Implementation and Analysis of the Tausworthe Pseudo-Random Number Generator

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Technology & Libraries

Technology: R

R Libraries: DescTools, EnvStats, grid, plotrix, randtests, SciViews, tidyverse

Tausworthe Generator

Implement the Tausworthe PRN generator (as described in Module 6) for reasonably large values of the parameters r, q, and l.

```
# Clear the R environment
rm(list=ls())

# Set a random seed
set.seed(6644)
```

```
# Libraries
library(DescTools)
library(EnvStats)
```

```
##
## Attaching package: 'EnvStats'

## The following objects are masked from 'package:stats':
##
##     predict, predict.lm

## The following object is masked from 'package:base':
##
##     print.default
```

```
library(grid)
library(plotrix)
library(randtests)
library(SciViews)
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.1     v readr     2.1.4
## v forcats   1.0.0     v stringr   1.5.0
## v ggplot2   3.4.2     v tibble    3.2.1
## v lubridate 1.9.2     v tidyr     1.3.0
## v purrr     1.0.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

Tausworthe generator algorithm

```r
# Define a function for the Tausworthe generator
tausworthe_generator <- function(r, q, l) {

  # Check value for r
  # 0 < r < q
  stopifnot("`r` must be an integer." = is.integer(r))
  stopifnot("`r` must be greater than 0 but less than q" = r > 0 && r < q)

  # Check value for q, q_max
  # r < q < 15
  stopifnot("`q` must be an integer." = is.integer(q))
  stopifnot("`q` must be greater than r but less than or equal to 10" = q > r && q <= 15)

  # Check value for l, l_max
  # 0 < l < 15
  stopifnot("`l` must be an integer." = is.integer(l))
  stopifnot("`l` must be greater than 1 but less than or equal to 15" = l > 1 && l <= 15)

  # Define the period, max_period
  period <- (2^q) - 1
  max_period <- (2^15) - 1

  # Initialize the first q binary digits in the B_i sequence with 1s
  B_i <- c()
  for (i in 1:q) {
    B_i <- append(B_i, 1)
  }

  # B_i sequence from q to reasonably large number, i > q
  for (i in (q+1):(max_period-q)) {
    i_min_r <- i - r
    i_min_q <- i - q
    B_ir <- B_i[i_min_r]
    B_iq <- B_i[i_min_q]
    B_i[i] <- xor(B_ir, B_iq)
  }

  ### Convert B_i sequence to Unif(0,1)
```

```
  # Define Unif(0,1) sequence
  unif_seq <- c()

  # Define the denominator
  denominator <- 2^l

  # Go from base 2 to base 10
  for (i in 1:(max_period-l)) {
    if (i == 1) {
      l_bits <- B_i[i:(i+l-1)]
      l_bits_str <- as.character(l_bits)
      l_bits_combined <- paste(l_bits_str, collapse="")
      bintodec <- BinToDec(as.numeric(l_bits_combined)) # numerator: l-bits in base 2
      unif <- bintodec / denominator
      unif_seq <- append(unif_seq, unif)
    }
    else if ((i-1) %% l == 0) {
      l_bits <- B_i[i:(i+l-1)]
      l_bits_str <- as.character(l_bits)
      l_bits_combined <- paste(l_bits_str, collapse="")
      bintodec <- BinToDec(l_bits_combined)
      unif <- bintodec / denominator
      unif_seq <- append(unif_seq, unif)
    }
  else {}
  }

  # Return function output
  return(unif_seq)
}
```

```
# Test the Tausworthe generator

# Good values for r, q, and l
r = 9 # can be any number between 1 and 14, inclusive
q = 10 # can be any number greater than r and less than or equal to 15
l = 15 # can be any number between 2 and 15, inclusive

# Bad values for r, q, and l (uncomment to run)
#r=1
#q=2
#l=2
expected_length = ceiling(((2^15)-1 - l) / l)
t_gen <- tausworthe_generator(r=as.integer(r), q=as.integer(q), l=as.integer(l))

# Print the expected length and actual length of the generator's output
print(expected_length)
```

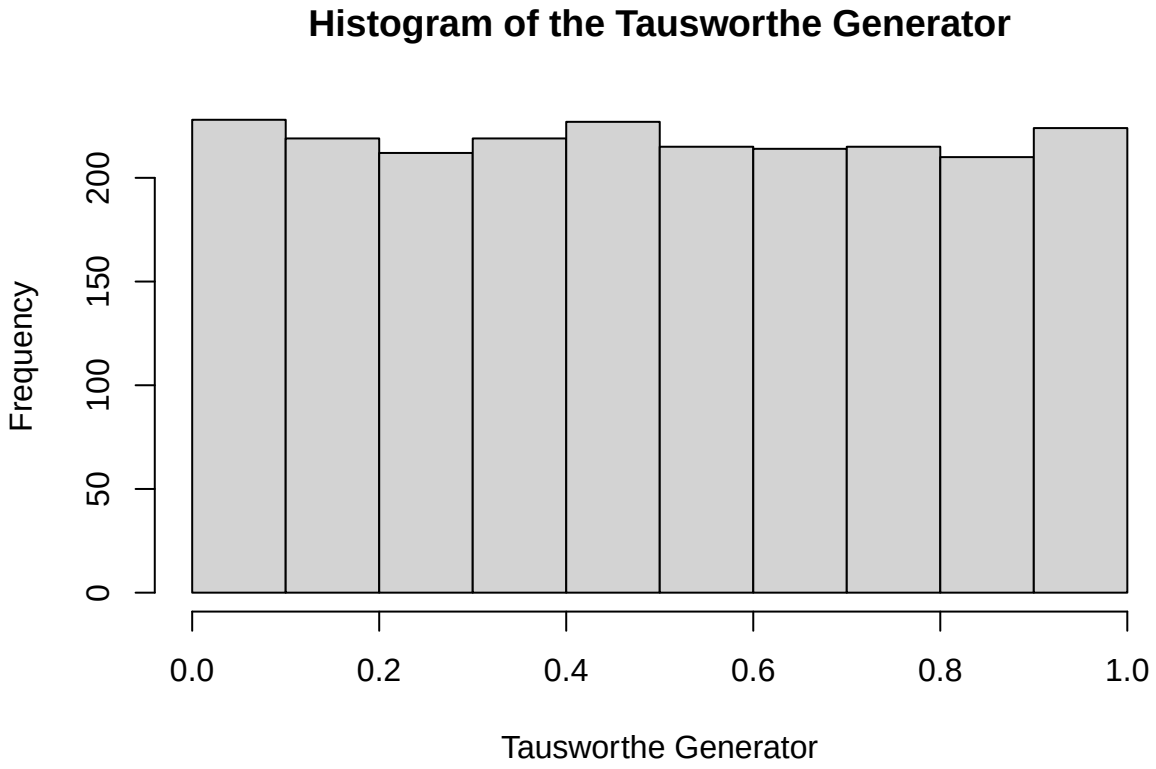
```
## [1] 2184
```

