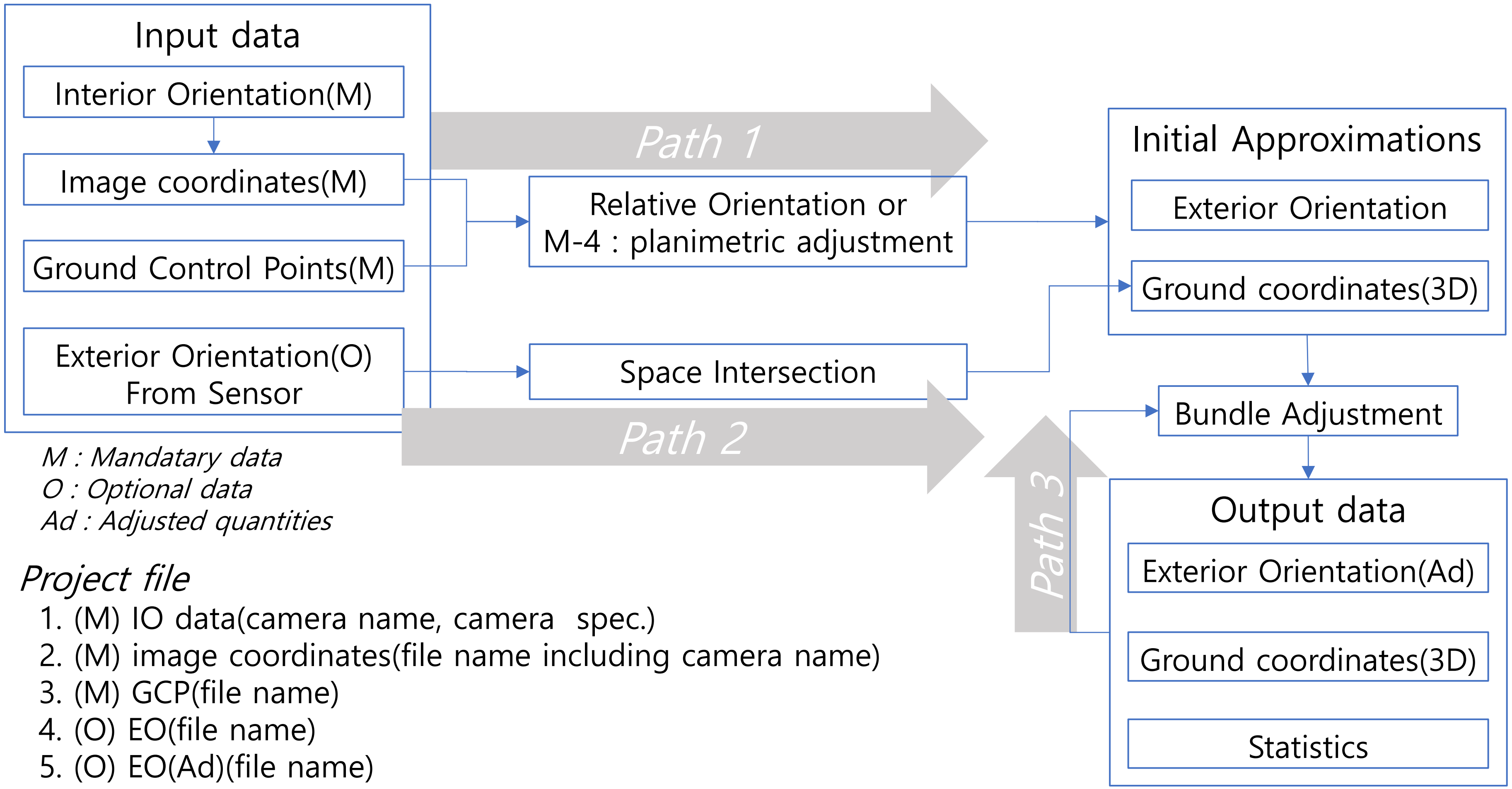
AT flow



M : Mandatary data
O : Optional data
Ad : Adjusted quantities

Project file

1. (M) IO data(camera name, camera spec.)
2. (M) image coordinates(file name including camera name)
3. (M) GCP(file name)
4. (O) EO(file name)
5. (O) EO(Ad)(file name)
6. (O) Ground coordinates(file name)

if 4. exists, go path 2.
if 5. and 6 exists, go path 3.

AT 1 : observation Equation

- Image coordinates of j point in i photographs

$$x_{ij} - x_0 = -f\,\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)} = -f\,\frac{r}{q} \quad \Rightarrow \quad J_{ij} = x_{ij} - x_0 + f\,\frac{r}{q}$$

$$y_{ij} - y_0 = -f\,\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)} = -f\,\frac{s}{q} \quad \Rightarrow \quad K_{ij} = y_{ij} - y_0 + f\,\frac{s}{q}$$

$$V_{ij} + \dot{B}_{ij}\dot{\Delta}_i + \ddot{B}_{ij}\ddot{\Delta}_j = \varepsilon_{ij}$$

$$\dot{B}_{ij} = \begin{bmatrix} b_{11_{ij}} & b_{12_{ij}} & b_{13_{ij}} & -b_{14_{ij}} & -b_{15_{ij}} & -b_{16_{ij}} \\ b_{21_{ij}} & b_{22_{ij}} & b_{23_{ij}} & -b_{24_{ij}} & -b_{25_{ij}} & -b_{26_{ij}} \end{bmatrix} \quad \ddot{B}_{ij} = \begin{bmatrix} b_{14_{ij}} & b_{15_{ij}} & b_{16_{ij}} \\ b_{24_{ij}} & b_{25_{ij}} & b_{25_{ij}} \end{bmatrix}$$

$$\dot{\Delta}_i = \begin{bmatrix} d\omega_i \\ d\varphi_i \\ d\kappa_i \\ dX_{L_i} \\ dY_{L_i} \\ dZ_{L_i} \end{bmatrix} \quad \ddot{\Delta}_j = \begin{bmatrix} dX_j \\ dY_j \\ dZ_j \end{bmatrix} \quad \varepsilon_{ij} = \begin{bmatrix} J_{ij} \\ K_{ij} \end{bmatrix} \quad V_{ij} = \begin{bmatrix} v_{x_{ij}} \\ v_{y_{ij}} \end{bmatrix}$$

$\dot{\Delta}_i$: orientation parameters of photo I , $\ddot{\Delta}_j$: ground coordinate of point j, V_{ij}: residual

AT 2 : observation Equation

- Image coordinates of j point in m photographs

$V_{ij} + \dot{B}_{ij}\dot{\Delta}_i + \ddot{B}_{ij}\Delta\ddot{\Delta}_j = \varepsilon_{ij}$

$$\begin{bmatrix} V_{1j} \\ V_{2j} \\ V_{3j} \\ \\ V_{mj} \end{bmatrix} + \begin{bmatrix} \dot{B}_{1j} & & & \\ & \dot{B}_{2j} & & \\ & & \dot{B}_{3j} & \\ & & & \dot{B}_{mj} \end{bmatrix} \begin{bmatrix} \dot{\Delta}_1 \\ \dot{\Delta}_2 \\ \dot{\Delta}_3 \\ \\ \dot{\Delta}_m \end{bmatrix} + \begin{bmatrix} \ddot{B}_{1j} \\ \ddot{B}_{2j} \\ \ddot{B}_{3j} \\ \\ \ddot{B}_{mj} \end{bmatrix} \ddot{\Delta}_j = \begin{bmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \varepsilon_{3j} \\ \\ \varepsilon_{mj} \end{bmatrix}$$

(2m, 1) (2m, 6m) (6m,1) (2m,3) (3,1) (2m,1)

