Gyalpozhing College of Information Technology
Royal University of Bhutan

MAT306

Math Assignment I

Name: Sonam
Enrollment Number: 12190024

Submission Date: 20/09/2021

Module Tutor: Mrs. Kezang Yuden

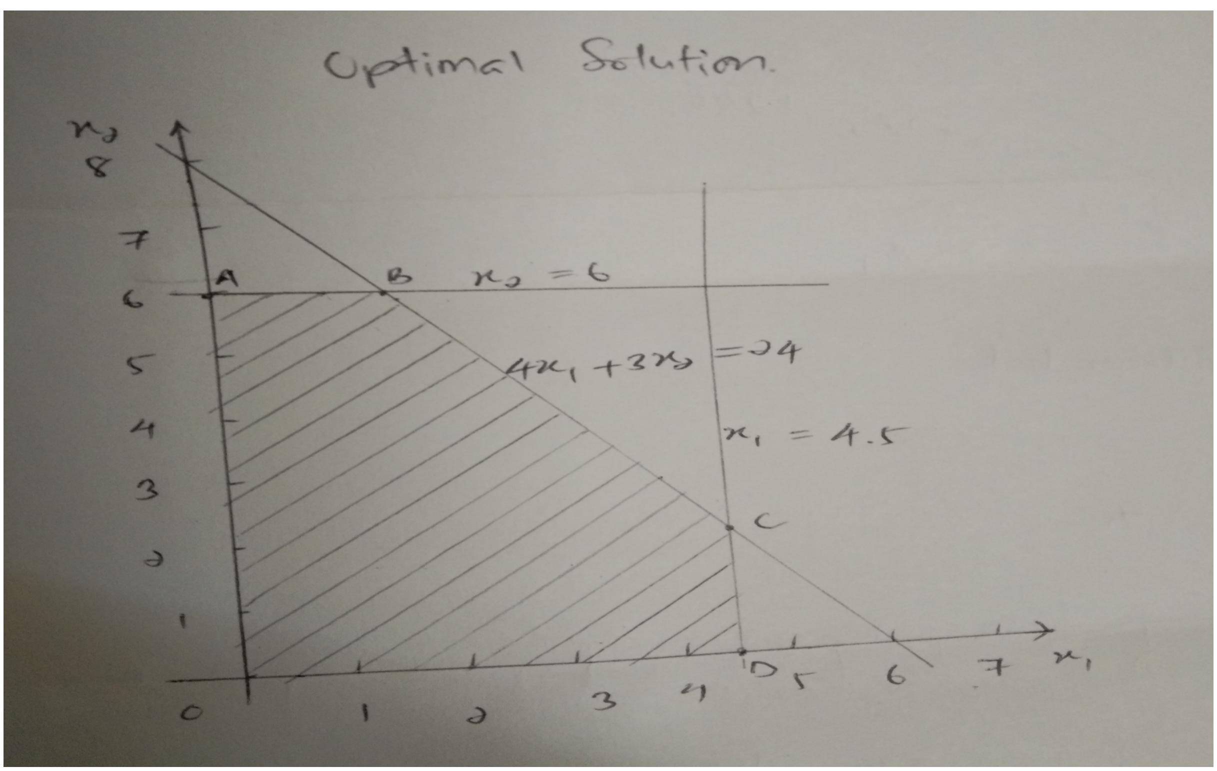
Question 1

Discuss the given special cases in the graphical method.

i) Multiple Optimal Solution

In Linear Programming Problem(LPP), the multiple optimal solution or alternative basic solution is occur when more than one set of basic solutions that can minimize or maximize the required objective function.
For example solve the following equation in graphical method.
Maximize $Z = 4x_1 + 3x_2$
Subject to,
$4x_1 + 3x_2 \leq 24$
$x_1 \leq 4.5$
$x_2 \leq 6$ and
$x_1, x_2 \geq 0$

