ICT600 Quiz:
Conditional Probability and Bayesian Networks

Instruction: Read each question carefully. Write **all** your works in the space provided. You won't get full credits even when your answer is right without your works all written down. You may use the back of the paper to continue your work. If you do so, write "continued on next page" or so to indicate that that is not the end of your solution. **This is NOT a group work.** You have to do it yourself.

Name:..... ID:.....

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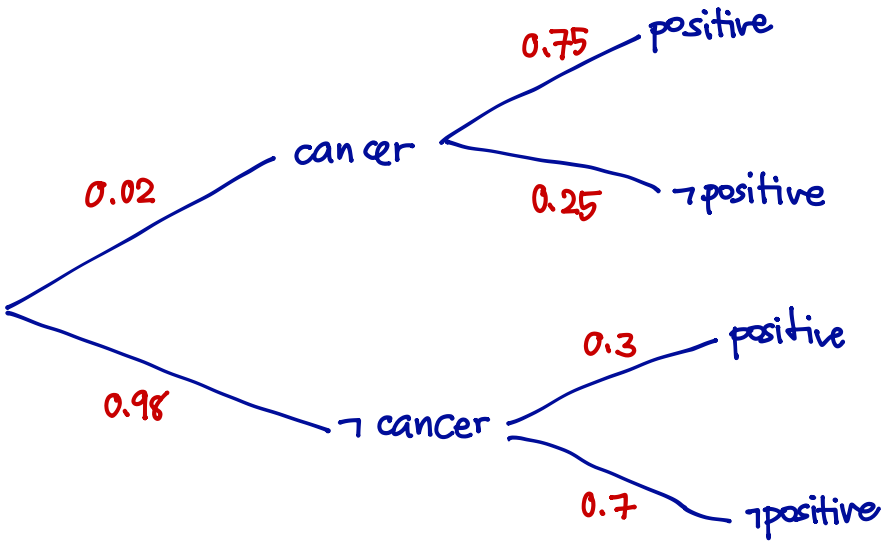
1. The New York Times of January 24, 1997, discusses the recommendation of a special panel concerning mammogram for women in their 40s. About 2% of women aged 40 to 49 years old develop breast cancer in their 40s. But the mammogram used for the woman in that age group has a high rate of false positive and false negatives; the false positive rate is 0.3 and the false negative rate is 0.25.

   (a) Draw a tree diagram of this scenario.



   (b) If a woman in her 40s has a positive mammogram, what is the probability that she actually has breast cancer?

$$P(\text{cancer} \mid \text{positive}) = \frac{P(\text{cancer}, \text{positive})}{P(\text{positive})}$$

$$= \frac{(0.02)(0.75)}{(0.02)(0.75) + (0.98)(0.3)}$$

$$= 0.049$$

2. Metastatic cancer is a possible cause of brain tumors and is also an explanation for increased total serum calcium. In turn, either of these could explain a patient falling into a coma. Severe headache is also associated with brain tumors. For simplicity, let

$$\begin{aligned} M &= \text{Metastatic cancer} \\ B &= \text{Brain tumors} \\ S &= \text{Increased total serum calcium} \\ C &= \text{Comas} \\ H &= \text{Severe Headaches} \end{aligned}$$

Suppose that:

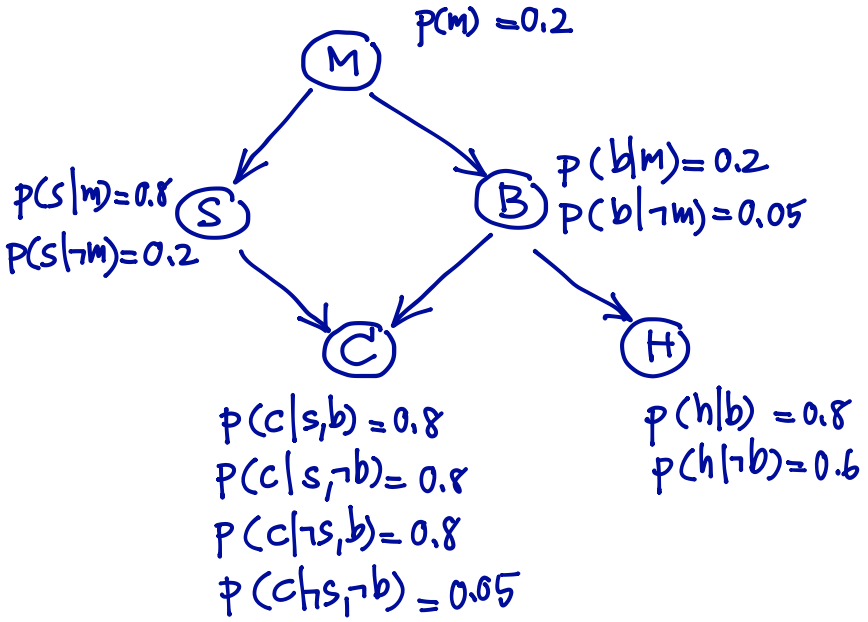
$p(m) = 0.2,$

$p(s|m) = 0.8,$ $p(s|\neg m) = 0.2,$ $p(b|m) = 0.2$ $p(b|\neg m) = 0.05$

$p(c|s,b) = 0.8$ $p(c|s,\neg b) = 0.8$ $p(c|\neg s,b) = 0.8$ $p(c|\neg s,\neg b) = 0.05$

$p(h|b) = 0.8$ $p(h|\neg b) = 0.6$

(a) Construct a Bayesian network to represent this scenario.



(b) What is a probability that a patient is diagnosed with Metastatic cancer, has brain tumors and severe headaches but has not fallen into a coma?

$$\begin{aligned} P(m, b, h, \neg c) &= \sum_S P(m, b, S, h, \neg c) \\ &= \sum_S P(m)P(S|m)P(b|m)P(h|b)P(\neg c|S, b) \\ &= P(m)P(b|m)P(h|b)\sum_S P(S|m)P(\neg c|S, b) \\ &= (0.2)(0.2)(0.8)\left[(0.8)(0.2) + (0.2)(0.2)\right] \\ &= 0.0064 \end{aligned}$$