```
print(length(t_gen))
```

```
## [1] 2184
```

```r
# Perform a decent number of statistical tests on the generator to see that it gives PRN's that are app

# Histogram to visualize distribution of data
hist(t_gen, 10, main = "Histogram of the Tausworthe Generator", xlab = "Tausworthe Generator");
```




```r
# Test #1: Chi-squared test for goodness-of-fit - Are the PRNs approximately Unif(0,1)?
gofTest(t_gen[!is.na(t_gen)], test='chisq', distribution = 'unif');
```

```
## $distribution
## [1] "Uniform"
##
## $dist.abb
## [1] "unif"
##
## $distribution.parameters
##          min          max
## 0.0009765625 0.9990234375
##
## $n.param.est
## [1] 2
##
## $estimation.method
## [1] "mle"
##
## $statistic
## Chi-square
##   8.353184
##
```

```
## $sample.size
## [1] 2183
##
## $parameters
## df
## 41
##
## $p.value
## [1] 1
##
## $alternative
## [1] "True cdf does not equal the\n                                    Uniform Distribution."
##
## $method
## [1] "Chi-square GOF"
##
## $data
##    [1] 0.9990234375 0.0313415527 0.0048828125 0.4692687988 0.0498657227
##    [6] 0.6640319824 0.2517089844 0.0704956055 0.7609863281 0.9308471680
##   [11] 0.3544006348 0.9746398926 0.3163452148 0.1273193359 0.2073669434
##   [16] 0.9024963379 0.1400146484 0.5523071289 0.9600524902 0.7854919434
##   [21] 0.4039611816 0.2181091309 0.0596313477 0.2248840332 0.3398132324
##   [26] 0.3765869141 0.0900573730 0.1341552734 0.8646240234 0.9892578125
##   [31] 0.9697570801 0.0968322754 0.1713562012 0.5474243164 0.5542602539
##   [36] 0.7727355957 0.8069458008 0.4673767090 0.1143798828 0.9814758301
##   [41] 0.7214660645 0.2297668457 0.9318237305 0.4478149414 0.9912109375
##   [46] 0.7825622559 0.1221923828 0.2307434082 0.6512756348 0.4761047363
##   [51] 0.2693176270 0.6356201172 0.9736633301 0.4721984863 0.1439514160
##   [56] 0.6786499023 0.8470764160 0.1791381836 0.8001098633 0.6874389648
##   [61] 0.0019531250 0.1878051758 0.0293273926 0.0656127930 0.1664733887
##   [66] 0.9532165527 0.5043945312 0.4225463867 0.5581665039 0.3971252441
##   [71] 0.8734130859 0.0822448730 0.8829345703 0.7629394531 0.7438964844
##   [76] 0.3212280273 0.7845153809 0.1849975586 0.2365722656 0.7127380371
##   [81] 0.3261108398 0.6912231445 0.1390380859 0.5837097168 0.9610290527
##   [86] 0.2556152344 0.4458618164 0.8040161133 0.8118286133 0.0606079102
##   [91] 0.0685424805 0.3231811523 0.9717102051 0.1596374512 0.1732788086
##   [96] 0.6129150391 0.6542053223 0.1946411133 0.3113403320 0.1075744629
##  [101] 0.5768432617 0.6207275391 0.4029846191 0.1869506836 0.0488891602
##  [106] 0.6951293945 0.2644042969 0.6031799316 0.8422546387 0.0793151855
##  [111] 0.6022033691 0.8108520508 0.0920104980 0.0714721680 0.8548889160
##  [116] 0.9279174805 0.8236694336 0.9249877930 0.6679382324 0.8751220703
##  [121] 0.0117187500 0.6268310547 0.0665893555 0.3853759766 0.9970703125
##  [126] 0.2190246582 0.0263977051 0.4098815918 0.1498107910 0.3670654297
##  [131] 0.7551269531 0.4926757812 0.2976684570 0.5464782715 0.5200500488
##  [136] 0.9240112305 0.6990356445 0.8897705078 0.5445251465 0.7078552246
##  [141] 0.9181823730 0.6336669922 0.7864685059 0.4975585938 0.2034606934
##  [146] 0.5278625488 0.1752319336 0.6757202148 0.6287841797 0.1292724609
##  [151] 0.3951721191 0.9357299805 0.0724487305 0.9483032227 0.9132995605
##  [156] 0.2908630371 0.8246459961 0.6444396973 0.6317138672 0.3485412598
##  [161] 0.4137878418 0.2751770020 0.3241577148 0.8780517578 0.1684265137
##  [166] 0.6415100098 0.5376892090 0.3651123047 0.9424438477 0.4751281738
##  [171] 0.3631591797 0.6307373047 0.4419555664 0.4284057617 0.8704833984
##  [176] 0.4264526367 0.9328002930 0.4167175293 0.9921875000 0.2507324219
##  [181] 0.0391540527 0.7541503906 0.3990783691 0.3123168945 0.0136718750
```

