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Q4

Exercise 8.2 of the textbook (page 279)

1) Estimate the average repair cost per saw for the past month.

Let the industry i denotes the ith cluster, the cluster sizes $m_i, i = 1, \ldots, 20$ are the number of saws, and y_i are the total repair cost.

Also $N = 96$ and $n = 20$, but M is unknown.

Then the average is

$\bar{y} = \frac{\sum yi}{\sum m_i} = \frac{2565}{130} \approx 19.73.$

Thus, the estimation of the average repair cost per saw for the past month is 19.73.

2) Place a bound on the error of estimation.

The estimated variance of $\hat{y}$ is $\hat{Var}(\bar{y}) = (\frac{N-n}{NnM^2})s_r^2$.

M is unknown but approximated by $\bar{m} = \frac{\sum m_i}{n} = \frac{130}{20} = 6.5.$

And s_r^2 is
$s_r^2 = \frac{1}{n-1}\sum(y_i - \bar{y}m_i)^2 = \frac{1}{n-1}(\sum y_i^2 - 2\bar{y}\sum m_i y_i + \bar{y}^2 \sum m_i^2) = \frac{1}{20-1}(460225 - 2 \cdot (\frac{2565}{130}) \cdot 22285 + (\frac{2565}{130})^2 \cdot 1118) \approx 845.56.$

Therefore, $\hat{Var}(\bar{y}) = 0.7922$ and the bound on the error of estimation is

$B = 2\sqrt{\hat{Var}(\bar{y})} = 1.78.$

Hence, the average repair cost per saw for the past month lies within 19.73 ± 1.78.