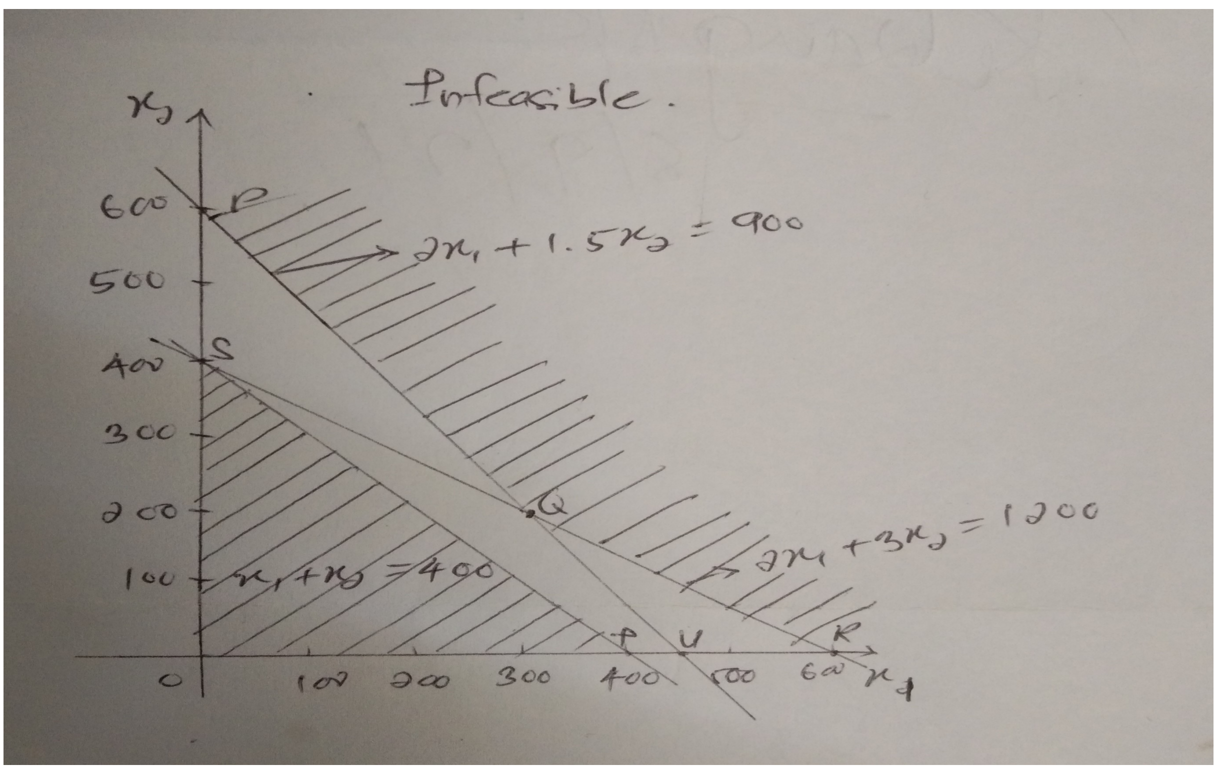
Solution:
The first constraint $4x_1 + 3x_2 \leq 24$ can be written as $4x_1 + 3x_2 = 24$.
Therefore when $x_1 = 0$, $x_2 = 8$
Similarly when $x_2 = 0$, $x_1 = 6$.
And hence the coordinates of the graph is (0,8) and (6,0)
The second constraint $x_1 \leq 4.5$ can also be written as $x_1 = 4.5$
And finally the third constraint $x_2 \leq 6$ can be written as $x_2 = 6$



From above graph the corner point of the feasible region are A, B, C and D. The Max Z = 24 which is achieve at both corner B and C. It can not only the point at B and C but also every points in between B and C, hence the given problem has **multiple optimal solution**.

ii) Infeasibility

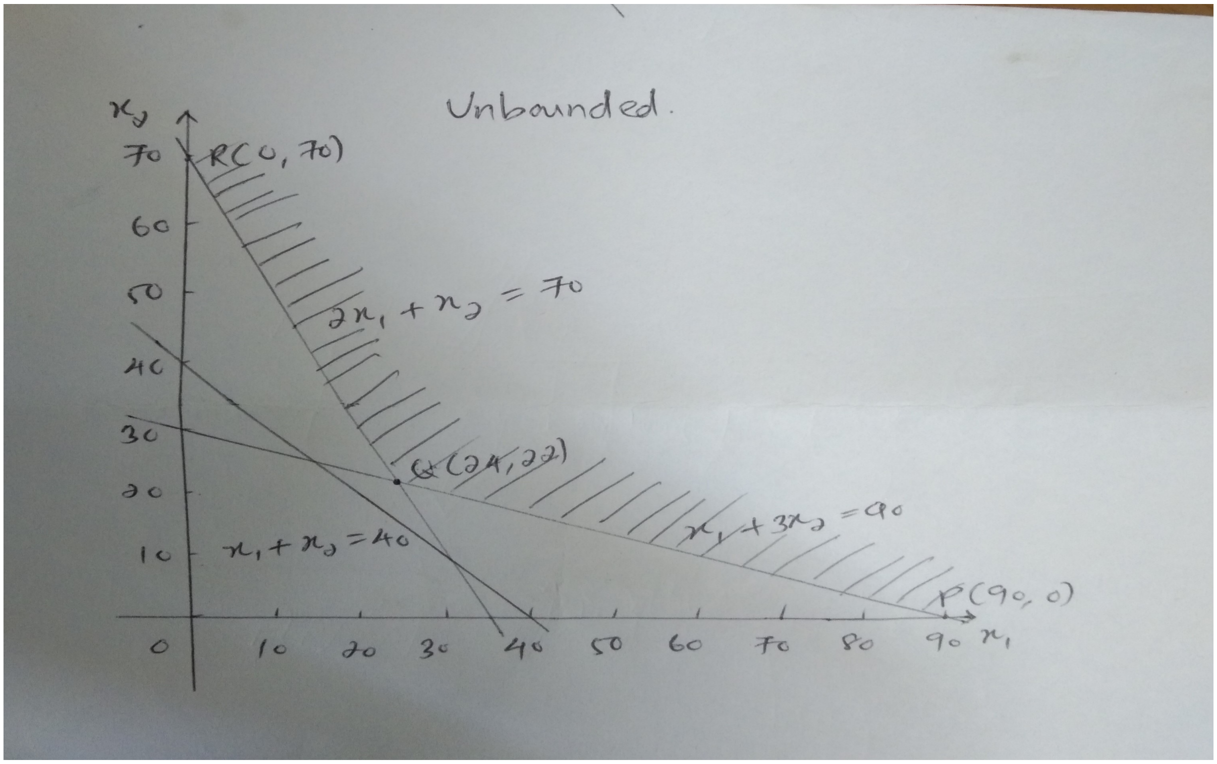
In some cases, there is no feasible solution area, i.e., there are no points that satisfy all constraints of the problem therefore such condition is said to be in infeasibility condition. An infeasible LP problem with two decision variables can be identified through its graph.
Consider the following LPP problem,
Minimize $Z = 200x_1 + 300x_2$
Subject to,
$2x_1 + 3x_2 \geq 1200$
$x_1 + x_2 \leq 400$
$2x_1 + 1.5x_2 \geq 900$ and
$x_1, x_2 \geq 0$
The first constraint $2x_1 + 3x_2 \geq 1200$ can be written as $2x_1 + 3x_2 = 1200$.
Therefore when $x_1 = 0$, $x_2 = 400$
Similarly when $x_2 = 0$, $x_1 = 600$.
And hence the coordinates of the graph is (0,400) and (600,0).
The second constraint $x_1 + x_2 \leq 400$ can also be written as $x_1 + x_2 = 400$
Therefore when $x_1 = 0$, $x_2 = 400$
Similarly when $x_2 = 0$, $x_1 = 400$.
And hence the coordinates of the graph is (0,400) and (400,0).
And finally the third constraint $2x_1 + 1.5x_2 \geq 900$ can be written as $2x_1 + 1.5x_2 = 900$
Where when $x_1 = 0$, $x_2 = 600$
And when $x_2 = 0$, $x_1 = 450$.
Hence the coordinates of the graph is (0,600) and (450,0).



