Tree for exercise 6.4.i

$$\frac{\dfrac{\overline{\rho\vdash 1:int}}{\rho[x\mapsto\alpha, f\mapsto\alpha\mapsto int]\vdash 1:int} \quad \dfrac{\rho[f\mapsto\forall\alpha.\alpha\mapsto int]f=\forall\alpha.\alpha\mapsto int}{\rho[f\mapsto\forall\alpha.\alpha\mapsto int]\vdash ff:int}}{\rho\vdash \text{let } fx=1 \text{ in } ff \text{ end} : int} \tag{1}$$

Tree for exercise 6.4.ii

$$\frac{\dfrac{\rho[x\mapsto int, f\mapsto int\mapsto int]x=int}{\rho[x\mapsto int, f\mapsto int\mapsto int]\vdash n:int} \quad \overline{\rho[\ldots]\vdash 10:int}}{\rho[x\mapsto int, f\mapsto int\mapsto int]\vdash x<10 : bool} \tag{2}$$

$$\frac{\dfrac{\rho[x\mapsto int, f\mapsto int\mapsto int]f=int\mapsto int}{\rho[x\mapsto int, f\mapsto int\mapsto int]\vdash f:int\mapsto int} \quad \dfrac{\dfrac{\rho[x\mapsto int, f\mapsto int\mapsto int]x=int}{\rho[x\mapsto int, f\mapsto int\mapsto int]\vdash x:int} \quad \overline{\rho[\ldots]\vdash 1:int}}{\rho[x\mapsto int, f\mapsto int\mapsto int]\vdash x+1:int}}{\rho[x\mapsto int, f\mapsto int\mapsto int]\vdash f(x+1) : int} \tag{3}$$

$$\frac{\text{(equation 2)} \quad \overline{\rho[\ldots]\vdash 42:int} \quad \overset{\text{(equation 3)}}{\overline{\rho[x\mapsto int, f\mapsto int\mapsto int]\vdash f(x+1):int}} \quad \dfrac{\dfrac{\rho[f\mapsto int\mapsto int]f=int\mapsto int}{\rho[f\mapsto int\mapsto int]\vdash f:int\mapsto int} \quad \overline{\rho[\ldots]\vdash 20:int}}{\rho[f\mapsto int\mapsto int]f20:int}}{\rho\vdash \text{let } fx= \text{ if } x<10 \text{ then } 42 \text{ else } f(x+1) \text{ in } f20 \text{ end} : int} \tag{4}$$