```
##  [186] 0.5641479492 0.0881042480 0.4468383789 0.8354187012 0.7971801758
##  [191] 0.5307922363 0.0185852051 0.6591491699 0.2200012207 0.1202392578
##  [196] 0.4186706543 0.6806030273 0.2840270996 0.2317199707 0.7448730469
##  [201] 0.4770812988 0.7991333008 0.7185974121 0.0126953125 0.7204895020
##  [206] 0.0734252930 0.9172058105 0.9142761230 0.7580566406 0.7746887207
##  [211] 0.3709716797 0.3795166016 0.4342651367 0.1820678711 0.4556579590
##  [216] 0.7390136719 0.9152526855 0.8519592285 0.7717590332 0.8383483887
##  [221] 0.4546813965 0.5827331543 0.9298706055 0.2604980469 0.9775695801
##  [226] 0.8460998535 0.2102966309 0.8088989258 0.1546936035 0.0851745605
##  [231] 0.7893981934 0.7776184082 0.1517639160 0.4293823242 0.7766418457
##  [236] 0.4332885742 0.4010314941 0.4995117188 0.0156555176 0.5024414062
##  [241] 0.2346191406 0.5249328613 0.3320007324 0.6258544922 0.0352478027
##  [246] 0.3804931641 0.4654235840 0.1771850586 0.9873046875 0.6581726074
##  [251] 0.0636596680 0.1036682129 0.9512329102 0.5700073242 0.2761535645
##  [256] 0.4800109863 0.8927307129 0.7019653320 0.6090393066 0.5298156738
##  [261] 0.1124267578 0.6698913574 0.6882934570 0.0450134277 0.5670776367
##  [266] 0.4323120117 0.4946289062 0.4848632812 0.5484008789 0.5856628418
##  [271] 0.7737121582 0.2771301270 0.3863525391 0.9034729004 0.2336730957
##  [276] 0.5571899414 0.4907226562 0.8607177734 0.6148681641 0.9659118652
##  [281] 0.2239074707 0.4956054688 0.3912658691 0.5610961914 0.1153564453
##  [286] 0.8256225586 0.7380371094 0.6346435547 0.8178100586 0.4868164062
##  [291] 0.7360839844 0.5719604492 0.8393249512 0.4235229492 0.5895690918
##  [296] 0.4000549316 0.3437194824 0.0009765625 0.0939025879 0.0146484375
##  [301] 0.5328063965 0.0832214355 0.9765930176 0.7521972656 0.2112731934
##  [306] 0.2790832520 0.1985473633 0.9367065430 0.0411071777 0.9414672852
##  [311] 0.3814697266 0.3719482422 0.1606140137 0.3922424316 0.5924987793
##  [316] 0.1182861328 0.3563537598 0.6630554199 0.3456115723 0.0695190430
##  [321] 0.2918395996 0.9804992676 0.6278076172 0.2229309082 0.4020080566
##  [326] 0.4059143066 0.0303039551 0.0342712402 0.1615905762 0.4858398438
##  [331] 0.5798034668 0.5866394043 0.3064575195 0.3270874023 0.5973205566
##  [336] 0.1556701660 0.0537719727 0.7884216309 0.3103637695 0.2014770508
##  [341] 0.5934753418 0.0244445801 0.3475646973 0.1322021484 0.3015747070
##  [346] 0.9211120605 0.5396423340 0.8010864258 0.9054260254 0.0459899902
##  [351] 0.5357360840 0.4274291992 0.9639587402 0.4118347168 0.4624938965
##  [356] 0.3339538574 0.9375610352 0.0058593750 0.3134155273 0.0332946777
##  [361] 0.1926879883 0.4985351562 0.1094970703 0.5131835938 0.7049255371
##  [366] 0.5748901367 0.6835327148 0.3775634766 0.2463378906 0.1488342285
##  [371] 0.2732238770 0.7600097656 0.9620056152 0.3495178223 0.4448852539
##  [376] 0.2722473145 0.8539123535 0.9590759277 0.8168334961 0.3932189941
##  [381] 0.7487792969 0.1017150879 0.7639160156 0.5876159668 0.3378601074
##  [386] 0.3143920898 0.0646362305 0.1975708008 0.9678649902 0.0362243652
##  [391] 0.4741516113 0.4566345215 0.6454162598 0.9123229980 0.3222045898
##  [396] 0.8158569336 0.1742553711 0.7068786621 0.6375732422 0.6620788574
##  [401] 0.4390258789 0.0841979980 0.8207397461 0.7688293457 0.6825561523
##  [406] 0.4712219238 0.2375488281 0.6815795898 0.3153686523 0.2209777832
##  [411] 0.2142028809 0.4352416992 0.2132263184 0.4664001465 0.2083435059
##  [416] 0.9960937500 0.1253662109 0.0195617676 0.8770751953 0.1995239258
##  [421] 0.6561584473 0.0068359375 0.2820739746 0.0440368652 0.7234191895
##  [426] 0.4176940918 0.8985900879 0.2653808594 0.5092773438 0.8295593262
##  [431] 0.6099853516 0.5601196289 0.2093200684 0.8403015137 0.1419982910
##  [436] 0.6158447266 0.8724365234 0.2385253906 0.8995666504 0.3592834473
##  [441] 0.5063476562 0.3602294922 0.5367126465 0.4585876465 0.9571228027
##  [446] 0.8790283203 0.3873291016 0.6854858398 0.1897583008 0.2171325684
##  [451] 0.0910339355 0.2278137207 0.8695068359 0.4576110840 0.9259643555
```

