Introduction

This report outlines and analyses an implementation of a parallel algorithm for the following problem: Given a grid, where each square has a positive integer as its cost, find the shortest 8 connected path from the top left most square to the bottom right most square, where the distance of a path is defined as the sum of costs of all squares on the path.

The algorithm I've devised is a modified version of the delta stepping algorithm from Mayer and Sanders [1], described in the algorithm 1 with its functions in algorithm 2.

Note that $\text{COST}(x, y)$ returns the cost of the cell on x_{th} row and y_{th}. Furthermore, while the cost takes a long time to calculate for the first time. This function caches the cost in a 2D array so subsequence call returns instantly. This has the additional effect of calculating the cost of each cell upfront in a parallel fashion during the FINDDELTA procedure. Not only does this allow delta to be calculated at the optimal value. It also allows the load of calculating the cost of each cell to be spread evenly among all processors, taking advantage of parallelization for the most expensive part of the algorithm.

Furthermore, to ensure that *relaxed* and *removed* can be queried in constant time as well as being made emptied in constant time, I implemented them as 2D int arrays where (x, y) is in the set if $set[x][y] = trueValue$. Here $trueValue$ is an integer. By incrementing $trueValue$ we can empty the set very quickly.

The implementation of the algorithm is less than ideal. Because I choose to use vector in the C++ standard library, which isn't thread safe, I have to heavily used locking and OpenMP critical region to ensure data races does not happen. This means the implementation of the algorithm isn't as parallel as it should be and further improvement on data structures and hence performance can be made.

Methodology

To confirm and measure the performance of my algorithm, I've

Experiments

Discussion

Conclusion

References

[1] U. Meyer and P. Sanders, ``Δ-stepping: A parallelizable shortest path algorithm,'' *Journal of Algorithms*, vol. 49, no. 1, pp. 114–152, 1998, 1998 European Symposium on Algorithms, ISSN: 0196-6774. DOI: https://doi.org/10.1016/S0196-6774(03)00076-2. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0196677403000762.

Algorithm 1

1: $x_{end} \leftarrow x_{size} - 1$
2: $y_{end} \leftarrow y_{size} - 1$
3: $delta \leftarrow \text{FINDDELTA}()$
4: **for** $x \leftarrow 0$ **to** x_{end} **do in parallel**
5:     **for** $y \leftarrow 0$ **to** y_{end} **do in parallel**
6:         $distance[x][y] \leftarrow \infty$
7:         $bmap[x][y] \leftarrow -1$ ▷ bmap map (x, y) to the which b[i] set they're in, hence speeding up the process.
8:     **end for**
9: **end for**
10: $distance[0][0] \leftarrow \text{COST}(0, 0)$
11: $\text{INSERT}(0, 0)$
12: $i_{dest} \leftarrow \infty$
13: $removed \leftarrow \emptyset$
14: $relaxed \leftarrow \emptyset$
15: $i \leftarrow 0$
16: **while** $i < i_{dest}$ **do**
17:     **while** b[i] is not empty **do**
18:         $\text{RELAX}(\text{b[i]}, relaxed, removed)$
19:         $b[i] \leftarrow \emptyset$
20:         **for each**$(x, y) \in relaxed$ **do in parallel**
21:             $\text{INSERT}(x, y)$
22:             **if** $x_{end} = x \wedge y_{end} = y$ **then**
23:                 $i_{dest} \leftarrow i$
24:             **end if**
25:         **end for**
26:     **end while**
27:     $relaxed \leftarrow \emptyset$
28:     $removed \leftarrow \emptyset$
29: **end while**
30: $x \leftarrow x_{end}$ ▷ Reconstruct the path from distance.
31: $y \leftarrow y_{end}$
32: $path \leftarrow []$
33: **while** $x \neq 0 \wedge y \neq 0$ **do**
34:     $x_{pred} \leftarrow x$
35:     $y_{pred} \leftarrow y$
36:     **for** $dx \leftarrow -1, 0, 1$ **do**
37:         **for** $dy \leftarrow -1, 0, 1$ **do**
38:             $x_{new} \leftarrow x + dx$
39:             $y_{new} \leftarrow y + dy$
40:             **if** (x_{new}, y_{new}) is valid coordinate $\wedge (x_{new}, y_{new}) \neq (x, y)$ **then**
41:                 **if** $distance[x_{new}][y_{new}] < distance[x_{pred}][y_{pred}]$ **then**
42:                     $x_{pred} \leftarrow x_{new}$
43:                     $y_{pred} \leftarrow y_{new}$
44:                 **end if**
45:             **end if**
46:         **end for**
47:     **end for**
48:     $pathprepend(x_{pred}, y_{pred})$
49: **end while**
50: **return** path

Algorithm 2 Functions of Algorithm 1

1: **function** FINDDELTA( )
2:     **for** $x \leftarrow 0$ **to** x_{end} **do in parallel**
3:         **for** $y \leftarrow 0$ **to** y_{end} **do in parallel**
4:             $cost_{max} \leftarrow$ MAX$(cost_{max}, \text{COST}(x, y))$
5:         **end for**
6:     **end for**
7:     **return** $cost_{max}/8$
8: **end function**
9: **function** REMOVE(x, y)
10:     **if** $bmap[x][y] \neq -1$ **then**
11:         $b[bmap[x][y]] \leftarrow b[bmap[x][y]] \setminus (x, y)$
12:     **end if**
13: **end function**
14: **function** INSERT(x, y)
15:     $i \leftarrow \lfloor distance[x][y]/delta \rfloor$
16:     $bmap[x][y] \leftarrow i$
17:     $b[i] \leftarrow b[i] \cup (x, y)$
18: **end function**
19: **function** RELAX(to-relax, relaxed, removed)   ▷ pass NULL as removed will cause the
    function to only Relax heavy edges. Pass a set as removed will cause the function to call REMOVE
    for each coordinate in to-relax and add them to removed.
20:     **for each** $(x, y) \in$ to-relax **do in parallel**
21:         **if** $removed \neq NULL$ **then**
22:             REMOVE(x, y)
23:             $removed \leftarrow removed \cap (x, y)$
24:         **end if**
25:         **for** $dx \leftarrow -1, 0, 1$ **do**
26:             **for** $dy \leftarrow -1, 0, 1$ **do**
27:                 $x_{next} \leftarrow x + dx$
28:                 $y_{next} \leftarrow y + dy$
29:                 **if** (x_{next}, y_{next}) is valid coordinate $\land (x_{next}, y_{next}) \neq (x, y)$ **then**
30:                     **if** $(\text{COST}(x_{next}, y_{next}) \leq delta) \oplus (removed = NULL)$ **then**   ▷ If removed is null,
    only relax heavy edges, otherwise only relax light edges.
31:                         $distance_{new} \leftarrow distance[x][y] + \text{COST}(x_{next}, y_{next})$
32:                         **if** $distance_{new} < distance[x_{next}][y_{next}]$ **then**
33:                             $distance[x_{next}][y_{next}] \leftarrow distance_{new}$
34:                             $relaxed \leftarrow relaxed \cap (x_{next}, y_{next})$
35:                         **end if**
36:                     **end if**
37:                 **end if**
38:             **end for**
39:         **end for**
40:     **end for**
41: **end function**