

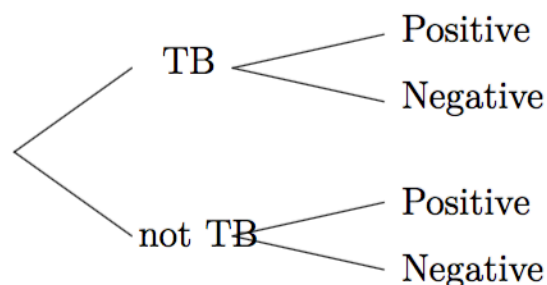
# Lecture 10

## Tree Diagrams and Bayes' Theorem

### 10.1 Tree Diagrams

In solving many probability problems, it is helpful to represent the various events and their associated probabilities by a tree diagram. To explain this useful notion, suppose that we wish to compute the probability of an event that results from performing a sequence of experiments. The various outcomes of each experiment are represented as branches emanating from a point.

**Example 10.1** Suppose that the reliability of a skin test for active pulmonary tuberculosis (TB) is specified as follows: Of people with TB, 98% have a positive reaction and 2% have a negative reaction; of people free of TB, 99% have a negative reaction and 1% have a positive reaction. From a large population of which 2 per 10,000 persons have TB, a person is selected at random and given a skin test, which turns out to be positive. What is the probability that the person has active pulmonary tuberculosis?



**Exercise 10.1** Fifty percent of the students enrolled in a business statistics course had previously taken a finite mathematics course. Thirty percent of these students received an A for the statistics course, where as 20% of the other students received an A for the statistics course.

1. Draw a tree diagram and label it with the appropriate probabilities.
2. What is the probability that a student selected at random previously took a finite mathematics course and did not received an A in the statistics course?

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**Exercise 10.2** Solve example 10.1 using Bayes' Theorem.

**Exercise 10.3** A population in which 3% are users of a drug is screened by a test that is 80% accurate in the sense that the probability of a false-positive (tested positive, given non-drug user) is 0.20 and the probability of a false-negative (tested negative, given drug user) is also 0.20. If a person test positive, calculate the probability that he or she is, in fact, not a user of the drug?

**Exercise 10.4** It is estimated that 10% of Olympic athletes use steroids. The test currently being used to detect steroids is said to be 93% effective in correctly detecting steroids in users. It yields false positives in only 2% of the tests. A country's best weightlifter tests positive. What is the probability that he actually takes steroids?

**Exercise 10.5** There are two cookie jars on the shelf in the kitchen. The red one has 10 chocolate-chip cookies and 15 gingersnaps. The blue jar has 20 chocolate-chip cookies and 10 gingersnaps. James goes down in the middle of the night and without turning on the light chooses a jar at random and then chooses a cookie at random. If the cookie is chocolate-chip, what is the probability that he got the cookie from the blue jar?



**Exercise 10.6** Ten percent of the pens made by Apex are defective. Only 5% of the pens made by its competitor, B-ink, are defective. Since Apex pens are cheaper than B-ink pens, an office order 70% of its stock from Apex and 30% from B-ink. A pen is chosen at random and found to be defective. What is the probability that it was produced by Apex?

## Credit

This lecture note is from Assoc. Prof. Dr. Nirattaya Khamsemanan's GTS111 partial note.