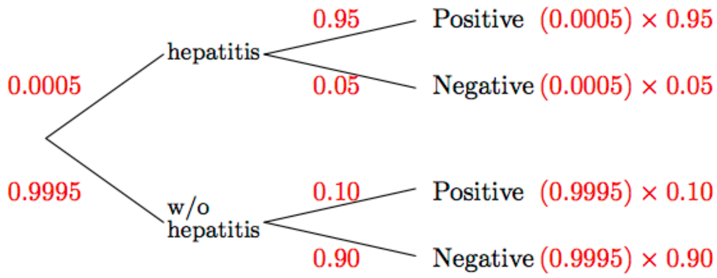
ICT600 Computational Mathematics

Homework 3

Disclaimer This is my attempt to help you learn the content of this course. There might be mistakes or typos. Do not use this material as your main study for the course.

Homework Assignment

For question (1) - (3): Dalton High Schools German club has 20 members.
(1) How many ways are there to form a committee of 3 members to represent the school at the Language Council?
Solution: $\binom{20}{3}$
(2) How many ways are there to select a president and treasurer for the club?
Solution: $\binom{20}{1}\binom{19}{1}$
(3) The German club from Dalton has 4 seniors. How many ways are there to select the three-people committee to include at least one senior?
Solution: $\binom{4}{1}\binom{16}{2}$
For problem (4) - (6): Suppose that the reliability of a test for hepatitis is specified as follows: Of people with hepatitis, 95% have a positive reaction and 5% have a negative reaction; of people free of hepatitis, 90% have a negative reaction and 10% have a positive reaction. From a large population of which 0.05% of the people have hepatitis, a person is selected at random and giving a test.
(4) Draw a tree diagram representing the reliability of this test for hepatitis.
Solution



(5) What is the probability that a person chosen at random will get the negative result from this test?
Solution

$$p(\text{negative}) = 0.0005 \times 0.05 + (0.9995) \times 0.90$$

(6) If the test is positive, what is the probability that the person actually has hepatitis?
Solution

$$\begin{aligned} p(\text{hepatitis}|\text{positive}) &= \frac{p(\text{hepatitis} \wedge \text{positive})}{p(\text{positive})} \\ &= \frac{(0.0005) \times 0.95}{(0.0005) \times 0.95 + (0.9995) \times 0.10} \end{aligned}$$

(7) The New York Times of January 24, 1997, discusses the recommendation of a special panel concerning mammogram for women in their 40s. About 2% of women aged 40 to 49 years old develop breast cancer in their 40s. But the mammogram used for the woman in that age group has a high rate of false

positive and false negatives; the false positive rate is 0.3 and the false negative rate is 0.25. If a woman in her 40s has a positive mammogram, what is the probability that she actually has breast cancer?
Solution This problem could be solved either by Bayes' Theorem or Tree Diagram.

Bayes'Theorem We want to find $p(\text{have breast cancer } | \text{tested positive})$. So we can let

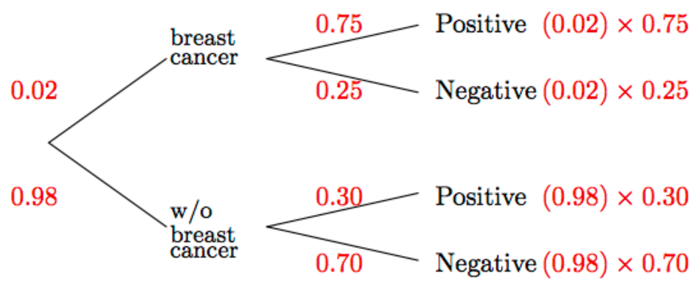
$A =$ tested positive
$B_1 =$ have breast cancer
$B_2 =$ do not have breast cancer

$p(B_1) = 0.02, p(B_2) = 0.98,$
$p(A|B_1) = 1 - (\text{false negative}) = 0.75,$
$p(A|B_2) = (\text{false positive}) = 0.3$

