Tree for exercise 6.4.i

$$(p8)\dfrac{(p1)\dfrac{}{\rho[x\mapsto\alpha, f\mapsto\alpha\to int]\vdash 1{:}int} \quad (p9)\dfrac{(p3)\dfrac{\rho[f\mapsto\forall\alpha.\alpha\to int]f=\forall\alpha.\alpha\to int}{\rho[f\mapsto\forall\alpha.\alpha\to int]\vdash ff{:}int} \quad (p3)\dfrac{\rho[f\mapsto\forall\alpha.\alpha\to int]f=\forall\alpha.\alpha\to int}{\rho[f\mapsto\forall\alpha.\alpha\to int]\vdash ff{:}int}}{\rho[f\mapsto\forall\alpha.\alpha\to int]\vdash ff{:}int}}{\rho\vdash \text{let } fx=1 \text{ in } ff \text{ end} : int}$$

Tree for exercise 6.4.ii

$$\text{condition:} \quad (p5)\dfrac{(p3)\dfrac{\rho[x\mapsto int, f\mapsto int\to int]x=int}{\rho[x\mapsto int, f\mapsto int\to int]\vdash n{:}int} \quad (p1)\dfrac{}{\rho[\ldots]\vdash 10{:}int}}{\rho[x\mapsto int, f\mapsto int\to int]\vdash x<10 : bool}$$

$$\text{recursive:} \quad (p9)\dfrac{(p3)\dfrac{\rho[x\mapsto int, f\mapsto int\to int]f=int\to int}{\rho[x\mapsto int, f\mapsto int\to int]\vdash f{:}int\to int} \quad (p4)\dfrac{(p3)\dfrac{\rho[x\mapsto int, f\mapsto int\to int]x=int}{\rho[x\mapsto int, f\mapsto int\to int]\vdash x{:}int} \quad (p1)\dfrac{}{\rho[\ldots]\vdash 1{:}int}}{\rho[x\mapsto int, f\mapsto int\to int]\vdash x+1{:}int}}{\rho[x\mapsto int, f\mapsto int\to int]\vdash f(x+1) : int}$$

$$(p8)\dfrac{(p7)\dfrac{\text{(condition)} \quad (p1)\dfrac{}{\rho[\ldots]\vdash 42{:}int} \quad \text{(recursive)}}{\rho[x\mapsto int, f\mapsto int\to int]\vdash \text{if } x<10 \text{ then } 42 \text{ else } f(x+1){:}int} \quad (p9)\dfrac{(p3)\dfrac{\rho[f\mapsto int\to int]f=int\to int}{\rho[f\mapsto int\to int]\vdash f{:}int\to int} \quad (p1)\dfrac{}{\rho[\ldots]\vdash 20{:}int}}{\rho[f\mapsto int\to int]f20{:}int}}{\rho\vdash \text{let } fx= \text{ if } x<10 \text{ then } 42 \text{ else } f(x+1) \text{ in } f20 \text{ end} : int}$$