```
##  [456] 0.8858642578 0.9191589355 0.7273254395 0.7913513184 0.9649353027
##  [461] 0.1302490234 0.4887695312 0.9230346680 0.6051330566 0.9044494629
##  [466] 0.0773315430 0.5425720215 0.8946838379 0.8887939453 0.5758666992
##  [471] 0.7146911621 0.3883056641 0.7166442871 0.2005004883 0.7497558594
##  [476] 0.0078125000 0.7512207031 0.1173095703 0.2624511719 0.6659851074
##  [481] 0.8129272461 0.0176086426 0.6902465820 0.2326965332 0.5885925293
##  [486] 0.4936523438 0.3290710449 0.5318298340 0.0518188477 0.9756164551
##  [491] 0.2850036621 0.1380615234 0.7399902344 0.9463500977 0.8509826660
##  [496] 0.3045043945 0.7648925781 0.5562133789 0.3349304199 0.8441467285
##  [501] 0.0224914551 0.7835388184 0.2161560059 0.2473144531 0.2424316406
##  [506] 0.2742004395 0.2928161621 0.8868408203 0.6385498047 0.6931762695
##  [511] 0.4517211914 0.6168212891 0.7785949707 0.2453613281 0.4303588867
##  [516] 0.3074340820 0.4829406738 0.6119384766 0.7478027344 0.1956176758
##  [521] 0.7805480957 0.0576782227 0.4128112793 0.3690185547 0.3173217773
##  [526] 0.4089050293 0.2434082031 0.3680419922 0.2859802246 0.4196472168
##  [531] 0.7117614746 0.2947692871 0.7000122070 0.6718444824 0.5004882812
##  [536] 0.0469360352 0.5073242188 0.2663879395 0.5415954590 0.9882812500
##  [541] 0.8760986328 0.1056213379 0.6395263672 0.5992736816 0.4683532715
##  [546] 0.0205383301 0.9707336426 0.1907348633 0.1859741211 0.0802917480
##  [551] 0.6961059570 0.7962341309 0.5591430664 0.1781616211 0.8315124512
##  [556] 0.6727905273 0.5347595215 0.1459045410 0.9902343750 0.8139038086
##  [561] 0.1114501953 0.7009887695 0.7029418945 0.5151367188 0.5171203613
##  [566] 0.5807800293 0.7429199219 0.2898864746 0.7933044434 0.6532287598
##  [571] 0.1635437012 0.2986450195 0.5778198242 0.5268859863 0.3941955566
##  [576] 0.6551818848 0.1007385254 0.2967224121 0.5122070312 0.6737670898
##  [581] 0.5661010742 0.1507873535 0.4605407715 0.7698059082 0.9005432129
##  [586] 0.4526977539 0.5229797363 0.7678527832 0.7137145996 0.4819641113
##  [591] 0.7059020996 0.7312316895 0.6669616699 0.9687805176 0.0029296875
##  [596] 0.1567077637 0.0166320801 0.5963439941 0.2492675781 0.0547485352
##  [601] 0.2565917969 0.3524475098 0.7874450684 0.3417663574 0.1887817383
##  [606] 0.1231689453 0.0744018555 0.6365966797 0.8800048828 0.4809875488
##  [611] 0.6747436523 0.7224426270 0.1361083984 0.9269409180 0.9795227051
##  [616] 0.9084167480 0.1965942383 0.8743896484 0.0508422852 0.8819580078
##  [621] 0.2937927246 0.6689147949 0.6571960449 0.0323181152 0.0987854004
##  [626] 0.4839172363 0.5180969238 0.7370605469 0.7283020020 0.8226928711
##  [631] 0.9561462402 0.6611022949 0.4079284668 0.0871276855 0.3534240723
##  [636] 0.8187866211 0.3310241699 0.7195129395 0.0420837402 0.9103698730
##  [641] 0.3843994141 0.8412780762 0.2355957031 0.6187744141 0.3407897949
##  [646] 0.1576843262 0.1104736328 0.6070861816 0.7176208496 0.1065979004
##  [651] 0.7331848145 0.6041564941 0.9980468750 0.0626831055 0.0097656250
##  [656] 0.9385375977 0.0997619629 0.3280639648 0.5034179688 0.1410217285
##  [661] 0.5220031738 0.8616943359 0.7088317871 0.9492797852 0.6326904297
##  [666] 0.2546386719 0.4147644043 0.8049926758 0.2800598145 0.1046447754
##  [671] 0.9201354980 0.5709838867 0.8079223633 0.4362182617 0.1192626953
##  [676] 0.4497680664 0.6796264648 0.7531738281 0.1801147461 0.2683410645
##  [681] 0.7292785645 0.9785461426 0.9395141602 0.1936645508 0.3427429199
##  [686] 0.0948791504 0.1085510254 0.5455017090 0.6138916016 0.9347534180
##  [691] 0.2287902832 0.9629821777 0.4429321289 0.4595642090 0.8636474609
##  [696] 0.8956604004 0.9824523926 0.5651245117 0.2443847656 0.4615173340
##  [701] 0.3025512695 0.9522094727 0.5386657715 0.2712707520 0.9473266602
##  [706] 0.9443969727 0.2879333496 0.3573303223 0.6941528320 0.3583068848
##  [711] 0.6002502441 0.3748779297 0.0039062500 0.3756103516 0.0586547852
##  [716] 0.1312255859 0.3329772949 0.9064636230 0.0087890625 0.8451232910
##  [721] 0.1163330078 0.7942810059 0.7468261719 0.1645202637 0.7658996582
```

