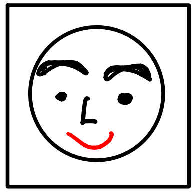
Linear Algebra

주재걸
고려대학교 컴퓨터학과



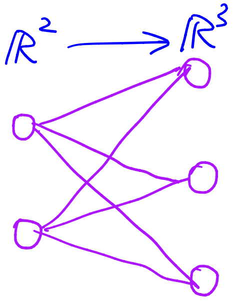
픽셀의 색에 따라
face detect

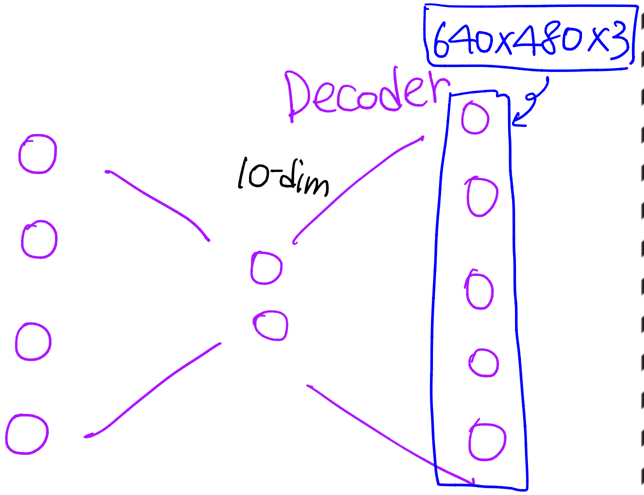
manifold

onto 불가

input수 < output 수

GAN





전사(전체 사영)　　　　　　화살표 무조건 1개씩

ONTO and ONE-TO-ONE

공역 = 치역　　　　　　　　일대일 함수(onto일 필요X)

$T: \mathbb{R}^3 \to \mathbb{R}^2$

$\boxed{T: \mathbb{R}^2 \to \mathbb{R}^3}$ → 절대 onto 불가능

$T: \mathbb{R}^2 \to \mathbb{R}^3$　공≠치

$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix}$　$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}4\\5\\6\end{bmatrix}$

