

Space intersection : Linear

$$\begin{aligned}
 \begin{bmatrix} x_a \\ y_a \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X_a - X_L \\ Y_a - Y_L \\ Z_a - Z_L \end{bmatrix} &\Leftrightarrow M^T \begin{bmatrix} x_a \\ y_a \\ -f \end{bmatrix} = \lambda \begin{bmatrix} X_a - X_L \\ Y_a - Y_L \\ Z_a - Z_L \end{bmatrix} \Leftrightarrow \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \lambda \begin{bmatrix} X_a - X_L \\ Y_a - Y_L \\ Z_a - Z_L \end{bmatrix} \\
 \Leftrightarrow c_1 = \frac{u}{w} = \frac{X_a - X_L}{Z_a - Z_L} &\Leftrightarrow c_1(Z_a - Z_L) = X_a - X_L \Leftrightarrow c_1 Z_a - X_a = -X_L + c_1 Z_L \\
 \Leftrightarrow c_2 = \frac{v}{w} = \frac{Y_a - Y_L}{Z_a - Z_L} &\Leftrightarrow c_2(Z_a - Z_L) = Y_a - Y_L \Leftrightarrow c_2 Z_a - Y_a = -Y_L + c_2 Z_L \\
 \Leftrightarrow \begin{bmatrix} -1 & 0 & c_1 \\ 0 & -1 & c_2 \end{bmatrix} \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} &= \begin{bmatrix} -X_L + c_1 Z_L \\ -Y_L + c_2 Z_L \end{bmatrix}
 \end{aligned}$$

- 2 "linear" equations per image
- 2n "linear" equations for n images
- matrix elements (c_i) are not constants
- Elements of right-hand side vector are not observations
- "Least Squares" is really pseudo least squares
- However if data is reasonably good then it works well enough to generate good initial approximations. Then nonlinear model with proper stochastic assignment can be iterated to convergence.
- We see this strategy on several occasions – use linear model to bootstrap yourself into the nonlinear model without agonizing over approximations (8-parameter transformation, DLT, etc.)