```
##  [726] 0.5259094238 0.4877929688 0.6424865723 0.5690307617 0.3699951172
##  [731] 0.4731750488 0.4254760742 0.6522521973 0.3824462891 0.2781066895
##  [736] 0.1674499512 0.9220581055 0.5112304688 0.8917541504 0.6080627441
##  [741] 0.6236572266 0.1212158203 0.1370849609 0.6463928223 0.9434204102
##  [746] 0.3192749023 0.3465881348 0.2258605957 0.3084106445 0.3892822266
##  [751] 0.6226806641 0.2151794434 0.1537170410 0.2414550781 0.8059692383
##  [756] 0.3739013672 0.0978088379 0.3902587891 0.5288391113 0.2063903809
##  [761] 0.6845092773 0.1586608887 0.2044372559 0.6217041016 0.1840209961
##  [766] 0.1429748535 0.7098083496 0.8558654785 0.6473693848 0.8500061035
##  [771] 0.3359069824 0.7502441406 0.0234680176 0.2536621094 0.1331787109
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## [1836] 0.1791381836 0.8001098633 0.6874389648 0.0019531250 0.1878051758
## [1841] 0.0293273926 0.0656127930 0.1664733887 0.9532165527 0.5043945312
## [1846] 0.4225463867 0.5581665039 0.3971252441 0.8734130859 0.0822448730
## [1851] 0.8829345703 0.7629394531 0.7438964844 0.3212280273 0.7845153809
## [1856] 0.1849975586 0.2365722656 0.7127380371 0.3261108398 0.6912231445
## [1861] 0.1390380859 0.5837097168 0.9610290527 0.2556152344 0.4458618164
## [1866] 0.8040161133 0.8118286133 0.0606079102 0.0685424805 0.3231811523
## [1871] 0.9717102051 0.1596374512 0.1732788086 0.6129150391 0.6542053223
## [1876] 0.1946411133 0.3113403320 0.1075744629 0.5768432617 0.6207275391
## [1881] 0.4029846191 0.1869506836 0.0488891602 0.6951293945 0.2644042969
## [1886] 0.6031799316 0.8422546387 0.0793151855 0.6022033691 0.8108520508
## [1891] 0.0920104980 0.0714721680 0.8548889160 0.9279174805 0.8236694336
## [1896] 0.9249877930 0.6679382324 0.8751220703 0.0117187500 0.6268310547
## [1901] 0.0665893555 0.3853759766 0.9970703125 0.2190246582 0.0263977051
## [1906] 0.4098815918 0.1498107910 0.3670654297 0.7551269531 0.4926757812
## [1911] 0.2976684570 0.5464782715 0.5200500488 0.9240112305 0.6990356445
## [1916] 0.8897705078 0.5445251465 0.7078552246 0.9181823730 0.6336669922
## [1921] 0.7864685059 0.4975585938 0.2034606934 0.5278625488 0.1752319336
## [1926] 0.6757202148 0.6287841797 0.1292724609 0.3951721191 0.9357299805
## [1931] 0.0724487305 0.9483032227 0.9132995605 0.2908630371 0.8246459961
## [1936] 0.6444396973 0.6317138672 0.3485412598 0.4137878418 0.2751770020
## [1941] 0.3241577148 0.8780517578 0.1684265137 0.6415100098 0.5376892090
## [1946] 0.3651123047 0.9424438477 0.4751281738 0.3631591797 0.6307373047
## [1951] 0.4419555664 0.4284057617 0.8704833984 0.4264526367 0.9328002930
## [1956] 0.4167175293 0.9921875000 0.2507324219 0.0391540527 0.7541503906
## [1961] 0.3990783691 0.3123168945 0.0136718750 0.5641479492 0.0881042480
## [1966] 0.4468383789 0.8354187012 0.7971801758 0.5307922363 0.0185852051
## [1971] 0.6591491699 0.2200012207 0.1202392578 0.4186706543 0.6806030273
## [1976] 0.2840270996 0.2317199707 0.7448730469 0.4770812988 0.7991333008
## [1981] 0.7185974121 0.0126953125 0.7204895020 0.0734252930 0.9172058105
## [1986] 0.9142761230 0.7580566406 0.7746887207 0.3709716797 0.3795166016
## [1991] 0.4342651367 0.1820678711 0.4556579590 0.7390136719 0.9152526855
## [1996] 0.8519592285 0.7717590332 0.8383483887 0.4546813965 0.5827331543
## [2001] 0.9298706055 0.2604980469 0.9775695801 0.8460998535 0.2102966309
## [2006] 0.8088989258 0.1546936035 0.0851745605 0.7893981934 0.7776184082
## [2011] 0.1517639160 0.4293823242 0.7766418457 0.4332885742 0.4010314941
## [2016] 0.4995117188 0.0156555176 0.5024414062 0.2346191406 0.5249328613
## [2021] 0.3320007324 0.6258544922 0.0352478027 0.3804931641 0.4654235840
## [2026] 0.1771850586 0.9873046875 0.6581726074 0.0636596680 0.1036682129
## [2031] 0.9512329102 0.5700073242 0.2761535645 0.4800109863 0.8927307129
## [2036] 0.7019653320 0.6090393066 0.5298156738 0.1124267578 0.6698913574
## [2041] 0.6882934570 0.0450134277 0.5670776367 0.4323120117 0.4946289062
## [2046] 0.4848632812 0.5484008789 0.5856628418 0.7737121582 0.2771301270
## [2051] 0.3863525391 0.9034729004 0.2336730957 0.5571899414 0.4907226562
## [2056] 0.8607177734 0.6148681641 0.9659118652 0.2239074707 0.4956054688
## [2061] 0.3912658691 0.5610961914 0.1153564453 0.8256225586 0.7380371094
## [2066] 0.6346435547 0.8178100586 0.4868164062 0.7360839844 0.5719604492
## [2071] 0.8393249512 0.4235229492 0.5895690918 0.4000549316 0.3437194824
```