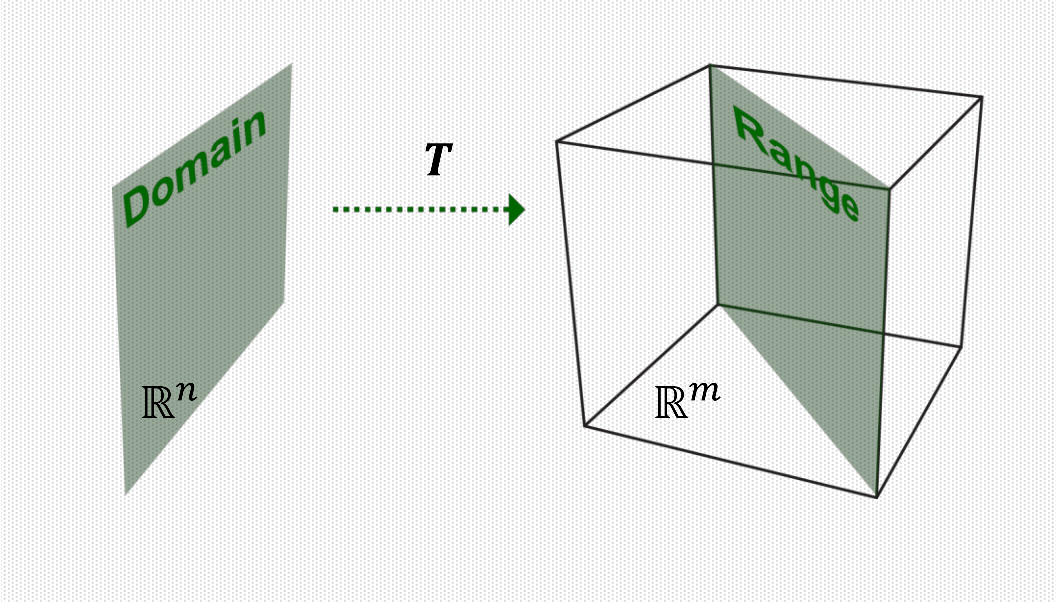
선형변환: 애네를 선형결합시킨것 = 함수에 넣어 선형결합시킨 것

$\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}$의 span (평면이 됨)이 치역

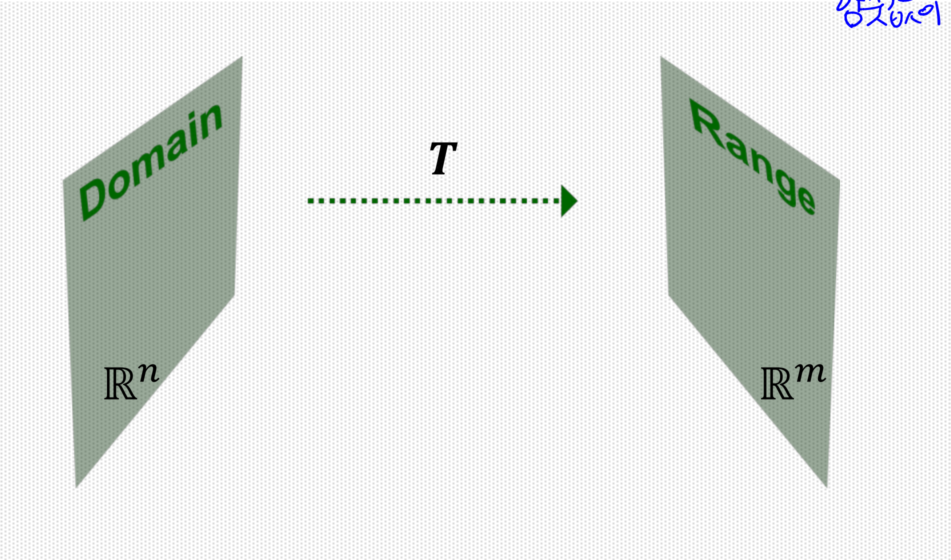
공역은 3차원이므로 ≠onto

- **Definition:** A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be onto $\mathbb{R}^m$ if each **b** ∈ $\mathbb{R}^m$ (공역) is the image of at least one **x** ∈ $\mathbb{R}^n$. That is, the range is equal to the co-domain.

onto: 공역에서 어떤 값을 뽑아도 정의역의 하나이상 값의 함숫값이 되어야 한다.

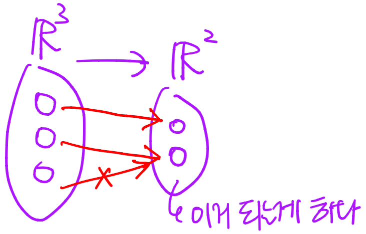


T is NOT onto $\mathbb{R}^m$



T is onto $\mathbb{R}^m$

ONTO and ONE-TO-ONE



$T: \mathbb{R}^2 \to \mathbb{R}^3$

$\boxed{T: \mathbb{R}^3 \to \mathbb{R}^2}$ → one-to-one 절대 불가

$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\end{bmatrix}$

$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}3\\4\end{bmatrix}$

$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}5\\8\end{bmatrix}$

$T: y = Ax = \begin{bmatrix}1&3&5\\2&4&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$

- **Definition:** A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be one-to-one if each $\mathbf{b} \in \mathbb{R}^m$ is the image of at most one $\mathbf{x} \in \mathbb{R}^n$. That is, each output vector in the range is mapped by only one input vector, no more than that.

$\begin{bmatrix}1&3&5\\2&4&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}5\\7\end{bmatrix}$

치역 中 원소 하나

↑ 하나 존재 → one-to-one

$Ax = b$

$x = A^{-1}b$

$b \in \text{Span}\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}, \begin{bmatrix}5\\8\end{bmatrix}\right\}$

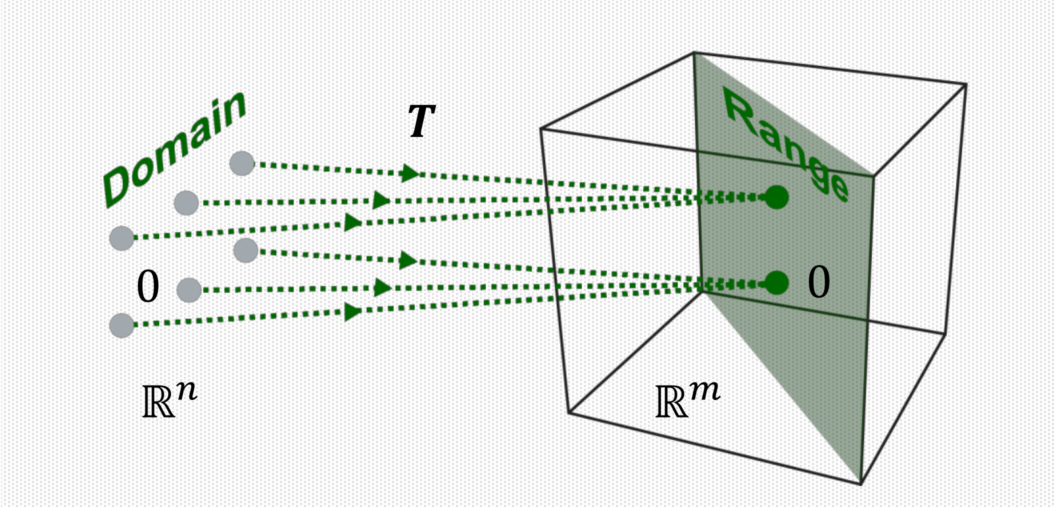
x가 unique?