The region located on the right of PQR includes all solutions, which satisfy the first and the third constraints. The region located on the left of ST includes all solutions, which satisfy the second constraint. Thus, the problem is infeasible because there is no set of points that satisfy all the three constraints altogether. So such type of problem is said to be **Infeasiblity**.

iii) Unboundedness

The solution become unboundedness when the objective function of the LPP becomes infinite which means when there is a solution that can be made infinitely large without violating any of its constraint in the problem.
Consider the following example,
Minimize $Z = 40x_1 + 60x_2$
Subject to,
$2x_1 + x_2 \geq 70$
$x_1 + x_2 \geq 40$
$x_1 + 3x_2 \geq 90$ and
$x_1, x_2 \geq 0$
The first constraint $2x_1 + x_2 \geq 70$ can be written as $2x_1 + x_2 = 70$.
Therefore when $x_1 = 0$, $x_2 = 70$
Similarly when $x_2 = 0$, $x_1 = 35$.
And hence the coordinates of the graph is (0,70) and (35,0).
The second constraint $x_1 + x_2 \geq 40$ can also be written as $x_1 + x_2 = 40$
Therefore when $x_1 = 0$, $x_2 = 40$
Similarly when $x_2 = 0$, $x_1 = 40$.
And hence the coordinates of the graph is (0,40) and (40,0).
And finally the third constraint $x_1 + 3x_2 \geq 90$ can be written as $x_1 + 3x_2 = 90$
Where when $x_1 = 0$, $x_2 = 30$
And when $x_2 = 0$, $x_1 = 90$.
Hence the coordinates of the graph is (0,30) and (90,0).



The point(x_1, x_2) must be somewhere in the solution space as shown in the shaded region. So there exists a infinite numbers of points in the feasible region which is unbounded which means there is a solution greater to infinity, therefore the given problem is **unboundedness**.

Question 2

i) Degeneracy

In basic feasible solution, the degeneracy means having fallen below a normal or desirable level. It occurs in the LPP when a basic variable acquires a zero value. It is the condition in the final solution, if either the number of basic variable is not equal to the number of constraint.
For the following example, solve LPP
Maximize $Z = 2x_1 + x_2$
Subject to,
$4x_1 + 3x_2 \leq 12$
$4x_1 + x_2 \leq 8$
$4x_1 + 2x_2 \leq 8$
$x_1, x_2 \geq 0$
Solution:
The standardize form of the above problem is expressed as follows;
Maximize $Z - 2x_1 - x_2 = 0$
Subject to,
$4x_1 + 3x_2 + s_1 = 12$
$4x_1 + x_2 + s_2 = 8$
$4x_1 + 2x_2 + s_3 = 8$
$x_1, x_2, s_1, s_2, s_3 \geq 0$
Therefore the simplex tableau of above is,

Basis	x_1	x_2	s_1	s_2	s_3	Solution
s_1	4	3	1	0	0	12
s_2	4	1	0	1	0	8
s_3	4	2	0	0	1	8
Z	-2	-1	0	0	0	0

From above table, the entering variable is x_1 because the value of -ve Z is least.
To see the leaving basis side, we need to find the ratio of leaving variable over the value in solution excluding the negative values and then select least ratio to leave. Now by solving row reduction form, we get

Basis	x_1	x_2	s_1	s_2	s_3	Solution
s_1	0	2	1	-1	0	4
x_1	1	1/4	0	1/4	0	2
s_3	0	1	0	-1	1	0
Z	0	-1/2	0	1/2	0	4

Solve the row reduction form for the above graph unless the value of Z becomes all positive value by finding entering and leaving value. After executing the leaving and entering value, we obtain the following,

Basis	x_1	x_2	s_1	s_2	s_3	Solution
s_1	0	0	1	1	-2	4
x_1	1	0	0	1/2	-1/4	2
x_2	0	1	0	-1	1	0
Z	0	0	0	0	1/2	4

The Optimal solution is, $x_1 = 2$, $x_2 = 0$ and $Z = 4$. Since for the above problem, it acquires one of the basic variable equals to **Zero** and hence it occurs the degeneracy condition.

ii) Alternative Optima

If the Z-row value for one or more non-basic variable is zero in the optimal tableau, then it is said that there exist a **alternative optimal solution**.
Graphically, it occurs when the objective function is parallel to non-redundant binding constraint(i.e, a constraint that is satisfied as an equation at the optimal solution).

