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Q3

Textbook page 159 Exercise 5.31

The population of the practicing anesthesiology was stratified into three groups:

- anesthesiologists (composing approximately 50% of the population),
- anesthesiology residents (composing approximately 10% of the population), and
- nurse anesthetists (composing approximately 40% of the population).

a) Estimate the population proportion of those who think they have worked beyond a safe limit.

- N_i : strata size for $i=1,2,3$
- n_i : sample size for $i=1,2,3$
- p_i : strata probability for $i=1,2,3$

To calculate the population proportion of anesthesiologists,

$$\hat{p} = \frac{\sum N_i p_i}{N} = \frac{(1330 \cdot 0.687) + (165 \cdot 0.824) + (1100 \cdot 0.782)}{2595} = \frac{1909.87}{2595} \approx 0.7360.$$

Thus the population proportion of anesthesiologists is 0.7360.

Calculate a bound on the error of estimation.

- $N = N_1 + N_2 + N_3 = 2595$

The variance of the estimated proportion is

$$\begin{aligned} V(\hat{p}) &= \frac{1}{N^2} \sum N_i^2 \left(1 - \frac{n_i}{N_i}\right) \left(\frac{p_i(1-p_i)}{n_i-1}\right) \\ &= \frac{1}{2595^2} \left(1330^2 \left(1 - \frac{913}{1330}\right) \left(\frac{0.687 \cdot (1-0.687)}{913-1}\right) + 165^2 \left(1 - \frac{136}{165}\right) \left(\frac{0.824 \cdot (1-0.824)}{136-1}\right) + 1100^2 \left(1 - \frac{860}{1100}\right) \left(\frac{0.782 \cdot (1-0.782)}{860-1}\right)\right) \\ &\approx 0.000028. \end{aligned}$$

The bound on the estimation is

$$B = 2\sqrt{V(\hat{p})} \approx 0.0106.$$

Hence, the bound on the error of estimation is 0.0106.

Then the estimate of proportion with a bound on the error of estimation, is given by 0.7360 ± 0.0106 .

b) Do anesthesiologists differ significantly from residents in this matter?

The difference between the percentage of anesthesiologists and residents who think they have worked beyond a safe limit is $0.824 - 0.687 = 0.137$.

The variance of the estimated proportion is

$$V(\hat{p}) = \frac{1}{1495^2} \left(1330^2 \left(1 - \frac{913}{1330}\right) \left(\frac{0.687 \cdot (1-0.687)}{913-1}\right) + 165^2 \left(1 - \frac{136}{165}\right) \left(\frac{0.824 \cdot (1-0.824)}{136-1}\right)\right) \approx 0.000061.$$

The bound on the estimation is

$$B = 2\sqrt{V(\hat{p})} \approx 0.0156.$$

Hence, the bound on the error of estimation is 0.0156.

Then the estimate of proportion with a bound on the error of estimation, is given by 0.137 ± 0.0156 .

Thus anesthesiologists differ from residents in this matter.

c) Do anesthesiologists differ significantly from nurse anesthetists in this matter?

The difference between the percentage of anesthesiologists and nurse anesthetists who think they have worked beyond a safe limit is $0.782 - 0.687 = 0.095$.

The variance of the estimated proportion is

$$V(\hat{p}) = \frac{1}{2430^2} \left(1330^2 \left(1 - \frac{913}{1330} \right) \left(\frac{0.687 \cdot (1 - 0.687)}{913 - 1} \right) + 1100^2 \left(1 - \frac{860}{1100} \right) \left(\frac{0.782 \cdot (1 - 0.782)}{860 - 1} \right) \right) \approx 0.000031.$$

The bound on the estimation is

$$B = 2\sqrt{V(\hat{p})} \approx 0.01114.$$

Hence, the bound on the error of estimation is 0.01114.

Then the estimate of proportion with a bound on the error of estimation, is given by 0.095 ± 0.01114 .

Thus anesthesiologists differ but not that much from nurse anesthetists in this matter.