↓

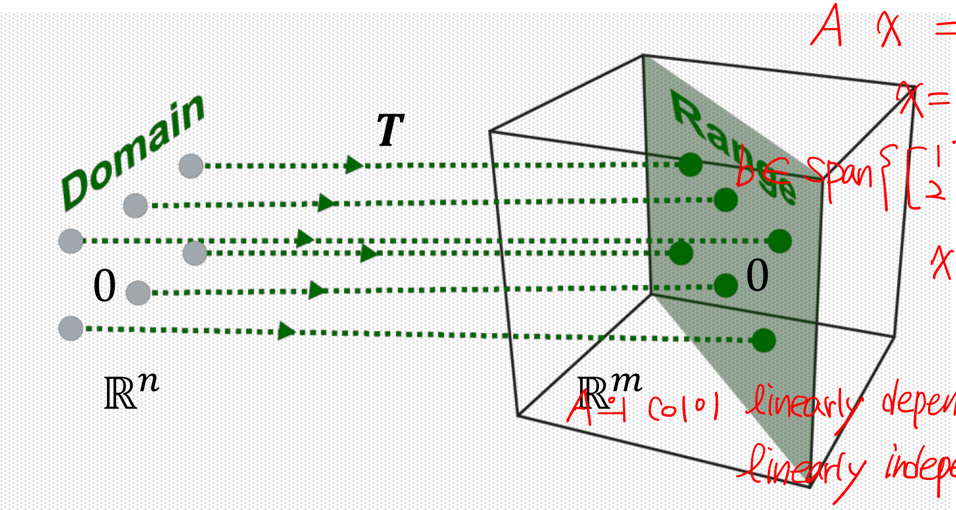
A의 col이 linearly dependent : 해 무수

linearly independent : 해 하나

근데 2차원 공간에서 벡터 3개 → 선형 종속 (해 무수)