The zero coefficient of non-basic variable x_1 indicates that it can be made basic, altering the value of basic variable without changing the value of Z.
For example, consider the following example,
Maximize $Z = 2x_1 + 4x_2$
Subject to,
$x_1 + 2x_2 \leq 5$
$x_1 + x_2 \leq 4$
$x_1, x_2 \geq 0$
Solution:
The standardize form of the above problem is expressed as follows;
Maximize $Z - 2x_1 - 4x_2 = 0$

Subject to,
$x_1 + 2x_2 + s_1 = 5$
$x_1 + x_2 + s_2 = 4$
$x_1, x_2, s_1, s_2 \geq 0$
Therefore the simplex tableau for above is,

Basis	x_1	x_2	s_1	s_2	**Solution**
s_1	1	2	1	0	5
s_2	1	1	0	1	4
Z	-2	-4	0	0	0

For above graph, since the value for Z is most negative in x_2 therefore it is the entering variable and least ratio in s_1 therefore it leaves. And the table I obtain is,

Basis	x_1	x_2	s_1	s_2	**Solution**
x_2	1/2	1	1/2	0	5/2
s_2	1/2	0	-1/2	1	3/2
Z	0	0	2	0	10

The value of Z is an optimum solution, but coefficient of non-basic variable x_1 is 0, indicates that it can made basic. Therefore by condition x_1 enters and that of s_2 leaves, giving the simplex tableau as,

Basis	x_1	x_2	s_1	s_2	**Solution**
x_2	0	1	1	-1	1
x_1	1	0	-1	2	3
Z	0	0	2	0	10

Therefore the alternative optimum solution of x_1, x_2 are basic variables that is
$x_1 = 3$, $x_2 = 1$ and
Maximize $Z = 10$

iii) Unbounded Solution

It is the solution whose objective function is infinite. OR when determining the leaving variable of any tableau, if there is no positive minimum ratio or all entries of pivot column are negative or zero, then that condition is said to be in unbounded solution. If the feasible region is unbounded then one or more decision variables will increase indefinitely without violating feasibility, and the value of the objective function can be made arbitrarily large. Consider the following example,
Maximize $Z = 2x_1 + x_2$
Subject to,
$-x_1 + x_2 \leq 10$
$-2x_1 \leq 40$
$x_1, x_2 \geq 0$

Solution:
The standardize form of the above problem is expressed as follows;
Maximize $Z - 2x_1 - x_2 = 0$

Subject to,
$-x_1 + x_2 + s_1 = 10$
$-2x_1 + s_2 = 40$
$x_1, x_2, s_1, s_2 \geq 0$
Therefore the simplex tableau for above is,

Basis	x_1	x_2	s_1	s_2	**Solution**
s_1	-1	1	1	0	10
s_2	-2	0	0	1	40
Z	-2	-1	0	0	0

From above table we can see x_1 is the entering variable but all the constraint coefficients under x_1 can be increased indefinitely, which gives the unbounded solution.

iv) Non-existing (or infeasible) solution

In linear programming the condition is said to be in infeasible situation if atleast one artificial variable is positive in the optimum iteration, then the LPP has no feasible solution.

This situation obviously does not occur when all the constraints are type $\leq$ with non-negative right-hand side because all slack variables provides an obvious feasible solution.
For example solve the following problem.
Maximize $Z = 3x_1 + 2x_2$
Subject to,
$2x_1 + x_2 \leq 2$
$3x_1 + 4x_2 \geq 12$ and
$x_1, x_2 \geq 0$
Since in the second constraint, due to the need of subtracting by slack variable in standardize form, we are introducing the artificial variable R_1 in my case to meet the condition that both the s_1 and s_2 should be $\geq$ zero.
Therefore the standardize form of above expression is now,
Maximize $Z - 3x_1 - 2x_2 + 100R_1 = 0$
Subject to,
$2x_1 + x_2 + s_1 = 2$
$3x_1 + 4x_2 - s_2 + R_1 = 12$ and
$x_1, x_2, s_1, s_2, R_1 \geq 0$
Solution:
The tableau form of above is,

Basis	x_1	x_2	s_1	s_2	R_1	**Solution**
s_1	2	1	1	0	0	2
R_1	3	4	0	-1	1	12
Z	-3	-2	0	0	100	0

New Z-row = old Z-row + (0 x s_1-row + (-100) x $R_1 - row$)

Basis	x_1	x_2	s_1	s_2	R_1	Solution
s_1	2	1	1	0	0	2
R_1	3	4	0	-1	1	12
Z	-303	-402	100	0	0	-1200

And the final table is,

Basis	x_1	x_2	s_1	s_2	R_1	Solution
x_1	2	1	1	0	0	2
R_1	-5	0	-4	-1	1	4
Z	501	0	402	100	0	-346

The optimal solution is, $x_2 = 2$, $R_1 = 4$ and $Z = -396$. Since the optimal solution for the artificial variable is positive or it still exist in the table, therefore the given problem is **Infeasible**.

