Judgements: s mparen - s lparen

Reference rules:

$$\frac{}{\epsilon \text{ mparen}} \text{Meps} \qquad \frac{s \text{ mparen}}{(s) \text{ mparen}} \text{Mpar} \qquad \frac{s_1 \text{ mparen} \quad s_2 \text{ mparen}}{s_1 s_2 \text{ mparen}} \text{Mseq}$$

$$\frac{}{\epsilon \text{ lparen}} \text{Leps} \qquad \frac{s_1 \text{ lparen} \quad s_2 \text{ lparen}}{(s_1) s_2 \text{ lparen}} \text{Lseq}$$

Question 1.

Theorem 1.1. If s lparen, then s mparen.

Proof) By rule induction on the judgement s lparen.

Case $\dfrac{}{\epsilon \text{ lparen}}$ Leps where $s = \epsilon$:

     ϵ mparen   by Meps.

Case $\dfrac{s_1 \text{ lparen} \quad s_2 \text{ lparen}}{(s_1) s_2 \text{ lparen}}$ Lseq where $s = (s_1) s_2$

    s_1 mparen   by Inductive hypothesis on s_1 lparen

    s_2 mparen   by Inductive hypothesis on s_2 lparen

    (s_1) mparen   by Mpar with s_1

    $(s_1) s_2$ mparen   by Mseq with (s_1) and s_2

                                            ∎

Judgement: s tparen

Inference rule:
$$\frac{}{\epsilon \text{ tparen}} T_{eps} \qquad \frac{s_1 \text{ tparen} \qquad s_2 \text{ tparen}}{s_1(s_2) \text{ tparen}} T_{seq}$$

Question 2.

Lemma 1.2 If s tparen and s' tparen, then ss' tparen

Proof) We can interpret this theorem as below:

    If s' tparen, then s tparen implies ss' tparen.

    By rule induction on s' tparen

      Case $\dfrac{}{\epsilon \text{ tparen}} T_{eps}$ where $s' = \epsilon$

s tparen	assumption
$ss' = s\epsilon = s$	
ss' tparen	from s tparen

      Case $\dfrac{s_1 \text{ tparen} \quad s_2 \text{ tparen}}{s_1(s_2) \text{ tparen}}$ where $s' = s_1(s_2)$

s tparen	assumption
$ss' = ss_1(s_2)$	
s implies ss_1 tparen	by induction hypothesis on s_1 tparen
ss_1 tparen	from assumption s tparen
$ss_1(s_2)$ tparen	by rule T_{seq} with ss_1 tparen and s_2 tparen

$\blacksquare$

Question 3.

Theorem 1.3. If s mparen, then s tparen.

Proof) By rule induction on s mparen

Case $\dfrac{}{\epsilon \text{ mparen}}$ Meps where $s = \epsilon$

ϵ tparen   by Teps

Case $\dfrac{s' \text{ mparen}}{(s') \text{ mparen}}$ Mpar where $s = (s')$

s' tparen   by Induction hypothesis

ϵ tparen   by Teps

$\epsilon(s')$ tparen   from Tseq with ϵ and s' $\left(\dfrac{\epsilon \text{ tparen} \quad s' \text{ tparen}}{\epsilon \cdot (s') \text{ tparen}} \text{ Tseq} \right)$

(s') tparen   from $\epsilon(s') = (s')$

Case $\dfrac{s_1 \text{ mparen} \quad s_2 \text{ mparen}}{s_1 \cdot s_2 \text{ mparen}}$ Mseq where $s = s_1 s_2$

s_1 tparen   by Induction hypothesis on s_1 mparen

s_2 tparen   by Induction hypothesis on s_2 mparen

$s_1 s_2$ tparen   by Lemma 1.2 ▢