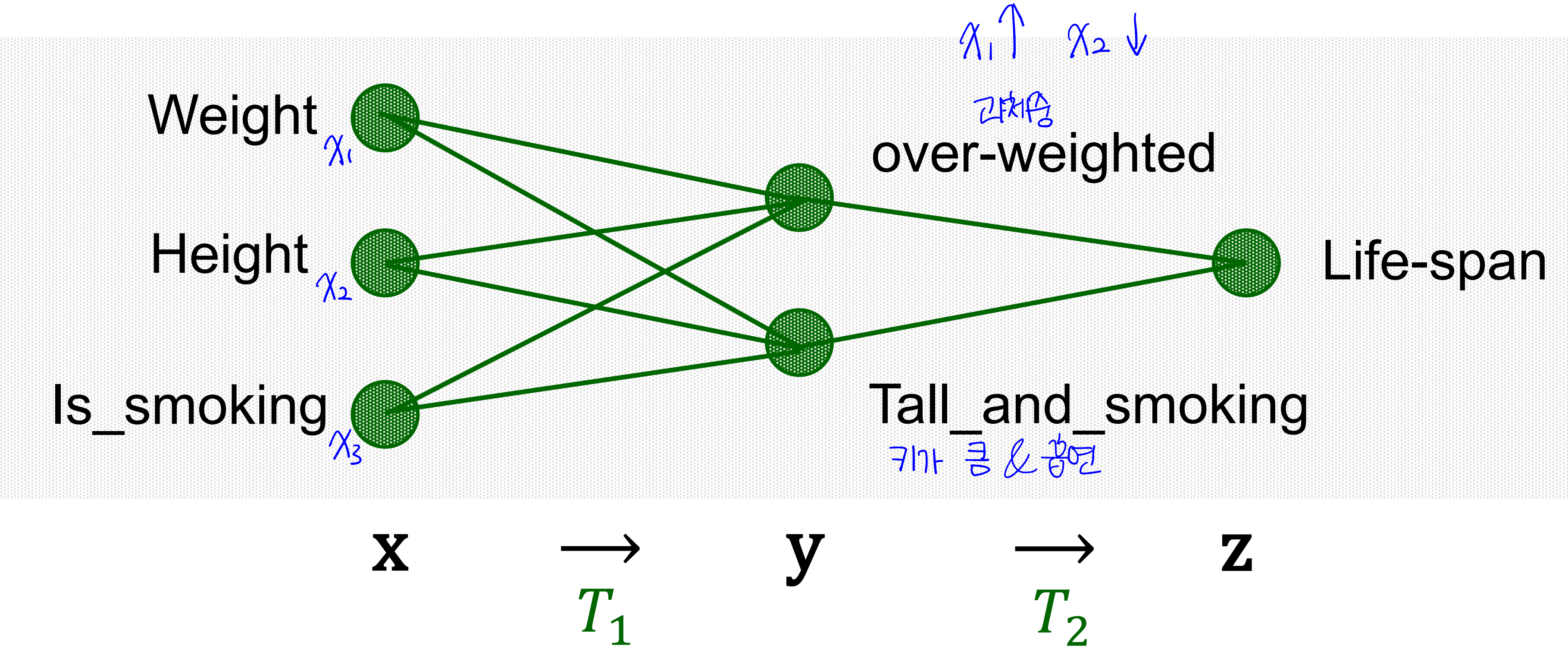
T is NOT one-to-one



T is one-to-one

Neural Network Example

- Fully-connected layers



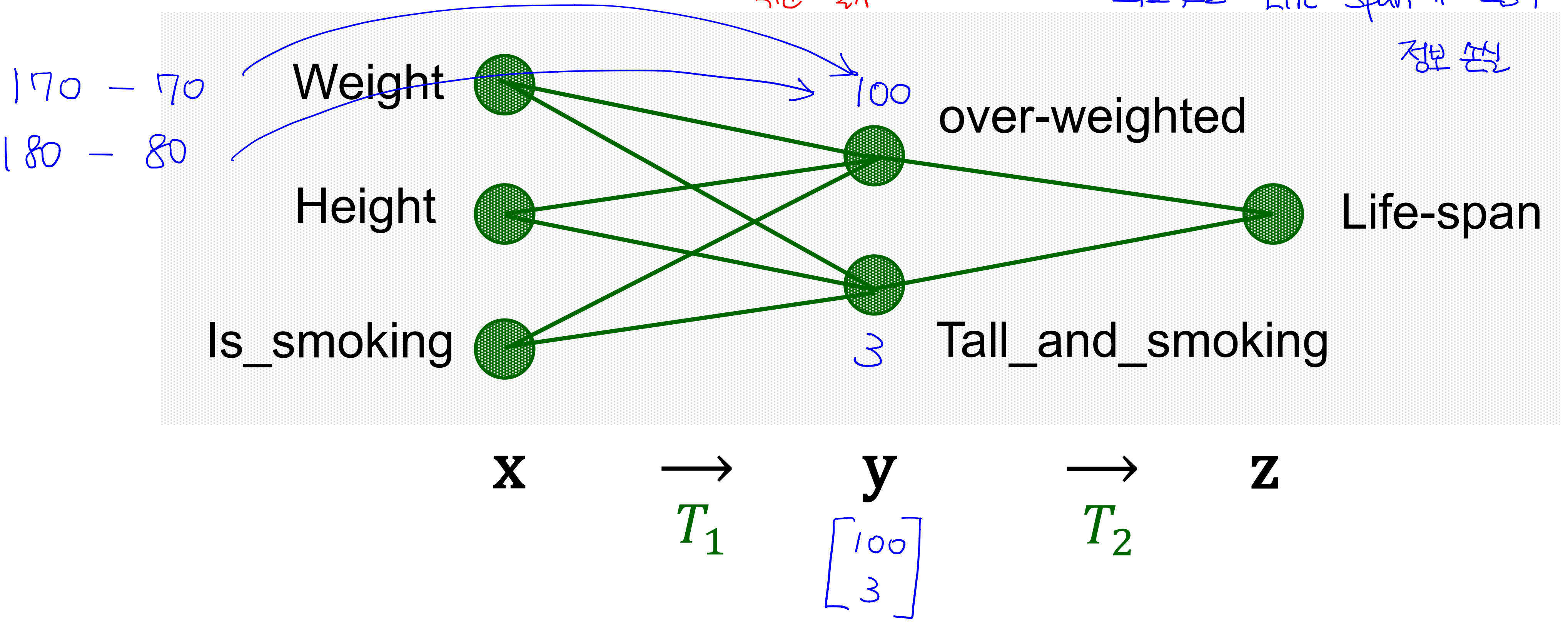
Neural Network Example: ONE-TO-ONE

- Will there be many (or unique) people mapped to the same (over_weighted, tall_and_smoking)?