- Image coordinates of n points m photographs

$V + \dot{B}\,\dot{\Delta} + \ddot{B}\ddot{\Delta} = \varepsilon$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \\ V_m \end{bmatrix} + \begin{bmatrix} \dot{B}_1 \\ \dot{B}_2 \\ \dot{B}_3 \\ \\ \dot{B}_m \end{bmatrix} \begin{bmatrix} \dot{\Delta}_1 \\ \dot{\Delta}_2 \\ \dot{\Delta}_3 \\ \\ \dot{\Delta}_m \end{bmatrix} + \begin{bmatrix} \ddot{B}_1 & & & \\ & \ddot{B}_1 & & \\ & & \ddot{B}_3 & \\ & & & \ddot{B}_n \end{bmatrix} \begin{bmatrix} \dot{\Delta}_1 \\ \ddot{\Delta}_j \\ \dot{\Delta}_3 \\ \\ \dot{\Delta}_m \end{bmatrix} \ddot{\Delta}_j = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \\ \varepsilon_m \end{bmatrix}$$

(2mn, 1) (2mn, 6m)(6m,1) (2mn,3) (3n,1) (2mn,1)

- Ground control

$X_j = X_j^{00} + v_{X_j}$

$Y_j = Y_j^{00} + v_{Y_j}$

$Z_j = Z_j^{00} + v_{Z_j}$

$\Longrightarrow$

$X_j^0 + \Delta X_j = X_j^{00} + v_{X_j}$

$Y_j^0 + \Delta Y_j = Y_j^{00} + v_{Y_j}$

$Z_j^0 + \Delta Z_j = Z_j^{00} + v_{Z_j}$

$$\begin{bmatrix} v_{x_j} \\ v_{y_j} \\ v_{z_j} \end{bmatrix} - \begin{bmatrix} \Delta X_j \\ \Delta Y_j \\ \Delta Z_j \end{bmatrix} = \begin{bmatrix} X_j^0 - X_j^{00} \\ Y_j^0 - Y_j^{00} \\ Z_j^0 - Z_j^{00} \end{bmatrix}$$

$\underline{\ddot{V}_j - \ddot{\Delta}_j = \ddot{C}_j}$

$X_j^{00},\ Y_j^{00},\ Z_j^{00}$: 측정값(control)

X_j^0, Y_j^0, Z_j^0 : 초기값

- Exterior Orientation parameter

$\omega_i = \omega_i^{00} + v_{\omega_i}$
$\varphi_i = \varphi_i^{00} + v_{\varphi_i}$
$\kappa_i = \kappa_i^{00} + v_{\kappa_i}$

$X_{L_i} = X_{L_i}^{00} + v_{X_{L_i}}$
$Y_{L_i} = Y_{L_i}^{00} + v_{Y_{L_i}}$
$Z_{L_i} = Z_{L_i}^{00} + v_{Z_{L_i}}$

$\omega_i^0 + \Delta\omega_i = \omega_i^{00} + v_{\omega_i}$
$\phi_i^0 + \Delta\phi_i = \phi_i^{00} + v_{\phi_i}$
$\kappa_i^0 + \Delta\kappa_i = \kappa_i^{00} + v_{\kappa_i}$
$X_{L_i}^0 + \Delta X_{L_i} = X_{L_i}^{00} + v_{X_{L_i}}$
$Y_{L_i}^0 + \Delta Y_{L_i} = Y_{L_i}^{00} + v_{Y_{L_i}}$
$Z_{L_i}^0 + \Delta Z_{L_i} = Z_{L_i}^{00} + Z_{X_{L_i}}$

$$\begin{bmatrix} v_{\omega_i} \\ v_{\phi_i} \\ v_{\kappa_i} \\ v_{X_{L_i}} \\ v_{Y_{L_i}} \\ Z_{X_{L_i}} \end{bmatrix} - \begin{bmatrix} \Delta\omega_i \\ \Delta\phi_i \\ \Delta\kappa_i \\ \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \end{bmatrix} = \begin{bmatrix} \omega_i^0 - \omega_i^{00} \\ \phi_i^0 - \phi_i^{00} \\ \kappa_i^0 - \kappa_i^{00} \\ X_{L_i}^0 - X_{L_i}^{00} \\ Y_{L_i}^0 - Y_{L_i}^{00} \\ Z_{L_i}^0 - Z_{L_i}^{00} \end{bmatrix}$$

$\underline{\dot{V}_i - \dot{\Delta}_i = \dot{C}_i}$

AT 2 : weight matrix

- The weight matrix of Image coordinates
  - $W_{ij} = \sigma_0 \begin{bmatrix} 1/\sigma_{x_{ij}} & \\ & 1/\sigma_{y_{ij}} \end{bmatrix}$ (2 X 2 matrix)
- for all the m image coordinates on point j

$$W_j = \begin{bmatrix} W_{1j} & & & & & \\ & W_{2j} & & & & \\ & & W_{3j} & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & W_{mj} \end{bmatrix}$$ (2m, 2m)

- for all n points on m images

$$W = \begin{bmatrix} W_1 & & & & & \\ & W_2 & & & & \\ & & W_3 & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & W_n \end{bmatrix}$$ (2mn, 2mn)

- The weight matrix of Exterior Orientation parameter

weight matrix for photograph i $W_i = \sigma_0 \dot{\sigma}_i^{-1}$ (6 X 6 matrix)

weight matrix for all m images

$$\dot{W} = \begin{bmatrix} \dot{W}_1 & & & & \\ & \dot{W}_2 & & & \\ & & \dot{W}_3 & & \\ & & & \ddots & \\ & & & & \dot{W}_m \end{bmatrix}$$ (6m X 6m)

- The weight matrix of ground coordinates

weight matrix for point j $W_j = \sigma_0 \dot{\sigma}_j^{-1}$ (3 X 3 matrix)
weight matrix for all n points

$$\ddot{W} = \begin{bmatrix} \ddot{W}_1 & & & & \\ & \ddot{W}_2 & & & \\ & & \ddot{W}_3 & & \\ & & & \ddots & \\ & & & & \ddot{W}_n \end{bmatrix}$$ (2n X 2n)

- **The weight matrix of bundle adjustment**

$$\overline{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{W} & & \\ & \dot{\boldsymbol{W}} & \\ & & \ddot{\boldsymbol{W}} \end{bmatrix}$$

AT 3 : normal equation

- Observation equation for image coordinates of n points m photographs

$$V + \dot{B}\,\dot{\Delta} + \ddot{B}\,\ddot{\Delta} = \varepsilon$$
$$\dot{V} - \dot{\Delta} = \dot{C}$$
$$\ddot{V} - \ddot{\Delta} = \ddot{C}$$

$$\begin{bmatrix} V \\ \dot{V} \\ \ddot{V} \end{bmatrix} + \begin{bmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \varepsilon \\ \dot{C} \\ \ddot{C} \end{bmatrix}$$

$$\underline{\boldsymbol{\bar{V}} + \boldsymbol{\bar{B}}\Delta = \boldsymbol{\bar{C}} \;\Rightarrow\; (\boldsymbol{\bar{B}^T\bar{W}\bar{B}})\Delta \;=\; \boldsymbol{\bar{B}^T\bar{W}\bar{C}}}$$

$$\underline{\;\Rightarrow \Delta = \left(\boldsymbol{\bar{B}^T\bar{W}\bar{B}}\right)^{-1}\boldsymbol{\bar{B}^T\bar{W}\bar{C}}}$$

$$\underline{\Rightarrow \quad \boldsymbol{N} \quad \Delta \quad = \quad \boldsymbol{K}}$$