```
## [2076] 0.0009765625 0.0939025879 0.0146484375 0.5328063965 0.0832214355
## [2081] 0.9765930176 0.7521972656 0.2112731934 0.2790832520 0.1985473633
## [2086] 0.9367065430 0.0411071777 0.9414672852 0.3814697266 0.3719482422
## [2091] 0.1606140137 0.3922424316 0.5924987793 0.1182861328 0.3563537598
## [2096] 0.6630554199 0.3456115723 0.0695190430 0.2918395996 0.9804992676
## [2101] 0.6278076172 0.2229309082 0.4020080566 0.4059143066 0.0303039551
## [2106] 0.0342712402 0.1615905762 0.4858398438 0.5798034668 0.5866394043
## [2111] 0.3064575195 0.3270874023 0.5973205566 0.1556701660 0.0537719727
## [2116] 0.7884216309 0.3103637695 0.2014770508 0.5934753418 0.0244445801
## [2121] 0.3475646973 0.1322021484 0.3015747070 0.9211120605 0.5396423340
## [2126] 0.8010864258 0.9054260254 0.0459899902 0.5357360840 0.4274291992
## [2131] 0.9639587402 0.4118347168 0.4624938965 0.3339538574 0.9375610352
## [2136] 0.0058593750 0.3134155273 0.0332946777 0.1926879883 0.4985351562
## [2141] 0.1094970703 0.5131835938 0.7049255371 0.5748901367 0.6835327148
## [2146] 0.3775634766 0.2463378906 0.1488342285 0.2732238770 0.7600097656
## [2151] 0.9620056152 0.3495178223 0.4448852539 0.2722473145 0.8539123535
## [2156] 0.9590759277 0.8168334961 0.3932189941 0.7487792969 0.1017150879
## [2161] 0.7639160156 0.5876159668 0.3378601074 0.3143920898 0.0646362305
## [2166] 0.1975708008 0.9678649902 0.0362243652 0.4741516113 0.4566345215
## [2171] 0.6454162598 0.9123229980 0.3222045898 0.8158569336 0.1742553711
## [2176] 0.7068786621 0.6375732422 0.6620788574 0.4390258789 0.0841979980
## [2181] 0.8207397461 0.7688293457 0.6825561523
##
## $data.name
## [1] "t_gen[!is.na(t_gen)]"
##
## $bad.obs
## [1] 0
##
## $cut.points
##  [1] 0.0009765625 0.0236594460 0.0463423295 0.0690252131 0.0917080966
##  [6] 0.1143909801 0.1370738636 0.1597567472 0.1824396307 0.2051225142
## [11] 0.2278053977 0.2504882812 0.2731711648 0.2958540483 0.3185369318
## [16] 0.3412198153 0.3639026989 0.3865855824 0.4092684659 0.4319513494
## [21] 0.4546342330 0.4773171165 0.5000000000 0.5226828835 0.5453657670
## [26] 0.5680486506 0.5907315341 0.6134144176 0.6360973011 0.6587801847
## [31] 0.6814630682 0.7041459517 0.7268288352 0.7495117188 0.7721946023
## [36] 0.7948774858 0.8175603693 0.8402432528 0.8629261364 0.8856090199
## [41] 0.9082919034 0.9309747869 0.9536576705 0.9763405540 0.9990234375
##
## $counts
##  [1] 54 53 50 52 51 46 53 49 49 47 49 47 50 51 48 48 51 54 49 47 56 51 44 49 50
## [26] 53 45 53 47 49 50 45 46 52 51 49 50 47 48 41 56 50 51 52
##
## $expected
##  [1] 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364
##  [9] 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364
## [17] 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364
## [25] 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364
## [33] 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364 49.61364
## [41] 49.61364 49.61364 49.61364 49.61364
##
## $X2.components
##  [1] 0.387800358 0.231135218 0.003008787 0.114781577 0.038739433 0.263201183
```

```
##  [7] 0.231135218 0.007589639 0.007589639 0.137685837 0.007589639 0.137685837
## [13] 0.003008787 0.038739433 0.052481989 0.052481989 0.038739433 0.387800358
## [19] 0.007589639 0.137685837 0.822065131 0.038739433 0.635166368 0.007589639
## [25] 0.003008787 0.231135218 0.429028026 0.231135218 0.137685837 0.007589639
## [31] 0.003008787 0.429028026 0.263201183 0.114781577 0.038739433 0.007589639
## [37] 0.003008787 0.137685837 0.052481989 1.495450381 0.822065131 0.003008787
## [43] 0.038739433 0.114781577
##
## attr(,"class")
## [1] "gof"
```

```
# Conclusion: The null hypothesis is that the data all have equal probabilities. The alternative is tha

