ICT600 Quiz:
CFG and Pushdown Automata

Instruction: Read each question carefully. Write **all** your works in the space provided. You won't get full credits even when your answer is right without your works all written down. You may use the back of the paper to continue your work. If you do so, write "continued on next page" or so to indicate that that is not the end of your solution. **This is NOT a group work.** You have to do it yourself.

Name:.. **ID:**....................................

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1. Let $L_1 = \{wtw | w, t \in \{0,1\}^+\}$. Prove by the pumping lemma that L_1 is not regular.

   Let p be a pumping length of L_1.

   First Trial:

   Let $s = 0^p10^p = xyz$ where $|xy| \leq p$ and $|y| > 0$, we have

   $$\begin{aligned} x &= 0^l & 0 &\leq l \leq (p-1) \\ y &= 0^k & 1 &\leq k \leq p,\ l+k \leq p \\ z &= 0^{p-l-k}10^p \end{aligned}$$

   Check if $xy^iz \in L_1, \forall i \geq 0$

   $i = 0;\qquad xy^0z = 0^l0^{p-l-k}10^p = 0^{p-k}10^p = \underbrace{0^{p-k}}_{w}\underbrace{10^k}_{t}\underbrace{0^{p-k}}_{w} \in L_1$

   $i = 2;\qquad xy^2z = 0^l0^k0^k0^{p-l-k}10^p = 0^{p+k}10^p = \underbrace{0^p}_{w}\underbrace{0^k1}_{t}\underbrace{0^p}_{w} \in L_1$

   ...

   Similarly, $xy^iz \in L_1, \forall i \geq 0$. Therefore, $s = 0^p10^p$ can be pumped. We cannot show that L_1 is not regular.

   Second Trial:

   Let $s = 0^p110^p1 = xyz$ where $|xy| \leq p$ and $|y| > 0$, we have

   $$\begin{aligned} x &= 0^l & 0 &\leq l \leq (p-1) \\ y &= 0^k & 1 &\leq k \leq p,\ l+k \leq p \\ z &= 0^{p-l-k}110^p1 \end{aligned}$$

   Check if $xy^iz \in L_1, \forall i \geq 0$

   $i = 0;\qquad xy^0z = 0^l0^{p-l-k}110^p1 = 0^{p-k}110^p1 = \underbrace{0^{p-k}1}_{w}\underbrace{10^k}_{t}\underbrace{0^{p-k}1}_{w} \in L_1$

   $i = 2;\qquad xy^2z = 0^l0^k0^k0^{p-l-k}110^p1 = 0^{p+k}110^p1 \notin L_1$

   Since we cannot split $0^{p+k}110^p1$ into wtw, $xy^2z \notin L_1$. s cannot be pumped. Thus, L_1 is not regular.

2. Convert the following CFG into the Chomsky normal form,

$$\begin{aligned} A &\to BAB \mid B \mid \varepsilon \\ B &\to 00 \mid \varepsilon \end{aligned}$$

Add a new start variable,

$$\begin{aligned} S_0 &\to A \\ A &\to BAB \mid B \mid \varepsilon \\ B &\to 00 \mid \varepsilon \end{aligned}$$

Remove $B \to \varepsilon$,

$$\begin{aligned} S_0 &\to A \\ A &\to BAB \mid B \mid BA \mid AB \mid \varepsilon \\ B &\to 00 \end{aligned}$$

Remove $A \to \varepsilon$,

$$\begin{aligned} S_0 &\to A \mid \varepsilon \\ A &\to BAB \mid B \mid BA \mid AB \mid BB \\ B &\to 00 \end{aligned}$$

Remove $A \to B$,

$$\begin{aligned} S_0 &\to A \mid \varepsilon \\ A &\to BAB \mid 00 \mid BA \mid AB \mid BB \\ B &\to 00 \end{aligned}$$

Remove $S_0 \to A$,

$$\begin{aligned} S_0 &\to BAB \mid 00 \mid BA \mid AB \mid BB \mid \varepsilon \\ A &\to BAB \mid 00 \mid BA \mid AB \mid BB \\ B &\to 00 \end{aligned}$$

Clean up

$$\begin{aligned} S_0 &\to CB \mid 00 \mid BA \mid AB \mid BB \mid \varepsilon \\ A &\to CB \mid DD \mid BA \mid AB \mid BB \\ B &\to DD \\ C &\to BA \\ D &\to 0 \end{aligned}$$

3. Let $L_2 = \{w|w$ starts and ends with the same symbol$\}$ with $\Sigma = \{0, 1\}$. Give a CFG generating L_2 and a PDA recognizing L_2.

Here are the CFG rules for L_2:

$$S \to 1A1|0A0$$
$$A \to 0A|1A|\varepsilon$$

PDA recognizing L_2 can be drawn as