(6m+3n, 6m+3n)(6m+3n,1))(6m+3n,1)

$$\dot{V} - \dot{\Delta} = \dot{C}$$

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \\ \vdots \\ \vdots \\ \dot{V}_m \end{bmatrix} - \begin{bmatrix} \dot{\Delta}_1 \\ \dot{\Delta}_2 \\ \dot{\Delta}_3 \\ \vdots \\ \vdots \\ \dot{\Delta}_m \end{bmatrix} = \begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \\ \dot{C}_3 \\ \vdots \\ \vdots \\ \dot{C}_m \end{bmatrix}$$

$$(A^TWA)\Delta = N\Delta = K$$

$$\begin{bmatrix}
\dot{N}_1 + \dot{W}_1 & {}_6 0^6 & \cdots & {}_6 0^6 & \bar{N}_{11} & \bar{N}_{12} & \cdots & \bar{N}_{1n} \\
{}_6 0^6 & \dot{N}_2 + \dot{W}_2 & \cdots & {}_6 0^6 & \bar{N}_{21} & \bar{N}_{22} & \cdots & \bar{N}_{2n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
{}_6 0^6 & {}_6 0^6 & {}_6 0^6 & \dot{N}_m + \dot{W}_m & \bar{N}_{m1} & \bar{N}_{m2} & \cdots & \bar{N}_{mn} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\bar{N}_{11}{}^T & \bar{N}_{21}{}^T & \cdots & \bar{N}_{m1}{}^T & \ddot{N}_1 + \ddot{W}_1 & {}_3 0^3 & \cdots & {}_3 0^3 \\
\bar{N}_{12}{}^T & \bar{N}_{22}{}^T & \cdots & \bar{N}_{m2}{}^T & {}_3 0^3 & \ddot{N}_2 + \ddot{W}_2 & \cdots & {}_3 0^3 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\bar{N}_{1n}{}^T & \bar{N}_{2n}{}^T & \cdots & \bar{N}_{mn}{}^T & {}_3 0^3 & {}_3 0^3 & \cdots & \ddot{N}_n + \ddot{W}_n
\end{bmatrix}
\begin{bmatrix} \dot{\Delta}_1 \\ \dot{\Delta}_2 \\ \vdots \\ \dot{\Delta}_m \\ \ddot{\Delta}_1 \\ \ddot{\Delta}_2 \\ \vdots \\ \ddot{\Delta}_n \end{bmatrix}
=
\begin{bmatrix} \dot{K}_1 - \dot{W}_1\dot{C}_1 \\ \dot{K}_2 - \dot{W}_2\dot{C}_2 \\ \vdots \\ \dot{K}_m - \dot{W}_m\dot{C}_m \\ \ddot{K}_1 - \ddot{W}_1\ddot{C}_1 \\ \ddot{K}_2 - \ddot{W}_2\ddot{C}_2 \\ \vdots \\ \ddot{K}_n - \ddot{W}_n\ddot{C}_n \end{bmatrix}$$

$$\dot{N}_i = \sum_{j=1}^{n}\left(\dot{B}_{ij}^T W_{ij} \dot{B}_{ij}\right) \qquad \bar{N}_{ij} = \dot{B}_{ij}^T W_{ij} \ddot{B}_{ij}$$

$$\ddot{N}_j = \sum_{i=1}^{m}\left(\ddot{B}_{ij}^T W_{ij} \ddot{B}_{ij}\right)$$

$$\dot{K}_i = \sum_{j=1}^{n}\left(\dot{B}_{ij}^T W_{ij} \epsilon_{ij}\right) \qquad \ddot{K}_j = \sum_{i=1}^{m} \ddot{B}_{ij}^T W_{ij} \varepsilon_{ij}$$

$$\Rightarrow \begin{bmatrix} \dot{N} & \bar{N} \\ \bar{N}^T & \ddot{N} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{K} \\ \ddot{K} \end{bmatrix} \Rightarrow \quad \begin{aligned} &\dot{N}\dot{\Delta} + \bar{N}\ddot{\Delta} = \dot{K} \\ &\bar{N}^T\dot{\Delta} + \ddot{N}\ddot{\Delta} = \ddot{K} \end{aligned}$$

$$\Rightarrow \ddot{\Delta} = \ddot{N}^{-1}(\ddot{K} - \bar{N}^T\dot{\Delta})$$

$$\Rightarrow \dot{N}\dot{\Delta} + \bar{N}\,\ddot{N}^{-1}(\ddot{K} - \bar{N}^T\dot{\Delta}) = \dot{K}$$

$$\Rightarrow (\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T)\dot{\Delta} = (\dot{K} - \bar{N}\ddot{N}^{-1}\ddot{K})$$

$$\Rightarrow \quad S \qquad \dot{\Delta} = \quad E$$

(6m,6m) (6m, 1) (6m,1)

- ✓ **EO parameters and ground points**
  - **관측방정식** $\dot{\Delta}, \dot{C}, \ddot{\Delta}, \ddot{C}$
  - **GPS/INS, GCP와 같이 측정된 값이 있을 때 적용**
  - **정규방정식을 만들 때** $\dot{W}_1\dot{C}_1$, $\ddot{W}_1\ddot{C}_1$**에 적용**

AT 3 : normal equation - 1

- Observation equation for image coordinates of n points m photographs

$$
\begin{aligned}
V + \dot{B}\,\dot{\Delta} + \ddot{B}\,\ddot{\Delta} &= \varepsilon \\
\dot{V} - \dot{\Delta} &= \dot{C} \\
\ddot{V} - \ddot{\Delta} &= \ddot{C}
\end{aligned}
$$

$$
\begin{bmatrix} V \\ \dot{V} \\ \ddot{V} \end{bmatrix} + \begin{bmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \varepsilon \\ \dot{C} \\ \ddot{C} \end{bmatrix}
$$

$\underline{(\bar{\boldsymbol{B}}^{\boldsymbol{T}}\bar{\boldsymbol{W}}\bar{\boldsymbol{B}})\Delta = \bar{\boldsymbol{B}}^{\boldsymbol{T}}\bar{\boldsymbol{W}}\bar{\boldsymbol{C}}}$

$$
\begin{bmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ 0 & -I \end{bmatrix}^{\boldsymbol{T}} \begin{bmatrix} W & 0 & 0 \\ 0 & \dot{W} & 0 \\ 0 & 0 & \ddot{W} \end{bmatrix} \begin{bmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ 0 & -I \end{bmatrix}^{\boldsymbol{T}} \begin{bmatrix} W & 0 & 0 \\ 0 & \dot{W} & 0 \\ 0 & 0 & \ddot{W} \end{bmatrix}^{T} \begin{bmatrix} \varepsilon \\ \dot{C} \\ \ddot{C} \end{bmatrix}
$$

$$
\begin{bmatrix} \dot{B}^{T}W & -\dot{W} & 0 \\ \ddot{B}^{T}W & 0 & -\ddot{W} \end{bmatrix} \begin{bmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{B}^{T}W & -\dot{W} & 0 \\ \ddot{B}^{T}W & 0 & -\ddot{W} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \dot{C} \\ \ddot{C} \end{bmatrix}
$$

$$
\begin{bmatrix} \dot{B}^{T}W\dot{B} + \dot{W} & \dot{B}^{T}W\ddot{B} \\ \ddot{B}^{T}W\dot{B} & \ddot{B}^{T}W\ddot{B} + \ddot{W} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{B}^{T}W\varepsilon - \dot{W}\dot{C} \\ \ddot{B}^{T}W\varepsilon - \ddot{W}\ddot{C} \end{bmatrix}
$$

$\underline{\Rightarrow \Delta = (\bar{\boldsymbol{B}}^{\boldsymbol{T}}\bar{\boldsymbol{W}}\bar{\boldsymbol{B}})^{-1}\bar{\boldsymbol{B}}^{\boldsymbol{T}}\bar{\boldsymbol{W}}\bar{\boldsymbol{C}}}$

$\underline{\Rightarrow \quad \boldsymbol{N} \quad \Delta \quad = \quad \boldsymbol{K}}$