# Test #2: Runs test for independence - Are the PRNs approximately independent?
runs.test(t_gen);
```

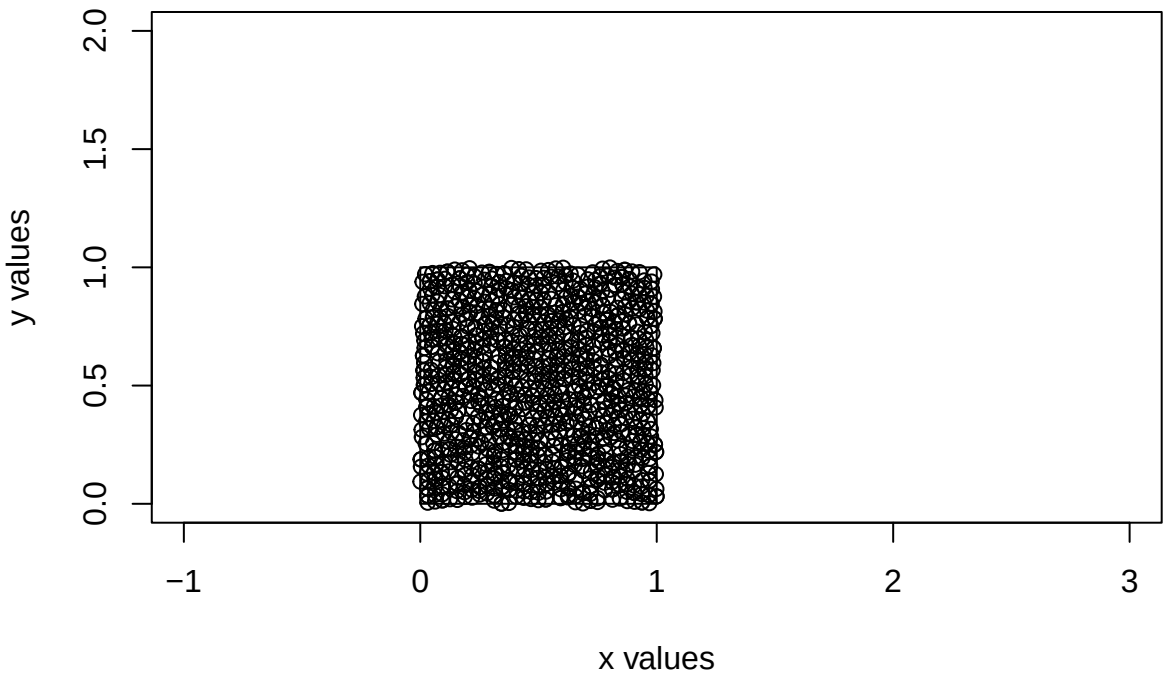
```
##
##  Runs Test
##
## data:  t_gen
## statistic = 0.8569, runs = 1111, n1 = 1090, n2 = 1090, n = 2180,
## p-value = 0.3915
## alternative hypothesis: nonrandomness
```

```
# Conclusion: The null hypothesis is that the data was produced randomly. The alternative hypothesis is

# Test #3: Kolmogorov_Smirnov test - Does the data come from a Uniform(0,1) distribution?
ks.test(x=t_gen, y="punif")
```

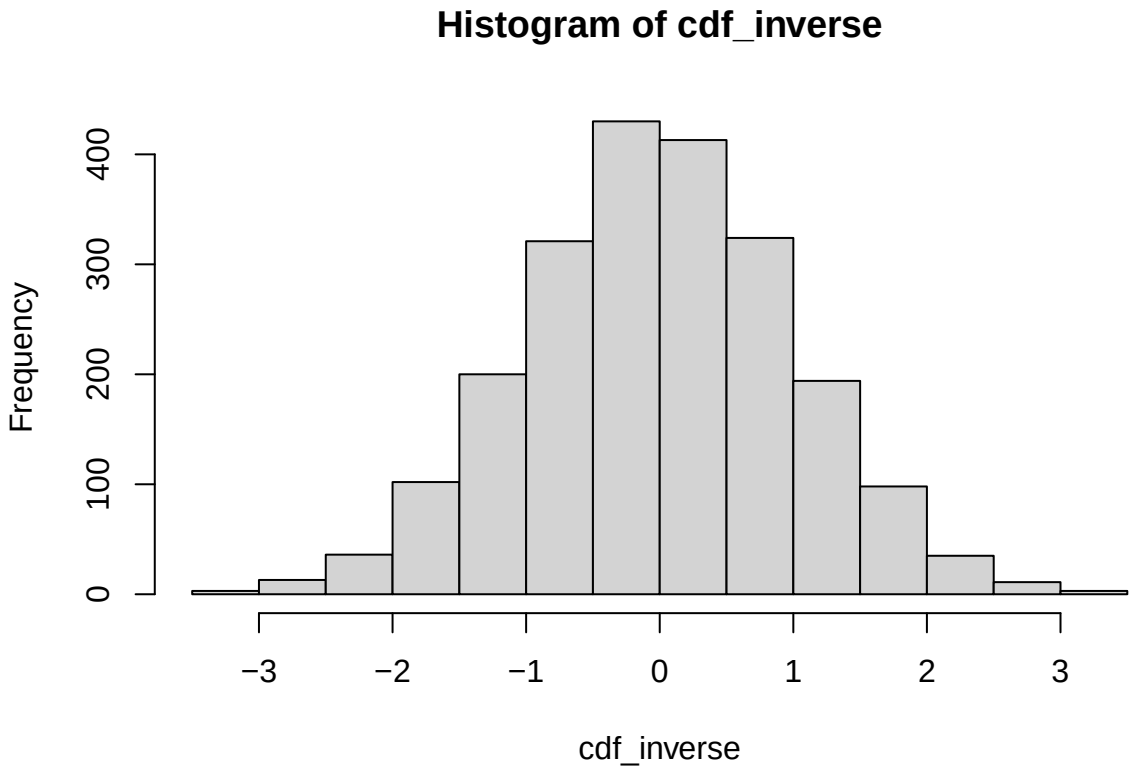
```
## Warning in ks.test.default(x = t_gen, y = "punif"): ties should not be present
## for the Kolmogorov-Smirnov test