그러나 복원 불가

의도적으로 Life-span에 도움이 되는 feature만 추출

정보 손실

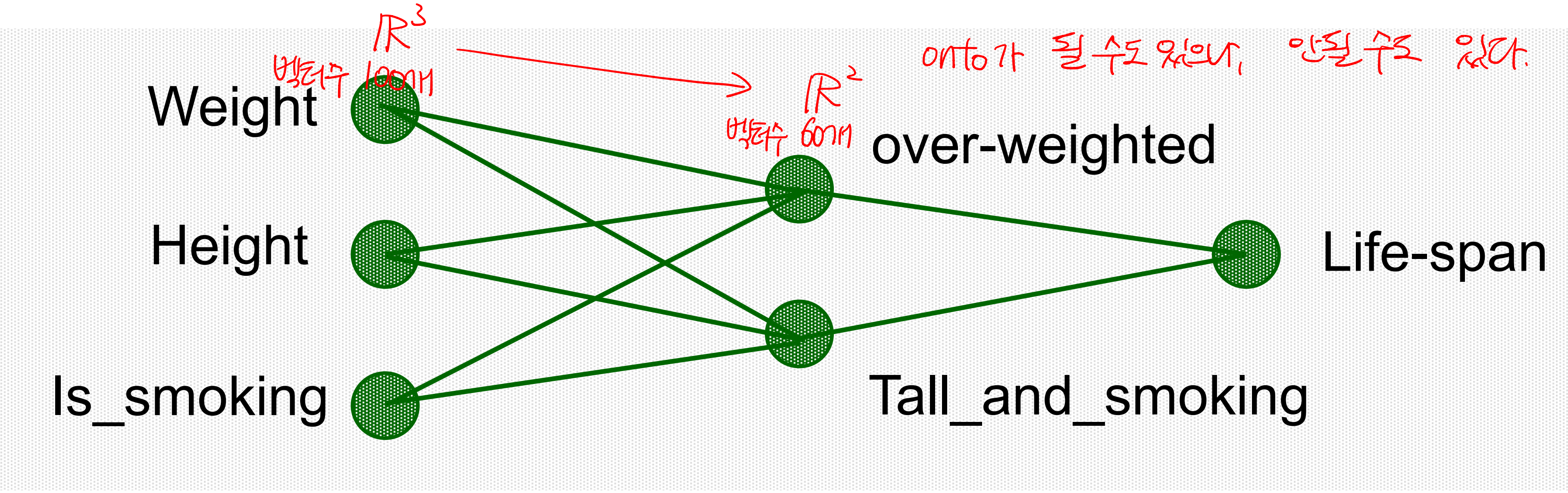


Neural Network Example: ONTO

- Is there any (over_weighted, tall_and_smoking) that does not exist at all?

$$\underbrace{\begin{bmatrix}0\\1\\0\end{bmatrix} \to \begin{bmatrix}2\\4\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\end{bmatrix} \to \begin{bmatrix}5\\10\end{bmatrix}}_{}$$

같은 값인



$$\mathbf{x} \xrightarrow[T_1]{} \mathbf{y} \xrightarrow[T_2]{} \mathbf{z}$$

ONTO and ONE-TO-ONE

- Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, i.e.,

$$T(\mathbf{x}) = A\mathbf{x} \text{ for all } \mathbf{x} \in \mathbb{R}^n.$$

- T is one-to-one if and only if the columns of A are linearly independent.

- T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if the columns of A span $\mathbb{R}^m$.

ONTO and ONE-TO-ONE

- **Example:**

  Let $T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

- Is T one-to-one?
- Does T map $\mathbb{R}^2$ onto $\mathbb{R}^3$?

ONTO and ONE-TO-ONE

- **Example:**

  Let $T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

- Is T one-to-one?
- Does T map $\mathbb{R}^3$ onto $\mathbb{R}^2$?

Further Study

- Gaussian elimination, row reduction, echelon form
  - Lay Ch1.2,

- LU factorization: efficiently solving linear systems
  - Lay Ch2.5

- Computing invertible matrices
  - Lay Ch2.2

- Invertible matrix theorem for square matrices
  - Lay Ch2.3, Ch2.9