(6m+3n, 6m+3n)(6m+3n,1))(6m+3n,1)

$$
(\bar{B}^{T}\bar{W}\bar{B})\Delta = \bar{B}^{T}\bar{W}\bar{C} \Leftrightarrow N\Delta = K
$$

$$
\begin{bmatrix}
\dot{N}_1 + \dot{W}_1 & {}_6 0^6 & \cdots & {}_6 0^6 & \bar{N}_{11} & \bar{N}_{12} & \cdots & \bar{N}_{1n} \\
{}_6 0^6 & \dot{N}_2 + \dot{W}_2 & \cdots & {}_6 0^6 & \bar{N}_{21} & \bar{N}_{22} & \cdots & \bar{N}_{2n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
{}_6 0^6 & {}_6 0^6 & {}_6 0^6 & \dot{N}_m + \dot{W}_m & \bar{N}_{m1} & \bar{N}_{m2} & \cdots & \bar{N}_{mn} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\bar{N}_{11}{}^{T} & \bar{N}_{21}{}^{T} & \cdots & \bar{N}_{m1}{}^{T} & \ddot{N}_1 + \ddot{W}_1 & {}_3 0^3 & \cdots & {}_3 0^3 \\
\bar{N}_{12}{}^{T} & \bar{N}_{22}{}^{T} & \cdots & \bar{N}_{m2}{}^{T} & {}_3 0^3 & \ddot{N}_2 + \ddot{W}_2 & \cdots & {}_3 0^3 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\bar{N}_{1n}{}^{T} & \bar{N}_{2n}{}^{T} & \cdots & \bar{N}_{mn}{}^{T} & {}_3 0^3 & {}_3 0^3 & \cdots & \ddot{N}_n + \ddot{W}_n
\end{bmatrix}
\begin{bmatrix} \dot{\Delta}_1 \\ \dot{\Delta}_2 \\ \vdots \\ \dot{\Delta}_m \\ \ddot{\Delta}_1 \\ \ddot{\Delta}_2 \\ \vdots \\ \ddot{\Delta}_n \end{bmatrix}
=
\begin{bmatrix} \dot{K}_1 - \dot{W}_1\dot{C}_1 \\ \dot{K}_2 - \dot{W}_2\dot{C}_2 \\ \vdots \\ \dot{K}_m - \dot{W}_m\dot{C}_m \\ \ddot{K}_1 - \ddot{W}_1\ddot{C}_1 \\ \ddot{K}_2 - \ddot{W}_2\ddot{C}_2 \\ \vdots \\ \ddot{K}_n - \ddot{W}_n\ddot{C}_n \end{bmatrix}
$$

$$
\dot{N}_i = \sum_{j=1}^{n}(\dot{B}_{ij}^{T}W_{ij}\dot{B}_{ij}) \quad \bar{N}_{ij} = \dot{B}_{ij}^{T}W_{ij}\ddot{B}_{ij}
$$

$$
\ddot{N}_j = \sum_{i=1}^{m}(\ddot{B}_{ij}^{T}W_{ij}\ddot{B}_{ij})
$$

$$
\dot{K}_i = \sum_{j=1}^{n}(\dot{B}_{ij}^{T}W_{ij}\epsilon_{ij}) \quad \ddot{K}_j = \sum_{i=1}^{m}\ddot{B}_{ij}^{T}W_{ij}\varepsilon_{ij}
$$

- **The weight matrix of bundle adjustment**

$$
\bar{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{W} & & \\ & \dot{\boldsymbol{W}} & \\ & & \ddot{\boldsymbol{W}} \end{bmatrix}
$$

$$
\Rightarrow \begin{bmatrix} \dot{N} & \bar{N} \\ \bar{N}^{T} & \ddot{N} \end{bmatrix} \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix} = \begin{bmatrix} \dot{K} \\ \ddot{K} \end{bmatrix} \Rightarrow
\begin{aligned}
&\dot{N}\dot{\Delta} + \bar{N}\ddot{\Delta} = \dot{K} \\
&\bar{N}^{T}\dot{\Delta} + \ddot{N}\ddot{\Delta} = \ddot{K} \\
&\Rightarrow \ddot{\Delta} = \ddot{N}^{-1}(\ddot{K} - \bar{N}^{T}\dot{\Delta}) \\
&\Rightarrow \dot{N}\dot{\Delta} + \bar{N}\,\ddot{N}^{-1}(\ddot{K} - \bar{N}^{T}\dot{\Delta}) = \dot{K} \\
&\Rightarrow (\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^{T})\dot{\Delta} = (\dot{K} - \bar{N}\ddot{N}^{-1}\ddot{K}) \\
&\Rightarrow \quad S \qquad \dot{\Delta} = \quad E
\end{aligned}
$$

(6m,6m) (6m, 1) (6m,1)

AT 4 : Reduced Normal Equation

$$(A^TWA)\Delta = N\Delta = K$$

$$\begin{bmatrix} \dot N_1+\dot W_1 & {}_6 0^6 & \cdots & {}_6 0^6 & \bar N_{11} & \bar N_{12} & \cdots & \bar N_{1n} \\ {}_6 0^6 & \dot N_2+\dot W_2 & \cdots & {}_6 0^6 & \bar N_{21} & \bar N_{22} & \cdots & \bar N_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ {}_6 0^6 & {}_6 0^6 & {}_6 0^6 & \dot N_m+\dot W_m & \bar N_{m1} & \bar N_{m2} & \cdots & \bar N_{mn} \\ \bar N_{11}{}^T & \bar N_{21}{}^T & \cdots & \bar N_{m1}{}^T & \ddot N_1+\ddot W_1 & {}_3 0^3 & \cdots & {}_3 0^3 \\ \bar N_{12}{}^T & \bar N_{22}{}^T & \cdots & \bar N_{m2}{}^T & {}_3 0^3 & \ddot N_2+\ddot W_2 & \cdots & {}_3 0^3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \bar N_{1n}{}^T & \bar N_{2n}{}^T & \cdots & \bar N_{mn}{}^T & {}_3 0^3 & {}_3 0^3 & \cdots & \ddot N_n+\ddot W_n \end{bmatrix} \begin{bmatrix} \dot\Delta_1 \\ \dot\Delta_2 \\ \\ \dot\Delta_m \\ \ddot\Delta_1 \\ \ddot\Delta_2 \\ \vdots \\ \ddot\Delta_n \end{bmatrix} = \begin{bmatrix} \dot K_1 - \dot W_1 \dot C_1 \\ \dot K_2 - \dot W_2 \dot C_2 \\ \vdots \\ \dot K_m - \dot W_m \dot C_m \\ \ddot K_1 - \ddot W_1 \ddot C_1 \\ \ddot K_2 - \ddot W_2 \ddot C_2 \\ \vdots \\ \ddot K_n - \ddot W_n \ddot C_n \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot N & \bar N \\ \bar N^T & \ddot N \end{bmatrix} \begin{bmatrix} \dot\Delta \\ \ddot\Delta \end{bmatrix} = \begin{bmatrix} \dot K \\ \ddot K \end{bmatrix}$$

$$\dot N\dot\Delta + \bar N\ddot\Delta = \dot K$$

$$\bar N^T\dot\Delta + \ddot N\ddot\Delta = \ddot K$$

$$\Rightarrow \ddot\Delta = \ddot N^{-1}(\ddot K - \bar N^T\dot\Delta)$$

$$\Rightarrow \dot N\dot\Delta + \bar N\ \ddot N^{-1}(\ddot K - \bar N^T\dot\Delta) = \dot K$$

$$\Rightarrow (\dot N - \bar N\ddot N^{-1}\bar N^T)\dot\Delta = (\dot K - \bar N\ddot N^{-1}\ddot K)$$

$$\Rightarrow \underset{(6m,6m)}{S}\ \ \underset{(6m,\ 1)}{\dot\Delta} = \underset{(6m,1)}{E}$$

$$S = (\dot N - \bar N\ddot N^{-1}\bar N^T)$$

$$E = \dot K - \bar N\ddot N^{-1}\ddot K$$

$$S\,\dot\Delta = E$$

$$\begin{bmatrix} \dot N_1+\dot W_1+S_1 & S_{12} & \cdots & \cdots & S_{1m} \\ S_{12}^T & \dot N_2+\dot W_2+S_2 & \cdots & \cdots & S_{2m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ S_{1m}^T & S_{2m}^T & \cdots & \cdots & \dot N_m+\dot W_m+S_m \end{bmatrix} \begin{bmatrix} \dot\Delta_1 \\ \dot\Delta_2 \\ \vdots \\ \vdots \\ \dot\Delta_m \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ \vdots \\ E_m \end{bmatrix}$$

$$S = \dot N_i + \dot W_i - \textstyle\sum_{j=1}^{n}\left[\bar N_j\left(\ddot N_j + \ddot W_j\right)^T\ \bar N_j^T\right] \qquad S_{ik} = -\textstyle\sum_{j=1}^{n}\bar N_j\left(\ddot N_j + \ddot W_j\right)^T\ \bar N_{kj}^T$$

$$E = \dot K_i - \dot W_i\dot C_i - \textstyle\sum_{j=1}^{n}\left[\bar N_j\left(\ddot N_j + \ddot W_j\right)^T\left(\ddot K_j - \ddot W_j\ddot C_j\right)\right] \qquad E_i = \dot K_i - \dot W_i\dot C_i - \textstyle\sum_{j=1}^{n}\left[\bar N_j\left(\ddot N_j + \ddot W_j\right)^T\left(\ddot K_j - \ddot W_j\ddot C_j\right)\right]$$