##
##  Asymptotic one-sample Kolmogorov-Smirnov test
##
## data:  t_gen
## D = 0.0072866, p-value = 0.9998
## alternative hypothesis: two-sided
```

```
# Conclusion: The null hypothesis is that the data comes from a Uniform(0,1) distribution, and the alte

# Plot adjacent PRN's (U_i, U_i+1), i = 1,2,..., on the unit square to see if there are any patterns

# Define variables
plot_points <- t_gen
x_vals <- c()
y_vals <- c()

# For-loop to create each adjacent pair
for (i in 1:length(plot_points)) {
  if (i %% 2 != 0) {
    x_vals <- append(x_vals, plot_points[i])
```

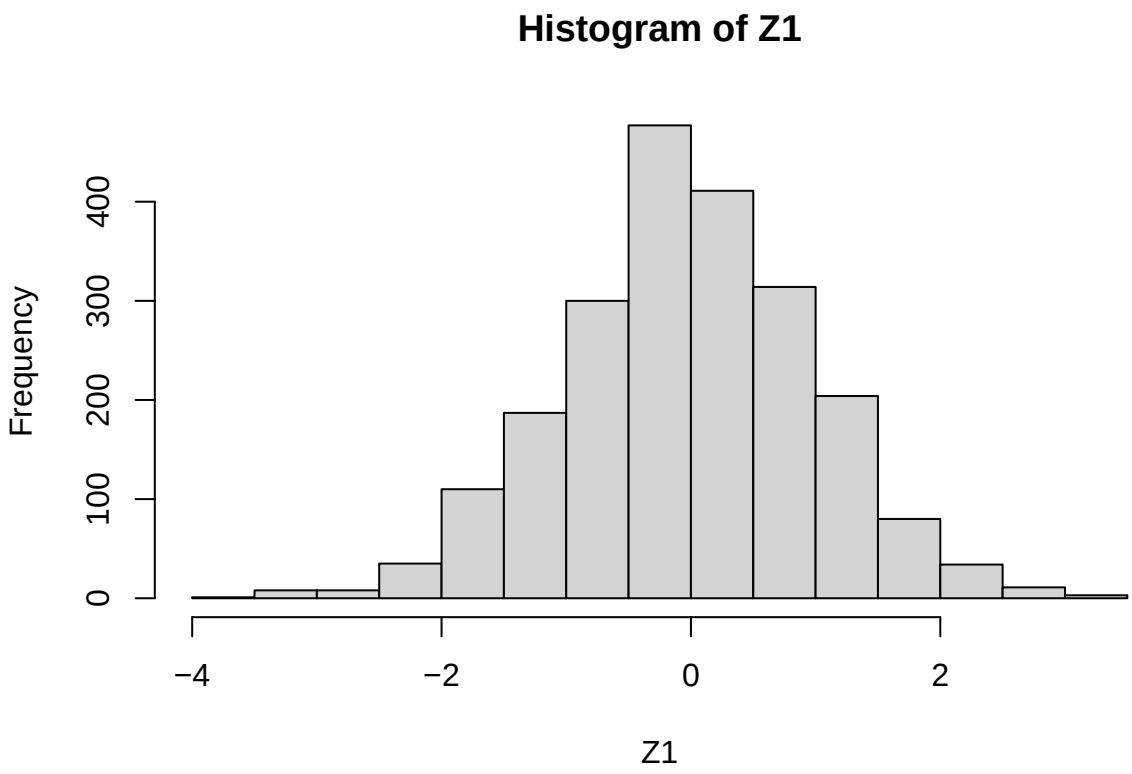
```
  }
  else {
    y_vals <- append(y_vals, plot_points[i])
  }
}

if (length(x_vals) != length(y_vals)) {
  x_vals <- head(x_vals, -1)
}

# Plot adjacent PRNs on the unit square
plot(c(0,2), c(0,2), type = "n", asp=1, xlim = c(0,2), ylim=c(0,2), xlab='x values', ylab='y values')
rect(0, 0, 1, 1)
points(x_vals, y_vals)
```




```
# Generate a few Nor(0,1) deviates (any way you want) using Unif(0,1)'s from your Tausworthe generator

# Method 1: Inverse of the CDF (inverse transform theorem)
cdf_inverse <- qnorm(p=t_gen, mean=0, sd=1)
hist(cdf_inverse);
```




```r
# Method 2: Box-Muller method for normal distribution
U1 <- tausworthe_generator(r=as.integer(9), q=as.integer(10), l=as.integer(15))
U2 <- tausworthe_generator(r=as.integer(7), q=as.integer(10), l=as.integer(15))

Z1 = sqrt(-2*ln(U1))*cos(2*pi*U2)
Z2 = sqrt(-2*ln(U1))*sin(2*pi*U2)

hist(Z1);
```


Histogram of Z1
Frequency
0
100
200
300
400
−4
−2
0
2
Z1

```
hist(Z2)
```


Histogram of Z2
Frequency
0
100
200
300
400
−4
−2
0
2
Z2

END