the two analyses

The statements of sampling theory are:
- vague
- obscure or misleading ("significant"?)
- heavily dependent on tacit assumptions
  gaussianity, stopping rules, ...
- subject to doubts and corrections
  "should I use this test or that test?" "Bonferroni correction?"

The predictions of Bayesian theory are:
- detailed
- quantitative
- easy to interpret and understand
  eg: "fraction x% of population will have effect y"
- calculated always in the same way
  for any sample size, stopping rule, number of hypotheses

Bayesian theory is used, for example, for all inferences about the black hole observed in the galaxy M87
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**First M87 Event Horizon Telescope Results. I.
The Shadow of the Supermassive Black Hole**

The Event Horizon Telescope Collaboration
(See the end matter for the full list of authors.)

define the joint model-parameter posterior distributions for the SSM model, following Bayes' theorem, by

$$P(\Theta, \mathcal{M}|D) = \sum_{\mathcal{I}} \frac{P(D|\Theta, \mathcal{I}) P(\mathcal{I}|\mathcal{M}) \pi(\Theta|\mathcal{M})}{P(D)}, \quad (62)$$

where $P(D|\Theta, \mathcal{I})$ is formally the likelihood, $\mathcal{I}$ refers to a particular snapshot image from $\mathcal{M}$ obtained with probability $P(\mathcal{I}|\mathcal{M})$, and $\pi(\Theta|\mathcal{M})$ are the priors on parameters Θ and the model $\mathcal{M}$, respectively. We now discuss each of these

This was just a simple example. Bayesian theory deals in the same way with multiple hypotheses, variates, and correlation questions

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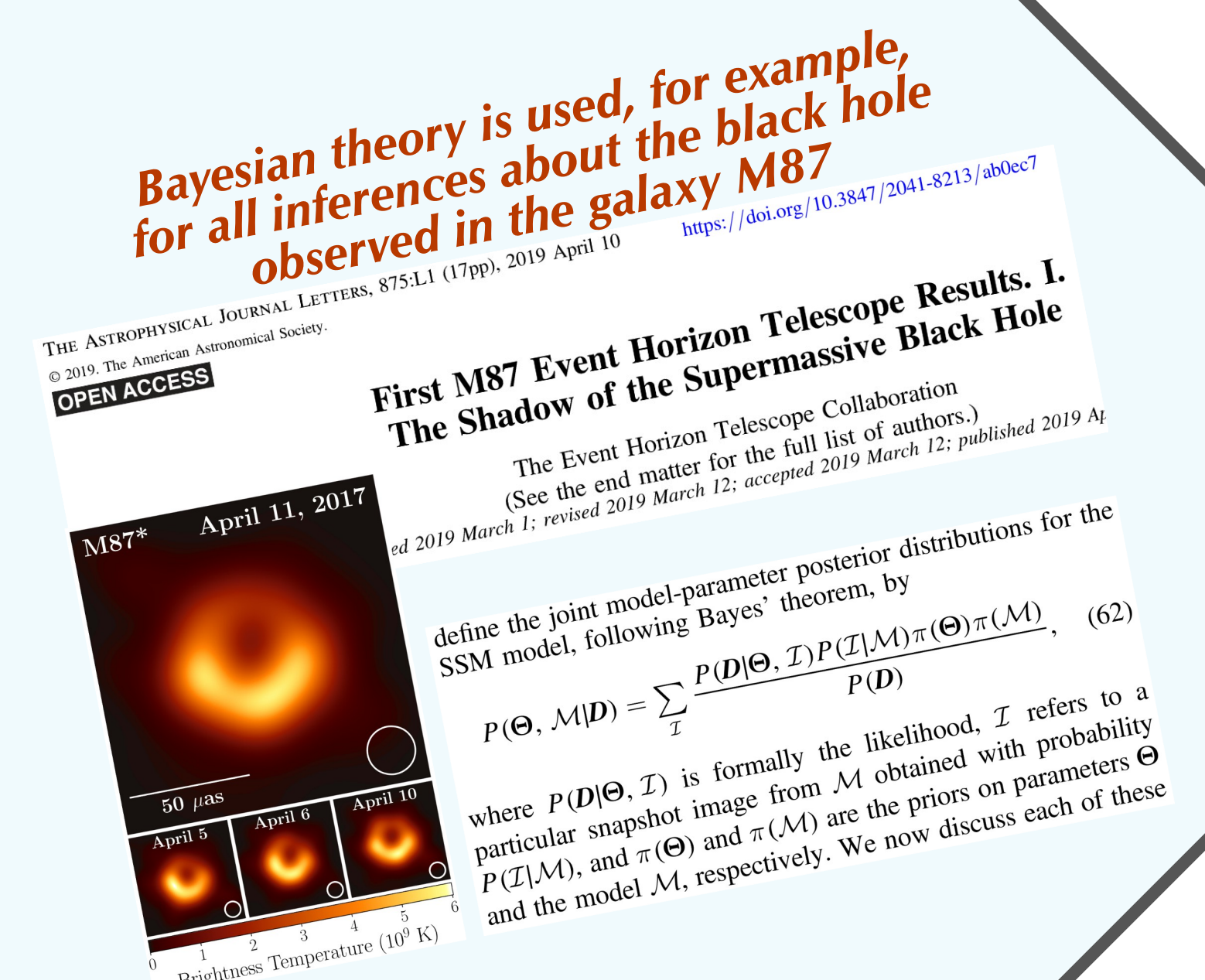
Sounds great, but there's very little friendly software for doing this!

True! That's why we are developing a user-friendly app to do Bayesian analysis on (non-imaging) medical data

Machine Learning

The maths will be taken care of under the hood
The app will suggest meaningful questions to be asked (in line with ASA's statement)
It works with mixtures of continuous, integer, categorical variables:
  it can make predictions about their future statistics, correlations, relevance, and more
Automatic imputation of missing data
No assumptions of gaussianity, linearity, or other (ie: nonparametric)
No need to wonder which test or formula to use, no corrections of any kind

We're already using a prototype version for drug-discovery and Alzheimer research

Please get in touch if you want to test it and help us making a great sofware!

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