AT 4 : derive SΔ=E (1)

$$\underline{\boldsymbol{\bar{N}\ddot{N}^{-1}\bar{N}^T}}$$

$$\begin{bmatrix} \bar{N}_{11} & \bar{N}_{12} & \cdots & \bar{N}_{1n} \\ \bar{N}_{21} & \bar{N}_{22} & \cdots & \bar{N}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \bar{N}_{m1} & \bar{N}_{m2} & \cdots & \bar{N}_{mn} \end{bmatrix} \begin{bmatrix} \ddot{N}_1 + \ddot{W}_1 & {}_3 0^3 & \cdots & {}_3 0^3 \\ {}_3 0^3 & \ddot{N}_2 + \ddot{W}_2 & \cdots & {}_3 0^3 \\ \cdots & \cdots & \cdots & \cdots \\ {}_3 0^3 & {}_3 0^3 & \cdots & \ddot{N}_n + \ddot{W}_n \end{bmatrix}^{-1} \begin{bmatrix} \bar{N}_{11}{}^T & \bar{N}_{21}{}^T & \cdots & \bar{N}_{m1}{}^T \\ \bar{N}_{12}{}^T & \bar{N}_{22}{}^T & \cdots & \bar{N}_{m2}{}^T \\ \cdots & \cdots & \cdots & \cdots \\ \bar{N}_{1n}{}^T & \bar{N}_{2n}{}^T & \cdots & \bar{N}_{mn}{}^T \end{bmatrix}$$

$$= \begin{bmatrix} \bar{N}_{11}(\ddot{N}_1 + \ddot{W}_1)^{-1} & \bar{N}_{12}(\ddot{N}_2 + \ddot{W}_2)^{-1} & \cdots & \bar{N}_{1n}(\ddot{N}_n + \ddot{W}_n)^{-1} \\ \bar{N}_{21}(\ddot{N}_1 + \ddot{W}_1)^{-1} & \bar{N}_{22}(\ddot{N}_2 + \ddot{W}_2)^{-1} & \cdots & \bar{N}_{2n}(\ddot{N}_n + \ddot{W}_n)^{-1} \\ \cdots & \cdots & \cdots & \cdots \\ \bar{N}_{m1}(\ddot{N}_1 + \ddot{W}_1)^{-1} & \bar{N}_{m2}(\ddot{N}_2 + \ddot{W}_2)^{-1} & \cdots & \bar{N}_{mn}(\ddot{N}_n + \ddot{W}_n)^{-1} \end{bmatrix} \begin{bmatrix} \bar{N}_{11}{}^T & \bar{N}_{21}{}^T & \cdots & \bar{N}_{m1}{}^T \\ \bar{N}_{12}{}^T & \bar{N}_{22}{}^T & \cdots & \bar{N}_{m2}{}^T \\ \cdots & \cdots & \cdots & \cdots \\ \bar{N}_{1n}{}^T & \bar{N}_{2n}{}^T & \cdots & \bar{N}_{mn}{}^T \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{j=1}^{n} \bar{N}_{1j}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{1j}{}^T & \sum_{j=1}^{n} \bar{N}_{1j}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{2j}{}^T & \cdots & \sum_{j=1}^{n} \bar{N}_{1j}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{mj}{}^T \\ \sum_{j=1}^{n} \bar{N}_{2j}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{1j}{}^T & \sum_{j=1}^{n} \bar{N}_{2j}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{2j}{}^T & \cdots & \sum_{j=1}^{n} \bar{N}_{2j}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{mj}{}^T \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{j=1}^{n} \bar{N}_{mj}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{1j}{}^T & \sum_{j=1}^{n} \bar{N}_{mj}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{2j}{}^T & \cdots & \sum_{j=1}^{n} \bar{N}_{mj}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{mj}{}^T \end{bmatrix}$$

$$S_{ik} = \dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T = \dot{N}_i + \dot{W}_i - \sum_{j=1}^{n} \bar{N}_{ij}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{kj}{}^T$$

** i = 1~m, k = 1~m*

If i =k, $S_{ik} = \dot{N}_i + \dot{W}_i - \sum_{j=1}^{n} \bar{N}_{ij}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{ij}{}^T$

If i ≠k, $S_{ik} = -\sum_{j=1}^{n} \bar{N}_{ij}(\ddot{N}_j + \ddot{W}_j)^{-1}\bar{N}_{kj}{}^T$

$$\dot{N}\dot{\Delta} + \bar{N}\ddot{\Delta} = \dot{K}$$
$$\bar{N}^T\dot{\Delta} + \ddot{N}\ddot{\Delta} = \ddot{K}$$
$$\Rightarrow \ddot{\Delta} = \ddot{N}^{-1}(\ddot{K} - \bar{N}^T\dot{\Delta})$$
$$\Rightarrow \dot{N}\dot{\Delta} + \bar{N}\ddot{N}^{-1}(\ddot{K} - \bar{N}^T\dot{\Delta}) = \dot{K}$$
$$\Rightarrow (\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T)\dot{\Delta} = (\dot{K} - \bar{N}\ddot{N}^{-1}\ddot{K})$$
$$\Rightarrow \underset{(6m,6m)}{S} \quad \underset{(6m,\ 1)}{\dot{\Delta}} = \underset{(6m,1)}{E}$$

$$S = (\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T)$$
$$E = \dot{K} - \bar{N}\ddot{N}^{-1}\ddot{K}$$

$$\dot{\Delta} = \dot{N}^{-1}(\dot{K} - \bar{N}\ddot{\Delta})$$
$$\bar{N}^T\dot{N}^{-1}(\dot{K} - \bar{N}\ddot{\Delta}) + \ddot{N}\ddot{\Delta} = \ddot{K}$$
$$(\ddot{N} - \bar{N}^T\dot{N}^{-1}\bar{N})\ddot{\Delta} = \ddot{K} - \bar{N}^T\dot{N}^{-1}\dot{K}$$

AT 4 : derive SΔ=E (2)

$$(A^TWA)\Delta = N\Delta = K$$

$$\begin{bmatrix} \dot N_1+\dot W_1 & {}_6 0^6 & \cdots & {}_6 0^6 & \bar N_{11} & \bar N_{12} & \cdots & \bar N_{1n} \\ {}_6 0^6 & \dot N_2+\dot W_2 & \cdots & {}_6 0^6 & \bar N_{21} & \bar N_{22} & \cdots & \bar N_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ {}_6 0^6 & {}_6 0^6 & {}_6 0^6 & \dot N_m+\dot W_m & \bar N_{m1} & \bar N_{m2} & \cdots & \bar N_{mn} \\ \bar N_{11}^T & \bar N_{21}^T & \cdots & \bar N_{m1}^T & \ddot N_1+\ddot W_1 & {}_3 0^3 & \cdots & {}_3 0^3 \\ \bar N_{12}^T & \bar N_{22}^T & \cdots & \bar N_{m2}^T & {}_3 0^3 & \ddot N_2+\ddot W_2 & \cdots & {}_3 0^3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \bar N_{1n}^T & \bar N_{2n}^T & \cdots & \bar N_{mn}^T & {}_3 0^3 & {}_3 0^3 & \cdots & \ddot N_n+\ddot W_n \end{bmatrix} \begin{bmatrix} \dot\Delta_1 \\ \dot\Delta_2 \\ \\ \dot\Delta_m \\ \ddot\Delta_1 \\ \ddot\Delta_2 \\ \vdots \\ \ddot\Delta_n \end{bmatrix} = \begin{bmatrix} \dot K_1-\dot W_1\dot C_1 \\ \dot K_2-\dot W_2\dot C_2 \\ \vdots \\ \dot K_m-\dot W_m\dot C_m \\ \ddot K_1-\ddot W_1\ddot C_1 \\ \ddot K_2-\ddot W_2\ddot C_2 \\ \vdots \\ \ddot K_n-\ddot W_n\ddot C_n \end{bmatrix}$$