Question 3

Machine	Product	Product	Available time (in mins)
	A	B	
I	30	20	300
II	5	10	110
Profit per unit (Nu)	6	8	

Solution:
Let $'Z'$ be the total profit generated by bakery.
Let $'x_1'$ and $'x_2'$ be the number of muffins A and B respectively.
Objective function,
Maximize $Z = 6x_1 + 8x_2$
Constraint:
$30x_1 + 20x_2 \leq 300$ .... (mins on oven I)
$5x_1 + 10x_2 \leq 110$ .... (mins on oven II)
$x_1, x_2 \geq 0$ .... (non-negativity condition).

The Linear Programming Problem(LPP) is as follows,
Maximize $Z = 6x_1 + 8x_2$
Subject to,
$30x_1 + 20x_2 \leq 300$
$5x_1 + 10x_2 \leq 110$ and
$x_1, x_2 \geq 0$
The standardize form of above constraint is;
Maximize $Z - 6x_1 - 8x_2 = 0$
Subject to,
$30x_1 + 20x_2 + s_1 = 300$
$5x_1 + 10x_2 + s_2 = 110$ and
$x_1, x_2, s_1, s_2 \geq 0$

Basis	x_1	x_2	s_1	s_2	**Solution**
s_1	30	20	1	0	300
s_2	5	10	0	1	110
Z	-6	-8	0	0	0

For above table, the entering variable is x_2 and that of leaving variable is s_2 after finding a ratio between solution over leaving variable.
To make pivot element into 1;

$R_2 \to R_2/10$

Basis	x_1	x_2	s_1	s_2	**Solution**
s_1	30	20	1	0	300
x_2	1/2	1	0	1/10	11
Z	-6	-8	0	0	0

By doing a **Row Reduction Form** by making a zero variable below and above pivot element, I got
$R_1 \to R_1 - 20R_2$
$R_3 \to R_3 + 8R_2$ i,e.

Basis	x_1	x_2	s_1	s_2	**Solution**
s_1	20	0	1	-2	80
x_2	1/2	1	0	1/10	11
Z	-2	0	0	4/5	88

In above, there still exists negative variables in Z, so the entering variable is x_1 and by finding a ratio, the leaving variable is s_1.

Basis	x_1	x_2	s_1	s_2	**Solution**
x_1	20	0	1	-2	80
x_2	1/2	1	0	1/10	11
Z	-2	0	0	4/5	88

In above the pivot element is 20, therefore
$R_1 \to R_1/20$

Basis	x_1	x_2	s_1	s_2	**Solution**
x_1	1	0	1/20	-1/10	4
x_2	1/2	1	0	1/10	11
Z	-2	0	0	4/5	88

Now,
$R_2 \to R_2 - (1/2)R_1$
$R_3 \to R_3 + 2R_1$ i,e.

Basis	x_1	x_2	s_1	s_2	Solution
x_1	1	0	1/20	-1/10	4
x_2	0	1	-1/40	1/20	9
Z	0	0	1/10	3/5	96

Now there is no any negative value for Z and hence the Optimal Solution at $x_1 = 4$ and $x_2 = 9$ yields the profit of Nu. 96

Question 4

a) Formulate the problem as an LP model. **Solution:**
Let 'Z' be the Cost.
Let 'x_1 and x_2' be a no. of grams of fresh food A and B respectively.
The Objective Function;
$Z = 3x_1 + 2x_2$
Constraints
$5x_1 + 4x_2 \geq 30$ .....Constraint for Vitamin A
$7x_1 + 2x_2 \geq 30$ .....Constraint for Vitamin B
$2x_1 + 8x_2 \geq 16$ .....Constraint for Vitamin C
$x_1, x_2 \geq 0$ .....Non-negativity constraint.
Therefore the LPP will be;

Minimize $Z = 3x_1 + 2x_2$
Subject to;
$5x_1 + 4x_2 \geq 30$
$7x_1 + 2x_2 \geq 30$
$2x_1 + 8x_2 \geq 16$
$x_1, x_2 \geq 0$

b) Find Optimal Solution using Graphical method.
Solution:
To find the coordinate, we are assuming x_1 as x-axis and x_2 as y-axis.

The first constraint $5x_1 + 4x_2 \geq 30$ can be written as $5x_1 + 4x_2 = 30$.
So when $x_1 = 0$, then $x_2 = 15/2$and
when $x_2 = 0$, then $x_1 = 6$. Therefore the coordinates are (0, 15/2) and (6, 0).

