ICT600 Exercise:
Basic Mathematics

Instruction: Read each question carefully. Write **all** your works in the space provided. You won't get full credits even when your answer is right without your works all written down. You may use the back of the paper to continue your work. If you do so, write "continued on next page" or so to indicate that that is not the end of your solution. **This is NOT a group work.** You have to do it yourself.

Name:.. **ID:**.........................

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1. Let $A = \{1, 2, 3, 4\}$, and $B = \{1, 3\}$. Evaluate the followings;

   (a) $|2^A| + |2^B|$

   16+4 = 20

   (b) $2^{A \cup B}$

   { {}, {1}, {2}, {3}, {4}, {1,2}, {1,3}, {1,4}, {2,3},
   {2,4}, {3,4}, {1,2,3}, {1,2,4}, {2,3,4}, {1,2,3,4} }

   (c) B^3

   { (1,1,1), (1,1,3), (1,3,1), (1,3,3),
   (3,1,1), (3,1,3), (3,3,1), (3,3,3) }

   (d) $(A - B)^2$

   { (2,2), (2,4), (4,2), (4,4) }

   (e) $(\mathbb{Z}_3)^3$

   { (0,0,0), (0,0,1), (0,0,2), (0,1,0), (0,1,1), (0,1,2),
   (0,2,0), (0,2,1), (0,2,2), (1,0,0), (1,0,1), (1,0,2),
   (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2),
   (2,0,0), (2,0,1), (2,0,2), (2,1,0), (2,1,1), (2,1,2),
   (2,2,0), (2,2,1), (2,2,2) }

2. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{0, 1, 2\}$, and $f : A \to B$ be a function defined by $\{(1, 2), (2, 2), (3, 1), (4, 2), (5, 0)\}$.

(a) What are the domain and range of f?

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Domain = {1,2,3,4,5}
Range = {0,1,2}
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(b) Is f injective, surjective, bijective?

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f is not injective.
f is surjective.
f is not bijective.
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3. For $a, b \in \mathbb{R}$, define $a \sim b$ to mean that $|a - b| < 5$. Prove or disprove each of the followings;

(a) The relation $\sim$ is reflexive.

$\forall a \in \mathbb{R} \quad |a-a| = 0 < 5$

(b) The relation $\sim$ is symmetric.

Assume that $a \sim b \Rightarrow |a-b| < 5 \Rightarrow -5 < a-b < 5$

$\Rightarrow 5 > b-a > -5 \Rightarrow |b-a| < 5 \Rightarrow b \sim a$

(c) The relation $\sim$ is transitive.

Assume that $a \sim b$ and $b \sim c$

$a \sim b \Rightarrow |a-b| < 5 \qquad b \sim c \Rightarrow |b-c| < 5$

$-5 < a-b < 5$ —— 1 $\qquad -5 < b-c < 5$ —— 2

1+2 $\qquad -10 < a-c < 10 \Rightarrow |a-c| < 10$

This disproves the transitive condition.

4. Examine the following formal descriptions of sets so that you can understand which members they contain. Write a short informal English description of each set.

   (a) $\{1, 3, 5, 7, \dots\}$

   Set of odd positive integers

   (b) $\{\dots, -4, -2, 0, 2, 4, \dots\}$

   set of even integers

   (c) $\{n \mid n = 2m \text{ for some } m \text{ in } \mathbb{N}\}$

   set of even positive integers

   (d) $\{n \mid n = 2m \text{ for some } m \text{ in } \mathbb{N}, \text{ and } n = 3k \text{ for some } k \text{ in } \mathbb{N}\}$

   set of positive integers that are divisible by 2 and 3

   (e) $\{w \mid w \text{ is a string of } \texttt{0}\text{s and } \texttt{1}\text{s and } w \text{ equals the reverse of } w\}$

   Set of palindrome

   (f) $\{n \mid n \text{ is an integer and } n = n + 1\}$

   Empty set

5. Let $S = \{xx, xxx\}$, how many ways can x^{19} be written as the product of words in S?

x^19 can be written as
  (1) 8 xx's and 1 xxx
  (2) 5 xx's and 3 xxx's
  (3) 2 xx's and 5 xxx's

From (1), there are $\frac{9!}{8!1!}$ ways to permute them.

From (2), there are $\frac{8!}{5!3!}$ ways to permute them.

From (3), there are $\frac{7!}{2!5!}$ ways to permute them.

Totally, there are 86 ways to write x^19.