$$\dot N\dot\Delta + \bar N\ddot\Delta = \dot K$$
$$\bar N^T\dot\Delta + \ddot N\ddot\Delta = \ddot K$$
$$\Rightarrow \ddot\Delta = \ddot N^{-1}(\ddot K - \bar N^T\dot\Delta)$$
$$\Rightarrow \dot N\dot\Delta + \bar N\ \ddot N^{-1}(\ddot K - \bar N^T\dot\Delta) = \dot K$$
$$\Rightarrow (\dot N - \bar N\ddot N^{-1}\bar N^T)\dot\Delta = (\dot K - \bar N\ddot N^{-1}\ddot K)$$
$$\Rightarrow \underset{(6m,6m)}{S}\ \underset{(6m,\ 1)}{\dot\Delta} = \underset{(6m,1)}{E}$$

$$S = (\dot N - \bar N\ddot N^{-1}\bar N^T)$$
$$E = \dot K - \bar N\ddot N^{-1}\ddot K$$

$$\boldsymbol{\bar N\ddot N^{-1}\ddot K}$$

$$\begin{bmatrix} \bar N_{11} & \bar N_{12} & \cdots & \bar N_{1n} \\ \bar N_{21} & \bar N_{22} & \cdots & \bar N_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \bar N_{m1} & \bar N_{m2} & \cdots & \bar N_{mn} \end{bmatrix} \begin{bmatrix} \ddot N_1+\ddot W_1 & {}_3 0^3 & \cdots & {}_3 0^3 \\ {}_3 0^3 & \ddot N_2+\ddot W_2 & \cdots & {}_3 0^3 \\ \cdots & \cdots & \cdots & \cdots \\ {}_3 0^3 & {}_3 0^3 & \cdots & \ddot N_n+\ddot W_n \end{bmatrix}^{-1} \begin{bmatrix} \ddot K_1-\ddot W_1\ddot C_1 \\ \ddot K_2-\ddot W_2\ddot C_2 \\ \vdots \\ \ddot K_n-\ddot W_n\ddot C_n \end{bmatrix}$$

$$= \begin{bmatrix} \bar N_{11}(\ddot N_1+\ddot W_1)^{-1} & \bar N_{12}(\ddot N_2+\ddot W_2)^{-1} & \cdots & \bar N_{1n}(\ddot N_n+\ddot W_n)^{-1} \\ \bar N_{21}(\ddot N_1+\ddot W_1)^{-1} & \bar N_{22}(\ddot N_2+\ddot W_2)^{-1} & \cdots & \bar N_{2n}(\ddot N_n+\ddot W_n)^{-1} \\ \cdots & \cdots & \cdots & \cdots \\ \bar N_{m1}(\ddot N_1+\ddot W_1)^{-1} & \bar N_{m2}(\ddot N_2+\ddot W_2)^{-1} & \cdots & \bar N_{mn}(\ddot N_n+\ddot W_n)^{-1} \end{bmatrix} \begin{bmatrix} \ddot K_1-\ddot W_1\ddot C_1 \\ \ddot K_2-\ddot W_2\ddot C_2 \\ \vdots \\ \ddot K_n-\ddot W_n\ddot C_n \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{j=1}^n \bar N_{1j}(\ddot N_j+\ddot W_j)^{-1}(\ddot K_j-\ddot W_j\ddot C_j) \\ \sum_{j=1}^n \bar N_{2j}(\ddot N_j+\ddot W_j)^{-1}(\ddot K_j-\ddot W_j\ddot C_j) \\ \cdots \\ \sum_{j=1}^n \bar N_{mj}(\ddot N_j+\ddot W_j)^{-1}(\ddot K_j-\ddot W_j\ddot C_j) \end{bmatrix}$$

$$\boxed{\begin{gathered}\boldsymbol{E_i = \dot K - \bar N\ddot N^{-1}\ddot K = \dot K_i - \dot W_i\dot C_i - \sum_{j=1}^n \bar N_{ij}(\ddot N_j+\ddot W_j)^{-1}(\ddot K_j-\ddot W_j\ddot C_j)} \\ \boldsymbol{*\ i = 1\sim m}\end{gathered}}$$

AT 4 : derive SΔ=E (3)

$$(A^TWA)\Delta = N\Delta = K$$

$$\begin{bmatrix} \dot{N}_1+\dot{W}_1 & {}_6 0^6 & \cdots & {}_6 0^6 & \bar{N}_{11} & \bar{N}_{12} & \cdots & \bar{N}_{1n} \\ {}_6 0^6 & \dot{N}_2+\dot{W}_2 & \cdots & {}_6 0^6 & \bar{N}_{21} & \bar{N}_{22} & \cdots & \bar{N}_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ {}_6 0^6 & {}_6 0^6 & {}_6 0^6 & \dot{N}_m+\dot{W}_m & \bar{N}_{m1} & \bar{N}_{m2} & \cdots & \bar{N}_{mn} \\ \bar{N}_{11}{}^T & \bar{N}_{21}{}^T & \cdots & \bar{N}_{m1}{}^T & \ddot{N}_1+\ddot{W}_1 & {}_3 0^3 & \cdots & {}_3 0^3 \\ \bar{N}_{12}{}^T & \bar{N}_{22}{}^T & \cdots & \bar{N}_{m2}{}^T & {}_3 0^3 & \ddot{N}_2+\ddot{W}_2 & \cdots & {}_3 0^3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \bar{N}_{1n}{}^T & \bar{N}_{2n}{}^T & \cdots & \bar{N}_{mn}{}^T & {}_3 0^3 & {}_3 0^3 & \cdots & \ddot{N}_n+\ddot{W}_n \end{bmatrix} \begin{bmatrix} \dot{\Delta}_1 \\ \dot{\Delta}_2 \\ \\ \dot{\Delta}_m \\ \ddot{\Delta}_1 \\ \ddot{\Delta}_2 \\ \vdots \\ \ddot{\Delta}_n \end{bmatrix} = \begin{bmatrix} \dot{K}_1-\dot{W}_1\dot{C}_1 \\ \dot{K}_2-\dot{W}_2\dot{C}_2 \\ \vdots \\ \dot{K}_m-\dot{W}_m\dot{C}_m \\ \ddot{K}_1-\ddot{W}_1\ddot{C}_1 \\ \ddot{K}_2-\ddot{W}_2\ddot{C}_2 \\ \vdots \\ \ddot{K}_n-\ddot{W}_n\ddot{C}_n \end{bmatrix}$$

$$\dot{N}\dot{\Delta} + \bar{N}\ddot{\Delta} = \dot{K}$$
$$\bar{N}^T\dot{\Delta} + \ddot{N}\ddot{\Delta} = \ddot{K}$$
$$\Rightarrow \ddot{\Delta} = \ddot{N}^{-1}(\ddot{K} - \bar{N}^T\dot{\Delta})$$
$$\Rightarrow \dot{N}\dot{\Delta} + \bar{N}\ \ddot{N}^{-1}(\ddot{K} - \bar{N}^T\dot{\Delta}) = \dot{K}$$
$$\Rightarrow (\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T)\dot{\Delta} = (\dot{K} - \bar{N}\ddot{N}^{-1}\ddot{K})$$
$$\Rightarrow \quad S \quad \dot{\Delta} = \quad E$$