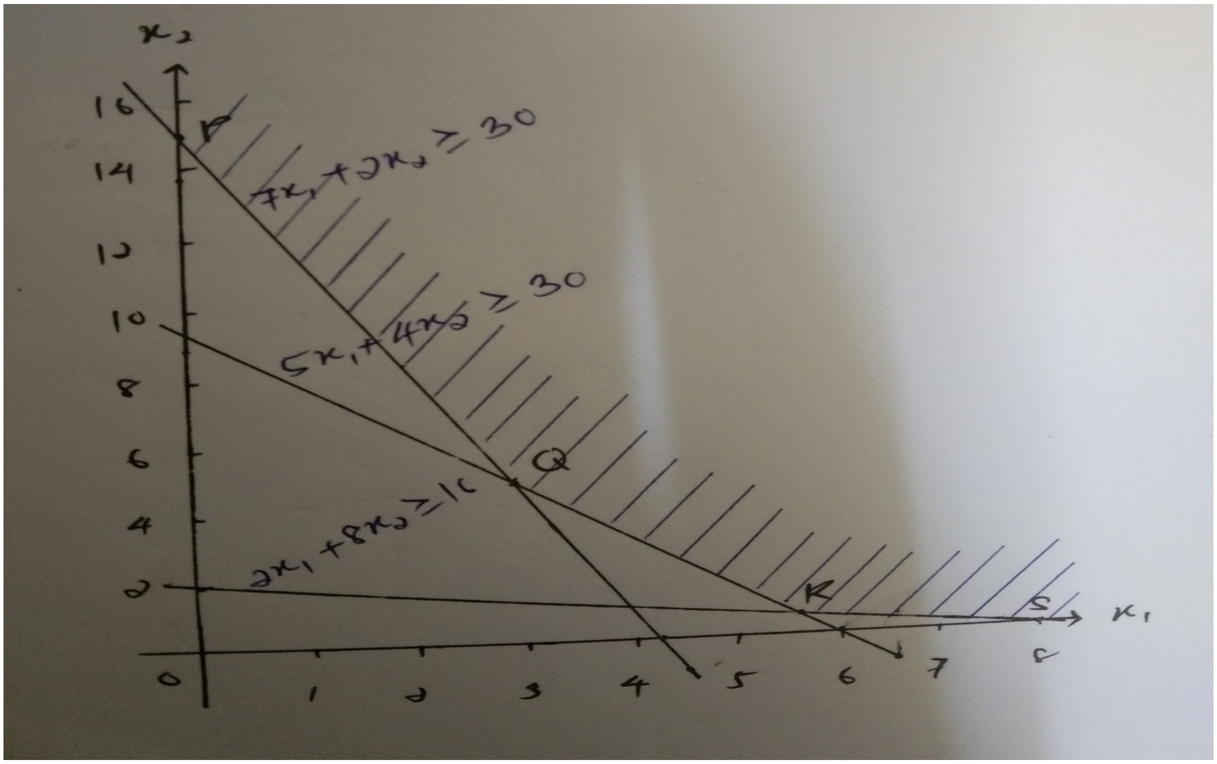
And for Second constraint $7x_1 + 2x_2 \geq 30$ can be written as $7x_1 + 2x_2 = 30$.
So when $x_1 = 0$, then $x_2 = 15$ and
when $x_2 = 0$, then $x_1 = 30/7$. Therefore the coordinates are (0, 15) and (30/7, 0).

For Third constraint $2x_1 + 8x_2 \geq 16$ can be written as $2x_1 + 8x_2 = 16$.
So when $x_1 = 0$, then $x_2 = 2$ and
when $x_2 = 0$, then $x_1 = 8$. Therefore the coordinates are (0, 2) and (8, 0).
Note:

- Putting a zero to every variable in the constraint.

- If the condition becomes **TRUE**, then shade towards origin.
- And if the condition becomes **FALSE**, shade the graph away from origin.

So putting a zero to x_1 and x_2 in above three, the constraint becomes all FALSE. Therefore shade the graph away from origin.



The corner points for the above graph is P,Q,R,S.
Coordinate of P(0, 15)
Coordinate of S(8, 0)
To find the coordinate of Q, solve **First** and **Second** constraint simultaneously, i.e,
$5x_1 + 4x_2 = 30$
$7x_1 + 2x_2 = 30$... multiply by 2

$5x_1 + 4x_2 = 30$....(1)
$14x_1 + 4x_2 = 60$....(2)
Now by subtracting equation (2) by equation (1), I get;
$9x_1 = 30$
Therefore; $x_1 = 3.33$
Now substituting value of x_1,
$x_2 = 3.33$
Therefore the coordinate is (3.33, 3.33)

To find the coordinate of R, solve **First** and **Third** constraint simultaneously, i.e,
$5x_1 + 4x_2 = 30$... multiply by 2
$2x_1 + 8x_2 = 16$

$5x_1 + 4x_2 = 30....(1)$
$14x_1 + 4x_2 = 60....(2)$
Now by subtracting equation (1) by (2), I get;
$8x_1 = 44$
Therefore; $x_1 = 5.5$
Now substituting value of x_1 in equation we get,
$x_2 = 0.625$
Therefore the coordinate is (5.5, 0.625)

Now the Objective Function at point P,Q,R,s is;
At Point P
Coordinates is (0, 15)
$Z = 3x_1 + 2x_2$
$Z = 3(0) + 2(15)$
$Z = 30$

At Point Q
Coordinates is (3.33, 3.33)
$Z = 3x_1 + 2x_2$
$Z = 3(3.33) + 2(3.33)$
$Z = 16.67$

At Point R
Coordinates is (5.5, 0.625)
$Z = 3x_1 + 2x_2$
$Z = 3(5.5) + 2(0.625)$
$Z = 17.75$

And at Point S
Coordinates is (8, 0)
$Z = 3x_1 + 2x_2$
$Z = 3(8) + 2(0)$
Therefore $Z = 24$
Form above cost at different point, we could see the minimum cost is 16.67 when $x_1 = 3.33$ and $x_2 = 3.33$. Hence we conclude that the Mother need to spent minimum cost if she can buy 3.33 grams of Fresh food A and 3.33 grams of Fresh food B.