Thus

$$p(B_1|A) = \frac{(0.02)(0.75)}{(0.02)(0.75) + (0.98)(0.3)}$$

Tree Diagram The tree diagram of the mammograms is



Thus,

$$\begin{aligned} p(\text{have breast cancer}|\text{ positive}) &= \frac{p(\text{have breast cancer } \wedge \text{ positive})}{p(\text{positive})} \\ &= \frac{(0.02) \times 0.75}{(0.02) \times 0.75 + (0.98) \times 0.30} \end{aligned}$$

For problem (8) - (10): Metastatic cancer is a possible cause of brain tumors and is also an explanation for increased total serum calcium. In turn, either of these could explain a patient falling into a coma. Severe headache is also associated with brain tumors. For simplicity, let

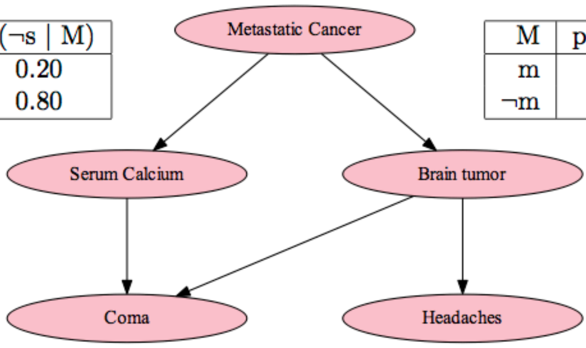
$$\begin{aligned} M &= \text{Metastatic cancer} \\ B &= \text{Brain tumors} \\ S &= \text{Increased total serum calcium} \\ C &= \text{Comas} \\ H &= \text{Severe Headaches} \end{aligned}$$

Suppose that:
$p(m) = 0.2$,
$p(s|m) = 0.8$, $p(s|\neg m) = 0.2$, $p(b|m) = 0.2$ $p(b|\neg m) = 0.05$
$p(c|s,b) = 0.8$ $p(c|s,\neg b) = 0.8$ $p(c|\neg s,b) = 0.8$ $p(c|\neg s,\neg b) = 0.05$
$p(h|b) = 0.8$ $p(h|\neg b) = 0.6$

(8) Construct a Bayesian network to represent this scenario.
Solution

p(m)	p(¬m)
0.20	0.80

M	p(s | M)	p(¬s | M)
m	0.80	0.20
¬m	0.20	0.80



M	p(b | M)	p(¬b | M)
m	0.20	0.80
¬m	0.05	0.95

S	B	p(c | S, B)	p(¬c | S, B)
s	b	0.80	0.20
s	¬b	0.80	0.20
¬s	b	0.80	0.20
¬s	¬b	0.05	0.95

B	p(h | B)	p(¬h | B)
b	0.80	0.20
¬b	0.60	0.40

(9) What is a probability that a patient is diagnosed with Metastatic cancer, has brain tumors and severe headaches but has not fallen into a coma?
Solution:

$$\begin{aligned}
p(m, b, h, \neg c) &= \sum_S p(m, b, S, h, \neg c) \\
&= \sum_S p(m)\ p(b|m)\ p(S|m)\ p(\neg c|S, b)\ p(h|b) \\
&= p(m)\ p(b|m)\ p(s|m)\ p(\neg c|s, b)\ p(h|b) + p(m)\ p(b|m)\ p(\neg s|m)\ p(\neg c|\neg s, b)\ p(h|b) \\
&= 0.00512 + 0.00128 \\
&= 0.0064
\end{aligned}$$

(10) Given that the patient is suffering from severe headaches but has not fallen into a coma, what is the probability that the patient has Metastatic cancer?
Solution:

$$\begin{aligned}
p(m|h, \neg c) &= \frac{p(m, \neg c, h)}{p(h, \neg c)} \\
&= \frac{\sum_{S,B} p(m, B, S, \neg c, h)}{\sum_{S,B,M} p(M, B, S, \neg c, h)} \\
&= \frac{\sum_{S,B} p(m)\ p(B|m)\ p(S|m)\ p(\neg c|S, B)\ p(h|B)}{\sum_{S,B,M} p(M)\ p(B|M)\ p(S|M)\ p(\neg c|S, B)\ p(h|B)} \\
&= \frac{0.04}{0.4112} \\
&= 0.0973
\end{aligned}$$