(6m,6m) (6m, 1) (6m,1)

$$\boldsymbol{\bar{N}^T}\dot{\Delta}$$

$$\begin{bmatrix} \bar{N}_{11}{}^T & \bar{N}_{21}{}^T & \cdots & \bar{N}_{m1}{}^T \\ \bar{N}_{12}{}^T & \bar{N}_{22}{}^T & \cdots & \bar{N}_{m2}{}^T \\ \cdots & \cdots & \cdots & \cdots \\ \bar{N}_{1n}{}^T & \bar{N}_{2n}{}^T & \cdots & \bar{N}_{mn}{}^T \end{bmatrix} \begin{bmatrix} \dot{\Delta}_1 \\ \dot{\Delta}_2 \\ \vdots \\ \dot{\Delta}_m \end{bmatrix} = = \begin{bmatrix} \sum_{i=1}^m \bar{N}_{i1}{}^T\dot{\Delta}_i \\ \sum_{i=1}^m \bar{N}_{i2}{}^T\dot{\Delta}_i \\ \cdots \\ \sum_{i=1}^m \bar{N}_{in}{}^T\dot{\Delta}_i \end{bmatrix}$$

$$S = (\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T)$$
$$E = \dot{K} - \bar{N}\ddot{N}^{-1}\ddot{K}$$

$$\dot{\Delta} = S^{-1}E$$
$$\ddot{\Delta} = \ddot{N}^{-1}(\ddot{K} - \bar{N}^T\dot{\Delta})$$

$$\dot{\Delta} = \dot{N}^{-1}(\dot{K} - \bar{N}\ddot{\Delta})$$
$$\bar{N}^T\dot{N}^{-1}\left(\dot{K} - \bar{N}\ddot{\Delta}\right) + \ddot{N}\ddot{\Delta} = \ddot{K}$$
$$\left(\ddot{N} - \bar{N}^T\dot{N}^{-1}\bar{N}\right)\ddot{\Delta} = \ddot{K} - \bar{N}^T\dot{N}^{-1}\ \dot{K}$$

$$\ddot{\Delta}_{\boldsymbol{j}} = (\boldsymbol{\ddot{N}_j} + \boldsymbol{\ddot{W}_j})^{-1}(\boldsymbol{\ddot{K}_j} - \boldsymbol{\ddot{W}_j\ddot{C}_j} - \textstyle\sum_{\boldsymbol{i=1}}^{\boldsymbol{m}} \boldsymbol{\bar{N}_{ij}}{}^{\boldsymbol{T}}\dot{\Delta}_{\boldsymbol{i}})$$

j = 1~n (number of points)

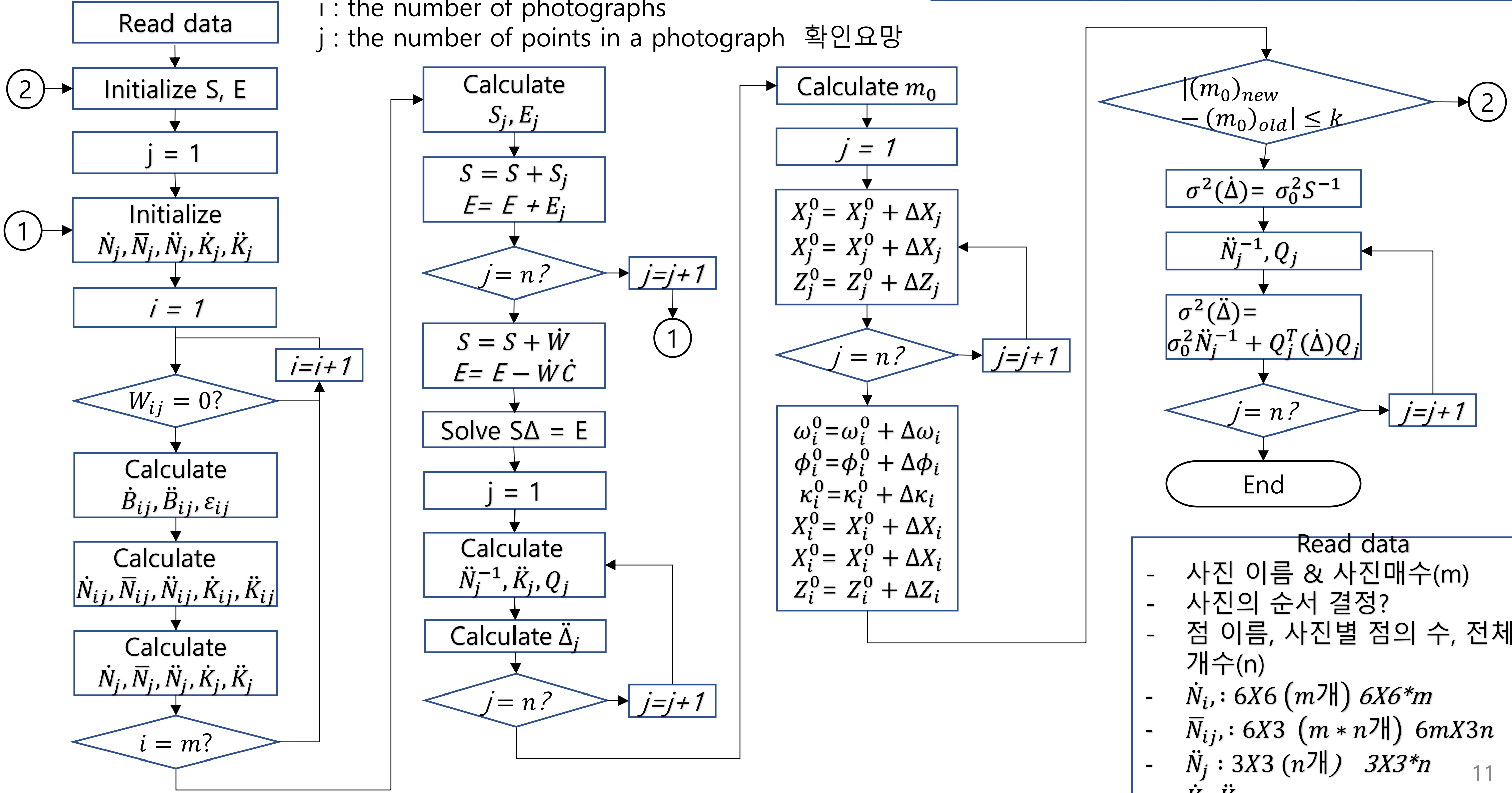
$$\dot{Q} = diagonal\left(\dot{N} - \bar{N}\ddot{N}^{-1}\bar{N}^T\right)^{-1}$$
$$\ddot{Q} = diagonal\left(\ddot{N} - \bar{N}^T\dot{N}^{-1}\bar{N}\right)^{-1}$$

```
QQ_xyz = np.linalg.inv(N_2dot - N_bar.T@np.linalg.inv(N_dot)@N_bar)
        print('### diagonal elem ', np.diag(NN_inv), np.diag(QQ_xyz))
```

AT 5 : Flow chart

- 사진매수(m)
- 전체 관측점 개수(n)(중복 포함)
- $(6m+3n)X2n*2nX(6m+3n) = (6m+3n)X2n*2nX1$
- $(6m+3n)X(6m+3n) = (6m+3n)X1$

i : the number of photographs
j : the number of points in a photograph 확인요망



Read data
- 사진 이름 & 사진매수(m)
- 사진의 순서 결정?
- 점 이름, 사진별 점의 수, 전체 점 개수(n)
- $\dot{N}_i$, : 6X6 (m개) $6X6*m$
- $\bar{N}_{ij}$, : 6X3 ($m*n$개) 6mX3n
- $\ddot{N}_j$: 3X3 (n개) $3X3*n$
- $\dot{K}_i, \ddot{K}_j$

AT 5 : The condition of iterations

$$V_{ij} + \dot{B}_{ij}\dot{\Delta}_i + \ddot{B}_{ij}\ddot{\Delta}_j = \varepsilon_{ij}$$
$$\dot{V}_i - \dot{\Delta}_i = \dot{C}_i$$
$$\ddot{V}_j - \ddot{\Delta}_j = \ddot{C}_j$$