Question 5

Consider the following problem.
Maximize $Z = 4x_1 + 5x_2 + 3x_3$
Subject to;
$x_1 + x_2 + 2x_3 \geq 20$
$15x_1 + 6x_2 - 5x_3 \leq 50$
$x_1 + 3x_2 + 5x_3 \leq 30$
$x_1, x_2, x_3 \geq 0$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint-1 is of type '$\geq$' we should subtract surplus variable S1 and add artificial variable A1
2. As the constraint-2 is of type '$\leq$' we should add slack variable S2
3. As the constraint-3 is of type '$\leq$' we should add slack variable S3

After introducing slack,surplus,artificial variables

Max Z = 4x1 + 5x2 + 3x3 - MA1

subject to

x1 + x2 + 2x3 - S1 + A1 = 20
6x1 - 5x3 + S2 = 50
x1 + 3x2 + 5x3 + S3 = 30
x1, x2, x3, S1, S2, S3, A1 $\geq$ 0

Basic	x1	x2	x3	s1	s2	s3	A1	
A1	1	1	2	-1	0	0	1	20
S2	6	0	-5	0	1	0	0	50
S3	1	3	5	0	0	1	0	30
Z	-4	-5	-3	0	0	0	M	0

After elimination of the M in the S2 column, we have the initial tableau:

Basic	x1	x2	x3	s1	s2	s3	A1		Min Ratio
A1	1	1	2	-1	0	0	1	20	10
S2	6	0	-5	0	1	0	0	50	-10
S3	1	3	5	0	0	1	0	30	6
Z	-4 - M	-5 - M	-3 -2M	M	0	0	0	-20M	

Negative minimum is -2M-3. So, the entering variable is x3.
Minimum ratio is 6. So, the leaving basis variable is S3.

$R_3 \longrightarrow R_3/5$

Basic	x1	x2	x3	s1	s2	s3	A1	
A1	1	1	2	-1	0	0	1	20
S2	6	0	-5	0	1	0	0	50
X3	1/5	3/5	1	0	0	1/5	0	6
Z	-4 - M	-5 - M	-3 -2M	M	0	0	0	-20M

$R_1 \longrightarrow R_1 - R_3$

$R_2 \longrightarrow R_2 + 5R_3$

$R_4 \longrightarrow R_4 - (-3 - 2M)R_3$

Basic	x1	x2	x3	s1	s2	s3	A1	
A1	3/5	-1/5	0	-1	0	-2/5	1	8
S2	7	3	0	0	1	1	0	80
X3	1/5	3/5	1	0	0	1/5	0	6
Z	-17/5 - 3/5M	-16/5 + 1/5M	0	M	0	3/5 + 2/5M	0	-8M + 18

Basic	x1	x2	x3	s1	s2	s3	A1		Min Ratio
A1	3/5	-1/5	0	-1	0	-2/5	1	8	13.3333
S2	7	3	0	0	1	1	0	80	11.4286
X3	1/5	3/5	1	0	0	1/5	0	6	30
Z	-17/5 - 3/5M	-16/5 + 1/5M	0	M	0	3/5 + 2/5M	0	-8M + 18	

Negative minimum is -17/5 - 3/5M. So, the entering variable is x3.
Minimum ratio is 11.4286. So, the leaving basis variable is S2.

$R_2 \longrightarrow R_2/7$

Basic	x1	x2	x3	s1	s2	s3	A1	
A1	3/5	-1/5	0	-1	0	-2/5	1	8
X1	1	3/7	0	0	1/7	1/7	0	80/7
X3	1/5	3/5	1	0	0	1/5	0	6
Z	-17/5 - 3/5M	-16/5 + 1/5M	0	M	0	3/5 + 2/5M	0	-8M + 18

$R_1 \longrightarrow R_1 - 3/5R_2$

$R_3 \longrightarrow R_3 - 1/5R_2$

$R_4 \longrightarrow R_4 - (-17/5 - 3/5M)R_2$

Basic	x1	x2	x3	s1	s2	s3	A1	
A1	0	-0.4571	0	-1	-0.0857	-0.4857	1	1.1429
X1	1	3/7	0	0	1/7	1/7	0	80/7
X3	0	0.5143	1	0	-0.0286	0.1714	0	3.7148
Z	0	0.4571M - 1.7429	0	M	0.0857M + 0.4857	0.4857M + 1.0857	0	-8M + 18

Hence, optimal solution is arrived with value of variables as :
x1 = 11.4286, x2 = 0, x3 = 3.7143, s1 = 0, s2 = 0, s3 = 0
Max Z=56.8571
But this solution is not feasible because the final solution violates the 1st constraint x1 + x2 + 2 x3 $\geq$ 20. and the artificial variable A1 appears in the basis with positive value 1.1429

