

Tree for exercise 6.4.i

$$(p8) \frac{(p1) \frac{}{\rho[x \mapsto \alpha, f \mapsto \alpha \rightarrow int] \vdash 1 : int} \quad (p9) \frac{(p3) \frac{\rho[f \mapsto \forall \alpha. \alpha \rightarrow int] f = \forall \alpha. \alpha \rightarrow int}{\rho[f \mapsto \forall \alpha. \alpha \rightarrow int] \vdash f f : int} \quad (p3) \frac{\rho[f \mapsto \forall \alpha. \alpha \rightarrow int] f = \forall \alpha. \alpha \rightarrow int}{\rho[f \mapsto \forall \alpha. \alpha \rightarrow int] \vdash f f : int}}{\rho[f \mapsto \forall \alpha. \alpha \rightarrow int] \vdash f f : int} \\ \rho \vdash \text{let } fx = 1 \text{ in } f f \text{ end} : int$$

Tree for exercise 6.4.ii

$$\text{condition:} \quad (p5) \frac{(p3) \frac{\rho[x \mapsto int, f \mapsto int \rightarrow int] x = int}{\rho[x \mapsto int, f \mapsto int \rightarrow int] \vdash n : int} \quad (p1) \frac{}{\rho[... ] \vdash 10 : int}}{\rho[x \mapsto int, f \mapsto int \rightarrow int] \vdash x < 10 : bool}$$

$$\text{recursive:} \quad (p9) \frac{(p3) \frac{\rho[x \mapsto int, f \mapsto int \rightarrow int] f = int \rightarrow int}{\rho[x \mapsto int, f \mapsto int \rightarrow int] \vdash f : int \rightarrow int} \quad (p4) \frac{(p3) \frac{\rho[x \mapsto int, f \mapsto int \rightarrow int] x = int}{\rho[x \mapsto int, f \mapsto int \rightarrow int] \vdash x : int} \quad (p1) \frac{}{\rho[... ] \vdash 1 : int}}{\rho[x \mapsto int, f \mapsto int \rightarrow int] \vdash x + 1 : int} \\ \rho[x \mapsto int, f \mapsto int \rightarrow int] \vdash f(x + 1) : int$$

$$(p8) \frac{(p7) \frac{(condition) \quad (p1) \frac{}{\rho[... ] \vdash 42 : int} \quad (recursive)}{\rho[x \mapsto int, f \mapsto int \rightarrow int] \vdash \text{if } x < 10 \text{ then } 42 \text{ else } f(x+1) : int} \quad (p9) \frac{(p3) \frac{\rho[f \mapsto int \rightarrow int] f = int \rightarrow int}{\rho[f \mapsto int \rightarrow int] \vdash f : int \rightarrow int} \quad (p1) \frac{}{\rho[... ] \vdash 20 : int}}{\rho[f \mapsto int \rightarrow int] \vdash f 20 : int} \\ \rho \vdash \text{let } fx = \text{if } x < 10 \text{ then } 42 \text{ else } f(x + 1) \text{ in } f 20 \text{ end} : int$$