- Convergency limits Δomega, Δphi, Δkappa
  - If Δomega, Δphi, Δkappa < k, then finish the iterations
  - k in arc seconds : 1 second = 4.85e-6 radian > 1.00e-6
- Difference of M_0 between successive Iterations
  - the standard error of unit weight $m_0 = \sqrt{\frac{\bar{V}^T \bar{W} \bar{V}}{n-u}} = \sqrt{\frac{\sum_j \sum_i V_{ij}^T W_{ij} V_{ij} + \sum_{i=1}^m \dot{V}_i^T \dot{W}_i \dot{V}_i + \sum_{j=1}^n \ddot{V}_j^T \ddot{W}_j \ddot{V}_j}{n-u}}$
  - If $\dot{\Delta}_i,\ \ddot{\Delta}_j \approx 0.0,\ then\, V_{ij} = \varepsilon_{ij},\ \dot{V}_i = \dot{C}_i,\ \ddot{V}_j = \ddot{C}_j \rightarrow m_0 = \sqrt{\frac{\bar{V}^T \bar{W} \bar{V}}{n-u}} = \sqrt{\frac{\sum_j \sum_i \varepsilon_{ij}^T W_{ij} \varepsilon_{ij} + \sum_{i=1}^m \dot{C}_i^T \dot{W}_i \dot{C}_i + \sum_{j=1}^n \ddot{C}_j^T \ddot{W}_j \ddot{C}_j}{n-u}}$
  - Make the square sum of $\varepsilon_{ij}, \dot{C}_i, \ddot{C}_j$ of the observation equations during each iteration.
  - The iteration finished, when m_{0P} - $m_{0K} < 0.1 m_{0c} \sim 0.01 m_{0c}$ $(0.1\sigma_0 \sim 0.01\sigma_0)$
  - $(m_{0P}$ $: previous\ iteration, m_{0c}: current\ iteration)$
- Fixed number of iterations : 5~10
- Divergency test
  - the correction values of EO parameters $\dot{\Delta}_i$
  - M_0 between successive Iterations

Blunder Detection : Adjustment computations

21.5 DETECTION OF OUTLIERS IN OBSERVATIONS

the standardized residual

$\bar{v}_i = v_i/\sqrt{q_{ii}}$

- v_i : the computed residual
- q_{ii} : the diagonal element of the Qvv matrix

the standard deviation in the residual $S_0\sqrt{q_{ii}}$

t statistic

$t_i = v_i/S_0\sqrt{q_{ii}} = v_i/S_v = \bar{v}_i/S_0$

- If the residual is significantly different from zero, the observation used to derive the statistic is considered to be a blunder.
- Baarda (1968) computed rejection criteria for various significance levels (see Table 21.1) determining the and levels for Type I and Type II errors.
- Type I error occurs when data are rejected that do not contain blunders
- Type II error occurs when a blunder is not detected in a data set where one is actually present.

$\frac{abs(v_i)}{S_0\sqrt{q_{ii}}} > rejection\ level$ or $\bar{v}_i = v_i/\sqrt{q_{ii}} > S_0 \times rejection\ level$

- the existence of any blunder in the data set will affect the remaining observations and since Equation (21.18) depends on S0 , whose value was computed from data containing blunders, all observations that are detected as blunders should not be removed in a single pass.
- Instead, only the largest or largest independent group of observations should be deleted.
- Furthermore, since Equation (21.18) depends on S0 , it is possible to rewrite the equation so that it can be computed during the final iteration of a nonlinear adjustment. In this case the appropriate equation is

The procedures for this manner of blunder detection

- Step 1: Locate all standardized residuals that meet the rejection criteria of Equation (21.17) or (21.18).
- Step 2: Remove the largest detected blunder or unrelated blunder groups.
- Step 3: Rerun the adjustment.
- Step 4: Continue steps 1 through 3 until all detected blunders are removed.
- Step 5: If more than one observation is removed in steps 1 through 4, reenter the observations in the adjustment one at a time. Check the observation after each adjustment to see if it is again detected as a blunder. If it is, remove it from the adjustment or have that observation reobserved.

Blunder Detection : Adjustment computations

21.4 Development of the covariance matrix for the residuals

$L + V = AX + C$
$\Longrightarrow V = AX - T \quad T = L - C$
$\Longrightarrow \mathrm{X} = (A^T WA)^{-1} A^T WT$

$L - \epsilon = A\bar{X} + C$
$\Longrightarrow \ T = L - C = A\bar{X} + \epsilon$

$V = AX - T = A\big((A^T WA)^{-1} A^T WT\big) - T$
$= A\big((A^T WA)^{-1} A^T W(A\bar{X} + \epsilon)\big) - (A\bar{X} + \epsilon)$
$= A(A^T WA)^{-1} A^T W\varepsilon - \varepsilon + A(A^T WA)^{-1} A^T WA\bar{X} - A\bar{X}$
$\Longrightarrow \ \mathrm{V} = \ A(A^T WA)^{-1} A^T W\varepsilon - \varepsilon$
$\Longrightarrow \ \mathrm{V} = -(\ W^{-1} - A(A^T WA)^{-1} A^T)W\varepsilon$
$\Longrightarrow \ \mathrm{V} = -(\ W^{-1} - AQ_{XX}A^T)W\varepsilon = -Q_{vv}W\varepsilon \quad (Q_{XX} = (A^T WA)^{-1})$

Q_{vv} matrix : singular and idempotent.

- Being singular, it has no inverse
- idempotent matrix
  - the square of the matrix is equal to the original matrix $Q_{vv}\ Q_{vv} = \ Q_{vv}$
  - every diagonal element is between zero and 1
  - the sum of the diagonal elements equals the degrees of freedom
    the trace of the matrix $\sum_{i=1}^{m} q_{ii}$ = the degrees of freedom
  - The sum of the square of the elements in any single row or column equals the diagonal element.
    $q_{ii} = q_{i1}^2 + q_{i2}^2 + \ q_{i3}^2 + \cdots + q_{im}^2 = q_{1i}^2 + q_{2i}^2 + \ q_{3i}^2 + \cdots + q_{mi}^2$

$\Delta v_i = -q_{ii} w_{ii} \Delta l_i = -r_i \Delta l_i$

- q_{ii} is the ith diagonal of the Q_{vv} matrix
- w_{ii} the ith diagonal term of the weight matrix, W,
- $r_i = q_{ii} w_{ii}$ is the observational redundancy number
  - the system has a unique solution, r_i will equal zero
  - if the observation is fully constrained, r_i would equal 1.
  - The redundancy numbers provide insight into the geometric strength of the adjustment
  - low redundancy numbers will have observations that lack sufficient checks to isolate blunders, and thus the chance for undetected blunders to exist in the observations is high
  - a high overall redundancy number enables a high level of internal checking of the observations and thus there is a lower chance of accepting observations that contain blunders
  - The quotient of r/m, where r is the total number of redundant observations in the system and m is the number of observations, is called the relative redundancy of the adjustment.

Space intersection : Linear

$$\begin{bmatrix} x_a \\ y_a \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X_a - X_L \\ Y_a - Y_L \\ Z_a - Z_L \end{bmatrix} \Leftrightarrow \; M^T \begin{bmatrix} x_a \\ y_a \\ -f \end{bmatrix} = \lambda \begin{bmatrix} X_a - X_L \\ Y_a - Y_L \\ Z_a - Z_L \end{bmatrix} \Leftrightarrow \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \lambda \begin{bmatrix} X_a - X_L \\ Y_a - Y_L \\ Z_a - Z_L \end{bmatrix}$$

$$\Leftrightarrow \; c_1 = \frac{u}{w} = \frac{X_a - X_L}{Z_a - Z_L} \Leftrightarrow \; c_1(Z_a - Z_L) = X_a - X_L \Leftrightarrow \; c_1 Z_a - X_a = -X_L + c_1 Z_L$$

$$\Leftrightarrow \; c_2 = \frac{v}{w} = \frac{Y_a - Y_L}{Z_a - Z_L} \; \Leftrightarrow c_2(Z_a - Z_L) = Y_a - Y_L \Leftrightarrow \; c_2 Z_a - Y_a = -Y_L + c_2 Z_L$$

$$\Leftrightarrow \begin{bmatrix} -1 & 0 & c_1 \\ 0 & -1 & c_2 \end{bmatrix} \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} = \begin{bmatrix} -X_L + c_1 Z_L \\ -Y_L + c_2 Z_L \end{bmatrix}$$

- 2 “linear” equations per image
- 2n “linear” equations for n images
- matrix elements (ci) are not constants
- Elements of right-hand side vector are not observations
- “Least Squares” is really pseudo least squares
- However if data is reasonably good then it works well enough <u>to generate good initial approximations</u>. Then nonlinear model with proper stochastic assignment can be iterated to convergence.
- We see this strategy on several occasions – use linear model to bootstrap yourself into the nonlinear model without agonizing over approximations (8- parameter transformation, DLT, etc.)