Question 6

Find the Dual of the LP:
Minimize $Z = 2x_1 + 3x_2$
Subject to,
$3x_1 + 2x_2 = 14$
$2x_1 - 4x_2 \geq 2$
$4x_1 + 3x_2 \leq 19$ and
$x_1, x_2 \geq 0$
Solution:
Minimize $Z = 2x_1 + 3x_2$
Subject to,
$3x_1 + 2x_2 = 14$.....Break into two equation
$2x_1 - 4x_2 \geq 2$
$4x_1 + 3x_2 \leq 19$.....multiply by -1 Now

Minimize $Z = 2x_1 + 3x_2$
Subject to,
$3x_1 + 2x_2 \geq 14$
$3x_1 + 2x_2 \leq 14$.....multiply by -1
$2x_1 - 4x_2 \geq 2$
$-4x_1 - 3x_2 \geq -19$

<u>Primal</u>
Minimize $Z = 2x_1 + 3x_2$
Subject to,
$3x_1 + 2x_2 \geq 14$
$-3x_1 - 2x_2 \geq -14$
$2x_1 - 4x_2 \geq 2$
$-4x_1 - 3x_2 \geq -19$

<u>Dual</u>
Let y_1, y_2 and y_3 are the Dual variables, therefore
Maximize $W = 14y_1 - 14y_2 + 2y_3 - 19y_4$

Subject to,
$3y_1 - 3y_2 + 2y_3 - 4y_4 \leq 2$
$2y_1 - 2y_2 - 4y_3 - 3y_4 \leq 3$
Let $y_1 - y_2 = y_5$
Now,
$3y_5 + 2y_3 - 4y_4 \leq 2$
$2y_5 - 4y_3 - 3y_4 \leq 3$
$y_3, y_4 \geq 0$, y_5 is unrestricted variable
Therefore the **Dual** can be
Maximize $W = 14y_1 - 14y_2 + 2y_3 - 19y_4$
Subject to,
$3y_5 + 2y_3 - 4y_4 \leq 2$
$2y_5 - 4y_3 - 3y_4 \leq 3$
$y_3, y_4 \geq 0$, y_5 unrestricted variable.

Question 7

Demonstrating the degeneracy problem in the following LPP using simplex method.
Maximize $Z = x_1 + 2x_2 + x_3$
Subject to,
$2x_1 + x_2 - x_3 \leq 2$
$-2x_1 + x_2 - 5x_3 \geq -6$
$4x_1 + x_2 + x_3 \leq 6$
$x_1, x_2, x_3 \geq 0$
Solution:
Maximize $Z = x_1 + 2x_2 + x_3$
Subject to,
$2x_1 + x_2 - x_3 \leq 2$
$-2x_1 + x_2 - 5x_3 \geq -6$...... multiply by -1
$4x_1 + x_2 + x_3 \leq 6$
$x_1, x_2, x_3 \geq 0$
Now,
Maximize $Z = x_1 + 2x_2 + x_3$
Subject to,
$2x_1 + x_2 - x_3 \leq 2$
$2x_1 - x_2 + 5x_3 \leq 6$
$4x_1 + x_2 + x_3 \leq 6$
$x_1, x_2, x_3 \geq 0$
The standardize form of the above problem is expressed as follows;
Maximize $Z - x_1 - 2x_2 - x_3 = 0$
Subject to,
$2x_1 + x_2 - x_3 + s_1 = 2$
$2x_1 - x_2 + 5x_3 + s_2 = 6$
$4x_1 + x_2 + x_3 + s_3 = 6$
$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$
Therefore the simplex tableau form of of above is,

Basis	x_1	x_2	x_3	s_1	s_2	s_3	Solution
s_1	2	1	-1	1	0	0	2
s_2	2	-1	5	0	1	0	6
s_3	4	1	1	0	0	1	6
Z	-1	-2	-1	0	0	0	0

For the entering variable, we have to see for the most negative value of Z that is x_2 in above table and least ratio between leaving and solution ignoring negative and zero. For above table the leaving variable is s_1. So by doing the **Row Reduction Form** for above table, I get,
$R_2 \to R_2 + R_1$
$R_3 \to R_3 - R_1$
$R_4 \to R_4 + 2R_1$

Basis	x_1	x_2	x_3	s_1	s_2	s_3	Solution
x_2	2	1	-1	1	0	0	2
s_2	4	0	4	1	1	0	8
s_3	2	0	2	-1	0	1	4
Z	3	0	-3	2	0	0	4

In above table the entering variable is x_3 and by finding ratio I find there is tie ratio. Further dividing that tie ratio with identity matrix, I find s_3 is leaving
$R_3 \to R_3/2$
$R_4 \to R_4 + 3R_3$
$R_2 \to R_2 - 4R_3$
$R_1 \to R_1 + R_3$

Basis	x_1	x_2	x_3	s_1	s_2	s_3	Solution
x_2	3	1	0	1/2	0	1/2	4
s_2	0	0	0	-1	1	-2	0
x_3	1	0	1	-1/2	0	1/2	2
Z	6	0	0	1/2	0	3/2	10

Therefore since there is no more negative value for Z, we don't have to move further and hence the value for
$x_1 = 0$, $x_2 = 4$, $x_3 = 2$ and $Z = 10$
Substituting the value in equation, I get,
The optimal solution; $Z = x_1 + 2x_2 + x_3$
$Z = 0 + 2.4 + 2$
$Z = 10$
Therefore since there is a value zero for the basic variable then the given problem is degeneracy.